here $g = pdf$

$$g(z) = \sum_k p_k(z) g(x_k) \tag{1}$$

$$p_k(x_j) = \delta_{jk}$$

$$|f\rangle = \sum_k c_k |g_k\rangle \tag{2}$$

$$c_k = f(x_k) \tag{3}$$

$$F(x) = \int_x^1 \frac{dz}{z} c(z) g(z) \tag{4}$$

$$F(x_j) = \sum_k \int_{x_j}^1 \frac{dz}{z} c(z) p_k(z) g(x_k) \tag{5}$$

here $g' = arbitrary fnc$

$$h_2(\xi) = \int_\xi^1 \frac{du}{u} g'(u) F(u) \tag{6}$$

$$h_2(\xi) = \sum_j \int_\xi^1 \frac{du}{u} g'(u) p_j(u) F(x_j) \tag{7}$$

$$h_2(\xi) = \sum_{j,k} \int_\xi^1 \frac{du}{u} g'(u) p_j(u) \int_{x_j}^1 \frac{dz}{z} c(z) p_k(z) \cdot g(x_k) \tag{8}$$

$$h_2(\xi) = \sum_k \left( \sum_j \int_\xi^1 \frac{du}{u} g'(u) p_j(u) \left( \int_{x_j}^1 \frac{dz}{z} c(z) p_k(z) \right) \right) \cdot g(x_k) \tag{9}$$