1 Problem sheet 2

The (well-posed) convection-diffusion equation is

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} - b\frac{\partial u}{\partial x}, \qquad 0 \le x \le 1,\ 0 \le t \le T, T > 0,$$

where $b > 0$ is given (a constant) and the boundary conditions are $u(0,t) = \varphi_0(t)$ and $u(1,t) = \varphi_1(t)$ for $t \in [0,T]$. Let

$$v_j' = \frac{1}{h^2}\left(v_{j-1} - 2v_j + v_{j+1}\right) - \frac{b}{2h}\left(v_{j+1} - v_{j-1}\right), \qquad j = 1, 2, \ldots, n_x,$$

where $h = \dfrac{1}{n_x + 1}$ and $v_j = v_j(t)$ be a semi-discrete method for the convection-diffusion equation.