Calculating Biological Quantities

CSCI 2897

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2021, Lecture 2

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Lecture 2 Plan

1. **One minute review of the basics:**
    1. Website
    2. Syllabus
    3. Canvas
    4. Slack
2. **Office Hours?**
3. **Asking “modeling” questions**
4. **Some vocabulary**
5. **Steps to modeling a biological problem (1-4)**

Last Time on CBQ…

Sean Taylor
FB Research

like
this
PDF

- Website: https://github.com/dblarremore/CSCI2897
  - Homework & reading posted, Code examples, Class notes
- Syllabus: https://github.com/dblarremore/CSCI2897#syllabus
- Canvas: Turn in homework, Lecture links, Check grades
- Slack: **Didn't get the invite? Stick around after class—we'll get you set up!**
- Textbook: See Slack.

Office Hours?

1. Post a link → book 15 min slots.

2. Drop-in. Set hours. Show up. I'll be on Zoom.

↓

Mix of both, why not.

Dynamical Models 101: Ask a question

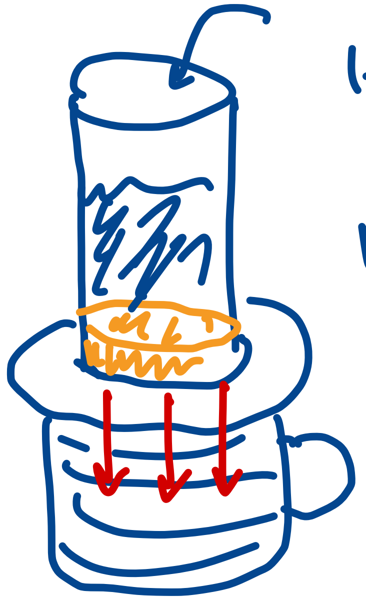
changes over time

- Think about a problem that puzzles you.
- Draw a "flow diagram" that illustrates the various processes at work.
- *Dynamical* models describe how a system changes over time.

Easily rephrased as dynamical models.

Examples

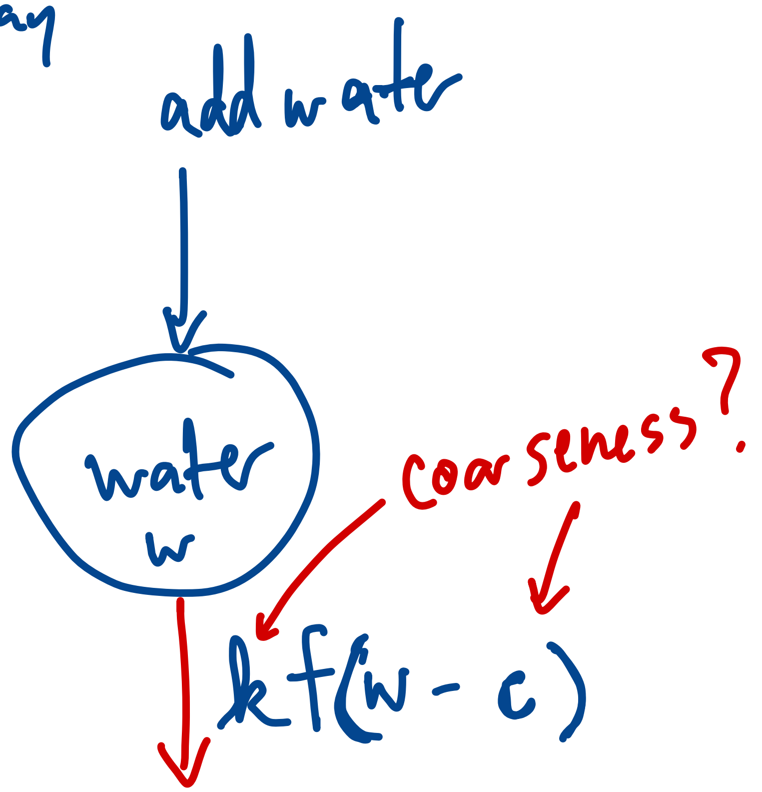
(1) Aeropress - makes coffee.



If I fill to top.
↓
water drops to middle.

If I fill it ½ way
↓
water doesn't percolate.

What determines the Δ in the water in aeropress over time?



(2) How does heat/oil affect how much of the egg is burned?

(3) How long does it take for the soil in my plant to dry out? (T, time since watering last, sun, humidity)

Deterministic vs Stochastic dynamical models

this course

- **Deterministic** models assume that the future is entirely predicted (i.e. determined) by the model.

Q: How much H_2O is in my coffee maker?

Model: Flow in / Flow out → deterministic.

- **Stochastic** models assume that random (stochastic) events affect the system.

Q: How much snow at Eldora? (base height?)

Model: Stochastic · snow fall,
· temperature

include a random variable
(source of that stochasticity)

Otto & Day: 7 steps to modeling a biological problem

1. Formulate the question

- What do you want to know?
- Describe that in the form of a question.
- Boil the question down.
- Start with the simplest, biologically reasonable description of the problem.

story.

"Explain it like I'm 5"

Otto & Day: 7 steps to modeling a biological problem

1. Formulate the question
2. Determine the basic ingredients

Define:
- variables
  - constraints?
  - interaction among variables.

Decide: time as discrete vs continuous?

time scale. $t=0$ vs $t=1$ ... day? year? season? second?

Define:
- parameters
  - constraints
    - fundamental
    - reasonable

Q: what's the diff? parameter vs. variable?

fixed

change over time

mathy stuff
autonomous vs. non-autonomouse
Diff. Eqns

preview!

Otto & Day: 7 steps to modeling a biological problem

1. Formulate the question
2. Determine the basic ingredients
3. Qualitatively describe the biological system

Life cycle diagrams

Flow diagrams

Event tables

Otto & Day: 7 steps to modeling a biological problem

LHS] lefthand side
RHS] righthand side.

1. Formulate the question
2. Determine the basic ingredients
3. Qualitatively describe the biological system
4. Quantitatively describe the biological system

Diagrams (guide) → Equations

Checks: • constraints hold?
• units on LHS vs RHS of equations

Big: Can the model actually answer Q in step 1?

Otto & Day: 7 steps to modeling a biological problem

1. Formulate the question
2. Determine the basic ingredients
3. Qualitatively describe the biological system
4. Quantitatively describe the biological system
5. Analyze the equations

- solve (analytical)
- simulate (numerical)
- analyze

Math Class

APPM
Diff. Eq.

Otto & Day: 7 steps to modeling a biological problem

1. Formulate the question
2. Determine the basic ingredients
3. Qualitatively describe the biological system
4. Quantitatively describe the biological system
5. Analyze the equations
6. Checks & balances

- check against known examples.
  e.g. if I don't water for 1 year → soil is v. dry.

- generalizability?
- alternatives to your model? (reflecting? repeating prev. steps)

Otto & Day: 7 steps to modeling a biological problem

1. Formulate the question
2. Determine the basic ingredients
3. Qualitatively describe the biological system
4. Quantitatively describe the biological system
5. Analyze the equations
6. Checks & balances
7. Relate the results back to the question

- Did your model help answer the Q?
- Intuitive? Counter intuitive?
- Tell a story to summarize the model. Insights?
- Experiments?

1. Formulate the question

Delta (capital) change

δ (lowercase delta)

- Find a living / biological object / thing / stuff
- Ask a Q about how it might Δ over time?

1. How does # branches on a tree change over time?
2. How does a cat change the # of mice in a barn?
3. How does # people w/ COVID change over a year?

pop. growth

immigration

interactions among variables

you can tell what the variable is!

2. Determine the basic ingredients

- **Variables**: what entities might change over time?
- Assign a letter to each variable. (Hint: use "intuitive" letters!)
- Write down *fundamental* constraints on your variables.
- Write down *reasonable* constraints on your variables.

\# branches $n(t) \geq 0$

\# mice $n(t) \geq 0$

\# infected ppl. $I(t) \geq 0$

\# recovered $R(t) \geq 0$

\# susceptible $S(t) \geq 0$

$I(t) + R(t) + S(t) =$ pop. size

notes:

- $n(t)$ explicitly saying "of t"
- alternatives:
  - n_t, n_{t+1}
  - n (no t)

conventions:

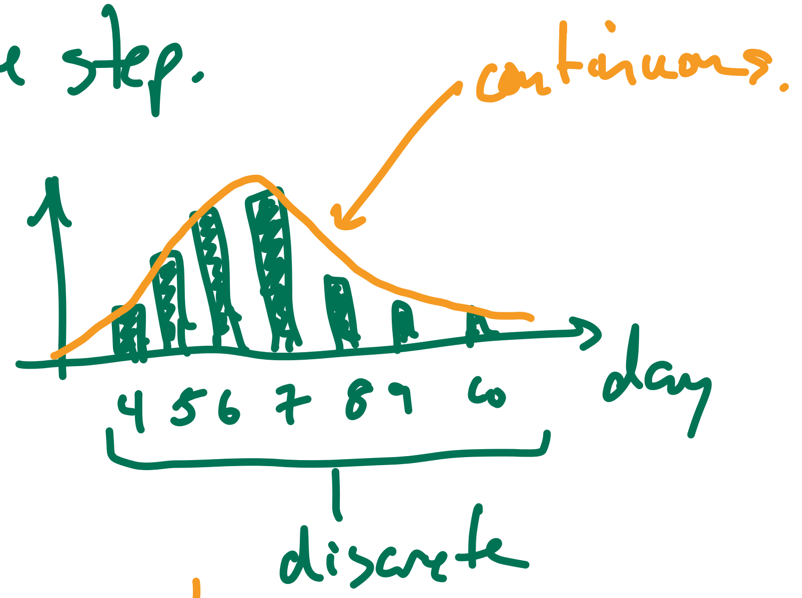
$n = \#$

$p =$ proportions

$n_1(t)$, $n_2(t)$] two species

Discrete time vs Continuous time

- **Discrete time models**: "jumpy"
  - assume that Δs cannot compound within a time step.
  - ↑ holds well if Δt is small/reasonable
  - Ex: viral load in SARS-CoV-2 infections



- **Continuous time models**: "smooth"
  - Assumes that variables can change at any point in time!
  - Seems better?

    But: could be unrealistic! Ex: You may need to grow to a certain size before branching.

- **Note:**

  Sometimes math is easier one way vs another!

Be clear about your time scale

- **Discrete time models**:
  - COVID models : day
  - Animal Pops over longer time : season/year
  - Soil Moisture : hours

  "tick" of the clock

- **Continuous time models**:
  - How long has passed between $t=1$ and $t=0$?

btw…

- You'll have to decide whether your variables are discrete or continuous too!

branches: integer ≥ 0

mice: integer ≥ 0

] discrete.

$\mathbb{R}$ = real #s.

↓

biomass mice: $\mathbb{R} \geq 0$

Infectious Disease:

S I R → integers ≥ 0 (people)

S I R → $0 \leq \mathbb{R} \leq 1$ (pop. proportions)

btw…

- You'll have to decide whether your variables are discrete or continuous too!

1. Often, values get large enough that you can model a discretized population as continuous (with little error).

2. Sometimes you can reinterpret a variable to go discrete → cont. (# mice → mouse biomass)

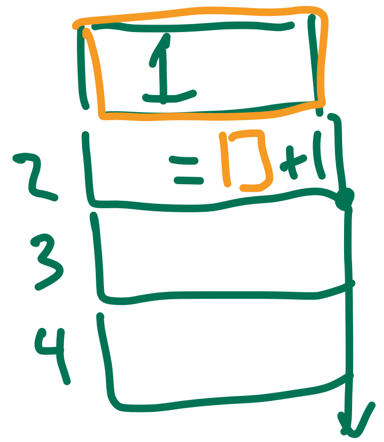
3. Easier Math

Recursion Equations

- A **recursion equation** describes the value of a variable in the next time step.

  $n(t+1) =$ "some function of $n(t)$"

- Examples.

- 

  Excel

- Bank Balance

  Dynamic?

- Conversion to binary (?)

- Fibonacci

  $n(t+1) = n(t) + n(t-1)$

note: "second order"

Difference Equations

- A **difference equation** describes the difference between a variable's values in two successive time steps

$$\Delta n = n(t+1) - n(t) = \text{"some function of } n(t)\text{"}$$

definition

- Examples.

• Excel: $\Delta n = 1$

interest rate

• Bank Interest: $\Delta x = r\,x(t)$

Differential Equations

[Q: $\frac{dx(t)}{dt} = r\,x(t) + b\,x(t-1)$
delay diff. equation. tough!]

- A **differential equation** describes the rate of change of the variable over time

$$\frac{dn(t)}{dt} = \text{"some function of } n(t)\text{"}$$

- Examples.

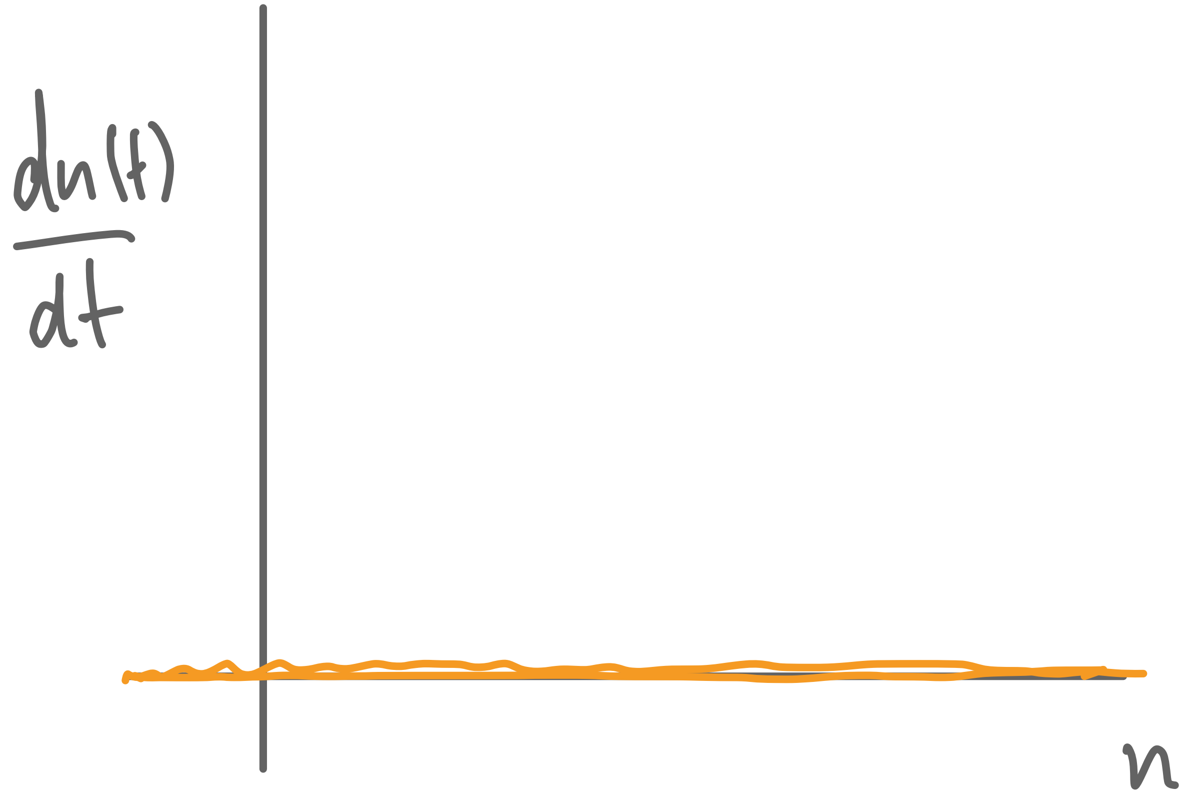
• Interest $\quad \frac{dn(t)}{dt} = r\,n(t)$

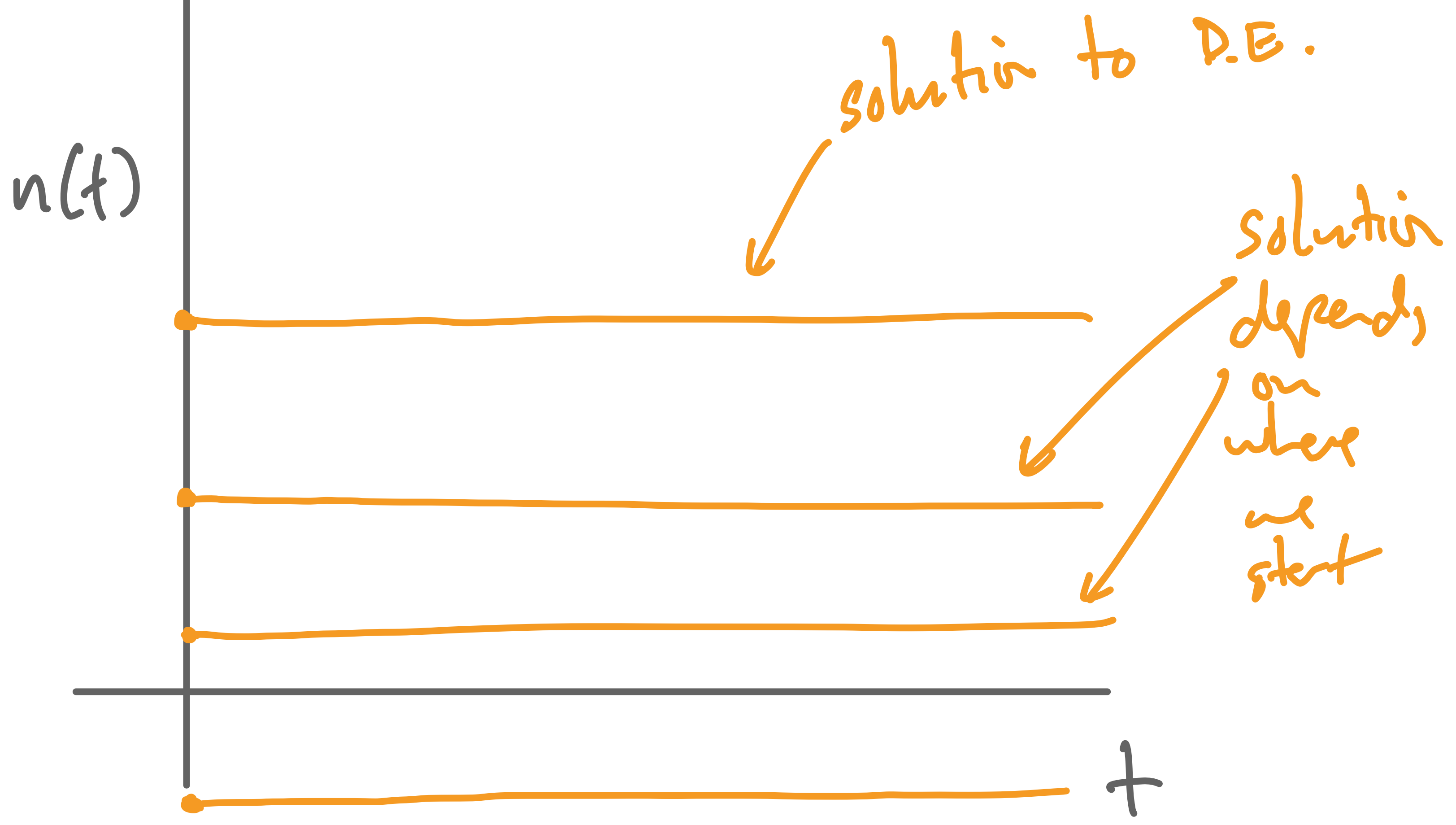
• Newton's Law of Cooling $\quad \frac{dT(t)}{dt} = -k\left(T(t) - T_{room}\right)$

Examples for Intuition

- Ex 1: (A) Sketch the derivative vs ~~time~~ n. (B) Sketch the variable vs time.

$$\frac{dn(t)}{dt} = 0$$

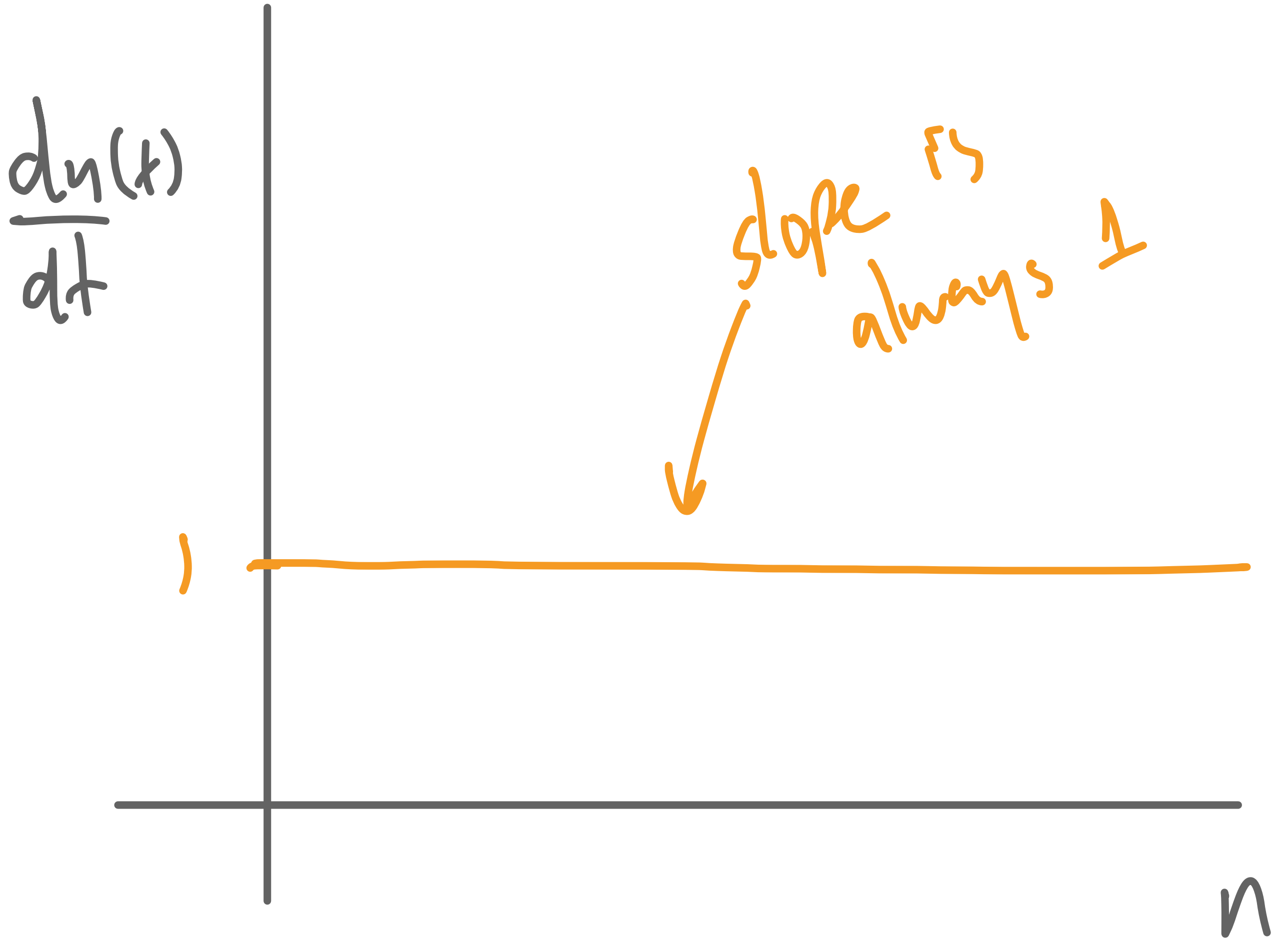


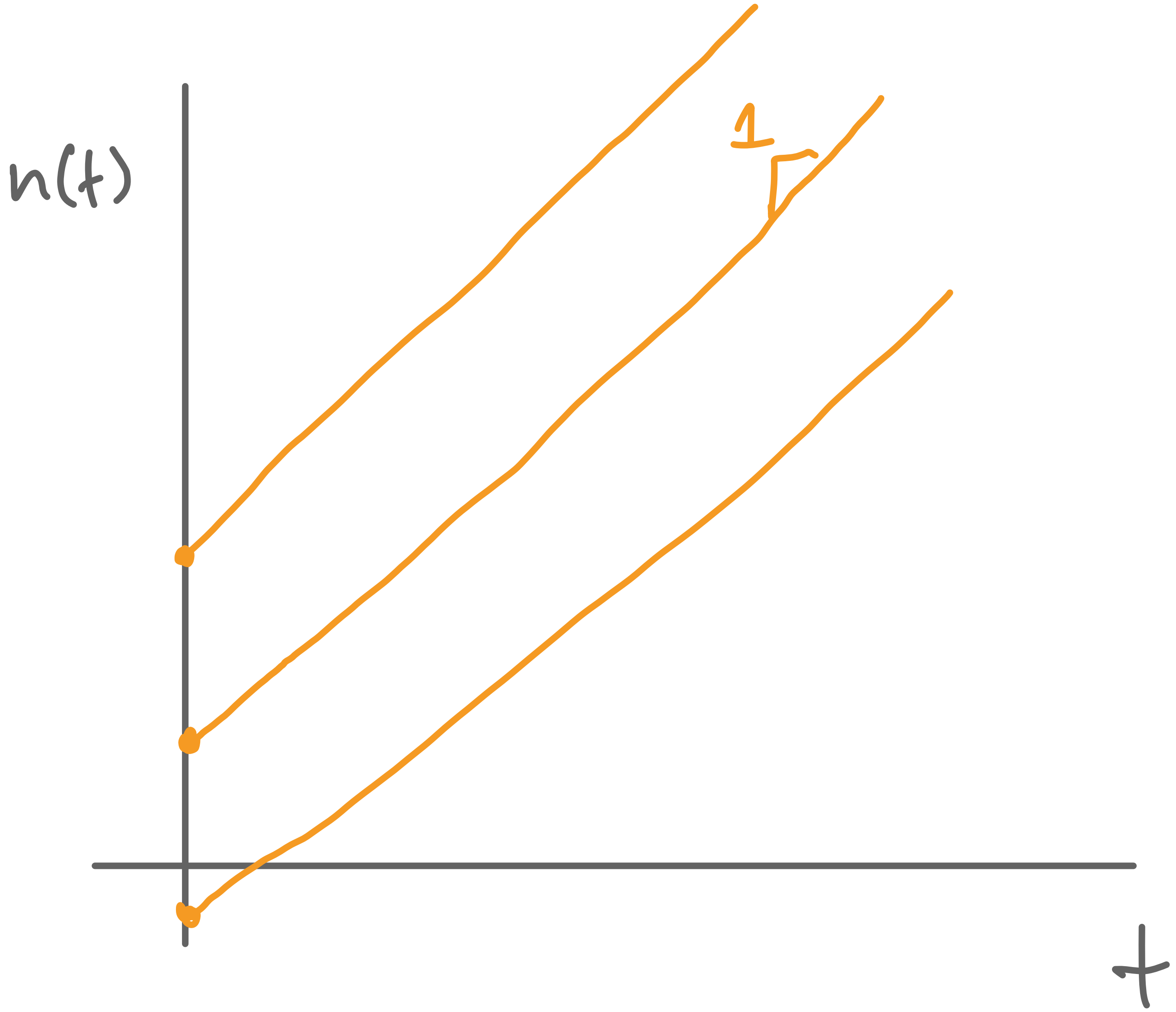


Examples for Intuition

- Ex 2: (A) Sketch the derivative vs ~~time~~ n. (B) Sketch the variable vs time.

$$\frac{dn(t)}{dt} = 1$$

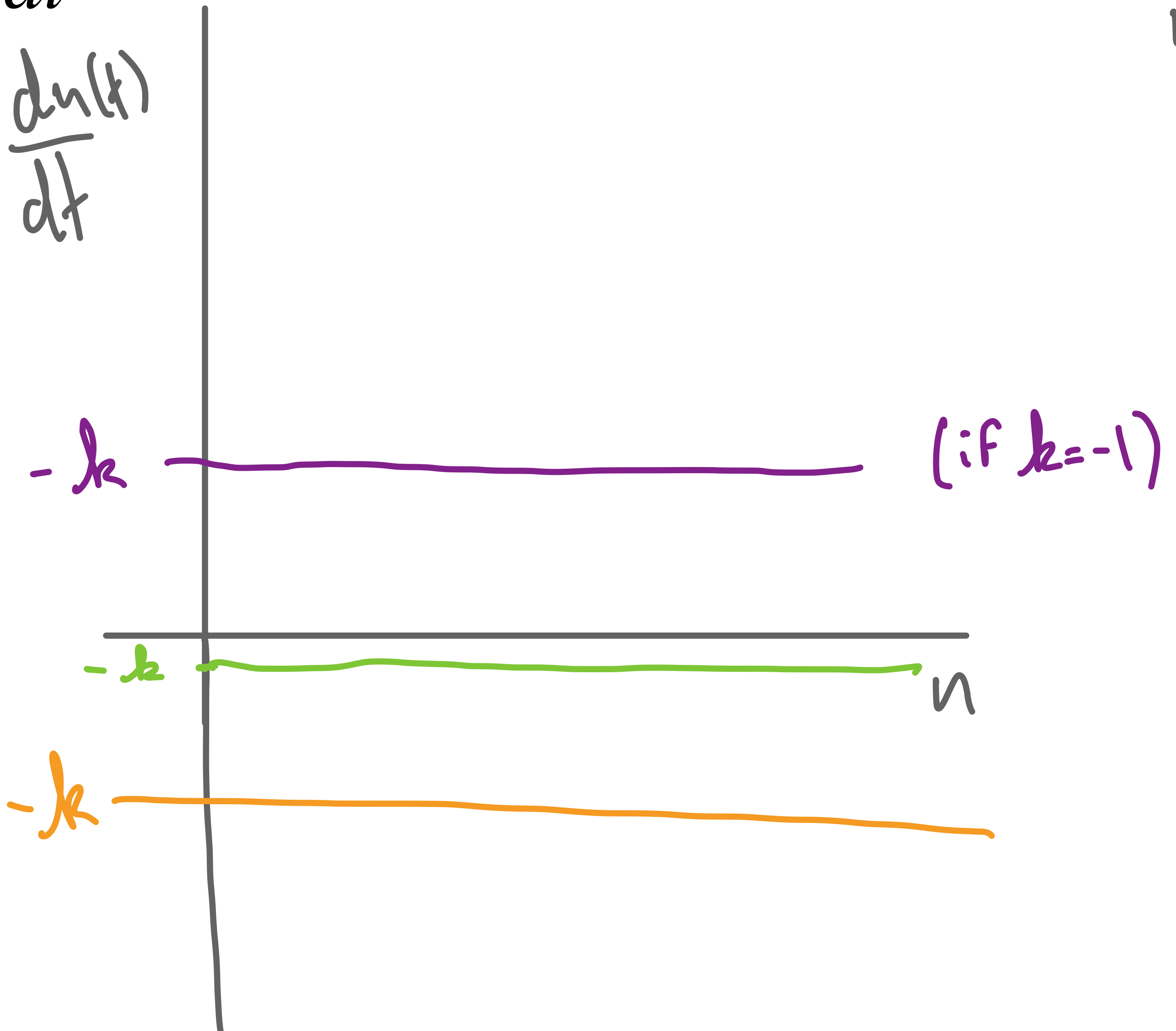


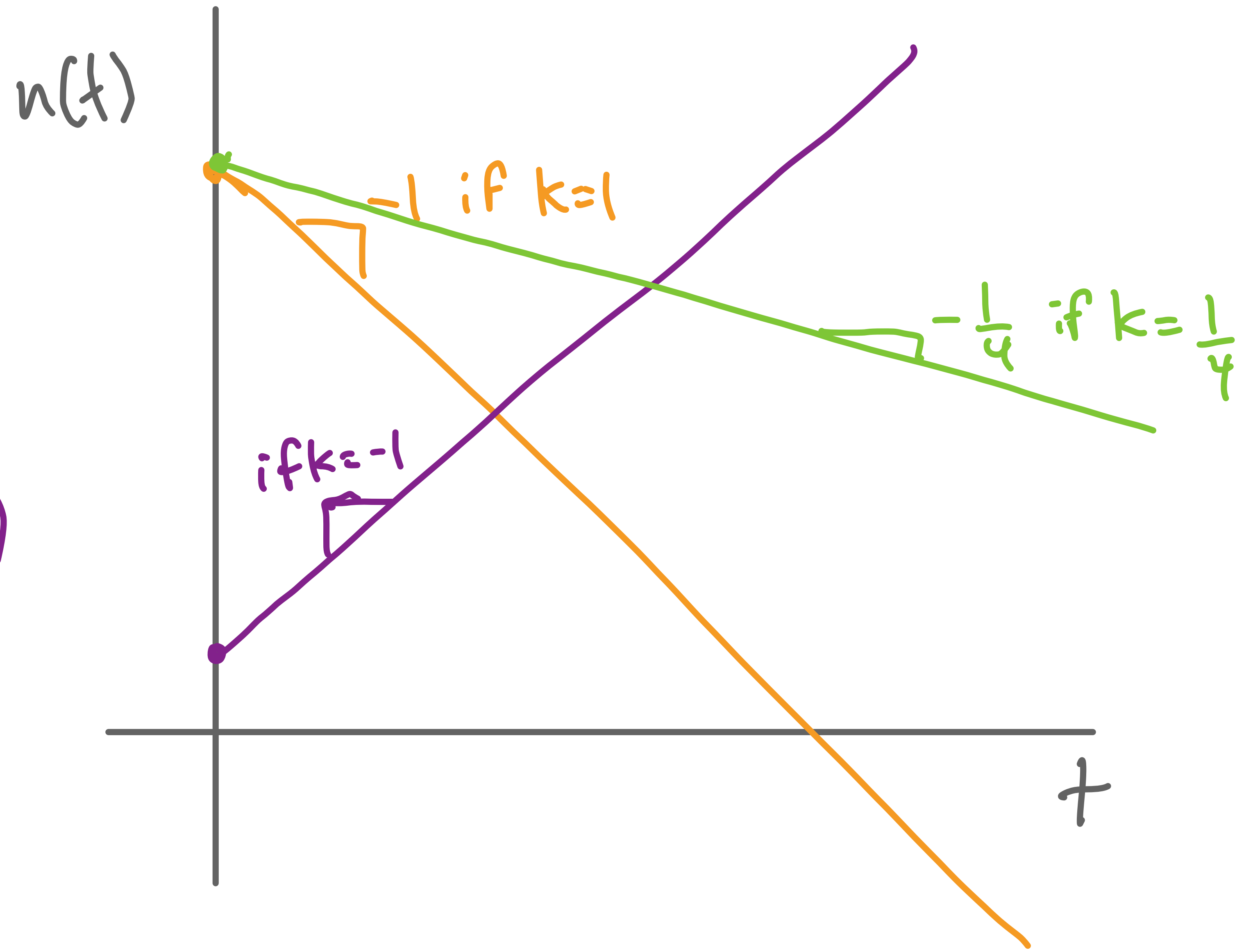


Examples for Intuition

- Ex 3: (A) Sketch the derivative vs ~~time~~ n. (B) Sketch the variable vs time.

$$\frac{dn(t)}{dt} = -k$$





Examples for Intuition

- Ex 4: (A) Sketch the derivative vs time. (B) Sketch the variable vs time.

$$\frac{dn(t)}{dt} = \sqrt{n}$$

start here next time.