Scalable Distributed Topologies

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MEIC SDLE 2021

Graphs

A graph $G(V, E)$ can be defined by a set of vertices, and a set of edges that connect pairs of vertices. For instance, a path $G(V = \{a, b, c, d\}, E = \{(a, b), (b, c), (c, d)\})$ can be made into a ring by adding one edge connecting its ends $E = E \cup \{(d, a)\}$.

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- Having an edge (x, y), we say that those vertices are adjacent (or neighbours).
- A Weighted Graph is obtained by assigning a weight to each edge.
- Path is a sequence of vertices, $\ldots, v_i, v_{i+1}, \ldots$, with edges connecting them $(v_i, v_{i+1}) \in E$.

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Walk Edges and vertices can be repeated

Trail Only vertices can be repeated

Path No repeated vertices or edges

Graphs

More topologies

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In a network context, graphs with cycles allow multi-path routing. This can be more robust but data handling can become more complex. Thus, distributed algorithms often construct trees to avoid cycles, while others try to work under multi-path.

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More concepts

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- Distance $d(v_i, v_j)$. Length of the shortest path connecting those nodes.
- Eccentricity of v_i. $ecc(v_i) = max(\{d(v_i, v_j) | v_j \in V\})$.
- Diameter. $D = max(\{ecc(v_i) | v_i \in V\})$
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- Radius. $R = min(\{ecc(v_i)|v_i \in V\})$
- Center. $\{v_i|ecc(v_i) == R\}$
- Periphery. $\{v_i|ecc(v_i) == D\}$

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Graphs
Questions

- How big is the center of a tree?
- How big is the center of a path?

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- How big is the center of a tree?
- How big is the center of a path?
- How big is the periphery of a ring? and the center?
- How to compute eccentricities?

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More complex topologies

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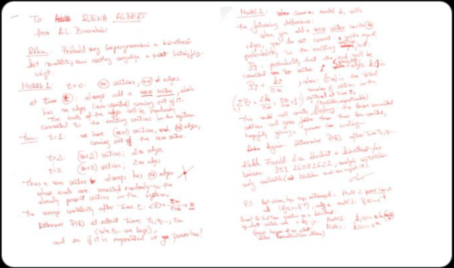
- Watts-Strogatz model. (Seen later on small world constructions).
- Barabasi-Albert model. Preferential attachment: the more connected a node is, the more likely it is to receive new links. Degree Distribution follows a power law.

Albert-László Barabási & Réka Albert

BA model was born in Porto

Albert-László Barabási
@barabasi

Happy XX Anniversary Scale-Free Model!
It was 20 years ago today that Réka and I had the email-fax exchange that resulted in the Barabasi-Albert model.
Luckily I saved the fax I sent to Reka from Porto, that, in a mixture of English and Hungarian, outlines the model.



3:14 PM · Jun 14, 2019 · Twitter Web Client

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- Distance from u to v is the length of the shortest path from u to v.
- A directed spanning tree with root node i is breadth first provided that each node at distance d from i in the graph appears at depth d in the tree.
- Every strongly connected graph has a breadth-first directed spanning tree.

Spanning Trees

Synchronous *SyncBFS* Algorithm

Processes communicate over directed edges. Unique UIDs are available, but network diameter and size is unknown.

Initial state in *SyncBFS*

- $parent = nil$
- $marked = False$ ($True$ in root node i_0)

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SyncBFS algorithm

- Process i_0 sends a *search* message in round 1.
- Unmarked processes receiving a *search* message from x do $marked = True$ and set $parent = x$, in the next round *search* messages are sent from these processes.

Complexity

- Time: At most *diam* rounds (depending on i_0 eccentricity).
- Message: $|E|$. Messages are sent across all edges E.

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Child Pointers

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Termination: Making i_0 know that the tree is constructed

All processes respond with *parent* or *nonparent*. Parent terminates when all children terminate. Responses are collected from leaves to tree root.

Applications of Breath First Spanning Trees.

Aggregation (Global Computation)

Input values in each process can be aggregated towards a sync node. Each value only contributes once, many functions can be used: Sums, Averages, Max, Voting.

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Leader Election

Largest *UID* wins. All process become root of their own tree and aggregate a $Max(UID)$. Each decide by comparing their own *UID* with $Max(UID)$.

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Broadcast

Message payload m can be attached to *SyncBFS* construction ($m|E|$ message load) or broadcasted once tree is formed ($m|V|$ message load).

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Computing Diameter

Each process constructs a *SyncBFS*. Then determines *maxdist*, longest tree path. Afterwards, all processes use their trees to aggregate $max(maxdist)$ from all roots/nodes.
Complexity: Time $O(diam)$ and messages $O(diam \times |E|)$.

Asynchronous setup

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We will now work on a “bare” asynchronous network model, avoiding, for now, useful abstractions like logical time and global snapshots.

System Model

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The lack of helping tools is compensated by a “generous” system model:

Faults

No faults.

Channels

Reliable FIFO send/receive channels.

Reliable FIFO send/receive channels
Automaton

Signature:
Input: $send(m)_{i,j}, m \in M$
Output: $receive(m)_{i,j}, m \in M$
States:
$queue = \langle\rangle$
Transitions:
$send(m)_{i,j}$
Effect:
$queue := queue + \langle m \rangle$
$receive(m)_{i,j}$
Precondition:
$queue.head = m$
Effect:
$queue.pophead()$

Reliable FIFO send/receive channels

Allowed trace behaviour

Let β be a sequence of actions, and *cause*() a function mapping each receive event $e \in \beta\mid_{receive}$ to a preceding send event $s \in \beta\mid_{send}$ such that:

1. $\forall receive(x) \in \beta\mid_{receive}: cause(receive(x)) = send(y) \Rightarrow x = y$. Messages don't come out of the blue.

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4. $receive <_\beta receive' \Rightarrow cause(receive) <_\beta cause(receive')$. Order is preserved.

Signature:

Input: $receive(''search'')_{i,j}$ $\quad j \in nbrs$

Output: $send(''search'')_{i,j}$ $\quad j \in nbrs$

Transitions:

$send(''search'')_{i,j}$

Precondition: $sendto(j) = yes$

Effect: $sendto(j) := no$

$receive(''search'')_{j,i}$

Effect:

if $i \neq i_0$ and $parent = null$ then

$parent := j$

for all $k \in nbrs \setminus \{j\}$ do

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Spanning Trees

AsynchSpanningTree automaton for node i

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Channel automaton

Consumes $send(''search'')_{f,t}$ and produces $receive(''search'')_{f,t}$ in reliable FIFO order.

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AsynchSpanningTree vs *SynchBFS*

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Faster longer paths will win over a slower direct path when setting up *parent*.

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While *AsynchSpanningTree* looks like an asynchronous translation of *SynchBFS*, the former does not necessarily produce a breadth first spanning tree.
Faster longer paths will win over a slower direct path when setting up *parent*.
One can however show that a spanning tree is constructed.

Invariant: A tree is gradually formed

In any reachable state, the edges defined by all *parent* variables form a spanning tree of a subgraph of G, containing i_0; moreover, if there is a message in any channel $C_{i,j}$ then i is in this spanning tree.

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In any reachable state, if $i = i_0$ or $parent \neq null$, and if $j \in nbrs_i \setminus \{i_0\}$, then either $parent_j \neq null$ or $C_{i,j}$ contains a search message or $sendto(j)_i$ is yes.

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Leading to:

Theorem

The *AsynchSpanningTree* algorithm constructs a spanning tree in the undirected graph G.

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Spanning Trees
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Altough a tree with height h, such that $h > diam$, can occur it only occurs if it does not take more time than a tree with $h = diam$.
Faster long paths must be faster!

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Child pointers and Broadcast

If nodes report *parent* or *nonparent* one can build a tree that broadcasts.

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No, because a fast path is not always fast. Complexity is $O(h(l+d))$, at most $O(n(l+d))$, where $n = |V|$.

Broadcast with Acks

Its is possible to build a *AsynchBcastAck* algorithm that collects acknowledgements as the tree is constructed. Upon incoming broadcast messages each node Acks if they already know the broadcast and Acks to parent once when all neighbours Ack to them.

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Leader Election with *AsynchBcastAck*

This algorithm includes termination and if all nodes initiate it and report their UIDs it can be used for Leader Election with unknown diameter and number of nodes.

Epidemic Broadcast Trees

Plumtree protocol

- Gossip broadcast
  - + highly scalable and resilient
  - - excessive message overhead
- Tree-based broadcast
  - + small message complexity
  - - fragile in the presence of failures

Can we get the best of both worlds?

- Eager push. Nodes immediately forward new messages
- Pull. Nodes periodically query for new messages
- Lazy push. Nodes push new message ids and accept pulls

In lazy push, there is a separation among payload and metadata

Epidemic Broadcast Trees

Gossiping into tree

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- Nodes sample random peers into a eagerPush set
- Neighbours should be stable and TCP can be used
- Links are kept reciprocal (towards undirected graph)
- First message reception puts origin in eagerPush
- Further duplicate receptions moves source to lazyPush
- Eager push of payload and lazy push of metadata

Epidemic Broadcast Trees

Repairing a broken tree

If the tree breaks, and graph stays connected, nodes get metadata but not payloads. This is detected by timer expiration and the metadata source in lazyPush is moved to eagerPush. Potential redundancy (due to cycles) is cleared later by the standard algorithm.

Small Worlds

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- The question was: "Why should there exist short chains of acquaintances linking together arbitrary pairs of strangers?"

Random graphs and clustering

- Consider a graph where a given number of edges is created uniformely at random among the graph vertices. See Erdős–Rényi model.
- The resulting random graph is known to depict a low diameter and thus could support small paths. $O(\log n)$.

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- Not so, since people's graphs have more clustering. If A is friend to B and C, then it is likely that B and C are also friends.
- Watts and Strogatz proposed a model that mixes short range and long range contacts.

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- The resulting random graph is known to depict a low diameter and thus could support small paths. $O(\log n)$.
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- Nodes establish k local contacts using some distance metric among vertices (say in a ring or lattice) and then a few long range contacts uniformly at random.

Random graphs and clustering

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Routing in Small Worlds

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- Long range contacts can be tuned to become more clustered in the vicinity. The target is to have uniformity across all distance scales, a property found in DHT designs like Chord, and locally find $O(\log^2 N)$ routes.

Uniformity across distance scales

Pieter Bruegel the Elder - The Hunters in the Snow

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