Math 201 **Name (Print):** ____________________

Fall 2019

Midterm2

11/08/2019

Time Limit: 50 minutes

This exam contains 10 pages (including this cover page) and 4 problems. Check to see if any pages are missing.

Statement of Ethics

I agree to complete this exam without unauthorized assistance from any person, materials, or device.

Signature ____________________ Date ____________________

1 Fuentes T 1:30 - 2:20 Bloomberg 278
2 Quinan T 3:00 - 3:50 Hodson 301
3 Quinan T 4:30 - 5:20 Hodson 216
4 Luo Th 1:30 - 2:20 Bloomberg 278
5 Luo Th 3:00 - 3:50 Maryland 309

Your section number: ____________________

You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided.
- **No partial credit for any 1 point problem**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this.
- **Any answer or solution on scratch paper will receive no credit**.

Do not write in the table to the right.

Problem	Points	Score
1	4	
2	5	
3	6	
4	5	
Total:	20	

1. Determine which of the following transformations with domain P_2, the space of all polynomials of degree at most 2, is a linear **transformation**. Remember to justify your answers.

   (a) (1 point) $T_1 : P_2 \to \mathbb{R}^4$ defined by $T_1(p) = (p(1), p(2), p(3), p(4))$.

   (b) (1 point) $T_2 : P_2 \to \mathbb{R}^3$ defined by $T_2(p) = (p(0), p'(0), p''(0))$.

(c) (1 point) $T_3 : P_2 \to \mathbb{R}^3$ defined by $T_3(p) = (p(1) + 2, (p(0))^3, p'(0))$.

(d) (1 point) $T_4 : P_2 \to P_2$ defined by $T_4(p)(x) = xp'(x)$.

2. (a) (3 points) Let $\mathbb{R}^{2\times 2}$ to be the space of 2×2 matrices and consider the ordered basis $\mathcal{B}$ of $\mathbb{R}^{2\times 2}$,

$$\mathcal{B} = \left(\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right).$$

For the linear transformation $T : \mathbb{R}^{2\times 2} \to \mathbb{R}^{2\times 2}$ defined by

$$T(A) = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} A - A \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix},$$

determine $[T]_{\mathcal{B}}$, the $\mathcal{B}$-matrix of T.

(b) (2 points) Find a basis of $\mathrm{im}(T) \subset \mathbb{R}^{2\times 2}$.

3. (a) (3 points) Perform the Gram-Schmidt process to $\boldsymbol{v}_1, \boldsymbol{v}_2 \in \mathbb{R}^4$, $\boldsymbol{v}_1 = (1,2,3,4)$, $\boldsymbol{v}_2 = (4,3,2,1)$, and find the corresponding QR factorization of the matrix

$$M = \begin{bmatrix} 1 & 4 \\ 2 & 3 \\ 3 & 2 \\ 4 & 1 \end{bmatrix}$$

(b) (3 points) Compute the 4×4 matrix (with respect to the standard basis) for the orthogonal projection onto the plane spanned by the vectors $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$ above.

4. (a) (3 points) Can you find a 2019×2019 matrix A such that $\mathrm{im}(A^2) = \ker(A^2)$? If so, please give an example; otherwise please explain why.

(b) (1 point) Can you find a 4×4 matrix A such that $\mathrm{im}(A^2) = \ker(A^2)$? If so, please give an example; otherwise please explain why.

(c) (1 point) Let $P_{=2}$ be the space of all polynomials of exactly degree 2. Prove or disprove: $P_{=2}$ is a linear space.

Please estimate your score. You can also provide your suggestion for the course here.