Math 202
Sparing 2019
Midterm
04/12/2019
Time Limit: 50 Minutes

Name (Print): ____________________

This exam contains 8 pages (including this cover page) and 5 problems. Check to see if any pages are missing.

Statement of Ethics

I agree to complete this exam without unauthorized assistance from any person, materials, or device.

Signature ____________________ Date ____________________

1 Lin T 1:30-2:20 Shriver 104
2 Lin T 3:00-3:50 Hodson 301
3 Sherwood Th 4:30-5:20 Gilman 119
4 Sherwood Th 3:00-3:50 Maryland 309
5 Koh T 4:30-5:20 Gilman 119
6 Stubis Th 1:30-2:20 Hodson 313
7 Stubis Th 3:00-3:50 Hodson 301
8 VanBlargan T 3:00-3:50 Gilman 119

Your section number: ____________________

You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this.

Do not write in the table to the right.

Problem	Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
Total:	100	

1. (a) (5 points) Compute the arc length of $\boldsymbol{c}(t) = (x(t), y(t)) = (t, t^2), 0 \leq t \leq 1$.
A $\frac{\sqrt{5}}{2} - \frac{\ln(-2+\sqrt{5})}{4}$.

(b) (5 points) Let $\boldsymbol{F}(x, y, z) = (zy, xz, xy)$. Find a function $f : \mathbb{R}^3 \to \mathbb{R}$ such that $\boldsymbol{F} = \nabla f$.
A: xyz.

(c) (5 points) Compute the curl of $V(x,y,z) = xy\boldsymbol{i} - x\boldsymbol{j} + 0\boldsymbol{k}$.
A: $-(x+1)\boldsymbol{k}$.

(d) (5 points) Let $\boldsymbol{F} = (x+y, z, x)$, compute $\nabla \cdot (\nabla \times \boldsymbol{F})$.
A: 0.

2. (20 points) Find the volume of the region bounded by $z = 0, x = 1, x = 2, y = 1, y = 2$ and the paraboloid $z = 9 - x^2 - y^2$.

   A: The region is $\{(x, y, z)|1 \leq x \leq 2, 1 \leq y \leq 2, 0 \leq z \leq 9 - x^2 - y^2\}$. The volume is

$$\int_1^2 \int_1^2 \int_0^{9-x^2-y^2} dzdydx = \frac{13}{3}.$$

3. (20 points) Let $D_a = \{(x, y)|x^2 + y^2 \le a^2\}$, compute

$$\iint_{D_a} dxdy.$$

A: $a^2\pi$.

4. (20 points) Calculate

$$\int_0^1 \int_{\sqrt{x}}^1 xe^{y^5}\, dydx.$$

A: $\int_0^1 \int_{\sqrt{x}}^1 xe^{y^5}\, dydx = \int_0^1 \int_0^{y^2} xe^{y^5}\, dxdy = \int_0^1 \frac{1}{2} y^4 e^{y^5}\, dy = \frac{1}{10} e^{y^5}|_0^1 = \frac{1}{10}(e-1).$

5. (20 points) Evaluate $\iint_D (x+y)dxdy$ where T is the triangle in the xy plane bounded by the lines $y=x, x+y=2, y=0$ using the linear change of variables $u=x+y, v=x-y$.

   A: The line $y=x$ maps to $v=0$, the line $x+y=2$ maps to $u=2$ and the line $y=0$ maps to $v=u$ hence $\iint_T (x+y)dA == \int_0^2 \int_0^u \frac{1}{2} u dv du = \frac{1}{2}\int_0^2 u^2 du = \frac{4}{3}$

Please estimate your score.