MOVE-R: OPTIMIZING THE R-INDEX

Symposium on Experimental Algorithms 2024 ·
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Text Indexing

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- $T = acb\underline{bc}ac\underline{bc}$
- **count**$(\underline{bc}) = 2$

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- **Compressed** Text Index: utilizes information redundancy in **repetitive** strings $\Longrightarrow$ **lower memory footprint**

Repetitive Strings

- $T_1 = bbccaaaaccbbaaaa$
- $T_2 = \mathtt{ATCGATCGATCGAT}$

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Repetitive Strings

- $T_1 = bbccaaaaccbbaaaa$
- $T_2 = \mathtt{ATCGATCGATCGAT}$
- in practice: DNA, log files, versioned documents, natural language

Queries

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Burrows Wheeler Matrix (BWM)

- $T = acbcbac\$$

i	SA	
1	8	
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3	1	
4	5	
5	3	
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4	5	b	ac$acb	c
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- Burrows Wheeler Transform (BWT) = last (L) column of the BWM
- SA-interval $[b, e]$ of P stores occurrences of P in T

SA-Interval

- $P = ac$ has SA-interval $[2, 3]$

$\Longrightarrow$ P occurs at $\mathrm{SA}[2, 3] = [6, 1]$ in T

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Text Indexing

Compressed BWT-based Text Indexes

- Let r = # equal-letter runs in L, σ = # distinct characters in T, ω = word-width of the word-RAM, m = length of the pattern
- r-index [6]:
  - $\mathcal{O}(r)$ space
  - Implements functions LF and Φ in $\underline{\mathcal{O}(\log\log_\omega n/r)\text{ time}}$
  - Count: $\mathcal{O}(m \log\log_\omega(\sigma + n/r))$ time
  - Locate: additional $\mathcal{O}(occ \log\log_\omega(n/r))$ time

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- OptBWTR [8] (not yet implemented):
  - $\mathcal{O}(r)$ space
  - Implements functions LF and Φ in $\underline{\mathcal{O}(1)}$ time using <u>move data structures</u>
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  - Count: $\mathcal{O}(m \log\log_\omega \sigma)$ time
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- Is the improved time complexity reflected in practice?

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Our Contribution

- Move-r: practically optimized implementation of OptBWTR
  - Practically optimized implementation and construction of the move data structure and other index data structures
  - Practically optimized count- and locate algorithms
  - More optimizations

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- Compared with the resp. fastest other index:
  - 2x-35x (typ. 15x) faster queries
  - 0.8x-2.5x (typ. 2x) larger index
  - 0.9-2x (typ. 2x) faster construction with 1-3 (typ. 3x) lower memory usage

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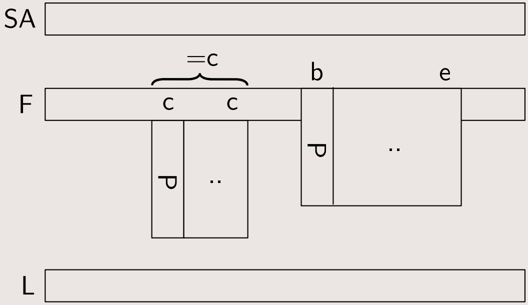
Compressed Text Indexes

Backward Search (Step)

- Given SA-interval $[b, e]$ of P
- Compute SA-interval of cP

Illustration



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- Given SA-interval $[b, e]$ of P
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- $\mathrm{LF}(i)$ = position of $\mathrm{SA}[i] - 1$ in SA (shift rotation down by 1)

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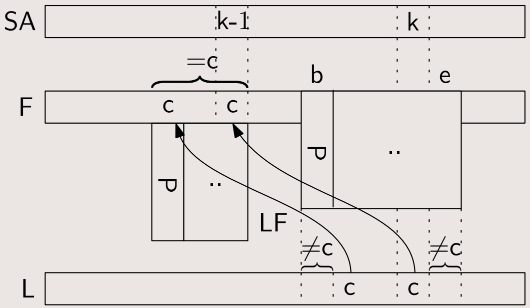

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Locate Query

- Compute values of SA in the SA-interval
- Implement function $\Phi(\mathrm{SA}[i]) = \mathrm{SA}[i-1]$
- Can be implemented in $\mathcal{O}(r)$ space [6]

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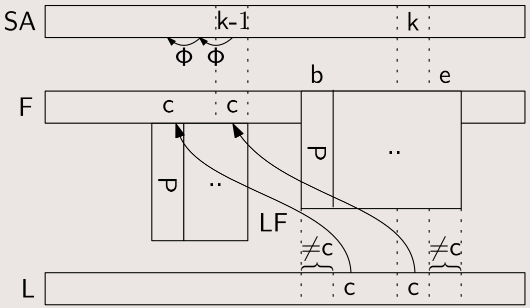

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LF in $O(r)$ space

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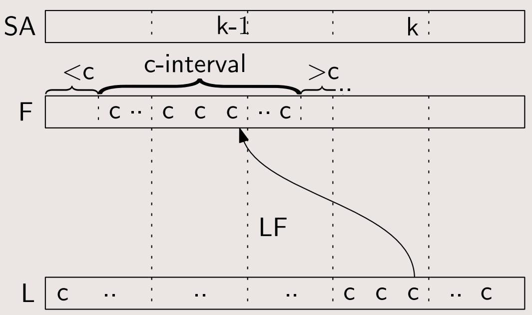

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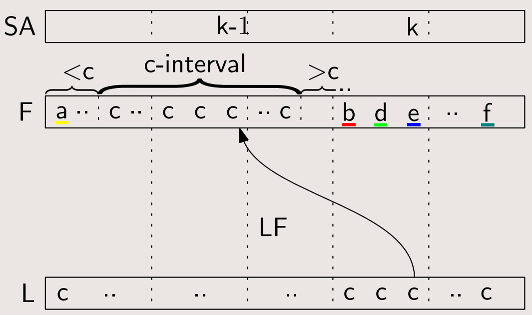

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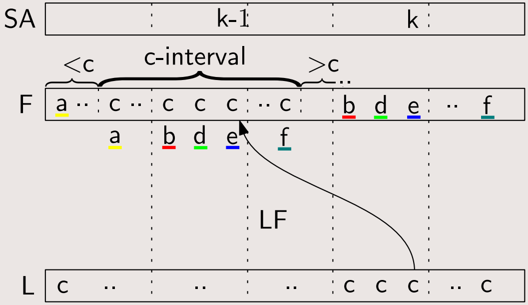

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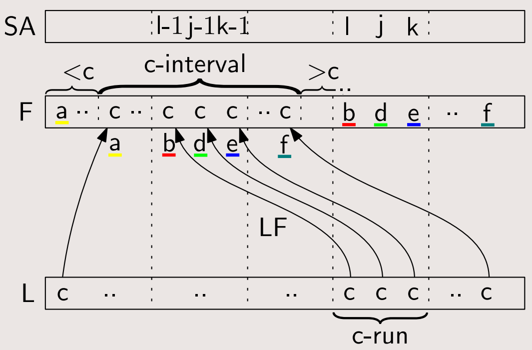

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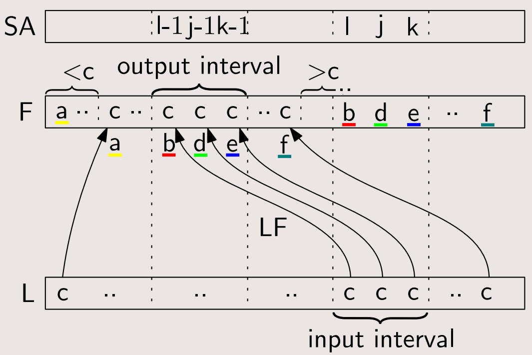

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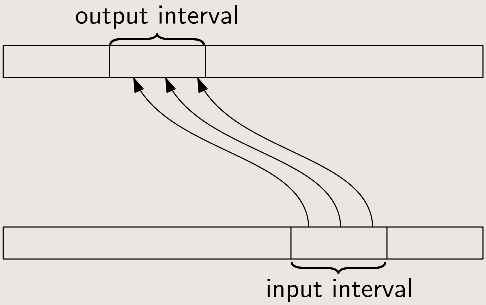

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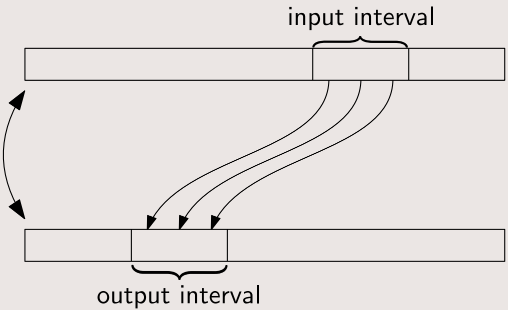

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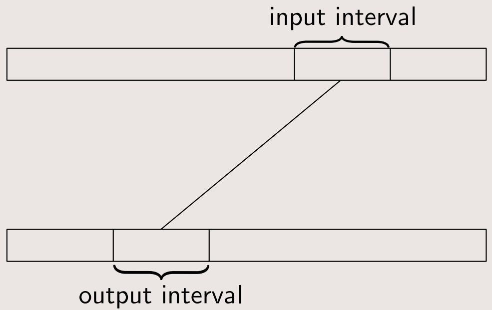

Compressed Text Indexes

Disjoint Interval Sequence

- $I = (p_1, q_1), (p_2, q_2), \ldots, (p_k, q_k)$ with $d_i = p_{i+1} - p_i$ and $n + 1 = p_k + d_k$

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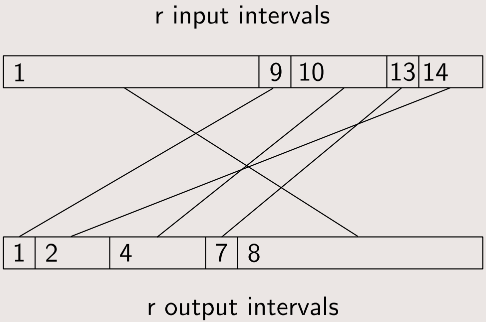

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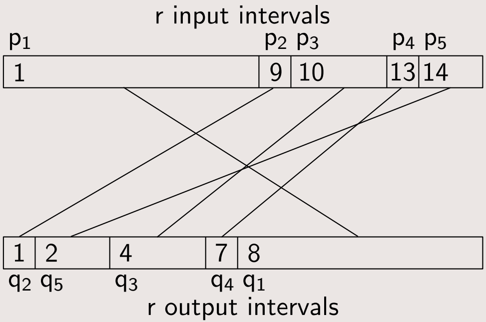

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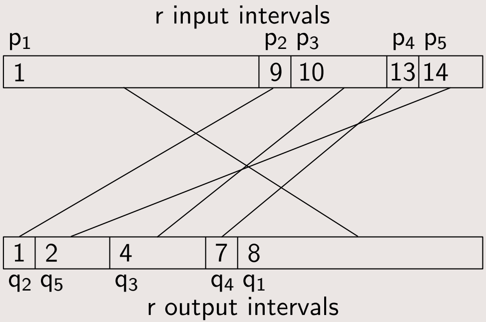

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$\Longrightarrow$ Represents function $f_I(i) = q_x + i - p_x$, where $i \in [p_x, p_x + d_x)$

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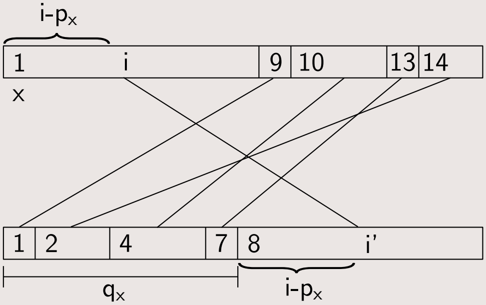

Compressed Text Indexes

Move Data Structure and Move Query

- $\mathrm{Move}(i, x) = (i', x')$ with $i' = f_I(i)$ and $i' \in [p_{x'}, p_{x'} + d_{x'})$

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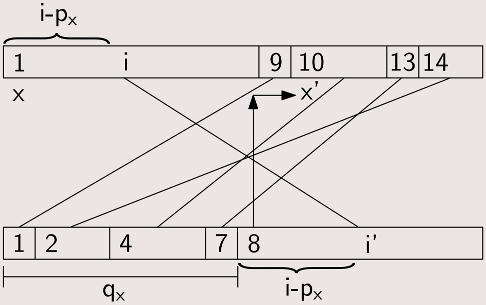

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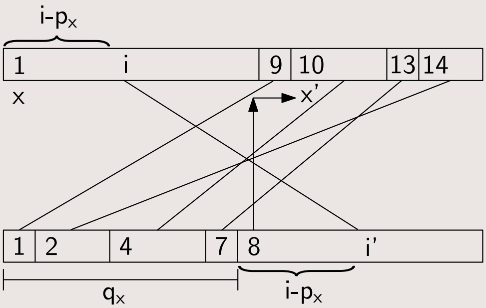

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Move Query

- $\mathrm{M_{idx}} = [1, 1, 1, 1, 1]$

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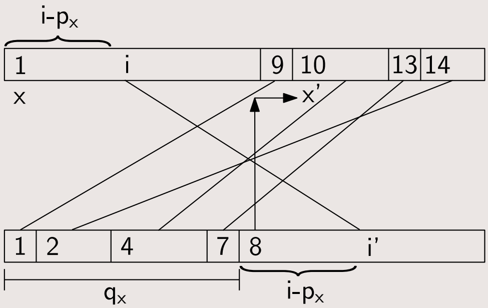

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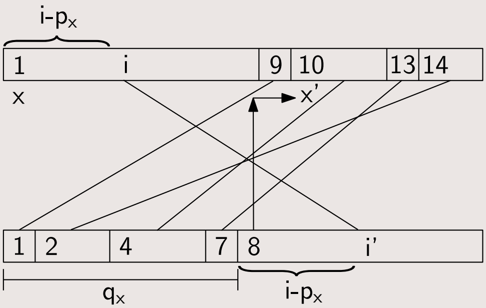

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- Runtime $\mathcal{O}$(#input intervals starting in $[q_x, q_x + d_x)) = \mathcal{O}(r)$

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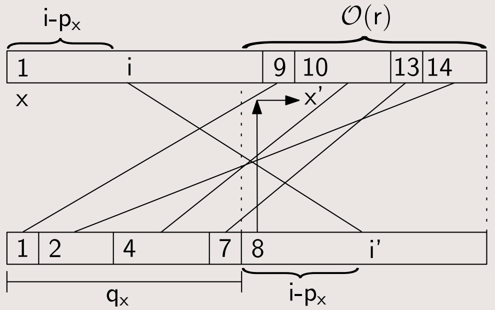

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$\Longrightarrow$ Limit to $\mathcal{O}(a)$ for $a \geq 2$

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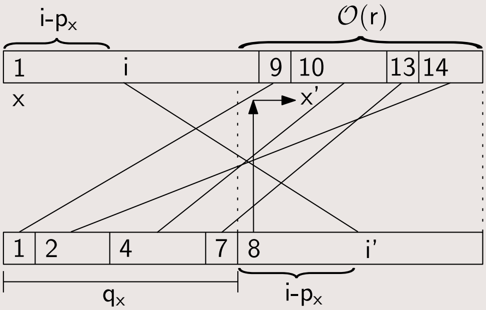

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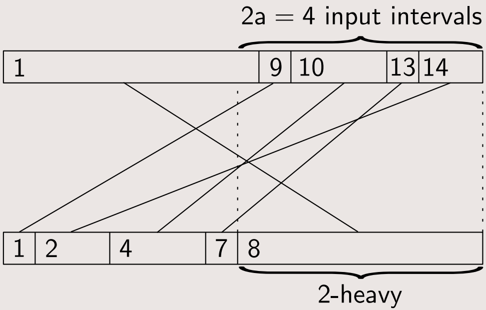
a-balanced Disjoint Interval Sequence

- Output interval $[q_x, q_x + d_x)$ is a-heavy $\Leftrightarrow$ $\geq 2a$ input intervals start in $[q_x, q_x + d_x)$

Illustration



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- a-balanced $\Leftrightarrow$ not a-heavy

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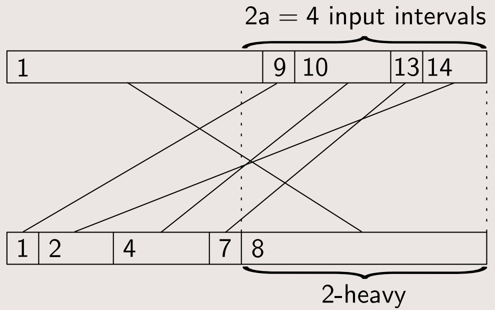

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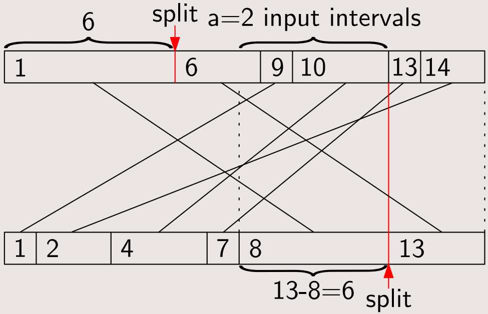
Balancing Algorithm

- Iteratively splits a-heavy output intervals
- Terminates as soon as I is a-balanced

Illustration



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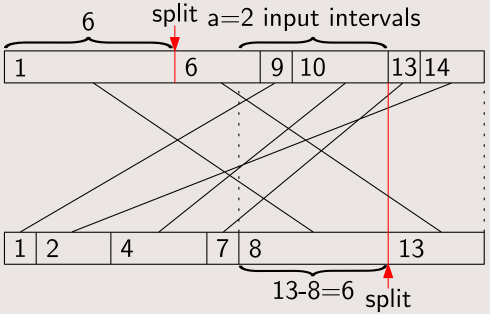
Balancing Algorithm

- Iteratively splits a-heavy output intervals
- Terminates as soon as I is a-balanced
- Let t' = number of splits, and $k' = k + t'$

$\Rightarrow$ $k' \leq k\frac{a}{a-1} \leq 2k = O(k)$

Illustration

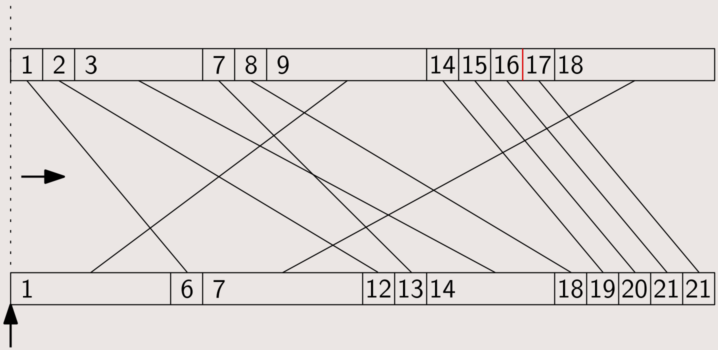


Fast Balancing Algorithm

General Approach

- Simultaneously iterate over input- and output intervals

Example Execution ($a = 2$)



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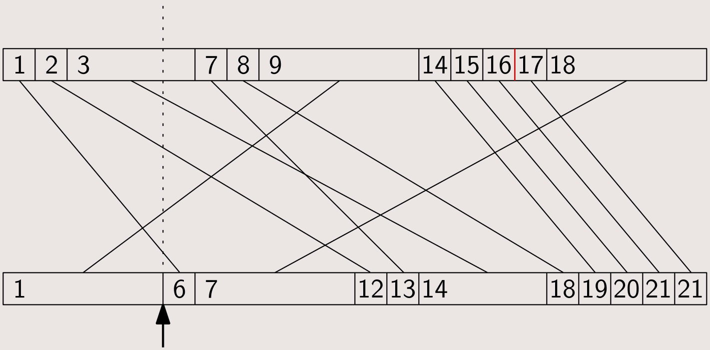

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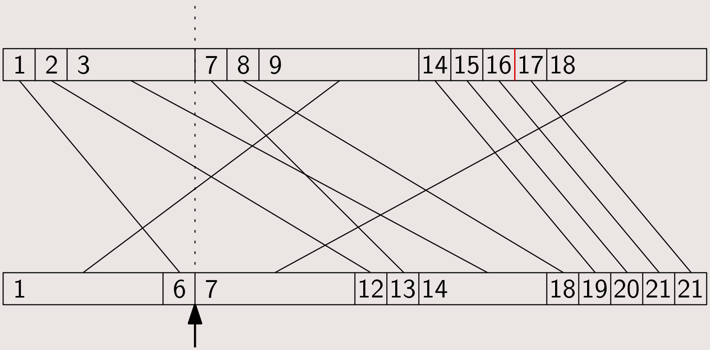

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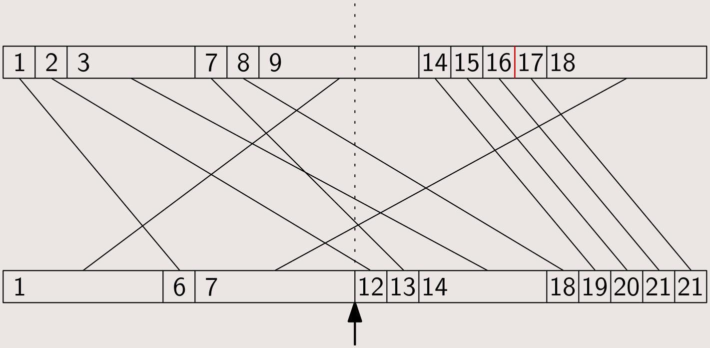

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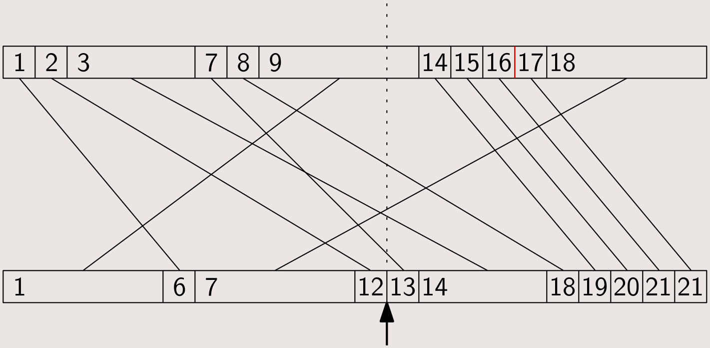

Fast Balancing Algorithm

General Approach

- Simultaneously iterate over input- and output intervals
- If an output interval is a-heavy:

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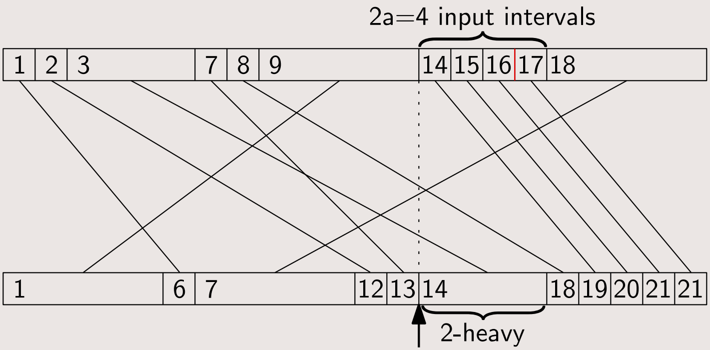

Fast Balancing Algorithm

General Approach

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  - Split and remember starting position u

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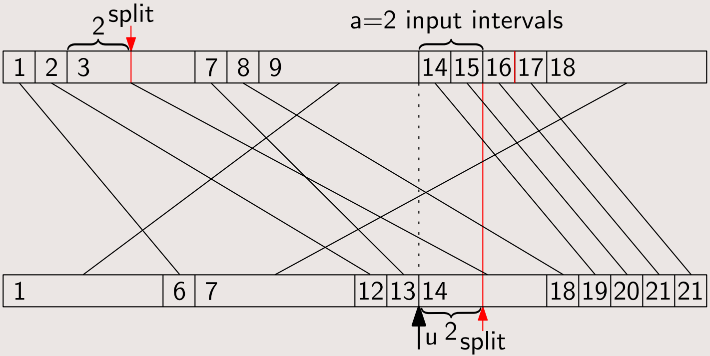

Fast Balancing Algorithm

General Approach

- Simultaneously iterate over input- and output intervals
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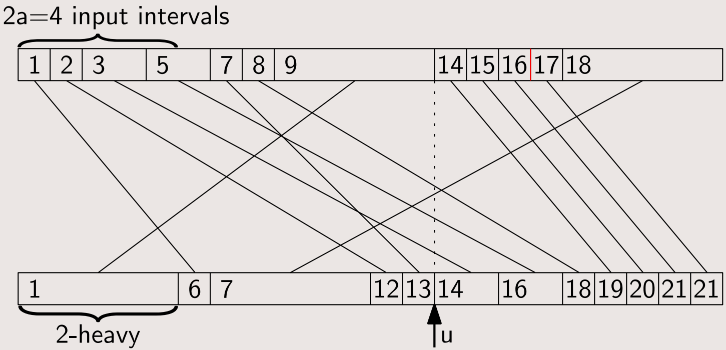

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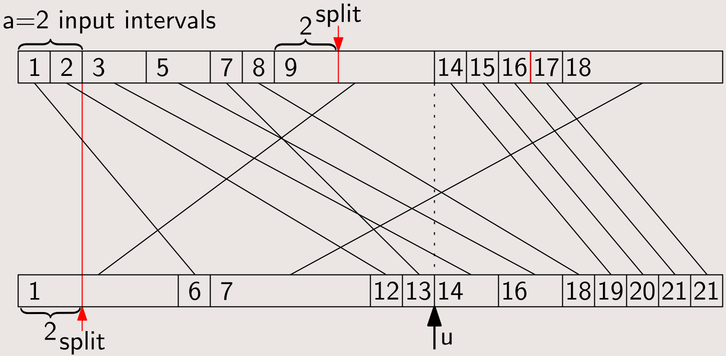

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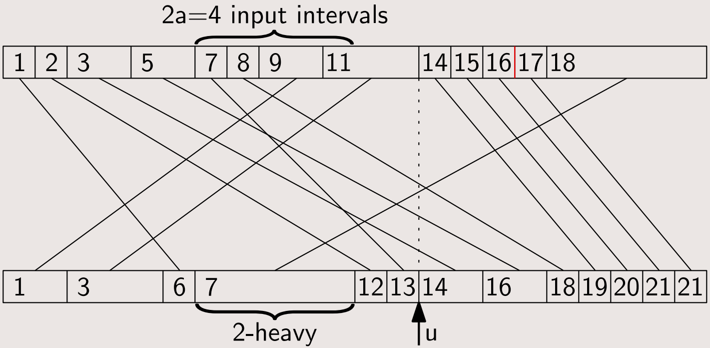

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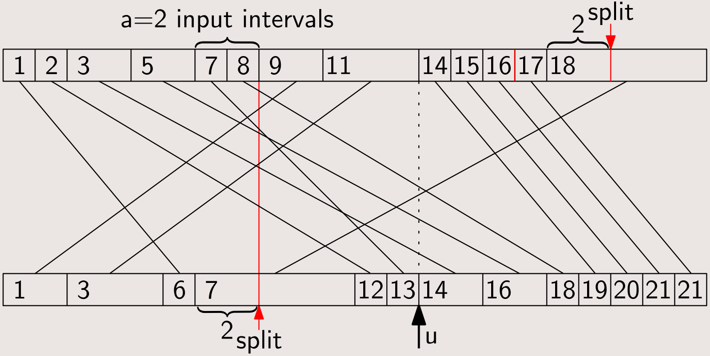

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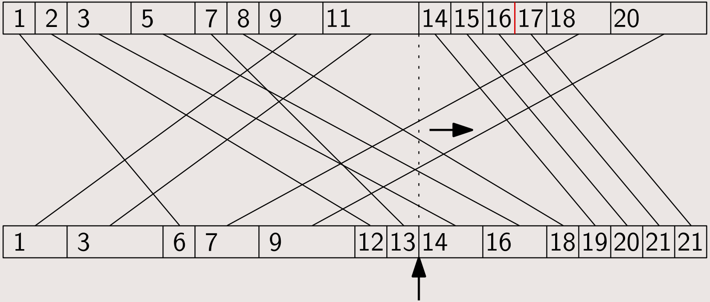

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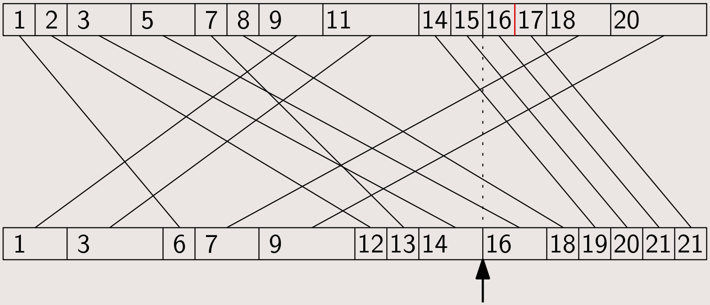

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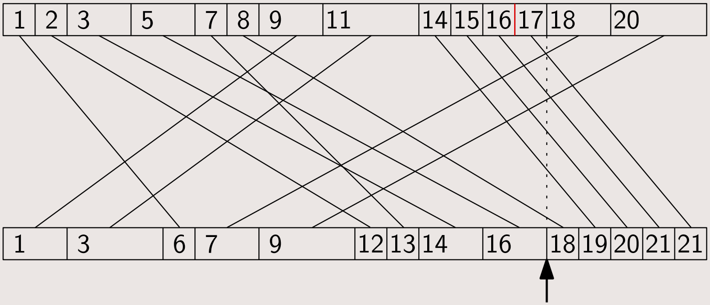

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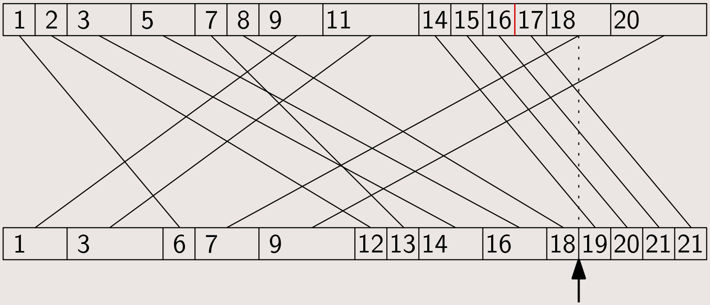

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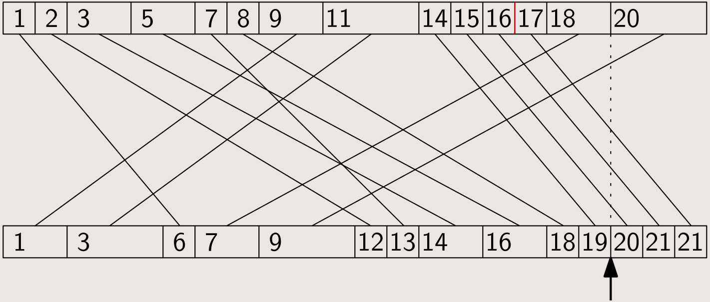

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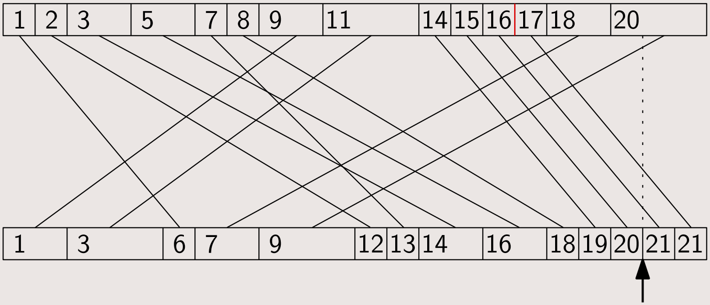

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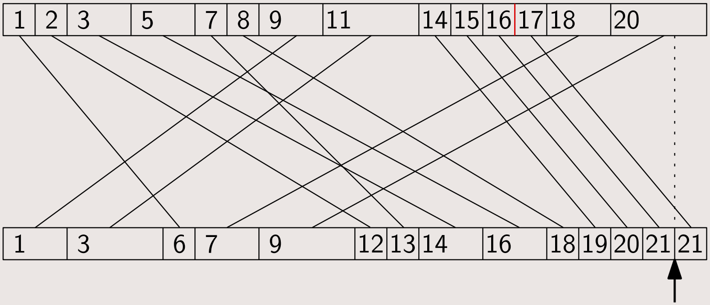

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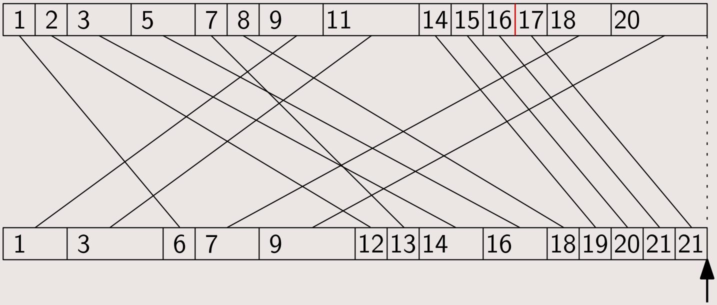

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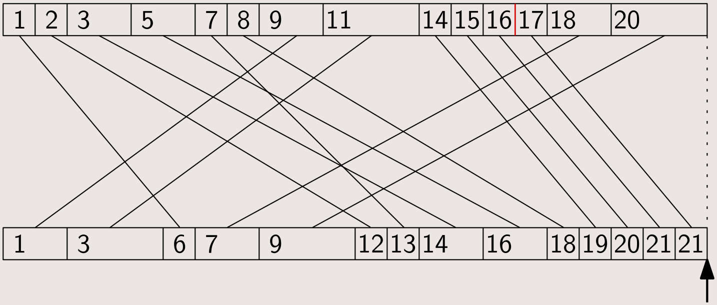
Details

- Use balanced search trees (B-Trees) for input and output intervals

$\Longrightarrow$ $\mathcal{O}(k \log k)$ time, $\mathcal{O}(1)$ space

Example Execution ($a = 2$)

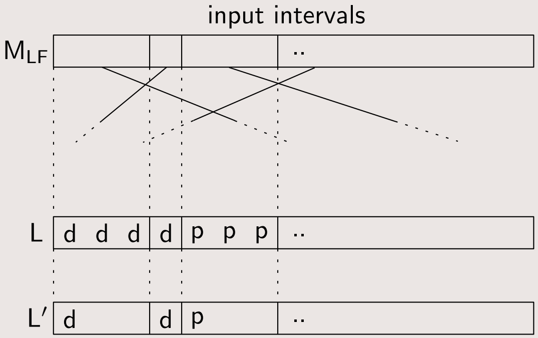


Move-r

Index Data Structures

- $|\mathrm{M}^{LF}| = r'$ move data structure for LF
- $L'[1..r']$ bwt characters of input intervals in M^{LF}
- $|RS_{L'}| = \mathcal{O}(r')$ rank-select data structure for L'

Backward Search



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- given SA-interval $[b, e]$ of P

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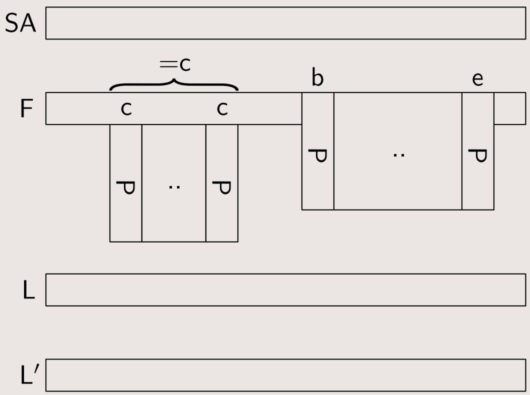

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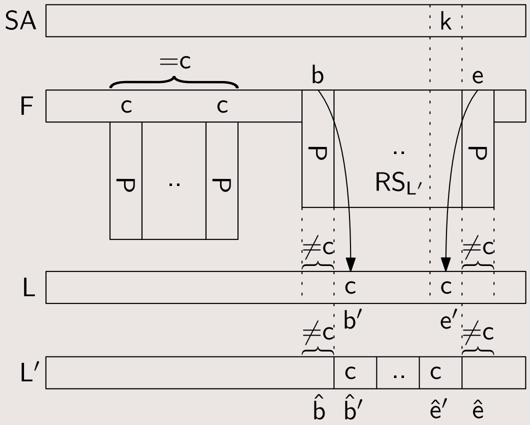
Backward Search (Step)

- given SA-interval $[b, e]$ of P

1. compute b' and e' with L' and $RS_{L'}$ (maintain input interval indexes $\hat{b}, \hat{e}, \hat{b}', \hat{e}'$ of b, e, b', e')

Backward Search



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2. compute SA-interval of cP with M^{LF}

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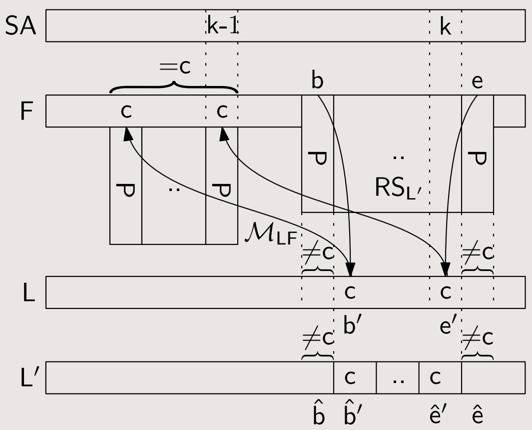

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- $|\mathsf{M}^{\Phi}| = r''$ move data structure for Φ
- $\mathsf{SA}_{\Phi}[1..r']$ (will be defined later)

$\Big\}$ for locate

Backward Search (Step)

- given SA-interval $[b, e]$ of P

1. compute b' and e' with L' and $RS_{L'}$ (maintain input interval indexes $\hat{b}, \hat{e}, \hat{b}', \hat{e}'$ of b, e, b', e')
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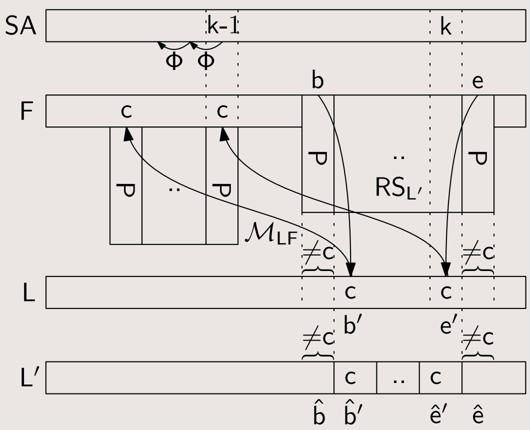
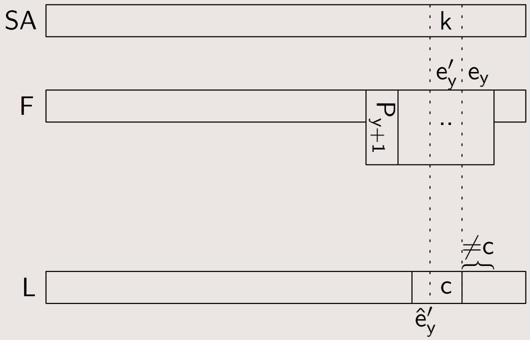


Optimized Locate Algorithm

Definitions

- e_i = end of SA-interval of P_{i+1}
- $e'_i = \mathrm{L.select}(P[i], \mathrm{L.rank}(P[i], e_i))$
- $\hat{e}'_i$ = index of M^{LF}-input interval containing e'_i

Backward Search



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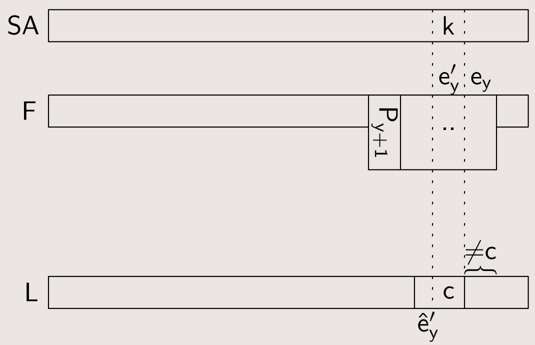

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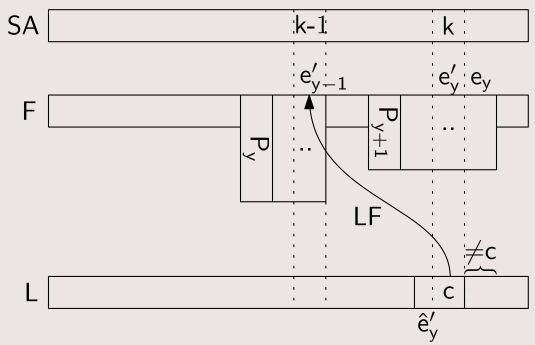
Observation

- For $i \in [0, y)$, $\text{L}[e_i] = P[i]$ implies $e'_i = e_i = \text{LF}(e'_{i+1})$

Backward Search



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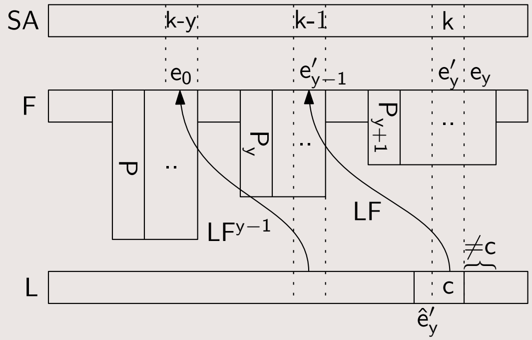
- For $i \in [0, y)$, $\mathrm{L}[e_i] = P[i]$ implies $e'_i = e_i = \mathrm{LF}(e'_{i+1})$

$\Longrightarrow$ By ind. $e_0 = \mathrm{LF}^y(e'_y) \Leftrightarrow \mathrm{SA}[e_0] = \mathrm{SA}[e'_y] - y$

Backward Search



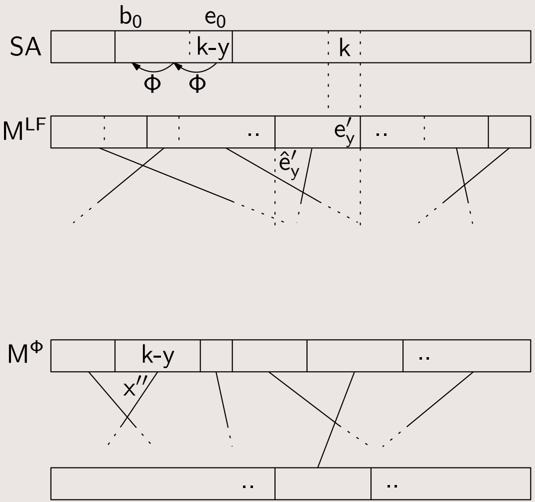
Optimized Locate Algorithm

General Procedure

- We want to compute $\mathrm{SA}[e_0] = k - y = \mathrm{SA}[e'_y] - y$ and the index x'' of the M^{Φ}-input interval containing $k - y$

$\Longrightarrow$ Then we can compute $\mathrm{SA}[b_0, e_0]$ with $e_0 - b_0$ Φ-move queries

Locate Phase



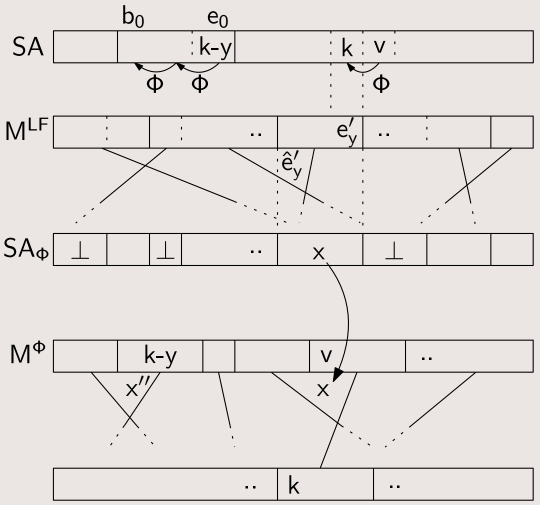
Optimized Locate Algorithm

Algorithm

- Obs.: there is an M^{Φ}-output interval starting with k

1. Compute index $x = SA_{\Phi}[\hat{e}'_y]$ of the M^{Φ}-input interval starting with v
2. Compute $k - y = M^{\Phi}_q[x] - y$

Locate Phase



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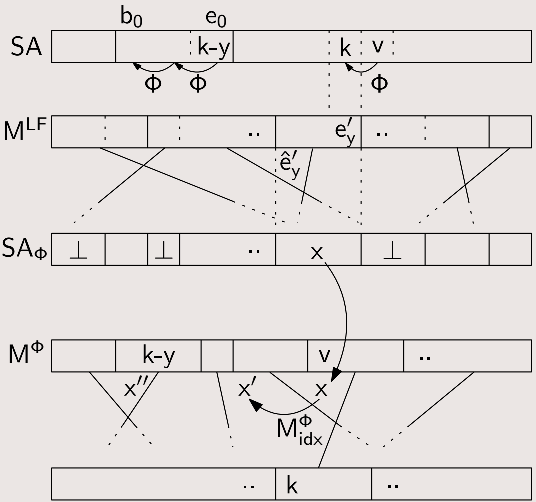

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4. Compute x'' with a $\mathcal{O}(\log y) = \mathcal{O}(\log m)$-time exponential search over the M^Φ-input intervals

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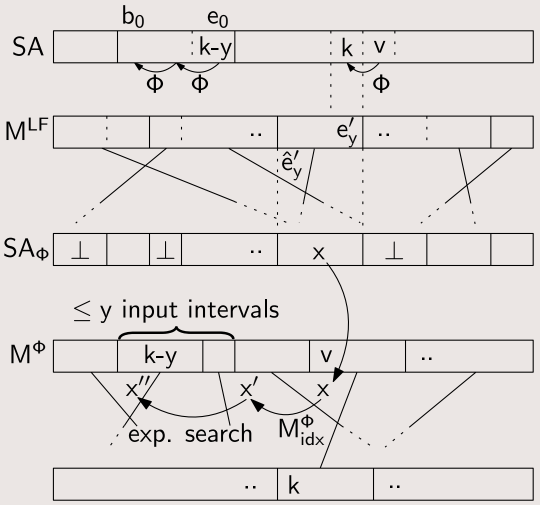

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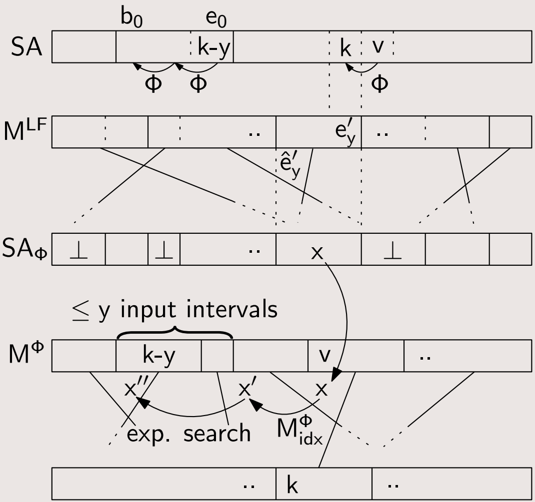

Optimized Locate Algorithm

Comparison with OptBWTR

- We can compute $\mathrm{SA}[e_0]$ and x'' in $\mathcal{O}(\log m)$ time and with $\mathcal{O}(\log m)$ cache misses
- OptBWTR: $\mathcal{O}(m)$ time and $3m$ cache misses

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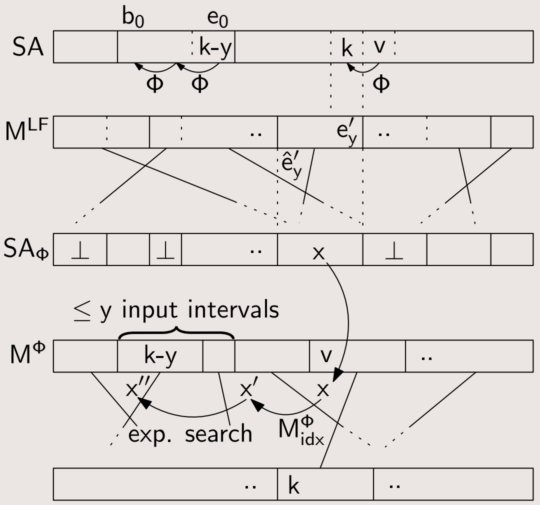

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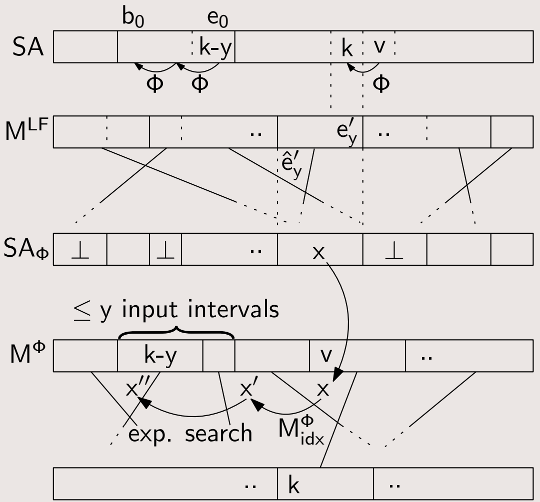
- We can compute $SA[e_0]$ and x'' in $\mathcal{O}(\log m)$ time and with $\mathcal{O}(\log m)$ cache misses
- OptBWTR: $\mathcal{O}(m)$ time and $3m$ cache misses
- $SA_\Phi[1..r']$ requires $r'\lceil \log r'' \rceil$ bits
- OptBWTR: stores two arrays $SA^+[1..r']$ and $SA^+_{\text{index}}[1..r']$ ($r'(\lceil \log n \rceil + \lceil \log r'' \rceil)$ bits)

Locate Phase

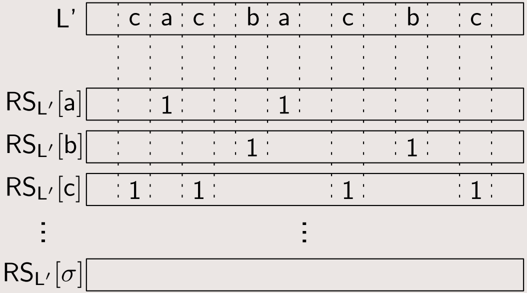


More Optimizations

Optimizations

- Faster rank-select data structure for $RS_{L'}$ using hybrid (compressed and uncompressed) bit-vectors with rank-select support

Illustration



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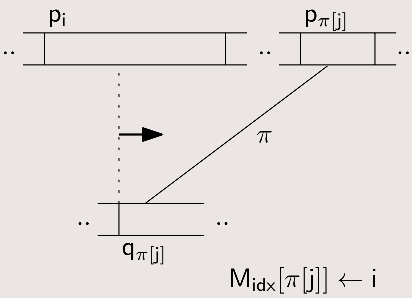

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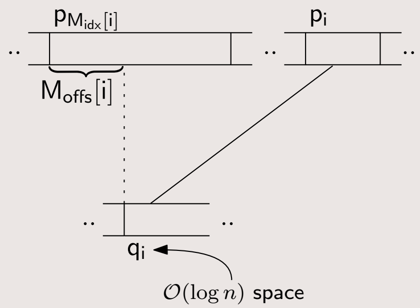

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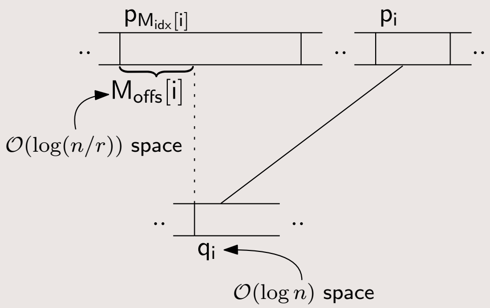

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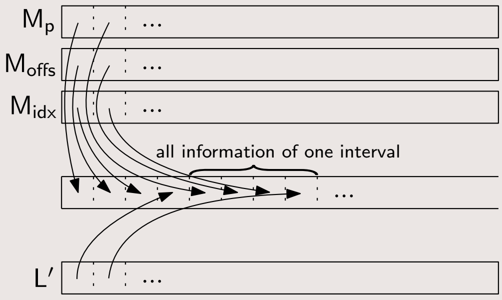

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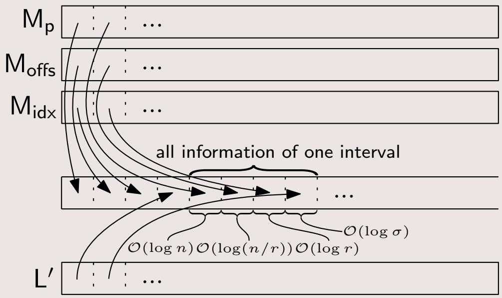

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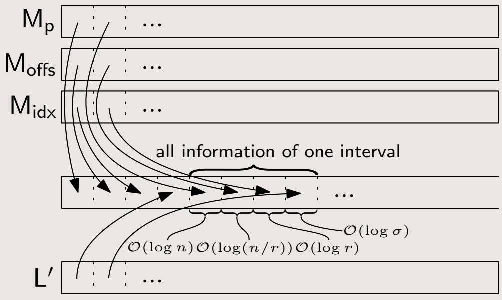

Experimental Setup

Tested Indexes

- ◆ move-r (static, Big-BWT [2])
- ■ r-index [6] (static, Big-BWT [2])
- ▲ online-rlbwt [1] (dynamic)
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Measured Texts

text	size [GB]	σ	n/r	r'/r	r''/r
einstein.en.txt	0.47	139	1611.18	1.23	1.49
sars2	84.19	80	686.57	1.38	1.06
dewiki	68.72	210	345.80	1.23	1.35
chr19	58.57	52	272.20	1.06	2.00
english	2.21	239	3.36	1.19	1.20

More Information

- $n/r \approx$ compressibility
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$\implies$ The following measurements use $a = 8$

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$\left.\begin{matrix}\text{r-index-f}\\ \text{block-rlbwt-2}\\ \text{block-rlbwt-v}\\ \text{block-rlbwt-r}\end{matrix}\right\}$ only count

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Test System

- 2x AMD EPYC 7452 (32/64x 2.35-3.35GHz, 2/16/128MB L1/2/3 cache)
- 1TB 3200 MT/s DDR4 RAM
- GCC 9.4.0 with flags `"-march=native -DNDEBUG -Ofast"`
- Ubuntu 18.04.6

More Information

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- σ = alphabet size
- r', r'' = number of intervals in M^{LF}/M^{Φ} (for $a = 2$)
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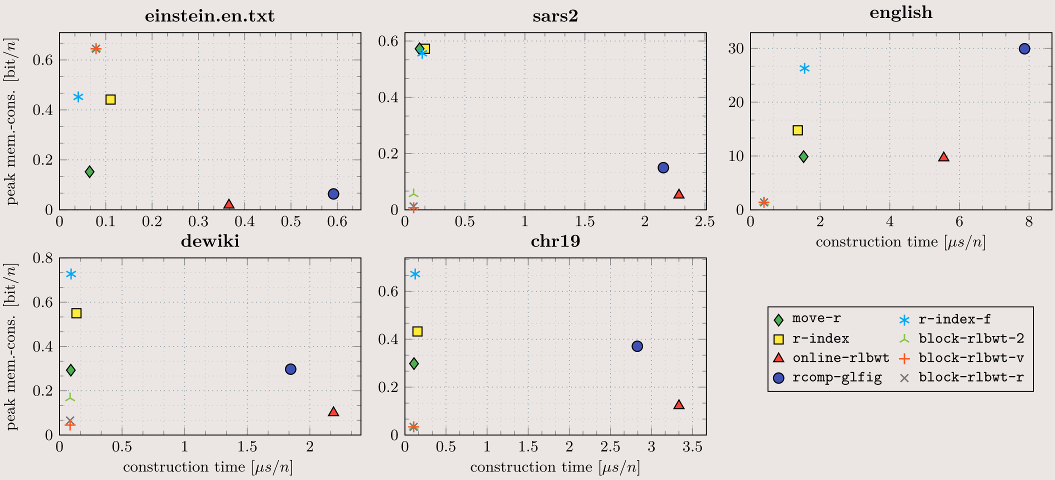
Results

Construction Performance



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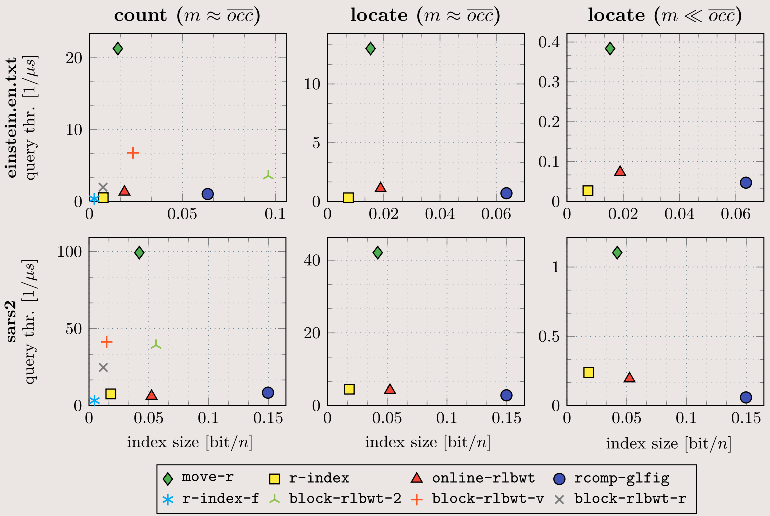

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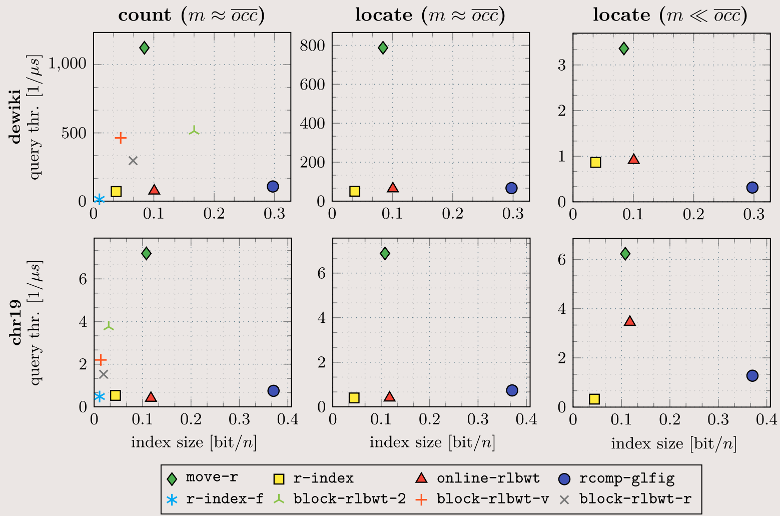

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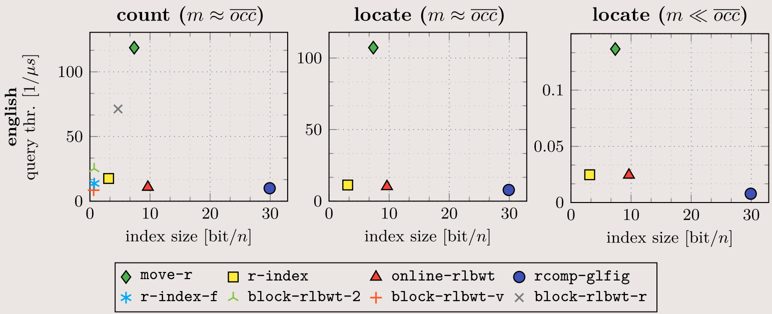

Summary

- 2x-35x (typ. 15x) faster queries
- 0.8x-2.5x (typ. 2x) larger index
- 0.9-2x (typ. 2x) faster construction with 1-3x (typ. 2x) lower memory usage

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