1 Chapter 3: Exercises

1. In the method of separation of variables for the wave equation (see the lecture notes for Chapter 3), consider the cases $\lambda > 0$ and $\lambda = 0$.

2. We showed, using the method of separation of variables, that

$$u(x,t) = (\sin \pi t + \cos \pi t) \sin \pi x$$

is a solution to the wave equation with zero boundary conditions. Use this solution to the wave equation to investigate the accuracy of the numerical method discussed in the notes, i.e.,

$$\mathbf{u}^{i+1} = 2\mathbf{u}^i - \mathbf{u}^{i-1} + \tau^2 D_n^2 \mathbf{u}^i, \qquad i = 0, \ldots, n_t - 1.$$

Let $\tau = 8/n_x^2$ and $n_t \tau = 2$, compute the maximum error of the method for $n_x = 2^3, 2^4, \ldots, 2^8$, plot it on a log-log scale and deduce the rate of decay of the error with n_x.