Number Theory Basics

Inclusion Exclusion principle

In many questions, we are required to include contributions of more than one term in our answer. This results in inclusion of the same term more than once, hence we use the principle of inclusion exclusion.

Question

Find the number of numbers in the interval [1,n] which are divisible by a or b.

Approach

We calculate the number of numbers divisible by a and b separately and then subtract the number of numbers which are divisible by both a and b.

Code

```
#include<iostream>
using namespace std;

int divisible(int n, int a, int b){

   int c1=n/a;
   int c2=n/b;
   int c3=n/(a*b);
   return c1+c2-c3;
}

int main(){

   int n, a, b;
   cin>>n>>a>>b;
   cout<<divisible(n,a,b)<<endl;
   return 0;
}
```

GCD (Greatest Common Divisor)

In our childhood, we used to call it HCF (Highest Common Factor). GCD of two numbers is the largest number that divides both of them.

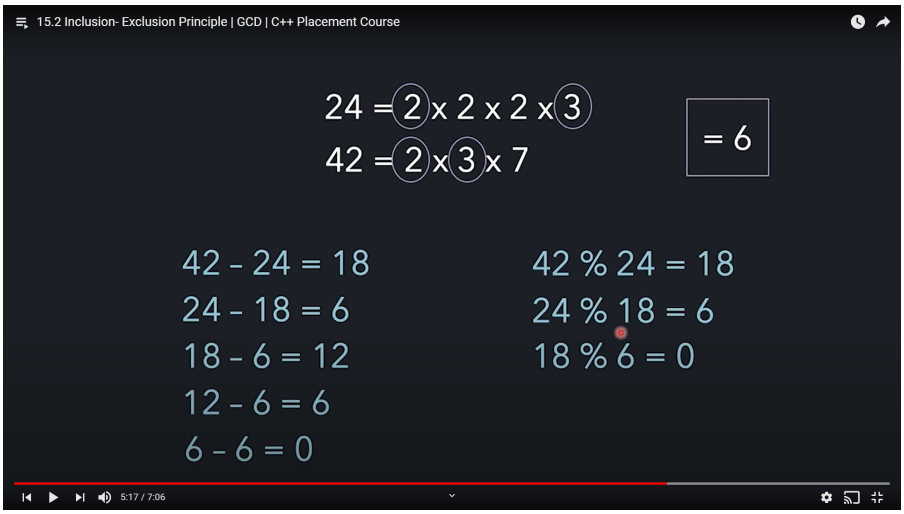
Idea

We use the logic that

$$gcd(a,b) = gcd(b,a\%b)$$

Code

```cpp
#include<iostream>
using namespace std;

int gcd(int a, int b){

   while(b!=0){
       int rem=a%b;
       a=b;
       b=rem;
   }
   return a;
}

int main(){

   int a,b;
   cin>>a>>b;

   cout<<gcd(a,b)<<endl;
   return 0;
}
```



GCD or HCF of two numbers does not change if we subtract one from other, until both of the are prime. We can do so till we get 0 and the last difference will give GCD.