

Josephus Problem Recursion

There are n people standing in a circle waiting to be executed. The counting out begins at some point in the circle and proceeds around the circle in a fixed direction. In each step, a certain number of people are skipped and the next person is executed. The elimination proceeds around the circle (which is becoming smaller and smaller as the executed people are removed), until only the last person remains, who is given freedom. Given the total number of persons n and a number k which indicates that $k-1$ persons are skipped and k th person is killed in circle. The task is to choose the place in the initial circle so that you are the last one remaining and so survive.

For example, if $n = 5$ and $k = 3$, then the safe position is 3. Firstly, the person at position 2 is killed, then person at position 4 is killed, then person at position 1 is killed. Finally, the person at position 5 is killed. So the person at position 3 survives. If $n = 7$ and $k = 4$, then the safe position is 4. The persons at positions 3, 6, 2, 7, 5, 1 are killed in order, and person at position 4 survives.

The problem has following recursive structure.

$$\text{ josephus}(n, k) = ((\text{ josephus}(n - 1, k) + k) \% n);$$

$$\text{ josephus}(1, k) = 0 \rightarrow \text{index } 0 \rightarrow \text{person } 1.$$

After the first person (k th from beginning) is killed, $n-1$ persons are left. So we call $\text{ josephus}(n - 1, k)$ to get the position with $n-1$ persons. But the position returned by $\text{ josephus}(n - 1, k)$ will consider the position starting from $k\%n$. So, we must make adjustments to the position returned by $\text{ josephus}(n - 1, k)$.

Advanced Recursion Challenges - Josephus Problem, Matrix Paths | DSA-One Course

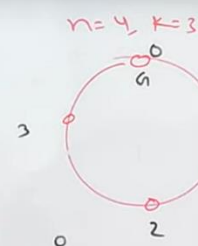
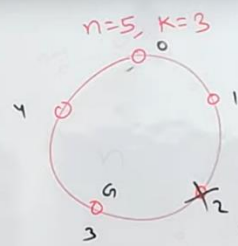
number of $n \times m$ matrix
Problem

$$j(n, k) = \left(j(n-1, k) + k \right) \% n$$

$$\begin{aligned} 0 &\rightarrow (k) \% n \\ 1 &\rightarrow (k+1) \% n \\ 2 &\rightarrow (k+2) \% n \\ &\vdots \end{aligned}$$

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and the number
ways in
Josephus



sub → orig.
0 → 3
1 → 4
2 → 0
3 → 1

$$j(n, k) = j(n-1, k) + \dots$$



$$1 + 3 = 4$$

$$2 + 3 = 5 \pmod 5 = 0$$

and H

$$\begin{aligned}
 & \uparrow 3 \\
 & j(5, 3) \\
 & \downarrow \uparrow 3 \\
 & (j(4, 3) + 3) \% 5 \\
 & \downarrow \uparrow 0 \\
 & (j(3, 3) + 3) \% 4 \\
 & \downarrow \uparrow 1 \\
 & (j(2, 3) + 3) \% 3 \\
 & \downarrow \uparrow 1 \\
 & (j(1, 3) + 3) \% 2 \\
 & \downarrow \uparrow 0 \\
 & 0
 \end{aligned}$$

$$j(n, k) = (j(n-1, k) + k) \% n$$

```

int jos (int n, int k) {
    if (n == 1)
        return 0;
    return (jos(n-1, k) + k) % n;
}
    
```

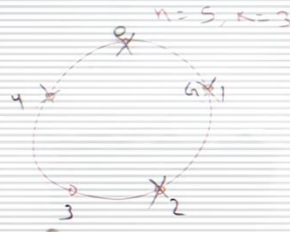
$j(5, 3)$

→ 3

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Backtracking | N-Queen Problem & Sudoku Solver | DSA-One Course #11

Backtracking

N-Queen Problem

```
boolean nQueen(int board[][ ], int row) {  
    if (row == N) return true;  
    for (int col = 0; col < N; col++) {  
        if (isSafe(board, row, col)) {  
            board[row][col] = 1;  
            if (nQueen(board, row+1))  
                return true;  
            board[row][col] = 0; ←  
        }  
    }  
    return false;  
}
```



15:16 / 23:38 • N-Queen Problem Code >