ii) Derive the steady state system size probabilities for a M/M/1/N/FCFS queueing model and hence obtain the mean number of customers in the queue. **(8)**

(OR)

b) i) Derive the steady-state system-size probabilities for a M/M/C/∞ FCFS queueing model and hence obtain the mean number of customers in the system. **(8)**

ii) Patients arrive at a clinic according to a Poisson process at a rate of 3 patients per hour. The waiting room cannot accommodate more than 6 patients. Examination time per patient is exponentially distributed random variable with rate of 4 per hour.

1) Find the effective arrival rate at the clinic.

2) What is the probability that an arriving patient will not wait ?

3) What is the expected waiting time $W_S$ in the system ? **(8)**

15. a) Discuss an M/G/1/∞ FCFS queueing system and hence obtain the Pollaczek-Khintchine (P-K) mean value formula for the system size. Deduce also the mean number of customers in the system for M/M/1/∞ FCFS queueing model from the P-K mean value formula. **(16)**

(OR)

b) Derive the system of differential difference equations for the joint probabilities of the system size of two-station tandem queueing system. Under the steady-state conditions, determine the steady-state probabilities of the system size and obtain 1) Expected number of customers in the system, 2) The mean waiting time in the system. **(16)**

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7/11/19/AN

Reg. No. : | | | | | | | | | | | | |

## Question Paper Code : 90342

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2019

Fourth Semester

Computer Science and Engineering

MA8402 – PROBABILITY AND QUEUEING THEORY

(Regulations 2017)

Time : Three Hours Maximum : 100 Marks

Answer ALL questions

PART – A (10×2=20 Marks)

1. A bag contains 8 white and 4 black balls. If 5 balls are drawn at random, what is the probability that 3 are white and 2 are black ?

2. Let $M_X(t) = \frac{1}{1-t}$, $|t| < 1$, be the moment generating function of a R.V. X. Find E(X) and E($X^2$).

3. If $f(x, y) = e^{-(x+y)}$, $x \geq 0, y \geq 0$, is the joint probability density function of (X, Y), Find $P(X + Y \leq 1)$.

4. Let X and Y be independent R.Vs with Var(X) = 9 and Var(Y) = 3. What is Var(4X – 2Y + 6) ?

5. Define : Markov process.

6. Let $\{X_n ; n \geq 0\}$ be a Markov chain having state space S = {1, 2} and one-step TPM $P = \begin{bmatrix} 1/2 & 1/2 \\ 0 & 1 \end{bmatrix}$. Find the stationary probabilities of the Markov chain.

7. In an M/M/1/∞/FCFS queue, the service rate, $\mu = \frac{1}{3}$/ minute and waiting time in the queue $W_q$ = 3 minute, compute the arrival rate, λ.

8. For a M/M/C/N/FCFS (C < N) queueing system, write the expressions for $P_0$ and $P_N$.

9. In an M/D/1 queueing system, an arrival rate of customers is 1/6 per minute and the server takes exactly 4 minutes to serve a customer. Calculate the mean number of customers in the system.

10. For an open Jackson queueing network, write the expression for traffic equations and stability condition of the system.

PART – B (5×16=80 Marks)

11. a) i) There are 3 boxes containing respectively, 1 white, 2 red, 3 black balls, 2 white, 3 red, 1 black balls; 3 white, 1 red, 2 black balls. A box is chosen at random and from it two balls are drawn at random. The two balls are 1 red and 1 white. What is the probability that they came from second box ? (8)

ii) The p.d.f. of a continuous R.V. X is given by $f(x) = \begin{cases} \frac{x}{2}e^{-\frac{x}{2}}, & x > 0 \\ 0, & x \le 0 \end{cases}$. Obtain

1) C.D.F. of X, F(x)
2) $P(X > 1)$
3) $P(1 < X < 2)$
4) $E(X^2)$. (8)

(OR)

b) i) Let X be a binomial R.V with $E(X) = 4$ and $Var(X) = 3$. Find: (1) $P(X = 5)$, (2) M.G.F. of X, $M_X(t)$, (3) $E(X^2 - 1)$, (4) $Var\left(-\frac{1}{2}X + 4\right)$. (8)

ii) A R.V. X is uniformly distributed on (–5, 15). Determine :

1) C.D.F. of X, F(x)
2) $P(X < 5/X > 0)$
3) $P(|X - 1| < 5)$
4) $E\left(e^{-\frac{X}{5}}\right)$. (8)

12. a) i) The joint p.d.f. of (X, Y) is given by $f(x, y) = \begin{cases} \frac{1}{240}, & 8.5 \le x \le 10.5, 120 \le y \le 240 \\ 0, & \text{otherwise} \end{cases}$

Obtain

1) The marginal p.d.fs of X and Y.
2) E(X) and E(Y)
3) E(XY)
4) Are X and Y independent R.Vs ? Justify. (8)

ii) Let X and Y be two continuous R.Vs with joint p.d.f. $f(x, y) = \begin{cases} 4xy, & 0 < x < 1, 0 < y < 1 \\ 0, & \text{otherwise} \end{cases}$. Determine the joint p.d.f. of the R.Vs $U = X^2$ and $V = XY$ and hence obtain the marginal p.d.f. of U. (8)

(OR)



b) i) The joint p.d.f. of R.V (X, Y) is given as $f(x, y) = \begin{cases} Ce^{-(2x+3y)}, & 0 \le y \le x < \infty \\ 0, & \text{otherwise} \end{cases}$.

Find :

1) The value of C.
2) Are the R.Vs X and Y independent ? (8)

ii) Let X and Y be random variables such that $E(X) = 1$, $E(Y) = 2$, $Var(X) = 6$, $Var(Y) = 9$ and the correlation coefficient $\rho_{XY} = -2/3$. Calculate :

1) The covariance, Cov(X, Y), of X and Y
2) E(XY)
3) $E(X^2)$ and $E(Y^2)$. (8)

13. a) i) Consider a random process $X(t) = Cos(t + \phi)$, where $\phi$ is a R.V. such that $P(\phi = 0) = P(\phi = \pi) = 1/2$. Determine 1) $E(X(t))$, 2) $E(X^2(t))$, 3) $R_{XX}(t, t + \tau)$. Is the process X(t) wide-sense stationary ? Justify. (8)

ii) State the postulates of a Poisson process $\{X(t) ; t \ge 0\}$ with parameter $\lambda$. Derive the system of differential difference equations and hence obtain the probability distribution, $P(X(t) = n) = \frac{e^{-\lambda t}(\lambda t)^n}{n!}$, n = 0, 1, 2, .... (8)

(OR)

b) i) Let $\{X_n ; n \ge 0\}$ be a Markov chain having state space S = {1, 2, 3} with one-step TPM. $P = \begin{bmatrix} 0 & 1 & 0 \\ 1/2 & 0 & 1/2 \\ 1 & 0 & 0 \end{bmatrix}$.

1) Draw a transition diagram.
2) Is the chain irreducible ? Explain.
3) Is the state – 2 ergodic ? Justify your answer. (8)

ii) Let X(t) and Y(t) be two independent Poisson processes with parameters $\lambda_1$ and $\lambda_2$ respectively. Obtain 1) $P(X(t) + Y(t) = n)$, n = 0, 1, 2, ...., 2) $P(X(t) - Y(t) = n)$, n = 0, ±1, ±2, .... (8)

14. a) i) A petrol station has one petrol pump. The cars arrive for service according to a Poisson process at a rate of 0.5 cars per minute and the service time for each car follows the exponential distribution with rate of 1 car per minute. compute :

1) The probability that the pump station is idle
2) The probability that 10 or more cars are in the system
3) The mean number, $L_S$ of cars in the system.
4) The mean waiting time, $W_q$, in the queue and the mean waiting time, $W_s$, in the system. (8)

Reg. No. [ ]

**Question Paper Code : 57508**

16/6/16
FN

**B.E./B.Tech. DEGREE EXAMINATION, MAY/JUNE 2016**

**Fourth Semester**

**Computer Science and Engineering**

**MA 6453 – PROBABILITY AND QUEUEING THEORY**

**(Common to Mechanical Engineering (Sandwich) and Information Technology)**

**(Regulations 2013)**

**Time : Three Hours** **Maximum : 100 Marks**

**Use of statistical tables may be permitted.**

**Answer ALL questions.**

**PART – A (10 × 2 = 20 Marks)**

1. Let X be a discrete R.V. with probability mass function

$$P(X=x)=\begin{cases}\frac{x}{10}, & x=1,2,3,4,\\ 0, & \text{otherwise}\end{cases}$$

Compute $P(X<3)$ and $E\left(\frac{1}{2}X\right)$.

2. If a R.V X has the moment generating function $M_x(t)=\frac{3}{3-t}$, compute $E(X^2)$.

3. The joint p.d.f. of R.V. (X,Y) is given as $f(x,y)=\begin{cases}\frac{1}{x}, & 0<y<x\le 1\\ 0, & \text{otherwise}\end{cases}$

Find the marginal p.d.f. of Y.



4. Let X and Y be two independent R.Vs with Var(X) = 9 and Var(Y) = 3. Find $Var(4X - 2Y + 6)$.

5. The random process X(t) is given by $X(t) = Y\cos(2\pi t)$, $t > 0$, where Y is a R.V. with $E(Y) = 1$. Is the process X(t) stationary ?

6. Derive the autocorrelation function for a Poisson process with rate $\lambda$.

7. For an M/M/C/N FCFS (C < N) queueing system, write the expressions for $P_0$ and $P_N$.

8. Define (i) balking and (ii) reneging of the customers in the queueing system.

9. An M/D/1 queue has an arrival rate of 10 customers per second and a service rate of 20 customers per second. Compute the mean number of customers in the system.

10. Write a expression for the traffic equation of the open Jackson queueing network.

**PART – B (5 × 16 = 80 Marks)**

11. (a) (i) The probability mass function of a discrete R.V X is given in the following table :

| X | –2 | –1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|
| P(X = x) | 0.1 | k | 0.2 | 2k | 0.3 | k |

Find (1) the value of k (2) $P(X < 1)$ (3) $P(-1 < X \le 2)$ (4) E(X). **(8)**

(ii) Let X be a continuous R.V with probability density function

$$f(x)=\begin{cases}xe^{-x}, & x>0\\ 0, & \text{otherwise}\end{cases}$$

Find (1) the cumulative distribution function of X

(2) Moment Generating Function $M_x(t)$ of X

(3) $P(X < 2)$

(4) E(X). **(8)**

OR



(b) (i) Let $P(X=x)=\left(\frac{3}{4}\right)\left(\frac{1}{4}\right)^{x-1}$, $x = 1, 2, 3, \ldots\ldots$, be the probability mass function of a R.V X Compute **(8)**

(1) $P(X > 4)$

(2) $P(X > 4/X > 2)$

(3) E(X)

(4) Var(X)

(ii) Let X be a uniformly distributed R.V. over [–5, 5]. Determine **(8)**

(1) $P(X \le 2)$

(2) $P(|X| > 2)$

(3) Cumulative distribution function of X

(4) Var (X)

12. (a) (i) Find the constant k such that

$$f(x,y)=\begin{cases}k(x+1)e^{-y}, & 0<x<1,\ y>0\\ 0, & \text{otherwise}\end{cases}$$

is a joint p.d.f. of the continuous R.V. (X, Y). Are X and Y independent R.Vs ? Explain. **(8)**

(ii) The joint p.d.f. of the continuous R.V. (X, Y) is given as

$$f(x,y)=\begin{cases}e^{-(x+y)}, & x>0,\ y>0\\ 0, & \text{otherwise}\end{cases}$$

Find the p.d.f. of the R.V $U = \frac{X}{Y}$. **(8)**

OR

(b) (i) Let the joint p.d.f. of R.V. (X, Y) be given as (8)

$f(x, y) = \begin{cases} Cxy^2 & , 0 \le x \le y \le 1 \\ 0 & , \text{otherwise} \end{cases}$ Determine (1) the value of C (2) the marginal p.d.fs of X and Y (3) the conditional p.d.f. $f(x/y)$ of X given $Y = y$

(ii) A joint probability mass function of the discrete R.Vs X and Y is given as

$$P(X = x, Y = y) = \begin{cases} \frac{x+y}{32} & , x = 1, 2, y = 1, 2, 3, 4 \\ 0 & , \text{otherwise} \end{cases}$$

Compute the covariance of X and Y.

13. (a) (i) Consider a random process $Y(t) = X(t)\cos(w_0 t + \theta)$, where X (t) is wide-sense stationary process, $\theta$ is a uniformly distributed R.V. over $(-\pi, \pi)$ and $w_0$ is a constant. It is assumed that X(t) and $\theta$ are independent. Show that Y(t) is a wide-sense stationary. (8)

(ii) Consider a Markov chain $\{X_n ; n = 0, 1, 2, \ldots\}$ having state space $S = \{1, 2\}$ and one-step

$$\text{TPM } P = \begin{bmatrix} \frac{4}{10} & \frac{6}{10} \\ \frac{8}{10} & \frac{2}{10} \end{bmatrix}$$ (8)

(1) Draw a transition diagram.

(2) Is the chain irreducible ?

(3) Is the state-1 ergodic ? Explain.

(4) Is the chain ergodic ? Explain.

OR

(b) (i) Let X(t) and Y(t) be two independent Poisson processes with parameters $\lambda_1$ and $\lambda_2$ respectively. Find

(1) $P(X(t) + Y(t)) = n$, $n = 0, 1, 2, 3, \ldots$,

(2) $P(X(t) - Y(t)) = n$, $n = 0, \pm 1, \pm 2, \ldots$. (8)



(ii) Consider a Markov chain $\{X_n; n = 0, 1, 2, \ldots\}$ having state space $S = \{1, 2, 3\}$ and one-step TPM $P = \begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} \\ 1 & 0 & 0 \end{bmatrix}$ and initial probability distribution $P(X_0 = i) = \frac{1}{3}, i = 1, 2, 3.$

Compute

(1) $P(X_3 = 2, X_2 = 1, X_1 = 2 / X_0 = 1)$

(2) $P(X_3 = 2, X_2 = 1/X_1 = 2, X_0 = 1)$

(3) $P(X_2 = 2/X_0 = 2)$

(4) Invariant probabilities of the Markov chain. (8)

14. (a) (i) Customers arrive at a watch repair shop according to a Poisson process at a rate of 1 per every 10 minutes, and the service time is an exponential random variable with mean 8 minutes. Compute (8)

(1) the mean number of customers $L_s$ in the system.

(2) the mean waiting time $W_s$ of a customer spends in the system,

(3) the mean waiting $W_q$ of a customer spends in the queue,

(4) the probability that the server is idle.

(ii) A petrol pump station has 4 petrol pumps. The service time follows an exponential distribution with mean of 6 minutes and cars arrive for service in a Poisson process at the rate of 30 cars per hour.

(1) Find the probability that no car is in the system.

(2) What is the probability that an arrival will have to wait in the queue ?

(3) Find the mean waiting time in the system. (8)

OR



(b) (i) A one person barber shop has 6 chairs to accommodate people waiting for a haircut. Assume that customers who arrive when all the 6 chairs are full leave without entering the barber shop. Customers arrive at the rate of 3 per hour and spend an average of 15 minutes in the barber's chair. Compute

(1) $P_0$

(2) $L_q$

(3) $P_7$

(4) $W_s$ (8)

(ii) Consider a single-server queue where the arrivals are Poisson with rate $\lambda = 10$ / hour. The service distribution is exponential with rate $\mu = 5$/hour. Suppose that customers balk at joining the queue when it is too long. Specifically, when there are 'n' in the system, an arriving customer joins the queue with probability $\frac{1}{(n+1)}$. Determine the steady-state probability that there are 'n' customers in the system. (8)

15. (a) Discuss an M/G/1/∞ FCFS queueing system and hence obtain the Pollaczek-Khintchine (P-K) mean value formula. Deduce also the mean system size for the M/M/1/∞ : FCFS queueing system from the P-K formula. (16)

OR



(b) (i) The police department has 5 patrol cars. A patrol car breaks down and repairs service one every 30 days. The police department has two repair workers, each of whom takes an average of 3 says to repair a car. Breakdown times and repair time are exponential. Determine the average number of patrol cars in good condition. Also find the average down time for a patrol car that needs repairs. (8)

(ii) A repair facility is shared by a large by a large number of machines for repair. The facility has two sequential stations with respective rates of service 1 per hour and 3 per hour. The cumulative failure rate of all the machines is 0.5 per hour. Assuming that the system behaviour may be approximated by a two-station tandem queue. Find

(1) the average number of customers in both station,

(2) the average repair time,

(3) the probability that both service stations are idle. (8)

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15. (a) In a network of 3 service stations 1,2,3 customers arrive at 1,2.3 from outside, in accordance with Poisson process having rates 5, 10, 15 respectively. The service times at the 3 stations are exponential with respective rates 10, 50, 100. A customer completing service at station 1 is equally likely to go to station 2 or go to station 3 or leave the system. A customer departing from service at station 2 always goes to station 3. A departure from service at station 3. 1 is equally likely to go to station 2 or leave the system. What is the average number of customers in the system? And what is the average time a customer spends in the system? (16)

Or

(b) Consider a queuing system where arrivals are according to a Poisson distribution with mean 5 per Hour. Find the expected waiting time in the system. If the service time distribution is (i) Uniform between: $t = 5$ minutes and $t = 15$ minutes (ii) Normal with mean 3 minutes and standard deviation 2 minutes. (16)

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Reg. No. :

**Question Paper Code : 72073**

30/05/2017 FN

B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2017.

Fourth Semester

Mechanical Engineering (Sandwich)

MA 6453 — PROBABILITY AND QUEUEING THEORY

(Common to Computer Science and Engineering/Information Technology)

(Regulations 2013)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. If the probability density function of a random variable $X$ is $f(x) = \frac{1}{4}$ in $-2 < x < 2$ find $P(|X| > 1)$.

2. If $X$ is a geometric variate, taking values 1, 2, 3, ...$\infty$, find P($X$ is odd).

3. Define conditional distribution for two- dimensional discrete and continuous random variables.

4. If $X = R\cos\phi$ and $Y = R\sin\phi$, how are the joint probability density function of $(X, Y)$ and $(R, \phi)$ are related?

5. Define a $k$th order stationary process. When will it become a strict sense stationary process?

6. State Chapman-Kolmogorov theorem.

7. State Little's formula for the queuing model $(M/M/S):(\infty/FIFO)$.

8. What are the values of $P_0$ and $P_n$ for the queuing model (M/M/1): (K/FIFO) when $\lambda = \mu$

9. State Pollaczek - Khintchine formula for (M/G/1) queuing model.

10. Write down the traffic equation for open Jackson network.

PART B — (5 × 16 = 80 marks)

11. (a) (i) The density function of a continuous random variable X is given by

$$f(x) = \begin{cases} ax, & 0 \le x \le 1 \\ a, & 1 \le x \le 2 \\ 3(a - x), & 2 \le x \le 3 \\ 0 & \text{otherwise} \end{cases}$$

(1) Find the value of 'a' (2) CDF of X. (8)

(ii) The probability of a man hitting a target is 1/4. If he fires 7 times, what is the probability of his hitting the target at least twice? And how many times must he fire so that the probability of his hitting the target at least once is greater than 2/3? (8)

Or

(b) (i) Find the MGF of the random variable X having the probability density function

$$f(x) = \begin{cases} \frac{x}{4} e^{-x/2} & x > 0 \\ 0 & \text{otherwise} \end{cases}$$

. Also find the first four moments about the origin. (8)

(ii) In a normal population with mean 15 and standard deviation 3.5, it is found that 647 observations exceed 16.25. What is the total number of observations in the population? (8)

12. (a) (i) The joint probability mass function of $(X, Y)$ is given by $p(x, y) = k(2x + 3y) x = 0, 1, 2;\ \ y = 1, 2, 3$. Find the value of $k$ and find all the marginal probability distributions. (6)

(ii) The probability density function of $(X, Y)$ is given by $f(x, y) = \begin{cases} 8xy, & 0 < x < y < 1 \\ 0 & otherwise \end{cases}$. Find $P\left(X < \frac{1}{2} \cap y < \frac{1}{4}\right)$. Are X and Y independent? Justify your answer. (10)

Or

(b) (i) If the joint pdf of $(X, Y)$ is given by $f(x, y) = e^{-(x+y)}$ $x > 0,\ y > 0$, prove that X and Y are uncorrelated. (8)

(ii) If X and Y follows an exponential distribution with parameter 2 and 3 respectively and are independent, find the probability density functions of $U = X + Y$. (8)



13. (a) (i) If the process $X(t) = P + Qt$ where P and Q are independent random variables with $E(P) = p$, $E(Q) = q$, $\text{var}(P) = \sigma_1^2$ and $\text{var}(Q) = \sigma_2^2$, find $E\{X(t)\}$ and $R(t_1, t_2)$. Is the process $\{X(t)\}$ stationary. (8)

(ii) On the average a submarine on patrol sights 6 enemy ships per hour. Assuming that the number of Ships sighted in a given length of time is a Poisson variate. Find the probability of sighting 6 ships in the next half-an-hour, 4 ships in the next 2 hours and at least 1 ship in the next 15 minutes. (8)

Or

(b) (i) Three boys A, B, and C are throwing a ball to each other. A always throws the ball to B and B always throws the ball to C, but C is just likely to throw the ball to B as to A. Show that the process is Markovian. Find the transition matrix and classify the states. (8)

(ii) Prove that the random process $X(t) = \cos(w_0 t + \theta)$ where $\theta$ is uniformly distributed in the interval $(-\pi, \pi)$ is wide sense stationary. (8)

14. (a) (i) Obtain $P_o$ and $P_n$ for the birth and death process. (8)

(ii) Arrives at a telephone booth are considered to be Poisson distribution, with an average time of 10 minutes between one arrival and the next. The length of phone call assumed to be distributed exponentially with mean 3 minutes. What is the probability that a person arriving at the booth will have to wait? And what is the average length of the queues that form from time to time? (8)

Or

(b) There are three typists in an office. Each typist can type an average of 6 letters per hour. If letters arrive for being typed at the rate of 15 letters per hour. What is the probability of all the typists will be busy? What is the average number of letters waiting to be typed? What is the average time a letter has to be spent for waiting and for being typed? And what is the probability that a letter will take longer than 20 minutes waiting to be typed and being typed? (16)

15. a) i) For an $(M/E_2/1)$ : $(FIFO/\infty/\infty)$ queueing model with $\lambda = \frac{6}{5}$ per hour and $\mu = \frac{3}{2}$ per hour, find the average waiting time of a customer. Also find the average time he spends in the system. **(8)**

ii) In a college canteen, it was observed that there is only one waiter who takes exactly 4 minutes to serve a cup of coffee once the order has been placed with him. If students arrive in the canteen at an average rate of 10 per hour, how much time one is expected to spend waiting for his turn to place the order. **(8)**

(OR)

b) i) Obtain the PollacZek-Khinchin formula for the $(M/G/1)$ : $(G_D/\infty/\infty)$ queueing model. **(10)**

ii) A repair facility shared by a large number of machines has 2 sequential stations with respective service rates of 2 per hour and 3 per hour. The cumulative failure rate of all the machines is 1 per hour. Assuming that the system behaviour may be approximated by the 2-stage tandem queue, find the average number of machines in service at both the stations and find the average repair time including the waiting time. **(6)**

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Reg. No. : | | | | | | | | | | | |

14/05/18

AN

**Question Paper Code : 41315**

B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2018

Fourth Semester

Mechanical Engineering (Sandwich)

MA 6453 – PROBABILITY AND QUEUEING THEORY

(Common to Computer Science and Engineering/Information Technology)

(Regulations 2013)

Time : Three Hours Maximum : 100 Marks

Answer ALL questions

PART – A **(10×2=20 Marks)**

1. If $M_X(t) = \frac{pe^t}{1-qe^t}$ is the Moment Generating function of X then find the mean and variance of X.

2. The mean of Binomial distribution is 20 and standard deviation is 4. Find the parameters of this distribution.

3. Let (X, Y) be a continuous Bivariate Random Variable. If X and Y are independent Random Variables then show that X and Y are uncorrelated.

4. Define conditional distributions.

5. State any two properties of Poisson Process.

6. If the transition probability matrix of a Markov chain is $P = \begin{pmatrix} 0 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$, find the stationary distribution of the chain.

7. What is the probability that a customer has to wait more than 15 minutes to get his service completed in $(M|M|1):(G_D/\infty/\infty)$ queueing system if $\lambda = 6$ per hour and $\mu = 10$ per hour ?

8. Define the following terms : Balking, Reneging and Jockeying.

9. Write the PollacZek-Khinchin formula for the case when the service time is constant.

10. Write an expression for traffic equation open Jackson Network.

PART – B (5×16=80 Marks)

11. a) i) Find the moment generating function of Gamma distribution and hence find its mean and variance. (10)

ii) The scores on an achievement test given to 1,00,000 students are normally distributed with mean 500 and standard deviation 100. What should the score of a student be to place him among the top 10% of all students ? (6)

(OR)

b) i) Determine the Moment Generating function of a normal random variable X with parameters $(\mu, \sigma^2)$, Hence find its mean and variance. (8)

ii) Let X be uniformly distributed random variable in the interval (a, 9) and $P[3 < x < 5] = 2/7$. Find the constant 'a' and compute $P[|x-5| < 2]$. (8)

12. a) The joint probability density function of the two dimensional random variable (X, Y) is given by $f(x, y) = \frac{x}{4}(1+3y^2)$, $0 < x < 2, 0 < y < 1$. Find

i) Conditional probability density functions of X given Y = y and Y given X = x. (8)

ii) $P[0.25 < X < 0.5 / Y = 0.33]$. (8)

(OR)

b) i) Given that X = 4Y + 5 and Y = kX + 4 are regression lines of X on Y and Y on X respectively. Show that $0 \le k \le \frac{1}{4}$. If $k = \frac{1}{16}$, find the means of X and Y and the correlation coefficient $r_{XY}$. (8)

ii) If the joint pdf of (X, Y) is given by $f(x, y) = x + y$, $0 \le x, y \le 1$, find the pdf of the R.V. U = XY. (8)



13. a) i) If X(t) is wide sense stationary process with autocorrelation $R(\tau) = Ae^{-\alpha|\tau|}$, determine Second order moment of the random variable X(8) – X(5). (8)

ii) Show that random process {X(t)} where $X(t) = A\cos(\omega_0 t + \theta)$ is a wide sense Stationary process if A and $\omega_0$ are constants and $\theta$ is uniformly distributed random variable over $(0, 2\pi)$. (8)

(OR)

b) i) Find the mean and auto correlation functions of Poisson Process. (8)

ii) Classify the states of the Markov chain for the one-step transition probability matrix $P = \begin{pmatrix} 0 & 1 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & 0 \end{pmatrix}$ with state space S = {1, 2, 3}. (8)

14. a) i) Derive the Steady state probabilities for $(M|M|1):(GD/N/\infty)$ queueing model and hence obtain the expressions for Ls and Ws. (10)

ii) A petrol pump station has 4 pumps. The service times follow the exponential disribution with a mean of 6 minutes and cars arrive for service in a Poisson process at the rate of 30 car per hour. What is the probability that an arrival would have to wait in line ? (6)

(OR)

b) i) Derive the formulae for Lq and Wq for $(M/M/C):(GD/\infty/\infty)$, C >1 queueing model. (8)

ii) Patients arrive at a clinic according to Poisson distribution at a rate of 30 patients per hour. The waiting room does not accommodate more than 14 patients. Examination time per patient is exponential with mean rate of 20 per hour.

1) What is the probability that an arriving patient will not wait ?

2) What is the expected waiting time until a patient is discharged from the clinic ? (8)

(ii) A car servicing station has 2 bays where service can be offered simultaneously. Because of space limitation, only 4 cars are accepted for servicing. The arrival pattern is Poisson with 12 cars per day. The service time in both the bays is exponentially distributed with $\mu$ = 8 cars per bay per day. Find the average number of cars in the service station, the average number of cars waiting for service. (8)

15. (a) (i) Automatic car wash facility operates with only one bay. Cars arrive according to Poisson distribution with a mean 4 cars per hour and may wait in the car parking lot if the bay is busy. The parking lot is large enough to accommodate any number of cars. If the service time for all cars is constant and equal to 10 minutes. determine the average numbers of cars waiting in the system, the average waiting time of a car in the queue. (8)

(ii) There are two salesmen in a departmental store, one is incharge of billing and receiving the payments and the other is incharge of weighing and delivering the items. After completing the service of billing and payment, the customers reach the shop according to Poisson process at the rate of 5 per hour and both the salesman take 6 minutes each to serve a customer on the average and the service times follow exponential distribution. Find the average number of customers in the shop. (8)

Or

(b) In a network of 3 service stations 1, 2, 3, customers arrive at stations 1 2, 3 from outside, in accordance with Poisson process having rates 5, 10, 15 respectively. The service times at the 3 stations are exponential with respective means 10, 50, 100. A customer completing services at station 1 is equally to go to station 2 or go to station 3 or leaving the station. A customer departing from service at station 2 always goes to station 3. A departure from service at station 3 is equally likely to go station 2 or leave the system. What is average number of customers in the system? What is the average time a customer spends in the system? (16)





17/05/19

Reg. No. : 

**Question Paper Code : 53251**

B.E./B.Tech. DEGREE EXAMINATIONS, APRIL/MAY 2019.

Fourth Semester

Mechanical Engineering (Sandwich)

MA 6453 — PROBABILITY AND QUEUEING THEORY

(Common to Computer Science and Engineering/Information Technology)

(Regulation 2013)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. A coin is tossed three times. If $X$ denotes the number of heads obtained, find the probability distribution of $X$.

2. State memory less property and which continuous and discrete distributions follow this property?

3. Determine the value of $k$ if $f(x,y)=\begin{cases} kxe^{-y}; & 0<x<2,\ y\geq 0 \\ 0; & \text{otherwise} \end{cases}$ is a joint probability density function of two dimensional random variable $(X, Y)$.

4. When will the two regression lines be (a) at right angles, (b) coincident?

5. Mention various types of random processes.

6. Define $n$ step transition probability in a Markov chain.

7. What are the characteristics of a Queueing system?

8. State the relationship between the average number of customers in the queue and in the system.

9. In (M/G/1) queue model, find the average number of customer in the system if $\lambda=\frac{1}{15}$ per minute, $\mu=\frac{1}{12}$ per minute, $Var(T)=9$ minute.

10. Define series queue model.

PART B — (5 × 16 = 80 marks)

11. (a) (i) If the random variable $X$ takes the values 1,2, 3, 4 such that $2P[X=1]=3P[X=2]=P[X=3]=5P[X=4]$, find the probability distribution and cumulative distribution function of $X$. (8)

(ii) Find the moment generating function of Binomial distribution and hence find its mean and variance. (8)

Or

(b) (i) The number of accidents in a year to taxi drivers in a city follows a Poisson distribution with mean equal to 3. Out of 1000 taxi drivers, find approximately the number of drivers with no accidents, with more than 3 accidents in a year. (8)

(ii) Trains arrives at a station at 15 minutes intervals starting at 4 am. If a passenger arrives at the station at a random time between 9 am and 9.30 am, find the probability that he has to wait for less than 6 minutes, more than 10 minutes. (8)

12. (a) (i) The joint probability density function of a two dimensional random variable (X, Y) is given by $f(x,y) = \begin{cases} 2: 0<x<1, 0<y<x \\ 0; \quad \text{elsewhere} \end{cases}$, find the marginal density functions of $X$ and $Y$. Also find the conditional density function of $Y$ given $X=x$ and the conditional density function of $X$ given $Y=y$. (8)

(ii) Estimate the coefficient of correlation between the two variables $x$ and $y$ from the data given below. (8)

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $x$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| $y$ | 9 | 8 | 10 | 12 | 11 | 13 | 14 |

Or

(b) (i) Two random variables $X$ and $Y$ have the following joint probability density function $f(x,y) = \begin{cases} 2-x-y; & 0 \le x \le 1, \ 0 \le y \le 1 \\ 0; & \text{otherwise} \end{cases}$. Find the covariance between $X$ and $Y$. (8)

(ii) If $X$ and $Y$ are independent random variables each uniformly distributed in (0, 1), find the probability density function of $U = XY$. (8)



13. (a) (i) If $X(t) = \sin(\omega t + Y)$, where $Y$ is uniformly distributed in $(0, 2\pi)$, prove that $\{X(t)\}$ is a wide sense stationary process. (8)

(ii) A radioactive source emits particles at a rate of 5 per minute in accordance with Poisson process. Each particle emitted has a probability 0.6 of being recorded. Find the Probability that 10 particles are recorded in 4 minutes period. (8)

Or

(b) (i) If $\{X(t)\}$ is a wide sense stationary process with autocorrelation function given by $R(\tau) = Ae^{-\alpha|\tau|}$, find the second order moment of the random variable $X(8) - X(5)$. (8)

(ii) The transition probability matrix of a Markov chain $\{X_n\}, n = 1, 2, 3, \ldots$ having three states 1, 2 and 3 is $p = \begin{pmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{pmatrix}$ and the initial probability distribution is $p^{(0)} = (0.7 \ \ 0.2 \ \ 0.1)$. Find $P[X_2 = 3]$ and $P[X_3 = 2, X_2 = 3, X_1 = 3, X_0 = 2]$. (8)

14. (a) (i) Customers arrive at a car service station follows a Poisson process at a rate of one per every 10 minutes and service time is an exponential random variable with mean 8 minutes. Find the average number of customers in the system, the average waiting time a customer spends in the system and the average time a customer spends in waiting for service. (8)

(ii) Patients arrive at a clinic according to Poisson distribution at a rate of 30 patients per hour. The waiting room does not accommodate more than 14 patients. Examination time per patient is exponential with mean rate of 20 per hour. Find the effective arrival rate. What is the probability that an arriving patient will not wait? What is the expected waiting time until a patient is discharged from the clinic? (8)

Or

(b) (i) A super market has two girls attending to sales at the counters. If the service time for each customer is exponential with mean 4 minutes and if people arrive in Poisson fashion at the rate of 10 per hour, what is the probability that a customer has to wait for service? What is the expected percentage of idle time for each girl? If the customer has to wait in the queue, what is the expected length of his waiting time? (8)

CSE 12/11/16 A.N

Reg. No. :

**Question Paper Code : 80611**

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2016.

Fourth Semester

Computer Science and Engineering

MA 6453 — PROBABILITY AND QUEUEING THEORY

(Common to Mechanical Engineering (Sandwich) and Information Technology)

(Regulations 2013)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. If a fair coin is tossed twice, find $P(X \le 1)$, where $X$ denotes the number of heads in each experiment.

2. Balls are tossed at random into 50 boxes. Find the expected number of tosses required to get the first ball in the fourth box.

3. The joint probability density function of a random variable $(X, Y)$ is $f(x, y) = ke^{-(2x+3y)}$, $x \ge 0$, $y \ge 0$. Find the value of $k$.

4. Write any two properties of joint cumulative distribution function.

5. A random process has the autocorrelation function $R_{xx}(\tau) = \frac{4\tau^2 + 6}{\tau^2 + 1}$, find the mean square value of the process.

6. Define Markov process.

7. What do you mean by balking, reneging of a queuing system?

8. State Little's formula for the queueing model $(M/M/1):(K/FIFO)$.

9. Write down the Pollaczek – Khintchine formula for $(M/G/1)$ queuing system.

10. Define bottle-neck of the system in queue networks.

PART B — (5 × 16 = 80 marks)

11. (a) (i) The probability distribution function of a random variable $X$ is given by $f(x)=\frac{4x\left(9-x^2\right)}{81}$, $0 \le x \le 3$. Find the mean, variance and third moment about origin. (10)

(ii) Messages arrive at a switch board in a Poisson manner at an average rate of six per hour. Find the probability that exactly two messages arrive within one hour, no message arrives within one hour and at least three messages arrive within one hour. (6)

Or

(b) (i) State and prove forgetfulness property of exponential distribution. Using this property, solve the following problem :

The length of the shower on a tropical island during rainy season has an exponential distribution with parameter 2, time being measured in minutes. What is the probability that a shower will last more than 3 minutes? (10)

(ii) The annual rainfall in inches in a certain region has a normal distribution with a mean of 40 and variance of 16. What is the probability that the rainfall in a given year is between 30 and 48 inches? (6)

12. (a) (i) The joint CDF of two discrete random variables $X$ and $Y$ is given by

$$F(x, y)=\begin{cases}\frac{1}{8}, & x=1,\ y=1\\ \frac{5}{8}, & x=1,\ y=2\\ \frac{1}{4}, & x=2,\ y=1\\ 1, & x=2,\ y=2\end{cases}$$

. Find the joint probability mass function and the marginal probability mass functions of $X$ and $Y$. (8)

(ii) If $X$ and $Y$ each follow an exponential distribution with parameter 1 and are independent, find the probability density function of $U=X-Y$. (8)

Or

(b) (i) The joint probability density function of a two dimensional random variable $(X, Y)$ is given by $f(x, y)=\frac{1}{8}x(x-y)$, $0<x<2$, $-x<y<x$ and 0 elsewhere. Find the marginal distributions of $X$ and $Y$ and the conditional distribution of $Y=y$ given that $X=x$. (8)

(ii) Calculate the correlation coefficient for the following data : (8)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $x$: | 65 | 66 | 67 | 67 | 68 | 69 | 70 | 72 |
| $y$: | 67 | 68 | 65 | 68 | 72 | 72 | 69 | 71 |



13. (a) (i) A random process $\{X(t)\}=K\cos\omega t$, $t \ge 0$ where $K$ is uniformly distributed in (0, 2). Determine the mean, autocorrelation and auto covariance function of the process $\{X(t)\}$. (10)

(ii) Prove that the inter arrival time of a Poisson process is an exponential distribution. (6)

Or

(b) (i) There are 2 white marbles in urn $A$ and 3 red marbles in urn $B$. At each step of the process, a marble is selected from each urn and the 2 marbles selected are interchanged. Let the state $a_i$ of the system be the number of red marbles in $A$ after $i$ changes. What is the probability that there are 2 red marbles in $A$ after 3 steps? In the long run, what is the probability that there are 2 red marbles in urn $A$? (10)

(ii) Find the nature of the states of the Markov with the tpm $P=\begin{bmatrix}0 & 1 & 0\\ 0.5 & 0 & 0.5\\ 0 & 1 & 0\end{bmatrix}$. (6)

14. (a) There are three typists in an office. Each typist can type an average of 6 letters per hour. If letters arrive for being typed at the rate of 15 letters per hour :

(i) What fraction of time all the typist will busy?

(ii) What is the average number of letters waiting to be typed?

(iii) What is the average time a letter has to spend for waiting and for being typed?

(iv) What is the probability that a letter will take longer than 20 minutes waiting to be typed and being typed? (16)

Or

(b) Patients arrive at a clinic according to Poisson distribution at a rate of 30 patients per hour. The waiting room does not accommodate more than 14 patients. Examination time per patient is exponential with mean rate of 20 per hour :

(i) Find the effective arrival rate at the clinic.

(ii) What is the probability that an arriving patient will not wait?

(iii) What is the expected waiting time until a patient is discharged from the clinic? (16)

15. (a) Consider a queuing system where arrivals are according to a Poisson distribution with mean 5 per hour. Find the expected waiting time in the system if the service time distribution is a uniform distribution between $t=5$ minutes and $t=15$ minutes. (16)

Or

(b) Consider a system of two servers where customers from outside the system arrive at server 1 at a Poisson rate 4 and at server 2 at a Poisson rate 5. The service rates in 1 and 2 are respectively 8 and 10. A customer upon completion of service at server 1 is equally likely to go to server 2 or to leave the system; whereas a departure from server 2 will go 25% of the time to server 1 and will depart the system otherwise. Determine the limiting probabilities, average number of customers and average waiting time of a customer in the system. (16)

———————

b) i) In a factory cafeteria, the customers have to pass through three counters. The customers buy coupons at the first counter, select and collect the snacks at the second counter and collect tea at the third. The server at each counter takes, on an average, 1.5 minutes, although the distribution of service time is approximately Poisson at an average rate of 6 per hour. Calculate

1) The average time a customer spends waiting in the cafeteria.
2) The average time of getting the service.
3) The most probable time in getting the service. **(10)**

ii) Consider a queueing system where a firm is engaged in both shipping and receiving activities. The management is always interested in improving the efficiency of new innovation in loading and unloading procedures. The arrival distribution of trucks is found to be Poisson with arrival rate of 3 trucks per hour. The service time distributions is exponential with unloading rate of 4 trucks per hour. Determine expected waiting time of the truck in the queue and what reductions in waiting time are possible if loading and unloading is standardized ? **(6)**

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13.11.17 AN

Reg. No. :

**Question Paper Code : 50784**

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2017

Fourth Semester

Mechanical Engineering (Sandwich)

MA 6453 – PROBABILITY AND QUEUEING THEORY

(Common to Computer Science and Engineering/Information Technology)

(Regulations 2013)

Time : Three Hours Maximum : 100 Marks

Answer ALL questions

PART – A (10×2=20 Marks)

1. A test engineer discovered that the cumulative distribution function of the lifetime of an equipment (in years) is given by $F_X(x) = 1 - e^{-\frac{x}{5}}$, $x \geq 0$. What is the expected lifetime of the equipment ?

2. If X is a normal random variable with mean 3 and variance 9, find the probability that X his between 2 and 5.

3. Let the joint probability density function of random variables X and Y be given by $f(x, y) = 8xy$, $0 \leq y \leq x \leq 1$. Calculate the marginal probability density function of X.

4. If X and Y are random variables having the joint density function $f(x, y) = \frac{1}{8}(6 - x - y)$, $0 < x < 2$, $2 < y < 4$, find $P[x + y < 3]$.

5. Define Markov Chain and one-step transition probability.

6. State any two properties of a Poisson process.

7. Arrivals at telephone booth are considered to be Poisson with an average time of 10 minutes between one arrival and the next. The length of phone calls is assumed to be distributed exponentially with a mean of 3 minutes. What is the average length of the queue that forms from time to time ?

8. Define the following terms : Balking, Reneging and Jockeying.

9. Find the length of the queue for an M/G/1 model if $\lambda = 5$, $\mu = 6$ and $\sigma = 1/20$.

10. Define series queue model.

PART – B (5×16=80 Marks)

11. a) i) Let X be a continuous random variable with the probability density function $f(x) = 1/4,\ 2 \le x \le 6$. Find the expected value and variance of X. (6)

ii) Find the moment generating function of a normal distribution and hence find its mean and variance. (10)

(OR)

b) i) Messages arrive at a switchboard in a Poisson manner at an average rate of six per hour. Find the probability that atleast three messages arrive within one hour. (6)

ii) For a Gamma random variable X with parameters $(K, \lambda)$, derive the moment generating function and hence obtain its mean and variance. (10)

12. a) Two random variables X and Y have the following joint probability density function $f(x, y) = xe^{-x(y+1)}$, $x \ge 0$, $y \ge 0$. Determine the conditional probability density function of X given Y and the conditional probability density function of Y given X. (16)

(OR)

b) If X and Y are two independent random variables each normally distributed with mean 0 and variance $\sigma^2$, then find the joint probability density function of $R = \sqrt{x^2 + y^2}$ and $\theta = \tan^{-1}\left(y/x\right)$ and hence find the probability density function of $\theta$. (16)

13. a) i) A random process {x(t)} has the probability distribution

$$P[X(t) = x] = \begin{cases} \dfrac{(at)^{x-1}}{(1+at)^{x+1}}, & x = 1,2,3\ldots \\ \dfrac{at}{1+at}, & x = 0 \end{cases}$$

Show that the process is non-stationary. (10)



ii) If {X(t)} is a Poisson process with parameter $\lambda t$, then prove that $P[X(t_1) = x/X(t_2) = n] = nc_x p^x q^{n-x}$ where $p = \dfrac{t_1}{t_2}$, $q = 1 - p$. (6)

(OR)

b) i) Suppose the arrival of calls at a switch board is modelled as a Poisson process with the rate of calls per minute being $\lambda = 0.1$. What is the probability that the number of calls arriving in a 10 minutes interval is less than 3 ? (6)

ii) Consider a Markov Chain with 3 states and transition probability matrix $P = \begin{pmatrix} 0.6 & 0.2 & 0.2 \\ 0.1 & 0.8 & 0.1 \\ 0.6 & 0 & 0.4 \end{pmatrix}$. Find the stationary probabilities of the chain. (10)

14. a) i) Consider a single server queueing system with Poisson input and exponential service times. Suppose the mean arrival rate is 3 calling limits per units, the expected service time is 0.25 hour and the maximum permissible calling units in the system is two. Calculate the expected number in the system. (8)

ii) Show that for a single service station, Poisson arrivals and exponential service time, the probability that exactly n calling units in the queueing system is $p_n = (1 - e)e^n$, $n \ge 0$, where e is the traffic intensity. Also, find the expected number of units in the system. (8)

(OR)

b) Let there be an automobile inspection situation with three inspection stalls. Assume that cars wait in such a way that when a stall becomes vacant, the car at the head of the line pulls up to it. The station can accommodate almost four cars waiting at one time. The arrival pattern is Poisson with a mean of one car every minute during the peak hours. The service time is exponential with a mean of 6 minutes. Find the average number of customers in the system during the peak hours, the average waiting time and the average number per hour that cannot enter the station because of full capacity. (16)

15. a) i) Derive Pollaczek-Khinchine formula for M/G/1 queueing system. (10)

ii) In a big factory, there are a large number of operating machines and sequential repair shops which do the service of the damaged machines exponentially with respective 1/hour and 2/hour. If the cumulative failure rate of all the machines in the factory is 0.5/hour, find (1) the probability that both repair shops are idle. (2) the average number of machines in the service. (6)

(OR)

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15. (a) Derive the Pollaczek — Khinchin formula for M/G/1 queue. Hence deduce the result for the queues M/D/1 and $M/E_k/1$ as special cases: (16)

Or

(b) Consider a system two servers where customers arrive from outside the system in a Poisson fashion at server 1 at a rate of 4 / hour and at server 2 at a rate of 5/hour. The customers are served at station 1 and station 2 at the rate of 8 / hour and 10 / hour respectively. A customer, after completion of service at server 1 is equally likely to go to server 2 or to leave the system. A departing customer from server 2 will go to server 1 twenty five percent of the time and will depart from the system otherwise. Find the total arrival rates at server 1 and server 2. Find the limiting probability of $n$ customers at server 1 and $m$ customers at server 2. Find the expected number of customers in the system.





Reg. No. :

**Question Paper Code : 20754**

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2018.

Fourth Semester

Mechanical Engineering (Sandwich)

MA 6453 — PROBABILITY AND QUEUEING THEORY

(Common to Computer Science and Engineering, Information Technology)

(Regulations 2013)

Time : Three hours Maximum : 100 marks

(Use of standard normal distribution table may be permitted)

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Check whether the function given by $f(x)=\frac{x+2}{25}$ for $x=1,2,3,4,5$ can serve as the probability distribution of a discrete random variable.

2. A random variable $X$ has the probability function $f(x)=\frac{1}{2^x}$ for $x=1,2,3,\ldots$ Find its moment generating function.

3. Let $X$ and $Y$ have the joint probability mass function

| | | X | | |
|---|---|---|---|---|
| | | 0 | 1 | 2 |
| Y | 0 | 0.1 | 0.4 | 0.1 |
| | 1 | 0.2 | 0.2 | 0 |

Find $P(X+Y>1)$ and $E(XY)$.

4. The joint probability distribution function of the random variable $(X,Y)$ is given by $f(x,y)=k\left(x^3y-xy^3\right)$, $0\le x\le 2, 0\le y\le 2$. Find the value of $k$.

5. Write the classification of random processes.

6. State any two properties of Poisson process.

7. State the basic characteristic of queueing system.

8. Write the Little's formula for queueing system.

9. State the formula for the probability that there are $n$ customers in the system of (M/M/1) :(FIFO/N/$\infty$).

10. Define: Open Jackson networks.

PART B — (5 × 16 = 80 marks)

11. (a) (i) A radar system has a probability of 0.1 of detecting a certain target during a single scan. Use Binomial distribution to find the probability that the target will be detected at least 2 times in four consecutive scans. Also compute the probability that the target will be detected at least once in twenty Scans. (8)

(ii) An electrical firm manufactures light bulbs that have a length of life which is normally distributed with mean $\mu = 800$ hours and standard deviation $\sigma = 40$ hours. Find the probability that a bulb burns between 778 and 834 hours. (8)

Or

(b) (i) The probability of an individual suffering a bad reaction from an injection of a certain antibiotic is 0.001. Out of 2000 individuals, use Poisson distribution to find the probability that exactly three suffer. Also, find the probability of more than two suffer from bad reaction. (8)

(ii) Electric trains in a particular route run every half an hour between 12. Midnight and 6 a.m. Using uniform distribution, find the probability that a passenger entering the station at any time between 1.00 a.m. and 1.30 a.m. will have to wait at least twenty minutes. (8)

12. (a) The joint probability distribution of a two dimensional random variable $(X,Y)$ is given by $f(x,y)=\frac{1}{3}(x+y),\ \ 0 \le x \le 1,\ \ 0 \le y \le 2$. Find the correlation coefficient. Also, find the equations of the two lines of regression. (16)

Or

(b) (i) Two random variables $X$ and $Y$ have the joint probability density function $f(x, y)=\begin{cases} 2-x-y, & 0 \le x \le 1, 0 \le y \le 1 \\ 0, & \text{otherwise} \end{cases}$. Find the marginal densities of $X$ and $Y$. Also, find the conditional density functions. (8)

(ii) The joint probability distribution of a two dimensional random variable $(X,Y)$ is given by $f(x,y)=x+y$, $0 \le x \le 1$, $0 \le y \le 1$. Find the probability distribution function of $U = XY$. (8)



13. (a) (i) Show that the random process $X(t) = A\cos\omega t + B\sin\omega t$ is wide sense stationary process if $A$ and $B$ are random variables such that $E(A)=E(B)=0$, $E(A^2)=E(B^2)$ and $E(AB)=0$. (6)

(ii) A machine goes out of order whenever a component part fails. The failure of this part is in accordance with a Poisson process with a mean rate of 1 per week. Find the probability that 2 weeks have lapsed since the last failure. If there are 5 spare parts of this component in an inventory and the next supply is not due in 10 weeks, find the probability that the machine will not be out of order in the next 10 weeks. (10)

Or

(b) Consider a Markov chain on (0,1,2) having the transition matrix given by $P=\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$. Show that the chain is irreducible. Find the period and the stationary distribution. (16)

14. (a) Derive the steady-state probabilities of the number of customers in M/M/1 queueing system from the birth and death processes and hence deduce that the average measures such as expected system size $L_s$, expected queue size $L_q$, expected waiting time in system $W_s$ and expected waiting time in queue $W_q$. (16)

Or

(b) A petrol pump station has 4 pumps. The service time follows the exponential distribution with a mean of 6 min. and cars arrive for service in a Poisson process at the rate of 30 cars per hour. What is the probability that an arrival would have to wait in line? Find the average waiting time in the queue, average time spent in the system and the average number of cars in the system. (16)

15. a) Derive Pollaczek – Khintchin formula for the average number of customers in the M/G/1 queueing system. **(16)**

(OR)

b) i) Write a short note on open queueing network. **(8)**

ii) Patients arrive at a clinic in a Poisson fashion at the rate of 3 per hour. Each arriving patients has to pass through two sections. The assistant in the first section take 15 minutes per patient and the doctor in the second section takes nearly 6 minutes per patient. If the service times in the two sections are approximately exponential, find the probability that there are 3 patients in the first sections and 2 patients in the second section. Find also average number of patients in the clinic and the average waiting time of a patient. **(8)**

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5/12/19/FN

Reg. No. :

**Question Paper Code : 91786**

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2019
Fourth Semester
Mechanical Engineering (Sandwich)
MA 6453 – PROBABILITY AND QUEUEING THEORY
(Common to Computer Science and Engineering/Information Technology)
(Regulations 2013)

Time : Three Hours Maximum : 100 Marks

Answer ALL questions

PART – A **(10×2=20 Marks)**

1. If the probability density function of a random variable X is $f(x)=\frac{1}{4}$ in $-2<x<2$ find $P(|X|>1)$.

2. If X is a geometric variate, taking values 1, 2, 3, ..., ∞, find P(X is odd).

3. The joint p.d.f. of R.V. (X, Y) is given as $f(x,y)=\begin{cases}\frac{1}{x}, & 0<y<x\le 1\\ 0, & \text{otherwise}\end{cases}$.
   Find the marginal p.d.f. of Y.

4. Let X and Y be two independent R.Vs with Var(X) = 9 and Var(Y) = 3. Find Var(4X – 2Y + 6).

5. Write the classification of random processes.

6. State any two properties of Poisson process.

7. What do the letters in the symbolic representation (a/b/c) (d/e) of a queueing model represent ?

8. What do you mean by balking and reneging ?

9. Write down Pollaczek-Khintchin formula.

10. What do you mean by bottle neck of a network ?

PART – B (5×16=80 Marks)

11. a) i) A radar system has a probability of 0.1 of detecting a certain target during a single scan. Use Binomial distribution to find the probability that the target will be detected at least 2 times in four consecutive scans. Also compute the probability that the target will be detected at least once in twenty scans. **(8)**

ii) An electrical firm manufactures light bulbs that have a length of life which is normally distributed with mean $\mu = 800$ hours and standard deviation $\sigma = 40$ hours. Find the probability that a bulb burns between 778 and 834 hours. **(8)**

(OR)

b) i) The probability of an individual suffering a bad reaction from an injection of a certain antibiotic is 0.001. Out of 2000 individuals, use Poisson distribution to find the probability that exactly three suffer. Also, find the probability of more than two suffer from bad reaction. **(8)**

ii) Electric trains in a particular route run every half an hour between 12. Midnight and 6 a.m. Using uniform distribution, find the probability that a passenger entering the station at any time between 1.00 a.m. and 1.30 a.m. will have to wait at least twenty minutes. **(8)**

12. a) Two random variables X and Y have the joint probability density function

$$f(x,y) = \begin{cases} c(4-x-y), & 0 \le x \le 2, 0 \le y \le 2 \\ 0, & \text{elsewhere} \end{cases}$$

Find the equations of two lines of regression. **(16)**

(OR)

b) i) The joint distribution of X and Y is given by $f(x,y) = \frac{x+y}{21}$, $x = 1, 2, 3, y = 1, 2$.

Find marginal distributions and conditional distributions. **(8)**

ii) If X and Y are independent random variables with probability density functions $e^{-x}, x \ge 0$ and $e^{-y}, y \ge 0$ respectively, find the density function of $U = \frac{X}{X+Y}$. **(8)**



13. a) i) Consider a random process $Y(t) = X(t) \cos(w_0 t + \theta)$, where X (t) is wide-sense stationary process, $\theta$ is a uniformly distributed R.V. over $(-\pi, \pi)$ and $w_0$ is a constant. It is assumed that X(t) and $\theta$ are independent. Show that Y(t) is a wide-sense stationary. **(8)**

ii) Consider a Markov chain $\{X_n ; n = 0, 1, 2, ....\}$ having state space $S = \{1, 2\}$ and one-step

$$\text{TPM } P = \begin{bmatrix} \frac{4}{10} & \frac{6}{10} \\ \frac{8}{10} & \frac{2}{10} \end{bmatrix}$$

1) Draw a transition diagram.
2) Is the chain irreducible ?
3) Is the state-1 ergodic ? Explain.
4) Is the chain ergodic ? Explain. **(8)**

(OR)

b) i) Let X(t) and Y(t) be two independent Poisson processes with parameters $\lambda_1$ and $\lambda_2$ respectively. Find

1) $P(X(t) + Y(t)) = n, n = 0, 1, 2, 3, ...,$
2) $P(X(t) - Y(t)) = n, n = 0, \pm 1, \pm 2, ....$ **(8)**

ii) Find the nature of the states of the Markov process with the transition probability matrix $P = \begin{bmatrix} 0 & 1 & 0 \\ 0.5 & 0 & 0.5 \\ 0 & 1 & 0 \end{bmatrix}$. **(8)**

14. a) i) Obtain $P_0$ and $P_n$ for the birth and death process. **(8)**

ii) Arrives at a telephone booth are considered to be Poisson distribution, with an average time of 10 minutes between one arrival and the next. The length of phone call assumed to be distributed exponentially with mean 3 minutes. What is the probability that a person arriving at the booth will have to wait ? And what is the average length of the queue that form from time to time ? **(8)**

(OR)

b) There are three typists in an office. Each typist can type an average of 6 letters per hour. If letters arrive for being typed at the rate of 15 letters per hour. What is the probability of all the typists will be busy ? What is the average number of letters waiting to be typed ? What is the average time a letter has to be spent for waiting and for being typed ? And what is the probability that a letter will take longer than 20 minutes waiting to be typed and being typed ? **(16)**