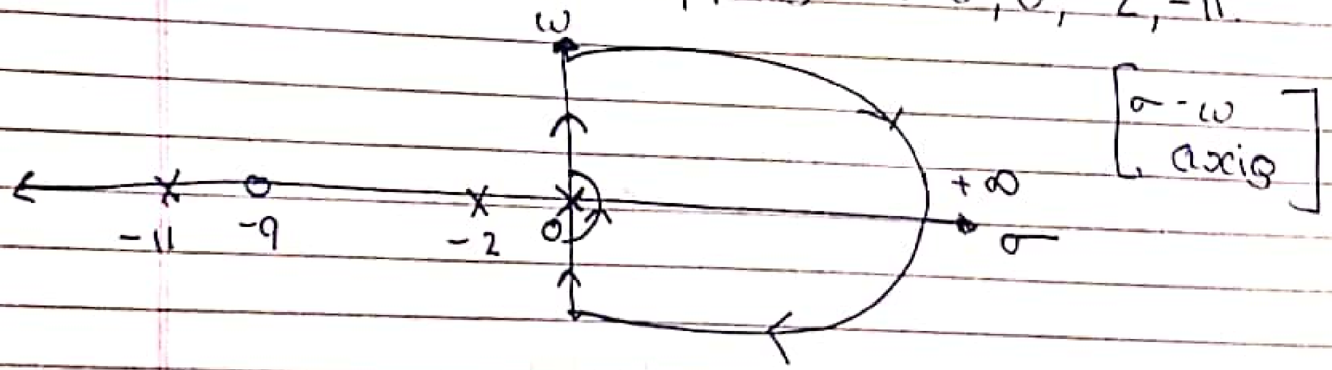
Consys Assignment

Name : JASH SHAH
Id : 2018A8PS0507P
Assignment Set : B
Instructor : Puneet Mishra

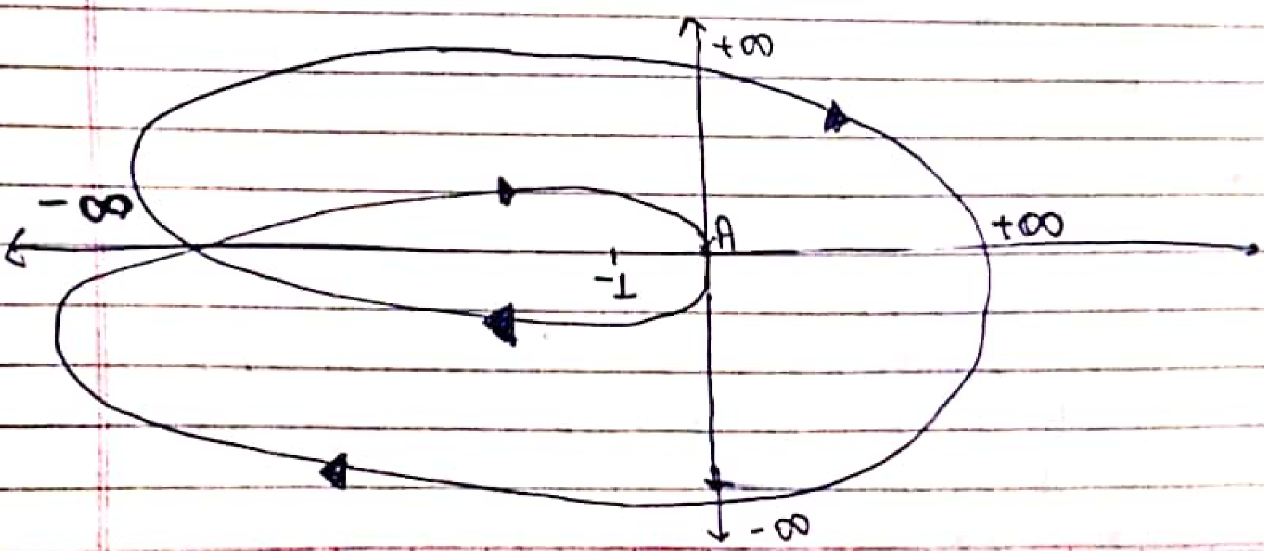
A1 T.F (open loop) $= \dfrac{K(s+9)}{s^2(s+2)(s+11)}$

S-plane plot

Zeroes = -9
Poles : $0, 0, -2, -11$.



By plotting the nyqust plot for/along given path,



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→ Point A in the graph $\neq (0,0)$
its a very small value that ≈ 0.
But, it becomes $\boxed{\lim_{a \to 0} K(a)}$ for given func.
The point departs at $+$ and $-$ $90°$.

→ •) From the T.F (open loop),

Poles $(0, -11, -2)$
So, no poles on RHP.

$\Rightarrow \boxed{P = 0}$

•) From the graph, the point $(-1, 0)$ is encircled twice. (clockwise)

$\boxed{N = +2}$

•) Using Nyquist theorem,

$$Z = P + N = +2$$
$$\neq 0$$

Ans So, closed-loop system is unstable

→ Matlab code :

```
s = tf('s');
sys = (s+9)/(s*s*(s+2)*(s+11));
nyqlog(sys), hold on
nyquist(sys)
hold off;
```

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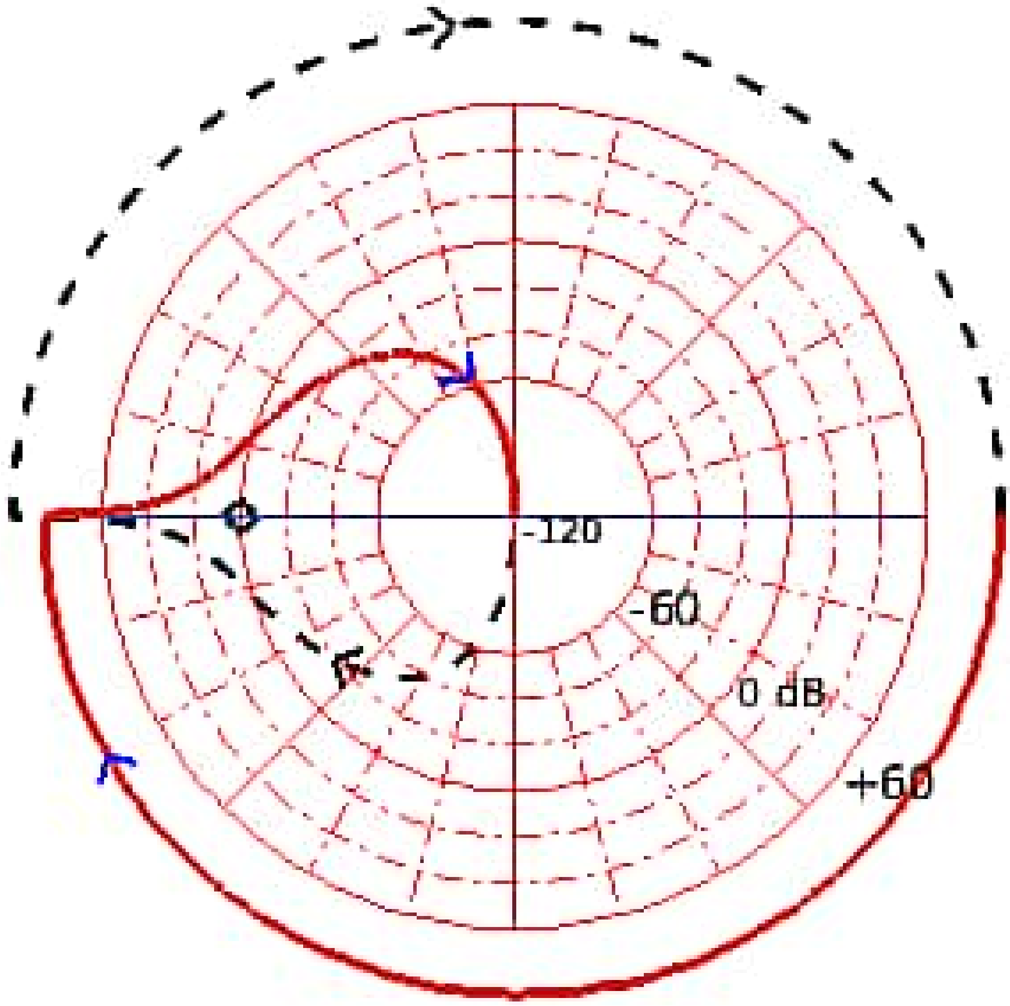
→ Now, From the matlab result, we can deduce the value of point 'A.

$A \Rrightarrow (-4.31 \times 10^{-6}, 0)$

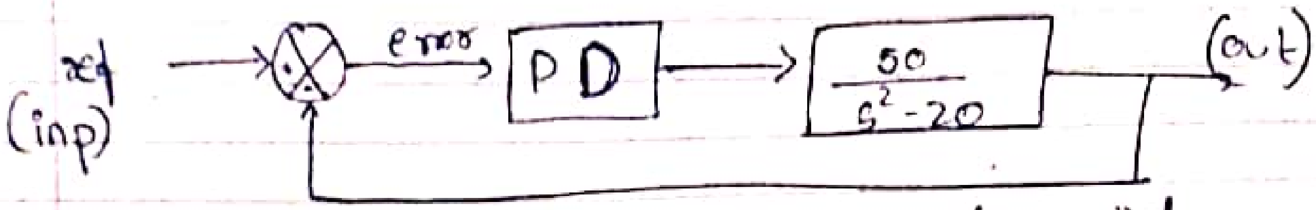
$|A| = (K)(4.31 \times 10^{-6})$

So, gain margin $= \frac{1}{|A|} = \frac{10^{6}}{K(4.31)}$

$\simeq \underline{\infty}$ (For small disturbance)


-120
-60
0 dB
+60

A2. The required system is as follows :-



(parrallel PID config)

and settling time $\leq$ 75 ms
Peak overshoot $\leq$ 6 %.

→ Random co-efficients K_p & K_D,

$$\underset{\text{(open loop)}}{T.F} = (K_p + sK_D)\left[\frac{50}{s^2 - 20}\right]$$

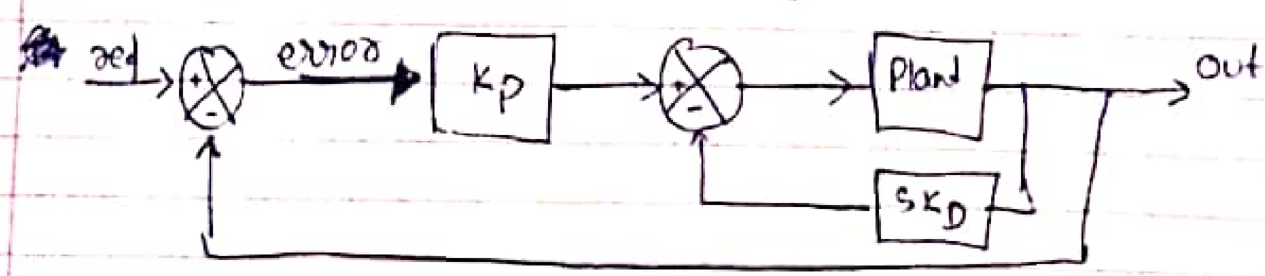
So for closed loop $\Rightarrow \boxed{\dfrac{50\,[K_p] + 50K_D s}{s^2 + (50K_D)s + (50K_p - 20)}}$

But this is not second order eqn and hence we need a different approach.

→ Now, we need (sum of): 1) error $= -(out - ref) \times K_p$

2) $\dfrac{d(error)}{dt} = \dfrac{d[ref - out]}{dt} = \boxed{-\dfrac{d(out)}{dt}}$ $\left\{ref \Rightarrow const\right\}$

So, we can use same system like :-



Here the Final T.F. we get is :-

$$\boxed{\frac{out(s)}{ref(s)} = \frac{50K_p}{s^2 + (50K_D s) + (50K_p - 20)}}$$

Also, we will take ref $\Rightarrow$ $\boxed{u(t) \text{ or } 1/s}$

→ Comparing the eqn with std eqn,

$$\omega_n = \sqrt{50K_p - 20} \qquad A = \left(\frac{50K_p}{(50K_p - 20)}\right)$$

$$\zeta = \left(\frac{25K_D}{\sqrt{50K_p - 20}}\right) \qquad \omega_d = \sqrt{50K_p - 20 - 625K_D^2}$$

Now, the second order response is :-

$$out(t) = A\left\{1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\sin(\omega_d t + \theta)\right\} \cdot inp(t)$$

Here, $\theta = \tan^{-1}\left[\frac{\sqrt{1-\zeta^2}}{\zeta}\right]$

& $int(t) = u(t)$

→ Tolerance band = 5% | Peak overshoot $\leq 6\%$

$\Rightarrow$ $\boxed{1.05 \text{ to } 0.95}$ | $\Rightarrow$ ≤ 0.06

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• For peak overshoot, $\boxed{t = \pi/\omega_d}$

So, putting this,

$$out(t) = \frac{50K_p}{50K_p - 20}\left[1 - \frac{e^{\frac{-\zeta\omega_n \pi}{\omega_n\sqrt{1-\zeta^2}}}}{\sqrt{1-\zeta^2}}\sin(\pi + \theta)\right]$$

$$= \left[\frac{50K_p}{50K_p - 20}\left\{1 + e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}}\right\} - 1\right] < 0.06$$

(1) — $$= \left[\left(\frac{5K_p}{5K_p - 2}\right)\left\{1 + e^{\frac{-25\pi K_D}{\sqrt{50K_p - 20 - 625K_D^2}}}\right\} - 1\right] \le 0.06$$

• For settling time,

$$1 + \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}} \le 1.05 \quad \text{or} \quad 1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}} \ge 0.95$$

$$\Rightarrow \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}} \le 0.05, \quad \text{and for this } t \le 75\text{ms}$$

(2) — $$\Rightarrow \frac{\ln \frac{1}{0.05\sqrt{1-\zeta^2}}}{\zeta\omega_n} \le \cancel{75\text{ ms}}\ 0.75$$

Solving the (1), (2) we have $\boxed{K_p = 10 \text{ (approx)}, \quad K_D = 3.3}$

Ans

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A3 Given $\boxed{T.F = \frac{K}{s(s^2+2s+9)}}$ This is For open loop.

For negative gain,

$$T.F = \frac{G(s)}{1+G(s)} \quad \left\{ G(s) = \frac{K}{(s^2+2s+9)s} \right\}$$

For root locus, we need to Focus on the poles of closed loop T.F.

i.e. $\boxed{1 + \frac{K}{s(s^2+2s+9)} = 0}$ ~~charac~~

So, the characteristic equation is

$$\boxed{s^3 + 2s^2 + 9s + K = 0}$$

Now, poles of $G(s) \Rightarrow -1 \pm (2\sqrt{2})i, 0$

zeroes of $G(s) \Rightarrow$ None.

1) No of lines to ∞ = No of asy = 3.

2) Angle of departure,

For $P_{(0)} = 180^\circ$

For $P_{(-1 \pm 2\sqrt{2}i)} = \pm\left(-90 + \tan^{-1}(2\sqrt{2})\right)$

3) Centroid of asymptote = $-2/3$

4) Angle of asymptote $\Rightarrow \left(\frac{2q+1}{3}\right)180^\circ$

$$= 60^\circ, 180^\circ, 300^\circ$$

5) Break away point,

$$\frac{dK}{ds} = -3s^2 - 4s - 9 = 0$$

$\boxed{\text{Not possible}}$

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6) Stability:

R-H array : — $s^3 + 2s^2 + 9s + K = 0$.

s^3	1	9
s^2	2	K
s^1	$\frac{K-18}{2}$	0
s^0	K	0

we have $K = \boxed{[0, 18]}$

But marginal stability $\Rightarrow \boxed{K = 18}$

Using auxillary eqn,

$$2s^2 + K = 0.$$
$$\Rightarrow s^2 = -9$$
$$\Rightarrow \boxed{s = \pm 3j}$$ Intercepts.

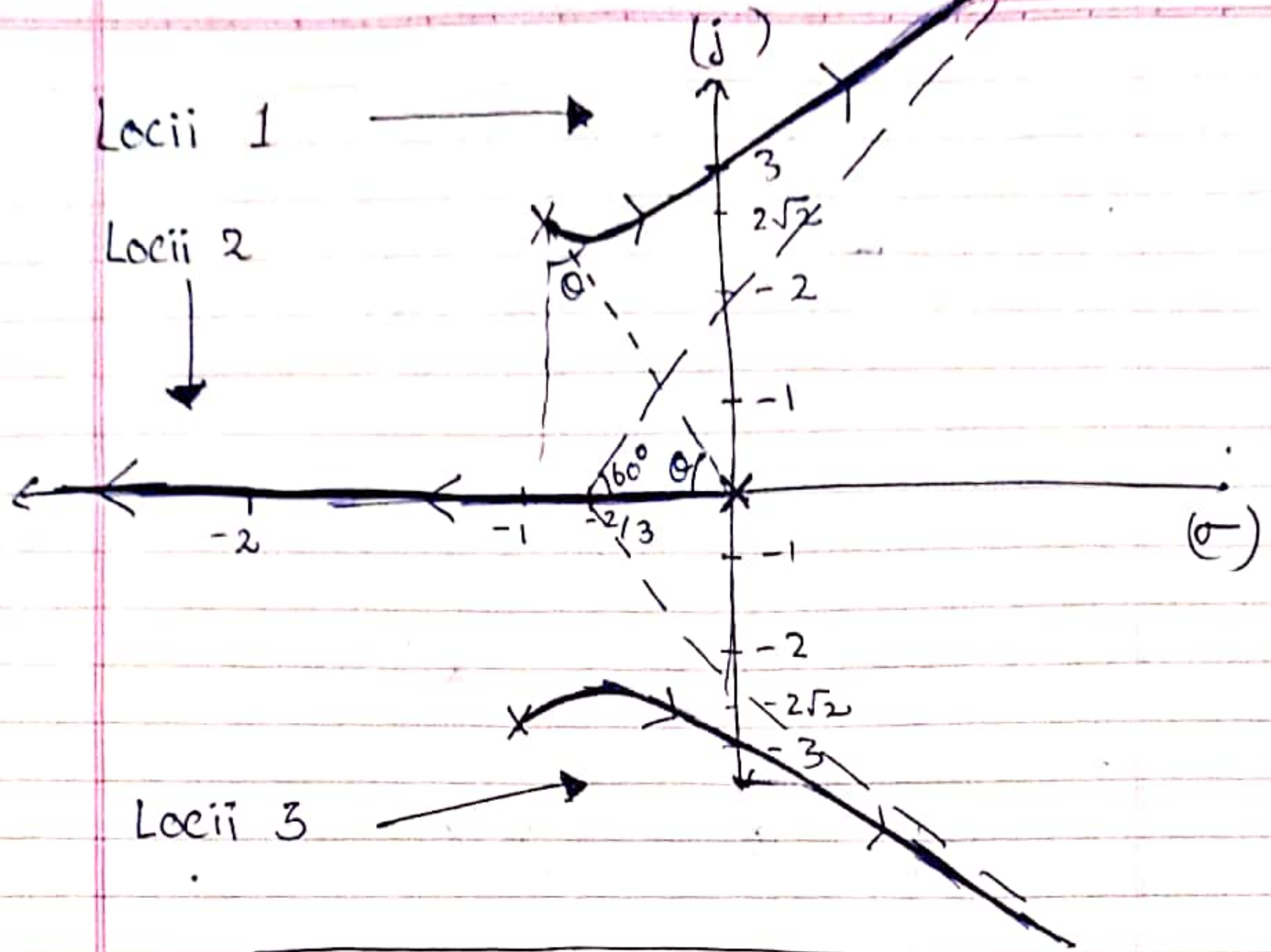
So, For stability, $\boxed{0 < K \leq 18}$ Ans.

where system is (closed loop) marginally stable at $K = 18$.

7) Plotting the root locus,

we have 2 imginary roots and would be symmetric.

Also, using all the parameters that we found we develop a locus.

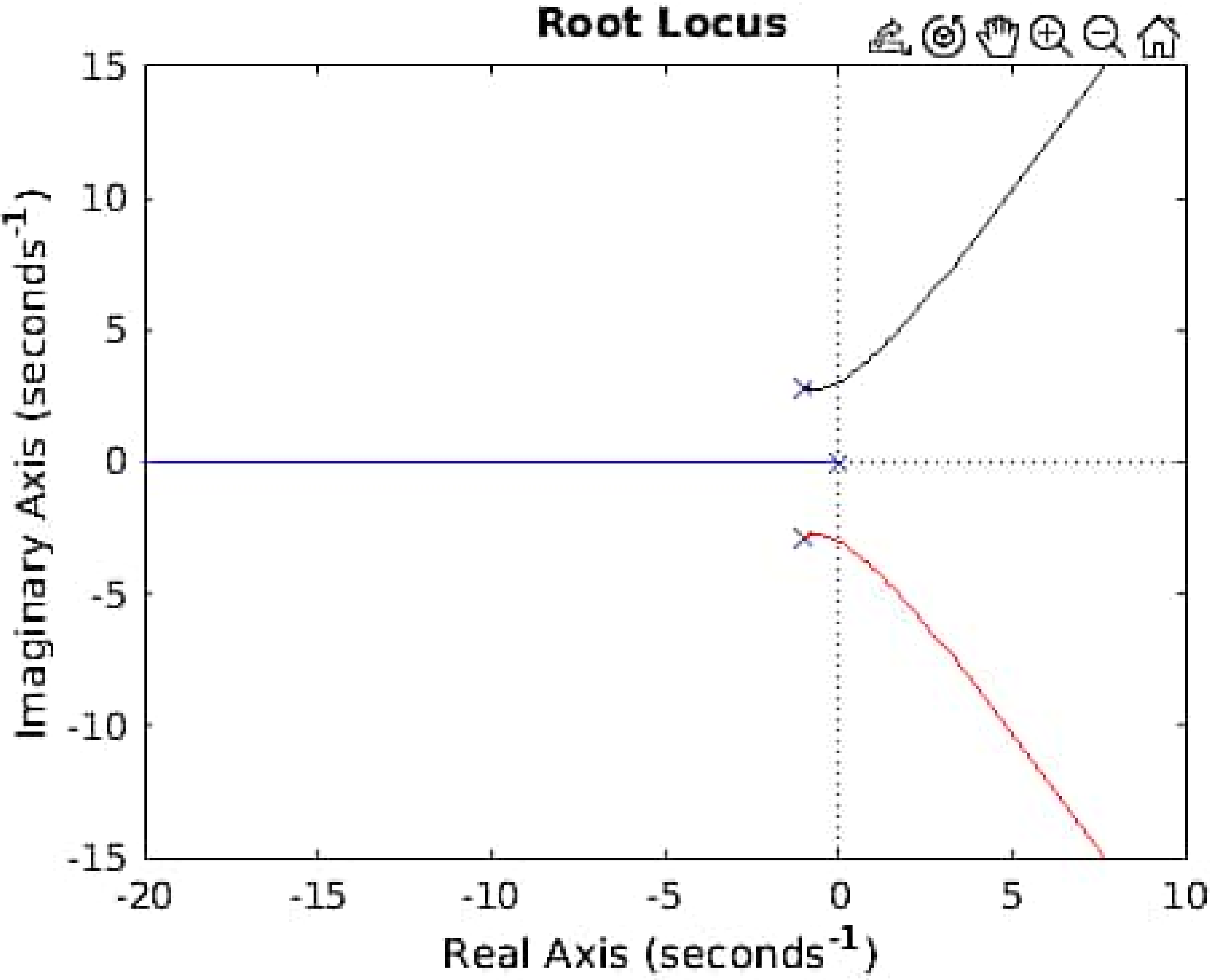


Root Locus of given closed loop system.

→ Matlab code.

```
S= tF('s');
G1 = 1/(s(s*s + 2*s + 9));
rlocus(G1);
```

Now, we can get $s = \pm 3j$ by clicking on points at $x = 0$ and checking the coordinates.
Put this into characteristic eqn to get $K = 18$ i.e. the marginal point.


Root Locus
15
10
5
0
-5
-10
-15
Imaginary Axis (seconds^{-1})
-20
-15
-10
-5
0
5
10
Real Axis (seconds^{-1})

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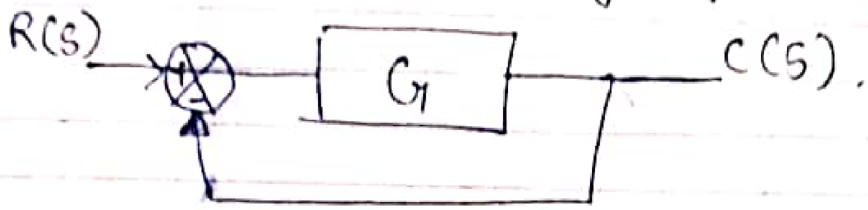
A4 $T.F. = \dfrac{K(s+1)}{(5s+1)(s^2+2s+4)}$ {open loop}

Also, the negative unity gain :-



$E = R(s) - C(s) = \left[\dfrac{R(s)}{1+G}\right]$

Steady state error $= \left\{ \lim_{s\to 0} \dfrac{s\,R(s)}{1+G(s)} \right\}$

[We have to keep $R(s) = 1/s$ otherwise the error goes to either 0 or ∞]

So, $\lim_{s\to 0} \dfrac{(s)(1/s)}{1+\dfrac{K(s+1)}{(5s+1)(s^2+2s+4)}} = 0.2$

$\Rightarrow \boxed{K = 16}$ Ans {for unit step input}

→ Now, we know that

$TF = \dfrac{16(s+1)}{5(s+1/5)[s+1+i\sqrt{3})(s+1-i\sqrt{3})}$ | Poles ⇒ $-1/5$, $-1 \pm i\sqrt{3}$ | zero ⇒ -1

~~For (S+1) ⇒ 1/(1/S+1)~~

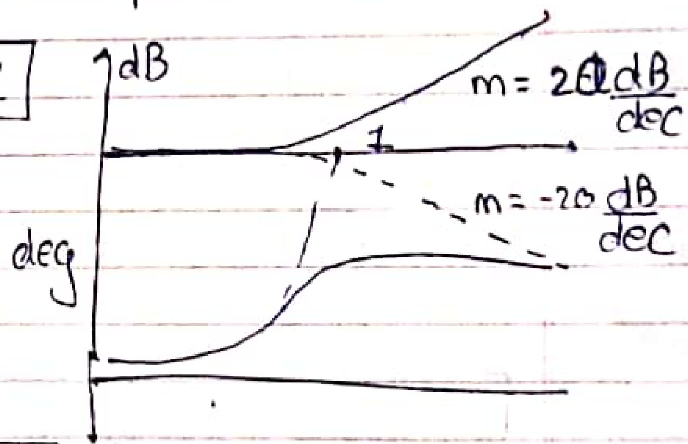
→ Plotting the bode - plot,

• $G \rightarrow \dfrac{16}{5}\,\dfrac{\sqrt{1+\omega^2}}{\sqrt{25\omega^2+1}\;\sqrt{\omega^4-4\omega^2+16}}$

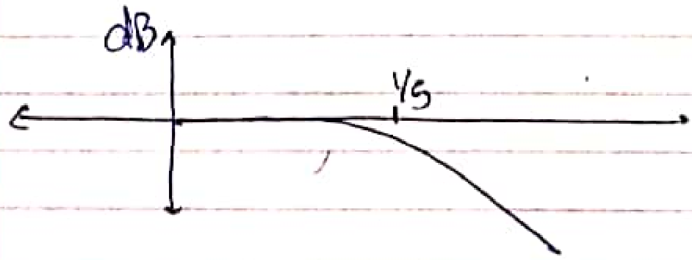
• Phase → $\tan^{-1}\left[\dfrac{5\omega^4-11\omega^2-4}{6\omega(\omega^2+3)}\right]$

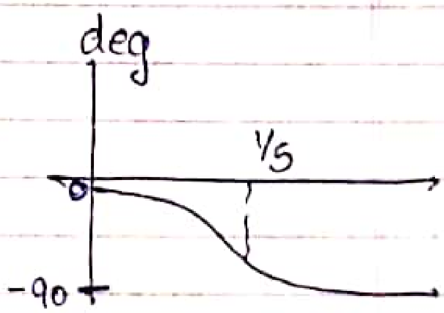
• Now, individual bode plots are :-

1) $(S+1) \rightarrow \boxed{\omega_0 = 1}$



2) $\dfrac{1}{1+5S} \rightarrow \boxed{\omega_0 = 1/5}$



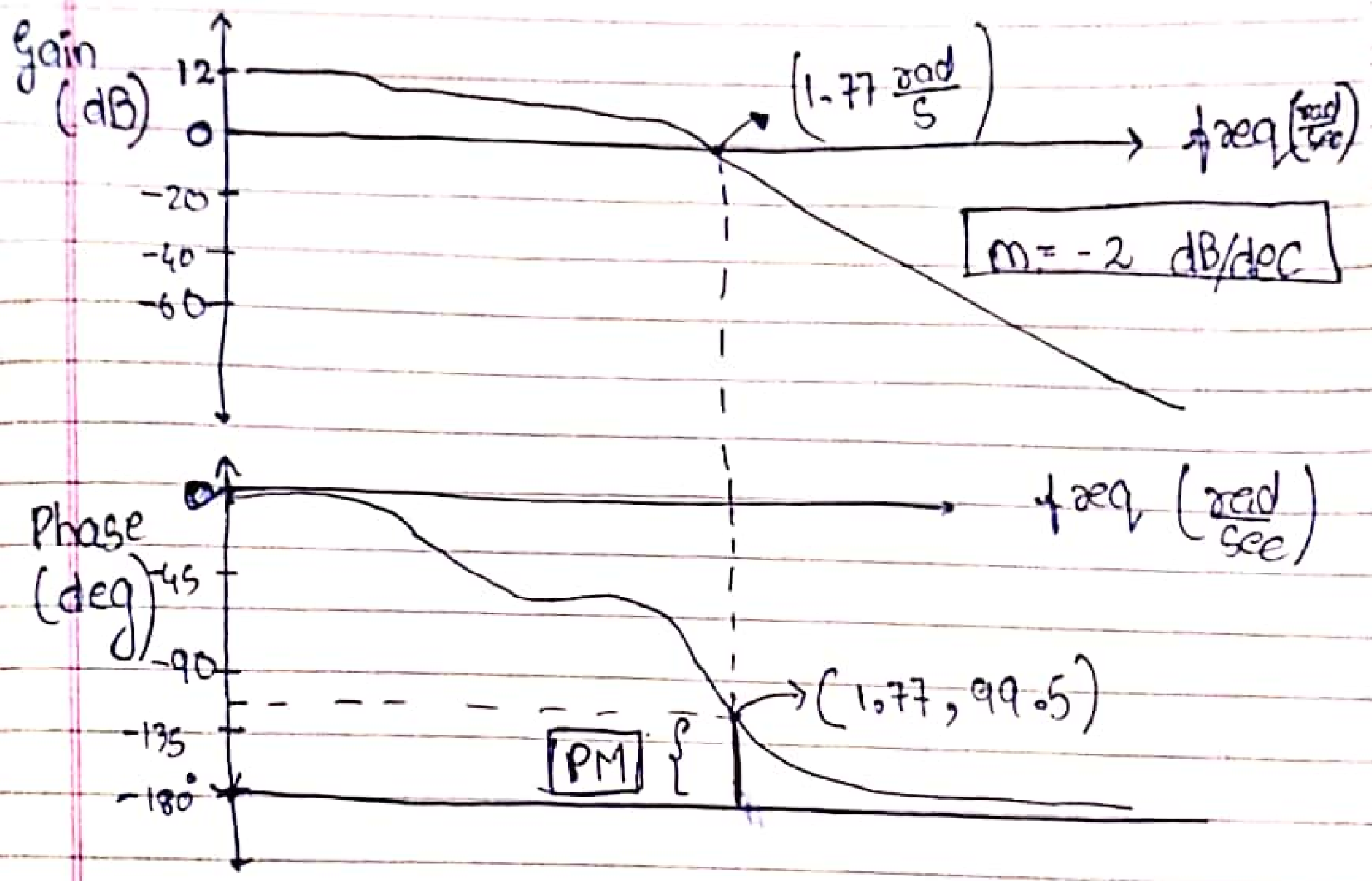


3) $\dfrac{1}{(S^2+2S+4)} \rightarrow \boxed{\text{Complex roots}}$ will have 2 spikes.

$\boxed{A = \frac{1}{2}}$; $\boxed{\zeta = \frac{2}{2\sqrt{4}} = \frac{1}{2}}$; $\boxed{\omega_0 = 2}$

So, because amplitude is in dB, we can add the 3 graphs. Same for $\tan^{-1}(\,)$.

→ Complete bode plot :-



→ Phase margin = ~~∞ dB~~ 80.5° [at $\omega = 1.77$]
Gain margin = ∞ dB

Ans: As both are positive, system is always stable.

→ MATLAB code :

```
sys = tf([16,16],[5,11,22,4]);
margin(sys);
```

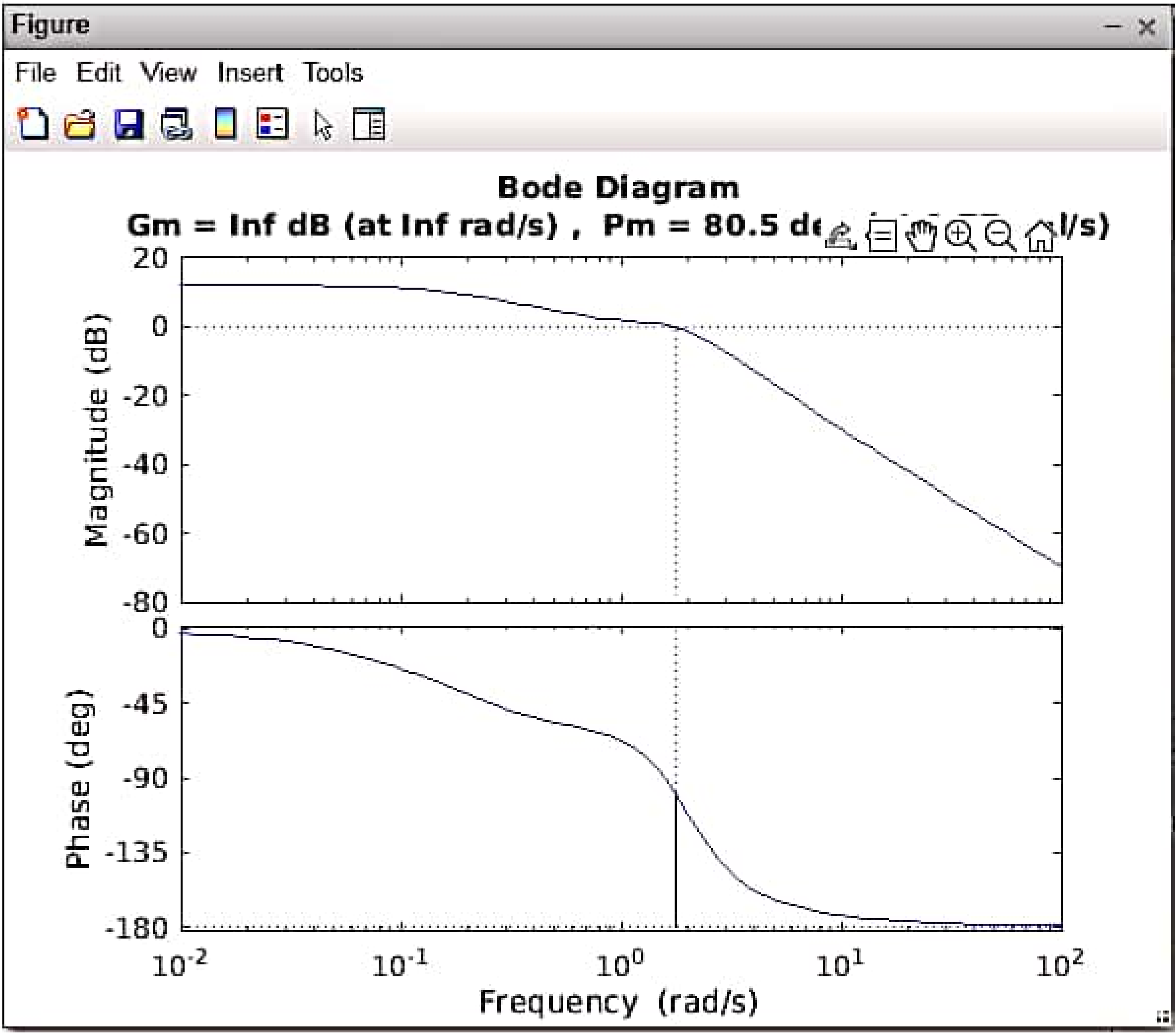


Figure
File Edit View Insert Tools
Bode Diagram
Gm = Inf dB (at Inf rad/s) , Pm = 80.5 d
l/s)
Magnitude (dB)
20
0
-20
-40
-60
-80
Phase (deg)
0
-45
-90
-135
-180
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
Frequency (rad/s)

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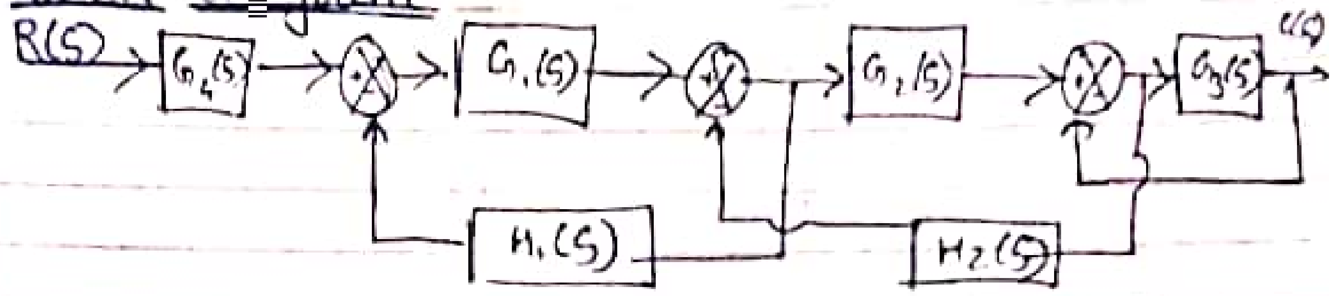
A5

∘) Block diagram



∘) Matlab code :-

```
% For G1(S)
n1 = 1 ; d1 = [1 1 1];

% For G2(S)
n2=1 ; d2 = [1 2];

% For G3(S)
n3 = 1; d3 = [1 2];

% For H2(S)
n4 = 1 ; d4=[1 3];

% For H1(S)
n5 = 100 ; d5= 1;

% For G4(S) [extra block for algorithm]
n6=1 ; d6 = 1

nblocks = 6;
blkbuild;
q = [1,6,-5; 2,1,-4; 3,-3,2; 4,2,-3; 5,-4,1; 6,0,0];

input = 6; output = 3;
```

[A, B, C, D] = connect(a, b, c, d, q, input, output);
[num, den] = ss2tf(A, B, C, D);
printsys (num, den, 'S');

Output displayed :-

State model [a, b, c, d] of the block diagram has 6 inputs and 6 outputs.

$$\text{num} / \text{dem} = \frac{S+3}{S^5 + 9S^4 + 131S^3 + 850S^2 + 2142S + 1820}$$

Result :-

Ans :

$$\boxed{\frac{C(S)}{R(S)} = \frac{S+3}{S^5 + 9S^4 + 131S^3 + 850S^2 + 2142S + 1820}}$$