Math 201
Fall 2019
Final
12/16/2019
Time Limit: 3 hours

Name (Print): ____________________

This exam contains 16 pages (including this cover page), 7 problems and **1 bonus problem**. Check to see if any pages are missing.

Statement of Ethics

I agree to complete this exam without unauthorized assistance from any person, materials, or device.

Signature ____________________ Date ____________________

1 Fuentes T 1:30 - 2:20 Bloomberg 278
2 Quinan T 3:00 - 3:50 Hodson 301
3 Quinan T 4:30 - 5:20 Hodson 216
4 Luo Th 1:30 - 2:20 Bloomberg 278
5 Luo Th 3:00 - 3:50 Maryland 309

Your section number: ____________________

You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided.
- **No partial credit for any 1 point problem**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this.
- **Any answer or solution on scratch paper will receive no credit**.

Do not write in the table to the right.

Problem	Points	Score
1	4	
2	4	
3	4	
4	6	
5	6	
6	6	
7	10	
8	0	
Total:	40	

1. Let $n \geq 2$ be a positive integer, P_n the set of polynomials of degree less than or equal to n. Prove or disprove if the following subset of P_n is a linear space. If the answer is yes, find a basis of the subspace.

   (a) (1 point) $\{a_1 t \mid a_1 \in \mathbb{R}\}$.

   (b) (1 point) $\{a_0 + a_1 t + \cdots + a_n t^n \mid a_0 + \cdots + a_n = 0, a_0, \ldots, a_n \in \mathbb{R}\}$.

(c) (1 point) $\{a_0 + a_1 t + \cdots + a_n t^n \mid a_0, \ldots, a_n \leq 0\}$.

(d) (1 point) $\{a_0 + a_1 t + \cdots + a_n t^n \mid a_1 = 0, a_0, a_2, \ldots, a_n \in \mathbb{R}\}$.

2. Consider the following system of linear equations,

$$\begin{array}{rrrl} x+ & y & & = 2 \\ & y & +2z & = 2\,. \\ 4x+ & & z & = 9 \end{array}$$

(a) (2 points) Use Gauss–Jordan elimination to solve the system of linear equations.

(b) (2 points) Use Carmer's rule to solve the system of linear equations.

3. (4 points) Find the quadratic polynomial of the form $f(x) = ax^2 + b$ which fits the data $(0, 0)$, $(1, 1)$, $(2, 2)$ best.

4. (a) (3 points) If A is a 4×4 matrix, is it true that there MUST ALWAYS BE a line L passing through the origin such that if $\boldsymbol{v} \in L$ then $A\boldsymbol{v} \in L$.

(b) (3 points) Let A be a square matrix, if A is diagonalizable, show that A^3 is diagonalizable.

5. Let
$$A = \begin{pmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{pmatrix}.$$

(a) (2 points) Find all the possible eigenvalues of A.

(b) (2 points) Find all the possible eigenvectors of A.

(c) (2 points) Is A diagonalizable? Explain why.

6. (a) (3 points) Compute the determinant of

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1^2 & 2^2 & 3^2 & 4^2 \\ 1^3 & 2^3 & 3^3 & 4^3 \\ 1^4 & 2^4 & 3^4 & 4^4 \end{pmatrix}.$$

(b) (3 points) Find the classical adjoint of the matrix

$$A = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 3 \end{pmatrix}.$$

and use the result to find A^{-1}.

7. (a) (5 points) Let $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, $p(\lambda) = \det(\lambda I_2 - A) = c_2\lambda^2 + c_1\lambda + c_0$. Compute c_2, c_1, c_0, and the matrix $c_2A^2 + c_1A + c_0I_2$.

(b) (5 points) Let B be an $n \times n$ diagonalizable matrix, $q(\lambda) = \det(\lambda I_n - B) = c_n\lambda^n + c_{n-1}\lambda^{n-1} + \cdots + c_0$. Compute the matrix $c_nB^n + c_{n-1}B^{n-1} + \cdots + c_0I_n$.

8. (bonus) Let A be an $n \times n$ matrix, $p(\lambda) = \det(\lambda I_n - A) = c_n\lambda^n + c_{n-1}\lambda^{n-1} + \cdots + c_0$. Compute $c_n A^n + c_{n-1}A^{n-1} + \cdots + c_0 I_n$.

Please estimate your score. You can also provide your suggestion for the course here.