MOVE-R: OPTIMIZING THE R-INDEX

Symposium on Experimental Algorithms 2024 ·
Nico Bertram, Johannes Fischer and Lukas Nalbach

Text Indexing

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- $T = acb\underline{bc}ac\underline{bc}$
- **count**$(\underline{bc}) = 2$

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- **Compressed** Text Index: utilizes information redundancy in **repetitive** strings $\Longrightarrow$ **lower memory footprint**

Repetitive Strings

- $T_1 = bbccaaaaccbbaaaa$
- $T_2 = \texttt{ATCGATCGATCGAT}$

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Repetitive Strings

- $T_1 = bbccaaaaccbbaaaa$
- $T_2 = \mathtt{ATCGATCGATCGAT}$
- in practice: DNA, log files, versioned documents, natural language

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BWT-Based Text Indexes

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- Suffix array $\mathsf{SA}[1..n]$, s.t. $T_{\mathsf{SA}[1]} \prec \ldots \prec T_{\mathsf{SA}[n]}$

Burrows Wheeler Matrix (BWM)

- $T = acbcbac\$$

i	SA
1	8
2	6
3	1
4	5
5	3
6	7
7	4
8	2

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- $\mathrm{rot}(T, i) = T[i, n]T[1, i)$

Burrows Wheeler Matrix (BWM)

- $T = acbcbac\$$
- i-th row $= \mathrm{rot}(T, \mathrm{SA}[i])$

i	SA	F		L
1	8	\$	acbcba	c
2	6	a	c\$acbc	b
3	1	a	cbcbac	\$
4	5	b	ac\$acb	c
5	3	b	cbac\$a	c
6	7	c	\$acbcb	a
7	4	c	bac\$ac	b
8	2	c	bcbac\$	a

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- $\mathsf{rot}(T, i) = T[i, n]T[1, i)$
- Burrows Wheeler Transform (BWT) = last (L) column of the BWM
- SA-interval $[b, e]$ of P stores occurrences of P in T

SA-Interval

- $P = \underline{ac}$ has SA-interval $[2, 3]$

$\Rightarrow$ $\mathbf{count}(P) = |[2, 3]| = 2$

$\Rightarrow$ $\mathbf{locate}(P) = \mathsf{SA}[2, 3] = \{6, 1\}$

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Compressed BWT-based Text Indexes

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- Let r = # equal-letter runs in L, σ = # distinct characters in T, ω = word-width of the word-RAM, m = length of the pattern
- r-index [Gagie et al. 2020]:
  - $\mathcal{O}(r)$ space
  - Implements functions LF and Φ in $\underline{\mathcal{O}(\log\log_\omega n/r) \text{ time}}$
  - Count: $\mathcal{O}(m \log\log_\omega(\sigma + n/r))$ time
  - Locate: additional $\mathcal{O}(occ \log\log_\omega(n/r))$ time

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- OptBWTR [Nishimoto et al. 2021] (not yet implemented):
  - $\mathcal{O}(r)$ space
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  - Count: $\mathcal{O}(m \log\log_\omega \sigma)$ time
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- Is the improved runtime reflected in practice?

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- Move-r: practically optimized implementation of OptBWTR
  - Practically optimized implementation and construction of the move data structure and other index data structures
  - Practically optimized count- and locate algorithms
  - More optimizations

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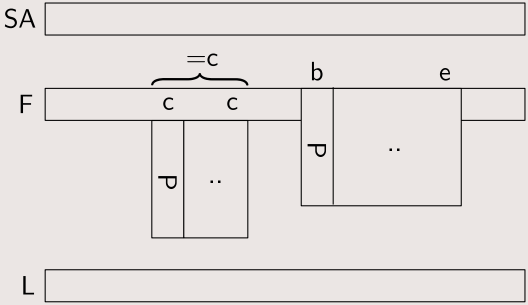
- Move-r: practically optimized implementation of OptBWTR
  - Practically optimized implementation and construction of the move data structure and other index data structures
  - Practically optimized count- and locate algorithms
  - More optimizations
- Compared with the resp. fastest other index:
  - 2x-35x (typ. 15x) faster queries
  - 0.9-2x (typ. 2x) faster construction with 1-3 (typ. 3x) lower memory usage, but
  - 0.8x-2.5x (typ. 2x) larger index

BWT-Runs Compressed Text Indexes

Backward Search (Step)

- Given SA-interval $[b, e]$ of P
- Compute SA-interval of cP

Illustration



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- $\mathrm{LF}(i)$ = position of $\mathrm{SA}[i] - 1$ in SA (shift rotation down by 1)

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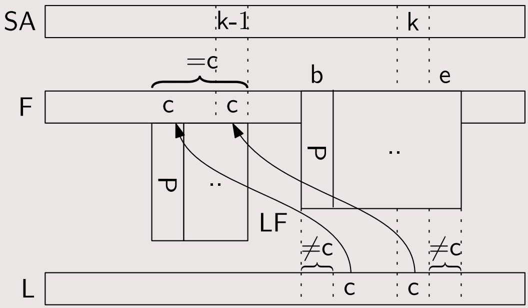

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Locate Query

- Compute values of SA in the SA-interval
- Implement function $\Phi(\mathsf{SA}[i]) = \mathsf{SA}[i-1]$
- Can be implemented in $\mathcal{O}(r)$ space

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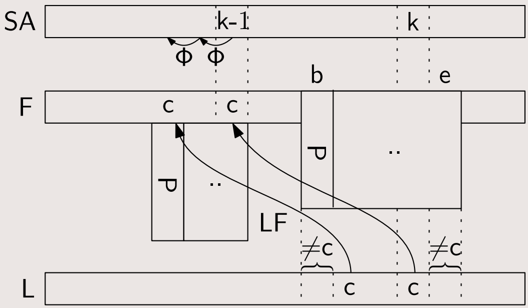

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LF in $\mathcal{O}(r)$ space

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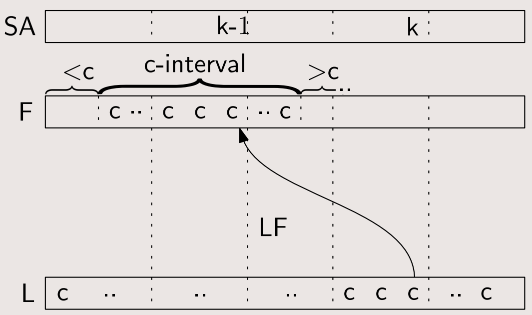

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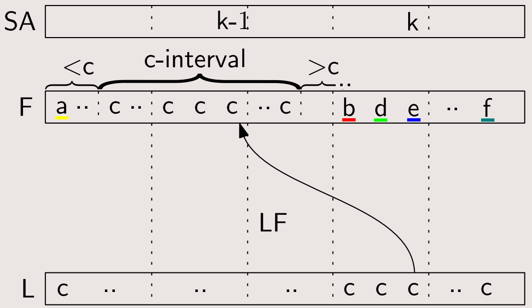

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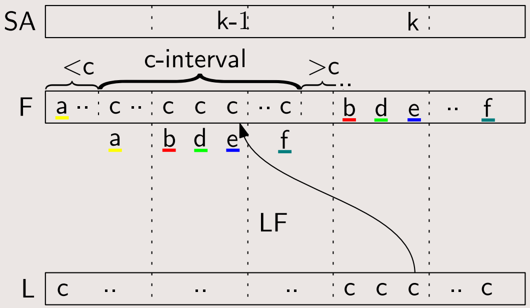

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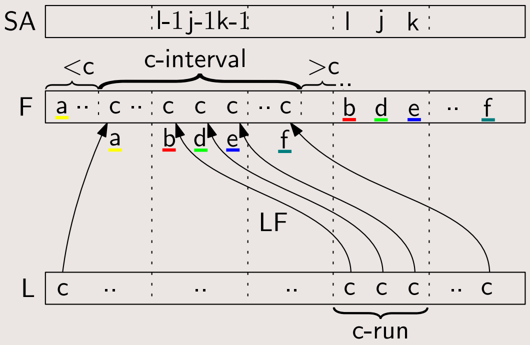

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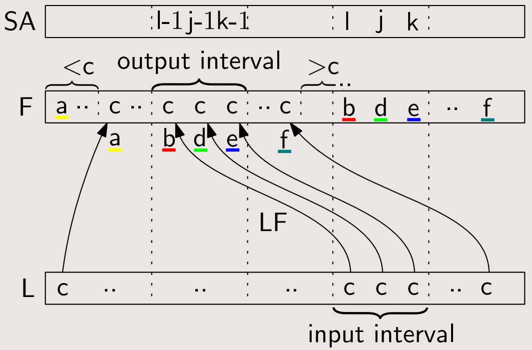

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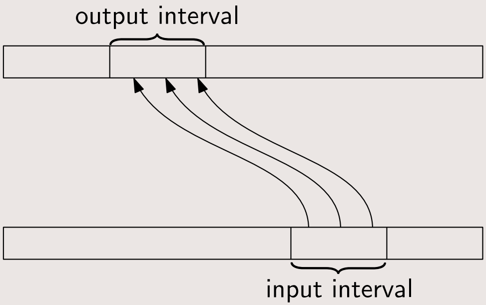

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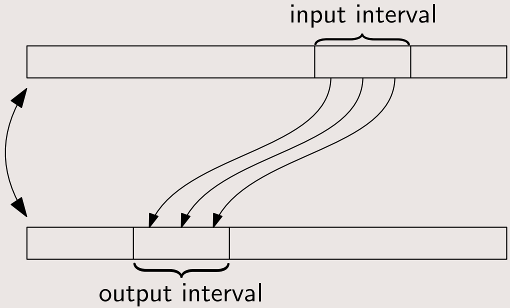

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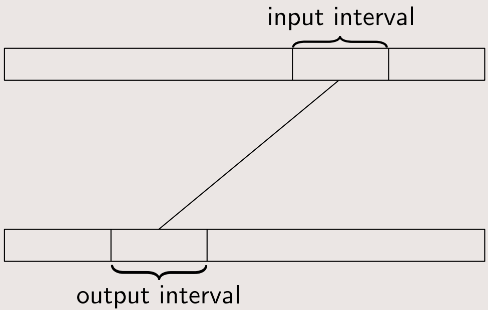

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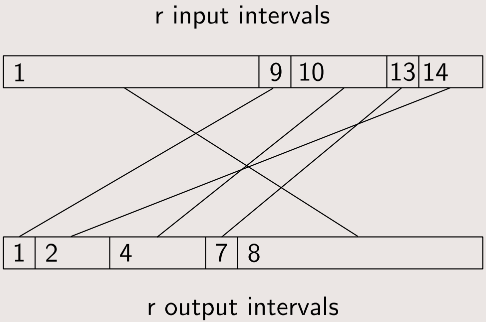

Move Data Structure

Disjoint Interval Sequence

- $I = (p_1, q_1), (p_2, q_2), \ldots, (p_k, q_k)$ with $d_i = p_{i+1} - p_i$ and $n + 1 = p_k + d_k$

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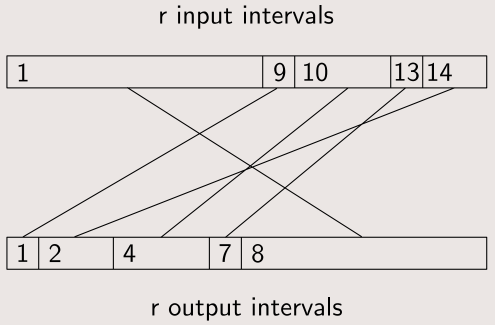

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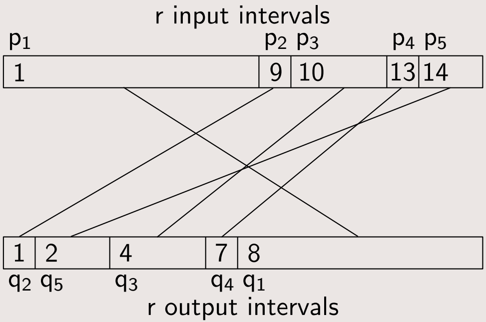

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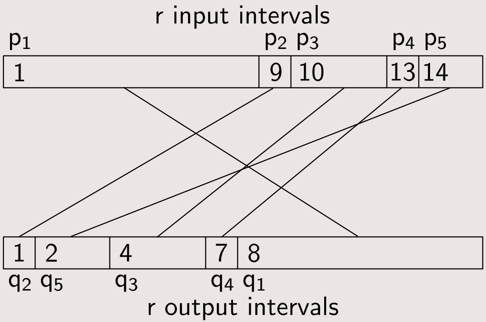

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$\Longrightarrow$ Represents function $f_I(i) = q_x + i - p_x$, where $i \in [p_x, p_x + d_x)$

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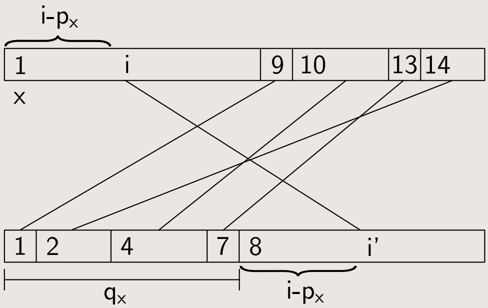

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Move Data Structure and Move Query

- Move(i, x) = (i', x') with $i' = f_I(i)$ and $i' \in [p_{x'}, p_{x'} + d_{x'})$

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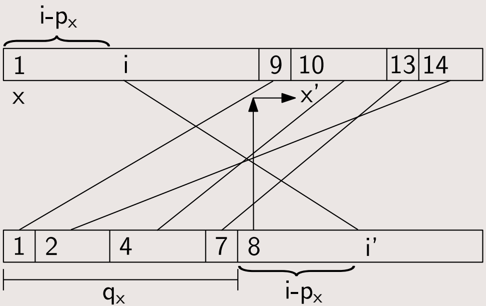

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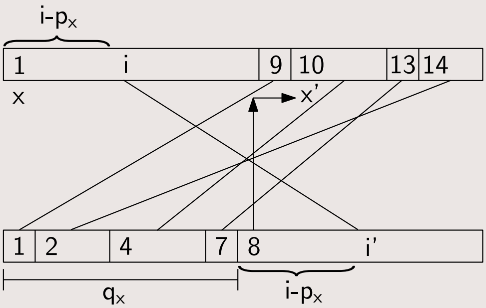

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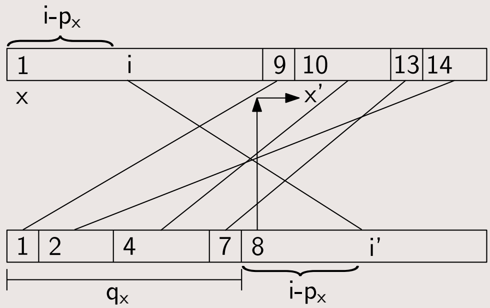

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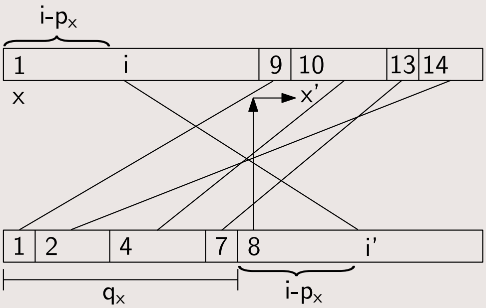

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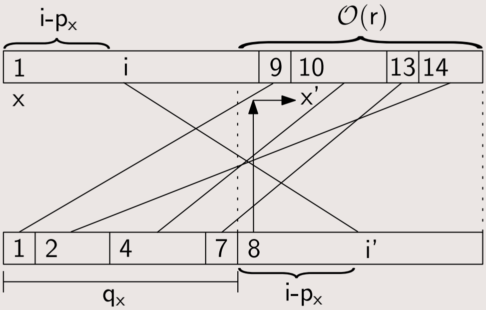

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$\Longrightarrow$ can be limited to $\mathcal{O}(1)$

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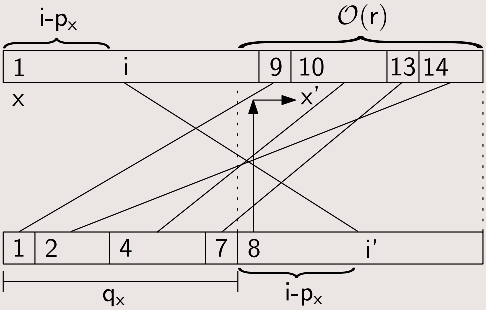
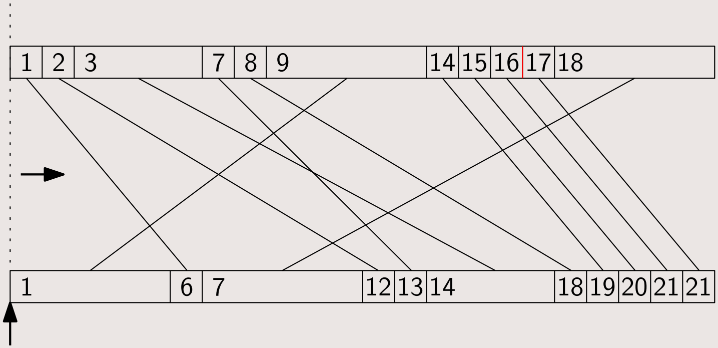


Fast Balancing Algorithm

General Approach

- Simultaneously iterate over input- and output intervals

Example Execution ($a = 2$)



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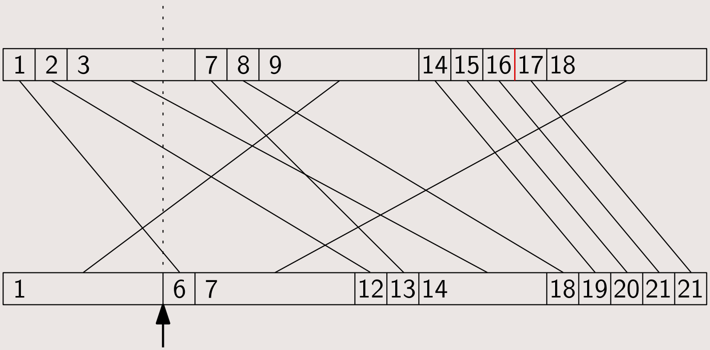

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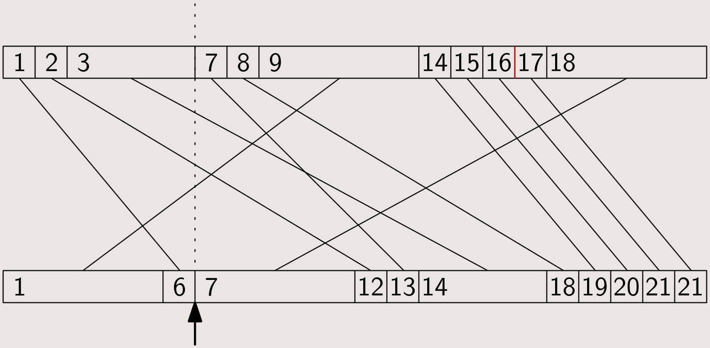

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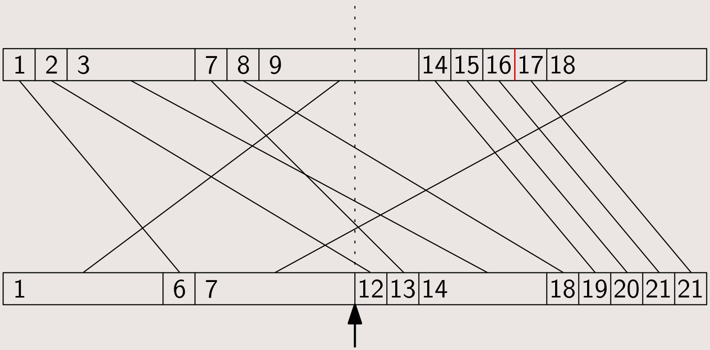

Fast Balancing Algorithm

General Approach

- Simultaneously iterate over input- and output intervals

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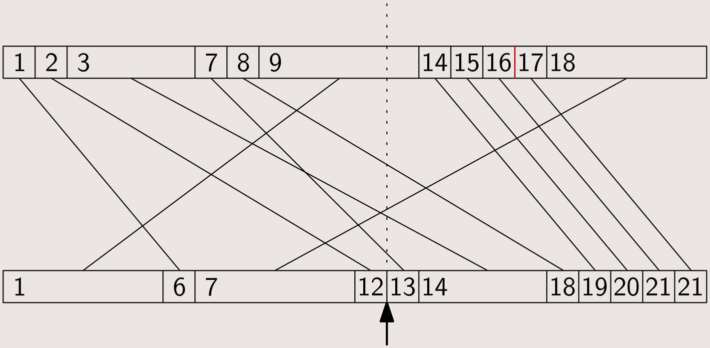

Fast Balancing Algorithm

General Approach

- Simultaneously iterate over input- and output intervals
- If an output interval is a-heavy:

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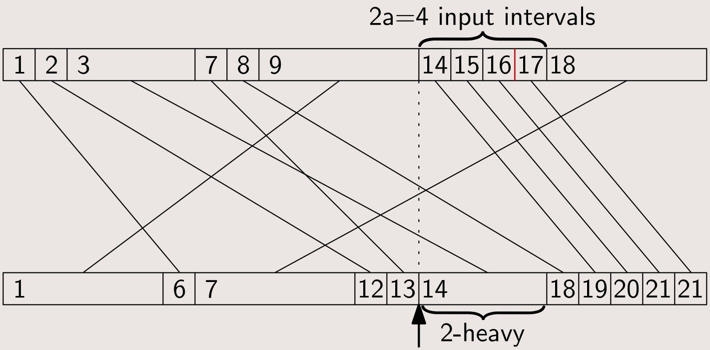

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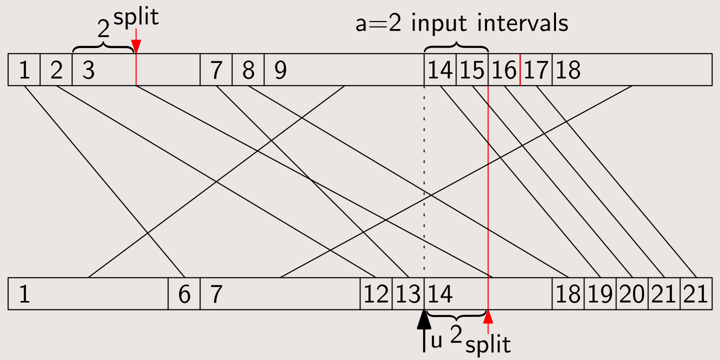

Fast Balancing Algorithm

General Approach

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- If an output interval is a-heavy:
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  - Check for new a-heavy output interval before u and recurse

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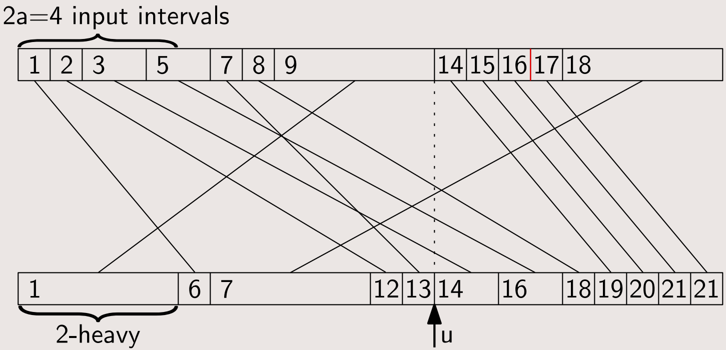

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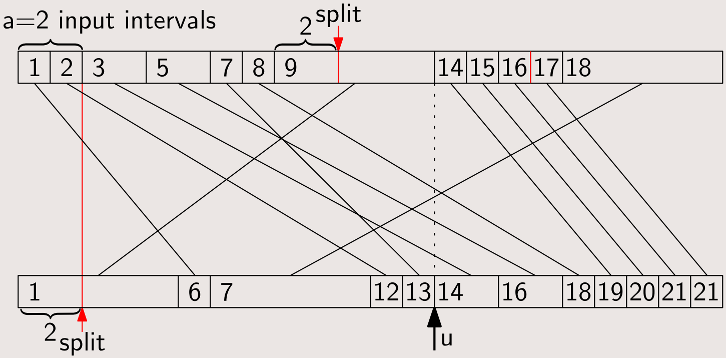

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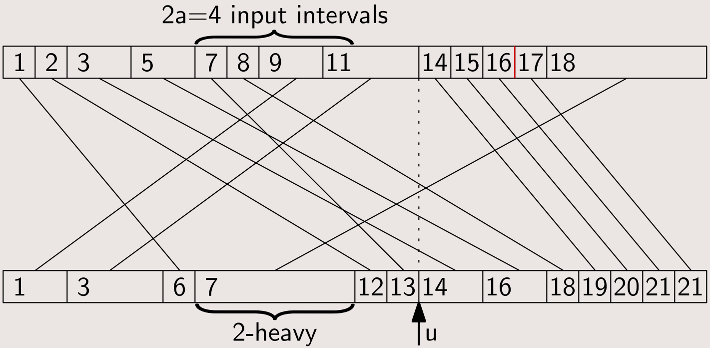

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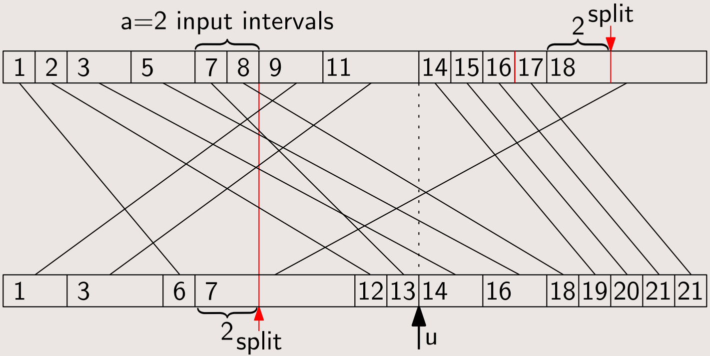

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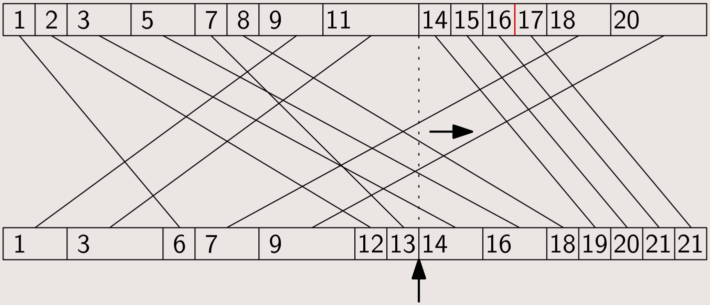

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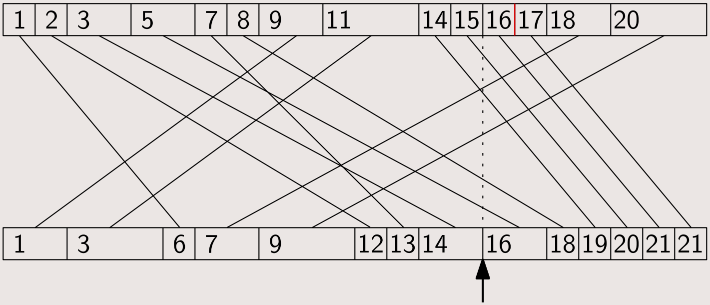

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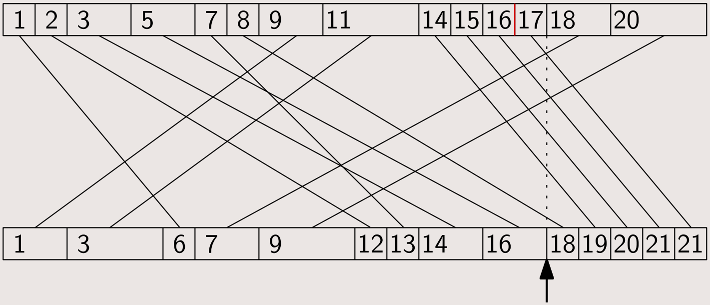

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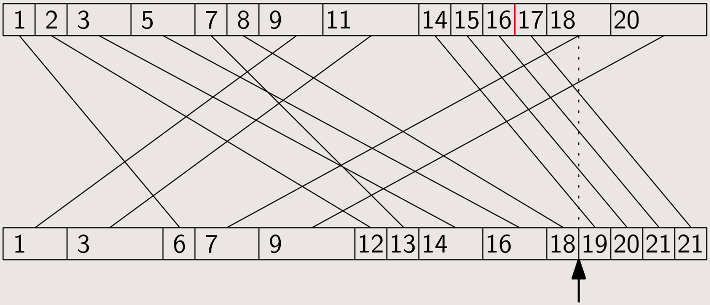

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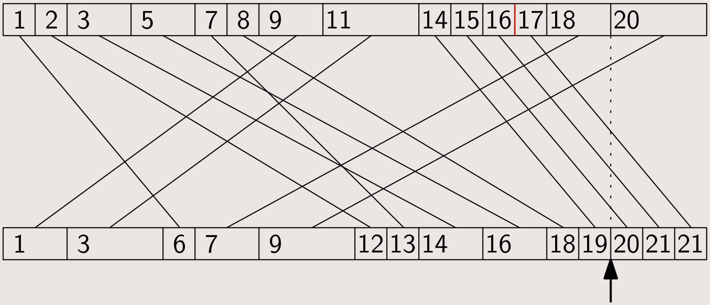

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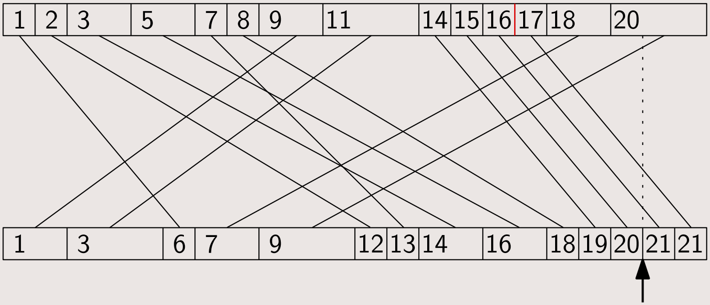

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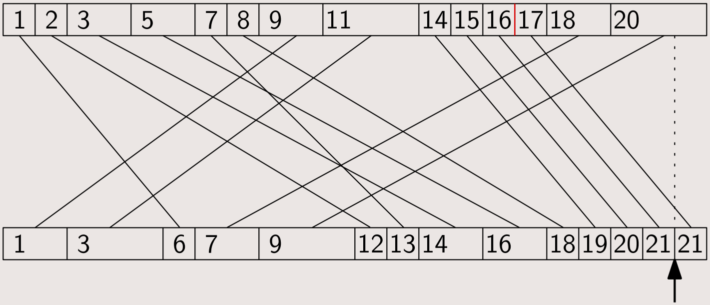

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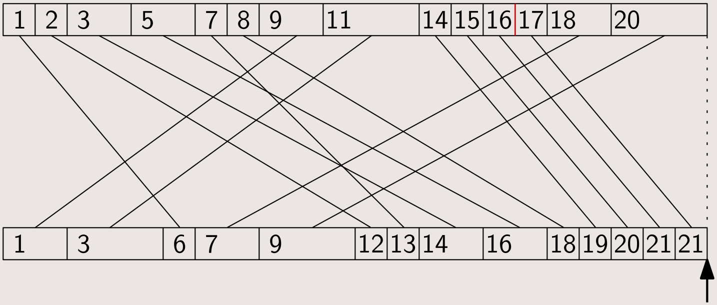

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Details

- Use balanced search trees for input and output intervals

$\Longrightarrow$ $\mathcal{O}(k \log k)$ time, $\mathcal{O}(1)$ additional space

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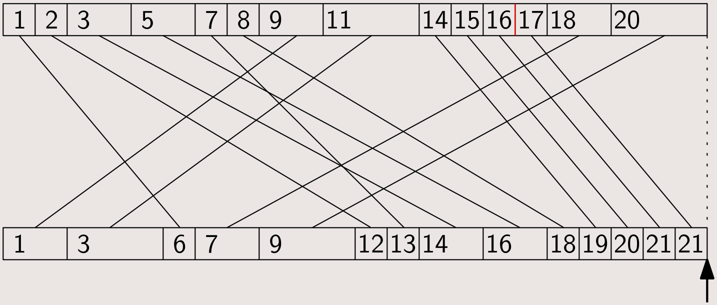

Experimental Setup

Tested Indexes

- move-r
- r-index [Gagie et al. 2020]
- online-rlbwt (dynamic) [Bannai et al. 2020]
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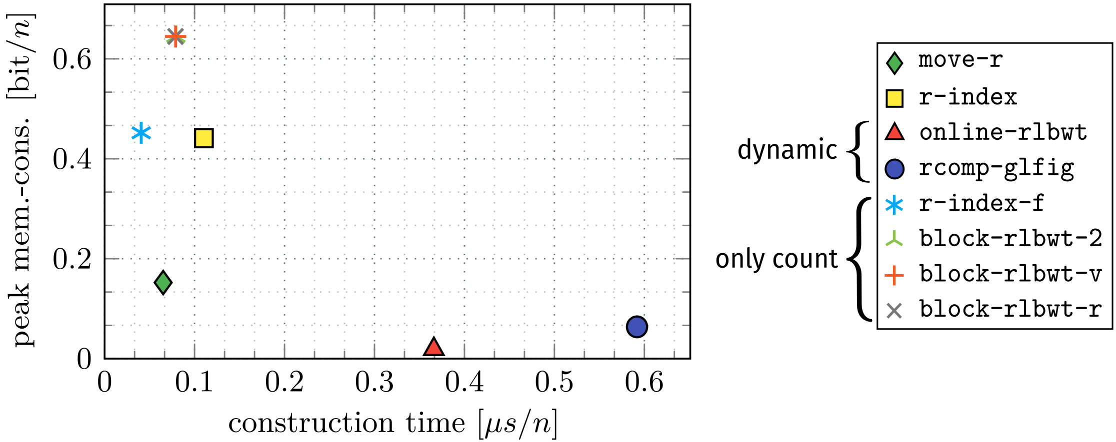
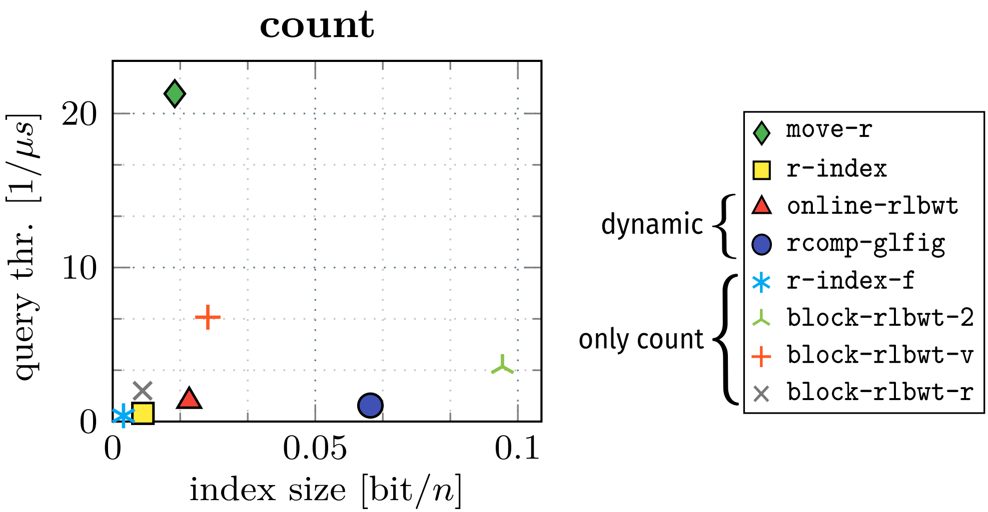
- ∗ r-index-f [Brown et al. 2022]
- ⅄ block-rlbwt-2 [Díaz-Domínguez et al. 2023]
- + block-rlbwt-v [Díaz-Domínguez et al. 2023]
- × block-rlbwt-r [Díaz-Domínguez et al. 2023]

only count

Construction Performance (einstein.en.txt)



Query Performance (einstein.en.txt)



Conclusion

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- The Move Data Structure can be constructed efficiently in practice
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- The Move Data Structure can be constructed efficiently in practice
- The improved $\mathcal{O}(1)$ time to answer LF- and Φ queries is reflected in practice
- Optimizing other aspects of the r-index further improved construction- and query performance
  - 2x-35x (typ. 15x) faster queries
  - 0.9-2x (typ. 2x) faster construction with 1-3 (typ. 3x) lower memory usage, but
  - 0.8x-2.5x (typ. 2x) larger index