

# 60009

Distributed Algorithms  
Imperial College London

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# Chapter 1

## Introduction

### 1.1 Course Structure & Logistics



**Dr Narankar Dulay**

The module is taught by Dr Narankar Dulay.

**Theory** For weeks 2  $\rightarrow$  10:

- Elixir (learning programming language)
- Introduction
- Reliable Broadcast
- FIFO, casual and total order Broadcast
- Consensus
- Flip Improbability Result
- Temporal Logic of Actions
- Modelling Broadcast
- Modelling Consensus

### 1.2 Course Resources

The course website contains all available slides and notes.

## 1.3 Distributed Systems

Distributed System	Definition 1.3.1
<p>A set of processes connected by a network, communicating by message passing and with no shared physical clock.</p> <ul style="list-style-type: none"><li>• No total order on events by time (no shared clock)</li><li>• No shared memory.</li><li>• Network is logical - processes may be on the same OS process, same VM, same machine different machines communicating over a physical network.</li></ul>	

Distributed systems must contend with the inherent uncertainty (failure, communication delay and an inconsistent view of the system's state) in communication between potentially physically independent processes (fallible machines, networks and software).

Leslie Lamport	Extra Fun! 1.3.1
<p>A computer scientist and mathematician, credited with creating TLA (used on this course), as well as being the initial developer of latex (used for these notes).</p> <p>” There has been considerable debate over the years about what constitutes a distributed system. It would appear that the following definition has been adopted at SRC:</p> <p>A distributed system is one in which the failure of a computer you didn't even know existed can render your own computer unusable. ”</p>	

## 1.4 Distributed Algorithms

Liveness Properties	Definition 1.4.1	Safety Properties	Definition 1.4.2
<p><i>Something good happens eventually</i> (Cannot be violated by finite computation)</p>		<p><i>Nothing bad happens</i> (Only violated by finite computations)</p>	

As liveness properties depend on computation, they can be constrained by a *fairness property*.

**unconditional fairness** Every process gets its turn infinitely often.

**strong fairness** Every process gets its turn infinitely often if it is enabled infinitely often.

**weak fairness** Every process gets its turn infinitely often if it is continuously enabled from a particular point in the execution.

### 1.4.1 Key Aspects

1. **The problem** Specified in terms of the *safety* and *liveness* properties of the algorithm.

2. **Assumptions made**

Bounds on process delays (timing assumption)

Types of process failures tolerated (failure assumption)

Use of reliable message passing (communication assumption)

3. **The algorithm** Expresses the solution to *the problem*, given *the assumptions*.

- Must prove the algorithm is correct (satisfies all *safety* and *liveness* properties)

- Time and space complexity of the algorithm

Mutual Exclusion Properties	Example Question 1.4.1
<p>What are the safety, liveness and fairness properties required for mutual exclusion of processes over some critical section?</p>	

<b>Safety</b>	At most one process accesses the critical section.	$(s \parallel t) \wedge (s \neq t) \Rightarrow \neg(cs(s) \wedge cs(t))$
<b>Liveness</b>	Every request for the critical section is eventually granted.	$req(s) \Rightarrow (\exists t : s \preceq t \wedge cs(t))$
<b>Fairness</b>	Requests are granted in the order.	$req\_start(s) \wedge req\_start(t) \wedge (s \rightarrow t) \Rightarrow (next\_cs(s) \rightarrow next\_cs(t))$

Note that  $\preceq$  is the *happens-before* relation.

## Consensus

## Definition 1.4.3

Processes Propose Values  $\rightarrow$  Processes decide on value  $\rightarrow$  Agreement Reached

<b>Agreement Property</b>	Two correct processes cannot decide on different values.
<b>Validity Property</b>	If all processes propose the same value, then the decided value is the proposed value.
<b>Termination Property</b>	System reaches agreement in finite time.

Consensus is impossible to solve for a fully asynchronous system, some timing assumptions are required.

It is difficult to prove the correctness of even simple distributed systems formally. By specifying an abstract model of an algorithm automatic model checkers can be used to verify properties.

### 1.4.2 Timing Assumptions

#### Asynchronous Systems

#### Definition 1.4.4

A system where process execution steps and inter-process communication take arbitrary time.

- No assumptions that processes have physical clocks.
- Sometimes useful to use *logical clocks* (used to capture a consistent ordering of events on a virtual timespan)

#### Synchronous Systems

#### Definition 1.4.5

A system containing assumptions on the upper bound timings for executing steps in a process.

- This means there are upper bounds for steps such as receiving messages, sending messages, arithmetic, etc.
- Easier to reason about.
- Implementation must ensure bounds are always met, this can potentially require very high bounds (so guarantee holds) which reduce performance. *Eventually synchronous models* were created to overcome this.

#### Eventually Synchronous Systems

#### Definition 1.4.6

Mostly synchronous systems. Do not have to *always* meet bounds, and can have periods of asynchronicity.

### 1.4.3 Failure Classes

Process Failure	Definition 1.4.7								
<p>A process internally fails and behaves incorrectly. Process sends messages it should not, or does not send messages it should.</p> <ul style="list-style-type: none"> <li>• Can be caused by a software bug, termination of process by user or OS, OS failure, hardware failure, cyber attack by adversary.</li> <li>• The process may be slowed down to the point it cannot send messages it needs to (or meet some timing assumption)</li> </ul> <table> <tr> <td><b>Fail-Stop</b></td><td>Failure can be reliably detected by other processes.</td></tr> <tr> <td><b>Fail-Silent</b></td><td>Not Fail-Stop.</td></tr> <tr> <td><b>Fail-Noisy</b></td><td>Failure can be detected, but takes time.</td></tr> <tr> <td><b>Fail-Recovery</b></td><td>Failing process can recover from failure.</td></tr> </table> <p>A process that is not faulty is a <b>Correct Process</b>.</p>		<b>Fail-Stop</b>	Failure can be reliably detected by other processes.	<b>Fail-Silent</b>	Not Fail-Stop.	<b>Fail-Noisy</b>	Failure can be detected, but takes time.	<b>Fail-Recovery</b>	Failing process can recover from failure.
<b>Fail-Stop</b>	Failure can be reliably detected by other processes.								
<b>Fail-Silent</b>	Not Fail-Stop.								
<b>Fail-Noisy</b>	Failure can be detected, but takes time.								
<b>Fail-Recovery</b>	Failing process can recover from failure.								

Link Failure	Definition 1.4.8	Byzantine Failure	Definition 1.4.9
<p>A link allowing for processes to communicate is disconnected and remains disconnected.</p> <p>A network connecting machines hosting processes may become partitioned due to a <i>link failure</i></p>		<p>Also called <b>Fail-Arbitrary</b>, a process exhibits some arbitrary behaviour (can be malicious).</p>	

Omission Failure	Definition 1.4.10
<b>Send Omission</b>	Fails to send all messages required by the algorithm.
<b>Receive Omission</b>	Fails to properly receive all messages required.

### 1.4.4 Communication Assumptions

#### Asynchronous Message Passing

Processes continue after sending messages, they do not wait for a message to be delivered. It is possible to build a synchronous message passing abstraction from asynchronous message passing.

#### Reliable Message Communication

Messages are assumed to be conveyed using a reliable medium.

- All sent messages are delivered.
- No duplicate messages are created.
- All delivered messages were sent.

Network failure is still a concern (breaks assumption), so TCP is used for messages, and more reliable message passing abstractions built on top.

Message delays are bounded, as a timeout is used.

### 1.4.5 Complexity

Complexity can be characterised using:

- Number of messages exchanged.
- Size of messages exchanged.
- Time taken from the perspective of an external observer, or some clock on a synchronous system.
- Memory, CPU time or energy used by processes.

# Chapter 2

## Elixir

### 2.1 learning Elixir

- [Introduction To Elixir & Installation](#)
- [Elixir Documentation and Standard Library](#)
- [Elixir Learning Resources](#)
- [Devhints Exlixir Cheatsheet](#)
- [Elixir Quick Reference](#)
- [Learn Elixir in Y Minutes](#)

#### Two Sum

#### Example Question 2.1.1

Write a program to provide the two indexes of numbers in a list that sum to a given target. (This is the famous leetcode problem two sum).

```
defmodule Solution do
  @spec two_sum(nums :: [integer], target :: integer) :: [integer]
  def two_sum(nums, target) do
    nums
    |> Enum.with_index()
    |> Enum.reduce_while(%{}, fn {num, idx}, acc ->
      case Map.get(acc, target - num) do
        nil ->
          {:cont, Map.put(acc, num, idx)}
        val ->
          {:halt, [idx, val]}
      end
    end)
  end
end
```

We could also write this recursively with a helper function

```
defmodule Solution do
  @spec two_sum(nums :: [integer], target :: integer) :: [integer]
  def two_sum(nums, target) do
    two_sum_aux(nums, target, %{}, 0)
  end

  defp two_sum_aux([next | rest], target, prevs, index) do
    val = Map.get(prevs, target - next)
    if val != nil do
      [val, index]
    else
      two_sum_aux(rest, target, Map.put(prevs, next, index), index + 1)
    end
  end
end
```



```

    end
  end
end

```

## Add two numbers

## Example Question 2.1.2

Given The following linked list structure, write a program taking two numbers (represented in reverse as linked lists), and produce a linked list of their sum. (This is leetcode problem add two numbers)

*# Definition for singly-linked list.*

```

defmodule ListNode do
  @type t :: %__MODULE__{
    val: integer,
    next: ListNode.t() | nil
  }
  defstruct val: 0, next: nil
end

```

```

defmodule Solution do
  @spec add_two_numbers(l1 :: ListNode.t | nil, l2 :: ListNode.t | nil) :: ListNode.t | nil
  def add_two_numbers(l1, l2) do
    x = get_list(l1) + get_list(l2)
    if x == 0 do
      %ListNode{val: 0, next: nil}
    else
      to_list(x)
    end
  end

  defp get_list(node) do
    case node do
      %ListNode{val: v, next: n} -> v + 10 * get_list(n)
      nil -> 0
    end
  end

  defp to_list(n) do
    case n do
      0 -> nil
      i -> %ListNode{val: rem(i,10), next: to_list(div(i,10))}
    end
  end
end

```

## 2.2 The Elixir System

### Elixir

### Definition 2.2.1

A concurrent (with actors) and functional programming language used for fault tolerant distributed systems.

- A modernized successor language to Erlang
- Runs using BEAM (Erlang's virtual machine) and hence compatible with erlang
- Has many additions over erlang (protocols, streams and metaprogramming)

A lightweight user level thread (green threads) managed by the runtime.

- Everything is a process.
- Processes are strongly isolated, when two processes interact it does not matter which nodes, or even machines they run on.
- Processes share no resources (cannot share variables), they can only interact through message passing.
- Process creation and destruction is fast.
- Processes interact by message passing.
- Processes have unique names, if a name is known it can be used to pass messages
- Error handling is non-local.
- Processes do what they are supposed to do or fail.

All elixir processes run within a node, a node can manage many processes (creation, scheduling, and garbage collection).

- A node runs as an OS process, potentially with several OS threads scheduled across several cores.
- Multiple nodes can run on a single machine (or virtual machine such as a docker container).
- A node can efficiently manage thousands to millions of elixir processes.

Communication between processes is implemented through shared memory on the same machine and TCP when over a network. However processes are not exposed to this - the same primitives are used for inter and intra node/machine communication.

## 2.3 Message Passing

The `send` and `receive` statements are used for message passing.

```
# send somedata (any type) to process p
send p, somedata

# Wait until a message that matches the pattern is added to the message queue
# (or a timeout occurs), then remove it (potentially skipping over messages
# that do not match)
receive do
  somepattern -> dosomething(somepattern)
  # ... some other patterns
end
```

- Each process has its own message queue.
- Messages received are appended to the message queue of the receiving process.
- The sender does not wait for the message to be appended, it continues immediately after sending.

We can implement a basic client-server system in this way. Here we are using a component-based approach (split the program into components, each asynchronously message pass), by convention each component is an elixir module, modules can be instantiated in many processes & (by convention) have a public `start()` function.

```
defmodule Cluster do
  def start do
    # Spawn two processes, with the function start
    # Server.ex and Client.ex are modules containing a public start function
    # (Assuming we have started a client_node and server_node)
    s = Node.spawn(:'server_node@172.19.0.2', Server, :start, [])
    c = Node.spawn(:'client_node@172.19.0.1', Client, :start, [])
  end
end
```

```

# We send the PIDs of the processes to each other, we can pattern match on
# atoms for convenience in receiving
send s, { :bind, c }
send c, { :bind, s }
end
end

```

```

defmodule Server do
  def start do
    receive do
      { :bind, c } -> next(c)
    end
  end

  # next is defined as private, here
  # recursion is used for iteration.
  # To avoid a stack overflow tail
  # recursion is required
  defp next(c) do
    receive do
      { :circle, radius } ->
        send c, { :result, 3.14 * radius
                  * radius }

      { :square, side } ->
        send c, { :result, side * side }
    end
    next(c)
  end
end

```

```

defmodule Client do
  def start do
    receive do
      { :bind, s } -> next(s)
    end
  end

  defp next(s) do
    send s, { :circle, 1.0 }
    receive do
      { :result, area } ->
        IO.puts "Area is #{area}"
    end
    Process.sleep(1000)
    next(s)
  end
end

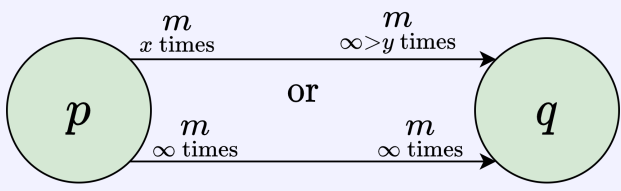
```

# Chapter 3

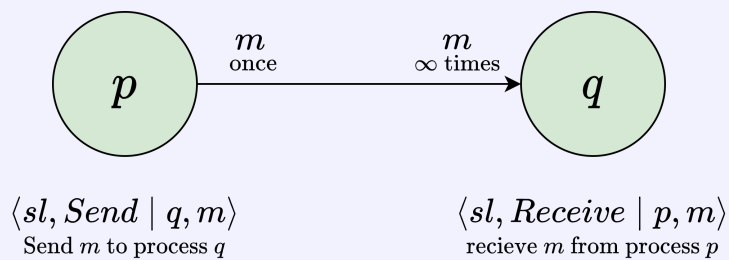
## Broadcast

### 3.1 Links (unassessed)

A link is a mechanism defining how two processes may interact by sending and receiving messages, and what properties hold for message passing.

Fair Loss Link		Definition 3.1.1
A weak link abstraction from which other links (e.g stubborn) can be built.		
		
$\langle fll, Send \mid q, m \rangle$ Send $m$ to process $q$		$\langle fll, Receive \mid p, m \rangle$ receive $m$ from process $p$
<b>Fair-Loss</b>	Liveness	Correct process $p$ infinitely sends message $m$ to correct process $q \Rightarrow q$ receives $m$ from $p$ infinitely many times.
<b>Finite Duplication</b>	Liveness	Correct process $p$ sends message $m$ a finite number of times to $q \Rightarrow m$ cannot be received infinitely many times from $p$ .
<b>No Creation</b>	Safety	Some process $q$ receives a message $m$ with sender $p \Rightarrow p$ previously sent $m$ to $q$ .

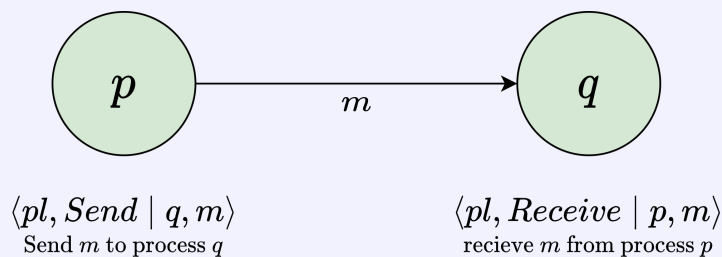
A link guaranteeing messages are received infinitely many times.



<b>Stubborn Delivery</b>	Liveness	Correct process $p$ sends message $m$ to correct process $q \Rightarrow q$ receives $m$ from $p$ infinitely many times.
<b>No Creation</b>	Safety	Some process $q$ receives a message $m$ with sender $p \Rightarrow p$ previously sent $m$ to $q$ .

Implement stubborn links with elixir using the fair loss link.

**UNFINISHED!!!**



- Also called *reliable message passing*

<b>Reliable Delivery</b>	Liveness	Correct process $p$ sends $m$ to correct process $q \Rightarrow q$ will eventually receive $m$ .
<b>No Duplication</b>	Safety	No message is received by a process more than once.
<b>No Creation</b>	Safety	Some process $q$ receives a message $m$ with sender $p \Rightarrow p$ previously sent $m$ to $q$ .

## 3.2 Failure Detection

A failure detector provides a process with a list of *suspected processes*.

- Failure detectors make, and encapsulate some timing assumptions in order to determine which processes are suspect.
- They are not fully accurate, and their specification allows for this.

A failure detector that is never incorrect / is entirely accurate.

- Never changes its view on failure  $\rightarrow$  once detected as crashed it cannot be *unsuspected*.
- Often represented as  $\mathcal{P}$

**Strong Completeness**    Liveness    Eventually every process that crashes is permanently detected as crashed by every correct process.

**Strong Accuracy**        Safety         $p$  detected  $\Rightarrow p$  has crashed. No process is suspected before it crashed.

We can implement a failure detector using timeouts and a heartbeat.

- Perfect links used to send requests for heartbeat.
- If reply is not received before timeout, the process is suspected to have crashed.
- **perfect links** are only reliable for correct processes.
- Timeout period has to be long enough to send the heartbeat to all processes and for the receiving processes to respond.

```
defmodule Perfect_Failure_Detector do
  def start do
    receive do
      { :bind, c, pl, processes, delay } ->
        # Send the first heartbeat request
        heartbeat_requests(delay)

        next(c, pl, processes, delay, processes, MapSet.new())
    end
  end

  defp next(c, pl, processes, delay, alive, crashed) do
    receive do
      # Send heartbeat requests over perfect link
      { :pl_deliver, from, :heartbeat_request } ->
        send pl, { :pl_send, from, :heartbeat_reply }
        next(c, pl, processes, delay, alive, crashed)

      # Receive heartbeat responses over perfect links
      { :pl_deliver, from, :heartbeat_reply } ->
        next(c, pl, processes, delay, MapSet.put(alive, from), crashed)

      # Timeout period expired
      # 1. Get all previously alive processes that did not respond (these have crashed)
      # 2. Send crashed to each
      :timeout ->
        newly_crashed =
          for p <- processes, p not in alive and p not in crashed, into: MapSet.new do p end

        # Inform process p of all newly crashed processes
        for p <- newly_crashed do send c, { :pfd_crash, p } end

        # Send new heartbeat requests over perfect links
        for p <- alive do send pl, { :pl_send, p, :heartbeat_request } end

        heartbeat_requests(delay)

        # Loop (empty set of alive, union set of old and newly crashed)
        next(c, pl, processes, delay, MapSet.new(), Mapset.union(crashed, newly_crashed))
    end
  end
end
```

```

end

defp heartbeat_requests(delay) do
  # after delay milliseconds, timeout will be received by this process
  Process.send_after(self(), :timeout, delay)
end
end

```

This implementation meets the properties of a *perfect failure detector* as:

- Strong Completeness** If a process crashes it will no longer reply to heartbeat messages, hence by *perfect links no-creation* property, no correct process will receive a heartbeat. So every correct process will detect a crash.
- Strong Accuracy** A process can only miss the timeout if it has crashed under our timing assumption.

Eventually Perfect Failure Detector		Definition 3.2.2
A failure detector that is not entirely accurate.		
<ul style="list-style-type: none"> <li>• Can restore processes (no longer suspected).</li> <li>• Often represented as <math>\Diamond\mathcal{P}</math></li> </ul>		
<b>Strong Completeness</b>	Liveness	Eventually every process that crashes is permanently detected as crashed by every correct process.
<b>Eventual Strong Accuracy</b>	Liveness	Eventually no correct process is suspected by any other correct process

### 3.3 Best Effort Broadcast

Best Effort Broadcast / BEB		Definition 3.3.1
A non-reliable, single-shot broadcast.		
<ul style="list-style-type: none"> <li>• Only reliable if the broadcasting process is correct during broadcast (if crashing during broadcast only some messages may be delivered, and processes may disagree on delivery)</li> <li>• No delivery agreement guarantee (correct processes may disagree on delivery)</li> <li>• Uses <i>Perfect Point-to-Point Link</i> and inherits properties from it.</li> </ul>		
<b>Validity</b>	Liveness	If a correct process broadcasts a message then every correct process eventually receives it.
<b>No Duplication</b>	Safety	No message is received by a process more than once.
<b>No Creation</b>	Safety	No broadcast is delivered unless it was broadcast.

We can implement this in elixir using the send and receive primitives as *Perfect Point-to-Point Link*.

```

# Broadcast using perfect point-to-point links
# processes <- the list of processes in the broadcast space
# pl         <- the perfect links process to use
# c         <- the object broadcasting & being delivered
defmodule Best_Effort_Broadcast do
  def start(processes) do
    receive do {:bind, pl, c} -> next(processes, pl, c)
  end

  defp next(processes, pl, c) do
    receive do
      {:beb_broadcast, msg} ->
        for dest <- processes do

```

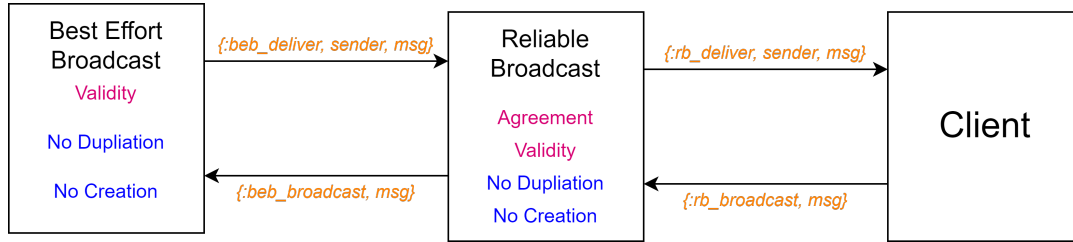
```

        send pl, {:pl_send, dest, msg}
    end
    {:pl_deliver, src, msg} ->
        send c, {:beb_deliver, src, msg}
    end
next (processes, pl, c)
end
end

```

## 3.4 Reliable Broadcast

Reliable Broadcast	Definition 3.4.1
Adds a delivery guarantee to <i>best effort broadcast</i>	
<b>Agreement</b>	Liveness If a correct process delivers message $m$ then all correct processes deliver $m$
<b>All Properties from Best Effort Broadcast</b>	
<ul style="list-style-type: none"> <li>The combination of <b>Validity</b> and <b>Agreement</b> form a <i>termination property</i> (system reaches agreement in finite time).</li> <li>Correct processes agree on messages delivered even if the broadcaster crashes while sending.</li> </ul>	



### 3.4.1 Eagre Reliable Broadcast

Eagre Reliable Broadcast	Definition 3.4.2
A <i>reliable broadcast</i> where every process re-broadcasts every message it delivers.	
<ul style="list-style-type: none"> <li>If the broadcasting process crashes, and only some correct processes deliver the message, then re-broadcast ensures eventually all will receive.</li> <li>This broadcast is <i>fail-silent</i></li> <li>Very inefficient to implement, broadcast to <math>n</math> processes results in <math>O(n^2)</math> messages from <math>O(n)</math> BEB broadcasts.</li> <li><b>Validity</b> property combined with retransmission provides <b>agreement</b>.</li> </ul>	
<b>All Properties from Reliable Broadcast</b>	

```

# Eagre reliable broadcast implemented using Best Effort Broadcast
# beb    <- the best effort broadcast process
# client <- the object broadcasting & being delivered
defmodule Eagre_Reliable_Broadcast do

  def start do
    receive do { :bind, client, beb } -> next(client, beb, MapSet.new) end
  end

  defp next(client, beb, delivered) do
    receive do
      { :rb_broadcast, msg } ->

```



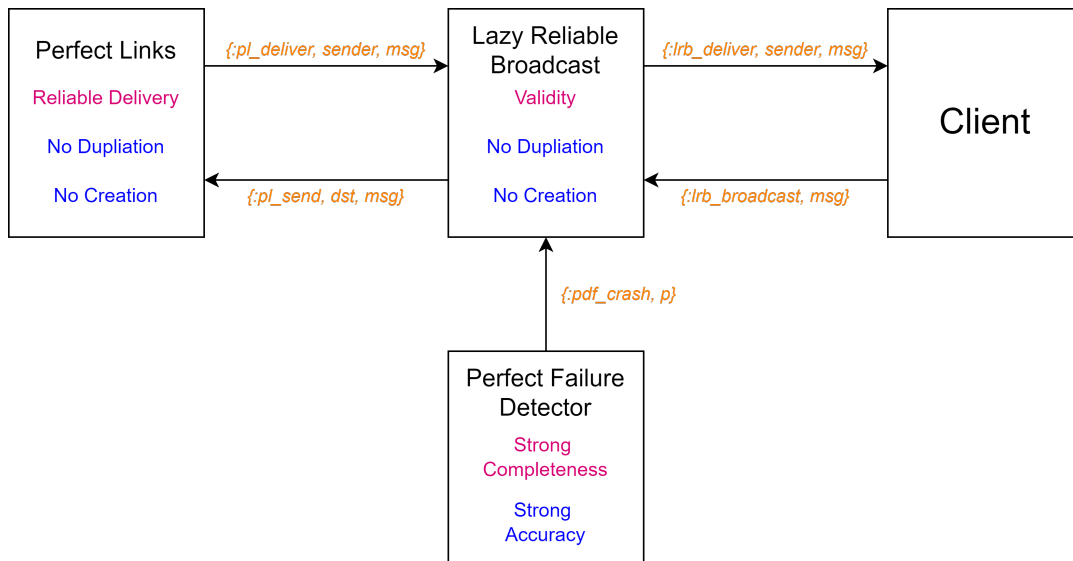
```

    send beb, { :beb_broadcast, { :rb_data, our_id(), msg } }
    next(client, beb, delivered)
  { :beb_deliver, from, { :rb_data, sender, msg } = rb_m } ->
    if msg in delivered do
      # Message was already delivered, so can be ignored
      next(client, beb, delivered)
    else
      # Message is new, so add to delivered, deliver to c & rebroadcast
      send client, { :rb_deliver, sender, msg }
      send beb, { :beb_broadcast, rb_m }
      next(client, beb, MapSet.put(delivered, msg))
    end
  end
end
end
end

```

### 3.4.2 Lazy Reliable Broadcast

Lazy Reliable Broadcast	Definition 3.4.3
<p>A reliable broadcast using <i>Best Effort Broadcast</i> with a <i>Failure Detector</i> to enforce agreement.</p> <ul style="list-style-type: none"> <li>• Uses a <i>perfect failure detector</i>.</li> <li>• When a process is detected to have crashed, all broadcasts delivered from the process are rebroadcasted</li> <li>• Agreement is derived from the <b>validity</b> of <i>best effort broadcast</i>, that every correct process broadcasts every message delivered from a crashed process and the properties of the <i>perfect failure detector</i>.</li> </ul>	



```

# Lazy Reliable Broadcast implemented using best effort broadcast
# beb    <- the best effort broadcast process
# client <- the object broadcasting & being delivered
defmodule Lazy_Reliable_Broadcast do
  def start do
    receive do
      { :bind, processes, client, beb } ->
        delivered = Map.new(processes, fn p -> {p, MapSet.new} end)
        next(client, beb, processes, delivered)
    end
  end

  defp next(client, beb, correct, delivered) do

```

```

receive do
  { :rb_broadcast, msg } ->
    # broadcast a message with our id
    send beb, { :beb_broadcast, { :rb_data, our_id(), msg } }
    next(client, beb, correct, delivered)

  { :pfd_crash, crashedP } ->
    # Failure detector has detected a crashed process
    # For each message delivered by the crashed process,
    # rebroadcast (from them)
    for msg <- delivered[crashedP] do
      send beb, { :beb_broadcast, { :rb_data, CrashedP, msg } }
    end
    next(c, beb, MapSet.delete(correct, crashedP), delivered) # cont

  { :beb_deliver, from, { :rb_data, sender, msg } = rb_m } ->
    # A message is delivered, if already received do nothing,
    # otherwise record the delivered message,
    if msg in delivered[sender] do
      next(c, beb, correct, delivered)
    else
      send c, { :rb_deliver, sender, msg }
      # add msg to the set of messages received from sender
      sender_msgs = MapSet.put(delivered[sender], msg)
      delivered = Map.put(delivered, sender, sender_msgs)

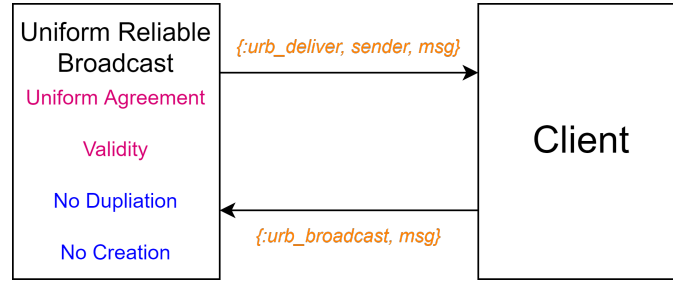
      # Due to transmission delay, the sender may have crashed
      # before this message is delivered, so we must check rebroadcast
      # if this is the case.
      if sender not in correct do
        send beb, { :beb_broadcast, rb_m }
      end

      next(c, beb, correct, delivered)
    end
  end
end
end
end

```

### 3.4.3 Uniform Reliable Broadcast

Uniform Reliable Broadcast / URB		Definition 3.4.4
<b>Uniform Agreement</b>	<b>Liveness</b>	If a process delivers a message, then all correct processes will deliver the message.
<b>All Properties from Best Effort Broadcast</b>		
<ul style="list-style-type: none"> <li>• Implies that faulty processes deliver a subset of messages delivered to correct processes (stronger than <b>agreement</b> - only for correct processes).</li> <li>• Avoids any scenario where a crashed process broadcasts and only a crashed process delivers (correct processes miss message).</li> </ul>		

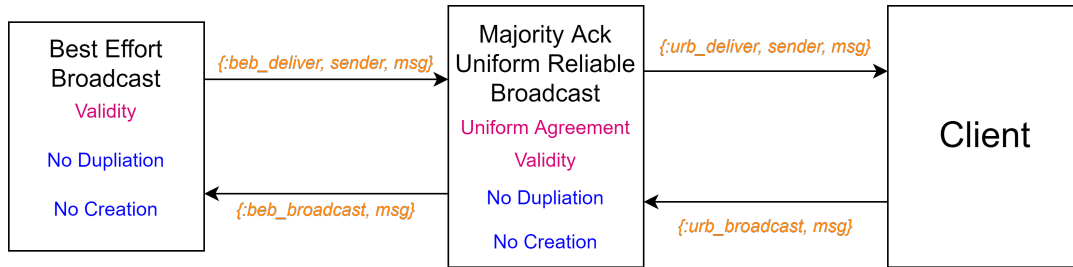


### Majority Ack Uniform Reliable Broadcast

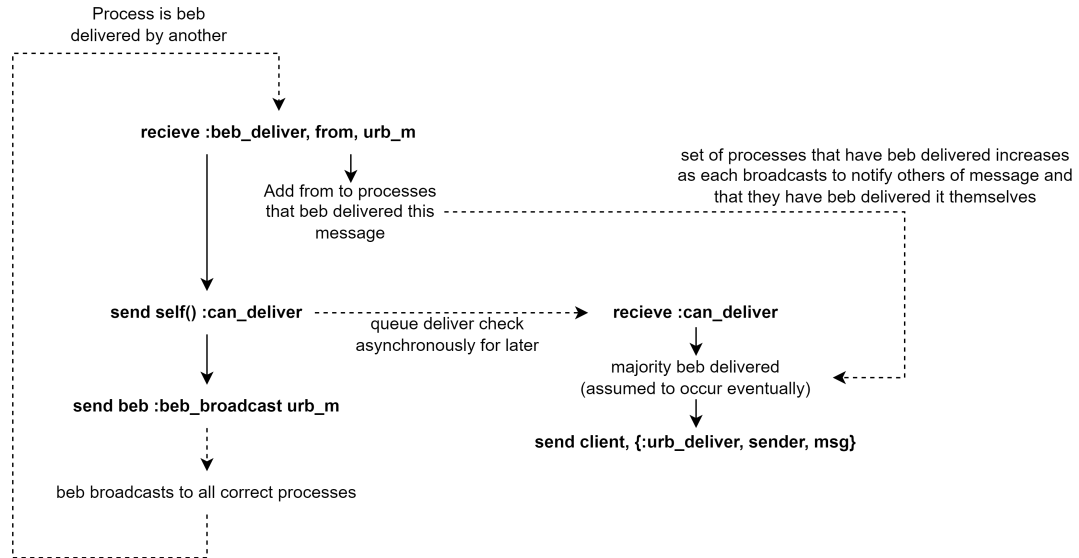
Definition 3.4.5

A *uniform reliable broadcast* implementation that assumes a majority of processes are correct.

- *Fail-silent* and does not use a *failure detector*.
- If  $n$  processes may crash, then  $2n + 1$  processes are needed with at least  $n + 1$  (majority) being correct



Each process tracks which other processes *BEB* them a specific message. Once the majority have done this, then can *URB* deliver the message.



**No Creation**  
**No Duplication**  
**Validity**  
**Uniform Agreement**

Provided by *BEB*.

Messages delivered are tracked in a **delivered** set.

As a *URB* sends via *BEB* (valid), and all messages *BEB* are eventually *URB* delivered.

If correct process  $Q$  *URB* delivers a message  $M$ , then  $Q$  was *BEB* delivered by a majority of processes (assumed correct), which means at least 1 correct process *BEB* broadcast  $M$ . Hence all correct processes eventually *BEB* deliver (and then *URB* deliver)  $M$ .

```

defmodule Majority_Ack_Uniform_Reliable_Broadcast do
  def start do
    receive do
      { :bind, client, beb, n_processes } ->
        next(client, beb, n_processes, MapSet.new, MapSet.new, Map.new)
    end
  end
end
  
```

```

end
end

# client      -> the client using uniform reliable broadcast
# beb        -> the best effort broadcast module used
# n_processes -> Need to know the number of processes to determine if more than half have delivered
# delivered  -> messages that been urb_delivered
# pending    -> messages that have been beb_broadcast but need to be urb-delivered
# bebd       -> foreach message, the set of processes that have beb-delivered (seen) it
defp next(client, beb, n_processes, delivered, pending, bebd) do
  receive do

    # Broadcast a message to all
    { :urb_broadcast, msg } ->
      # Use best effort broadcast to send message
      send beb, { :beb_broadcast, { :urb_data, our_id(), msg } }

      # Asynchronously check if the message can be delivered
      send self(), :can_deliver

      # Mark message as pending
      new_pending = MapSet.put(pending, { our_id(), msg })

      next(client, beb, n_processes, delivered, new_pending, bebd)

    # Receive via best effort broadcast
    { :beb_deliver, from, { :urb_data, sender, msg } = urb_m } ->
      # Get the processes that have seen this message, and add from to that set
      msg_pset = Map.get(bebd, msg, MapSet.new)
      next_bebd = Map.put(bebd, msg, MapSet.put(msg_pset, from))

      # Asynchronously check if the message can be delivered
      send self(), :can_deliver

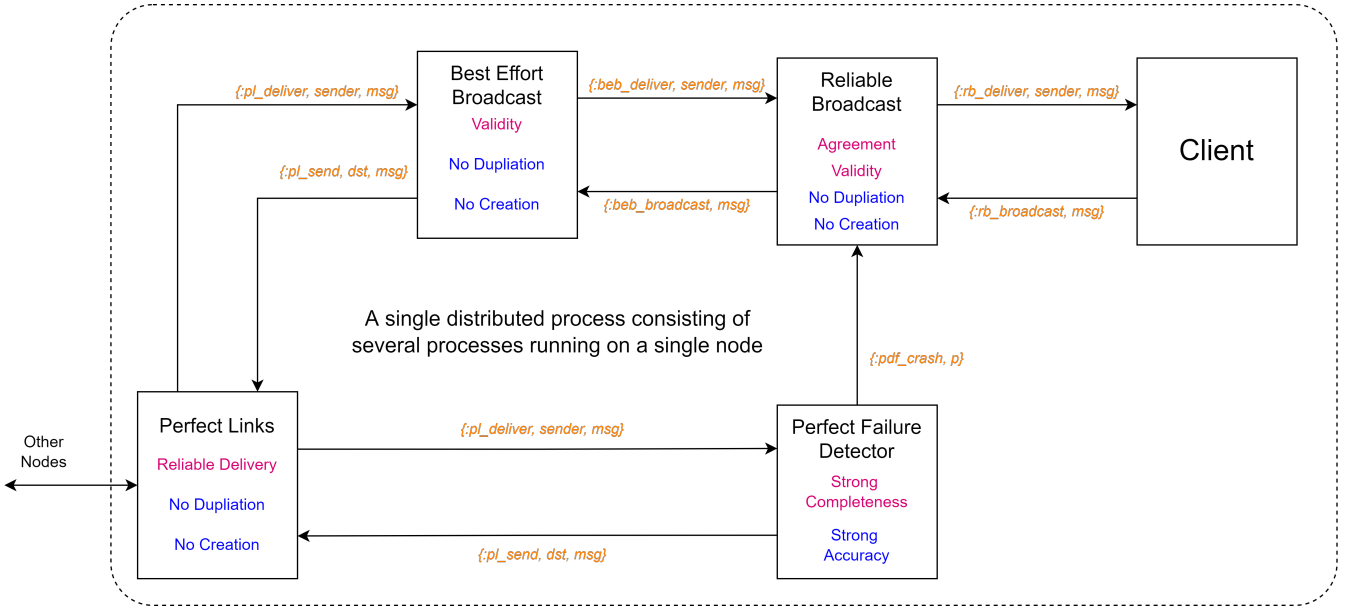
      # If the message has previously been recieved & placed in pending (do
      # nothing), else we must add it to pending.
      if { sender, msg } in pending do
        next (client, beb, n_processes, delivered, pending, next_bebd)
      else
        send beb, { :beb_broadcast, urb_m }
        new_pending = MapSet.put(pending, { sender, msg })
        next(client, beb, n_processes, delivered, new_pending, next_bebd)
      end

    # Determine if a best effort broadcast delivery can be uniform reliably delivered
    :can_deliver ->
      # Can only deliver if
      # - Message not already delivered
      # - Message has been delivered by a majority of other processes
      new_delivered_msgs =
        for { sender, msg } <- pending,
            msg not in delivered and
            MapSet.size(bebd[msg]) > n_processes/2
        into: MapSet.new
      do send client, { :urb_deliver, sender, msg }
      msg
    end
    new_delivered = MapSet.union(delivered, new_delivered_msgs)
    next(client, beb, n_processes, new_delivered, pending, bebd)
  end
end

```

end  
end

### 3.4.4 Process Configuration



## 3.5 Message Ordering

### 3.5.1 FIFO Message Delivery

#### First In First Out/FIFO Reliable Broadcast (FRB)

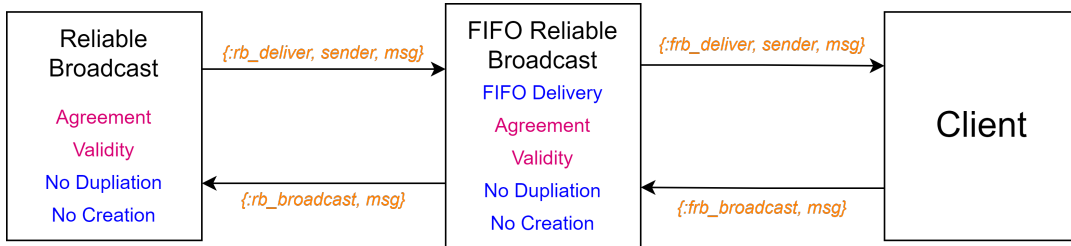
#### Definition 3.5.1

Messages delivered in broadcast order.

**FIFO Delivery Safety** If a process broadcasts  $M_1 \prec M_2$  then all correct processes will deliver  $M_1 \prec M_2$ .

#### All Properties from Reliable Broadcast

- Only applies per-sender, this is analogous to sequential consistency in concurrency.
- The same scheme can be applied to *uniform reliable broadcast (FIFO-URB)*.
- Same number of messages as the underlying reliable broadcast implementation.



```
defmodule FIFO_Reliable_Broadcast do # uses RB and sequence no's
  @initial_seq 0

  def start do
    receive do
      { :bind, client, rb } -> next(client, rb, @initial_seq, Map.new, [ ])
    end
  end
end
```

```

# pseqno -> for each process holds the seq_num of the next
#           message to be frb-delivered from that process
# pending {> messages that have been rb-delivered to this process and
#           awaiting to be frb-delivered to the client
#
# Message formats:
# { :frb_broadcast, msg }
# { :rb_deliver, from, { :frb_data, {sender, msg, seq } } }
defp next(client, rb, seq_num, pseqno, pending) do
  receive do
    { :frb_broadcast, msg } ->
      send rb, { :rb_broadcast, { :rb_data, {self(), msg, seq_num}} }
      next(client, rb, seq_num + 1, pseqno, pending)
    { :rb_deliver, _, { :frb_data, {sender, _, _} = frb_msg } } ->
      {new_pseqno, new_pending} = check_pending_and_deliver(client, sender, pseqno, pending ++ [frb_msg])
      next(client, rb, seq_num, new_pseqno, new_pending)
  end
end

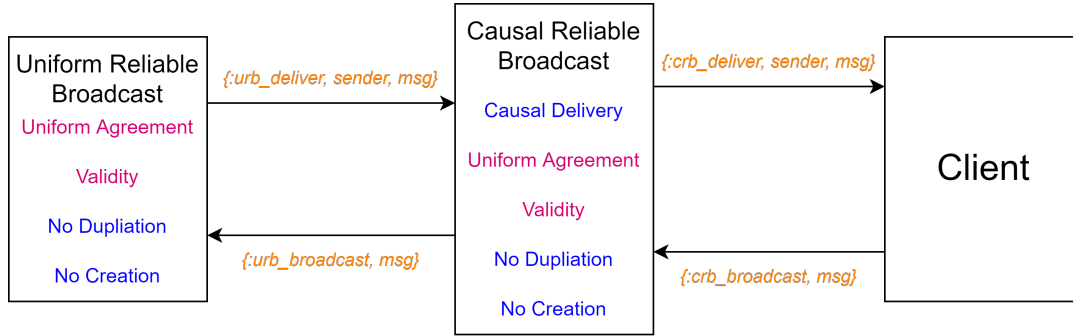
defp check_pending_and_deliver(client, sender, pseqno, pending) do
  # returns the first frb message from sender where the process seq matches the message seq
  # If no sequence number exists in pseqno, we assume it is the first (0)
  case Enum.find(pending, fn {from, _, seq} -> from == sender and seq == Map.get(pseqno, from, @initial))
  do
    {_, msg, seq} = data ->
      send client, { :fdb_deliver, msg }
      new_pseqno = Map.put(pseqno, sender, seq + 1)
      new_pending = List.delete(pending, data)
      check_pending_and_deliver(client, sender, new_pseqno, new_pending)
  end
  -> {pseqno, pending}
end
end
end

```

### 3.5.2 Causal Order Message Delivery

Causal Order Relation		Definition 3.5.2
A relation over messages $M_1 \rightarrow M_2$ when $M_1$ causes $M_2$ . A causal relation between messages is determined by:		
<b>FIFO Order</b>	Process message broadcast order	$\{\text{broadcast}, M_1\} \prec \{\text{broadcast}, M_2\} \Rightarrow M_1 \rightarrow M_2$ .
<b>Local Order</b>	Process delivers and then broadcasts	$\{\text{deliver}, M_1\} \prec \{\text{broadcast}, M_2\}$
<b>Transitivity</b>		$M_1 \rightarrow M_2 \wedge M_2 \rightarrow M_3 \Rightarrow M_1 \rightarrow M_3$

Causal Order/CO Message Delivery		Definition 3.5.3
Messages are delivered in an order respecting the causal order relation.		
<b>Causal Delivery Property</b>	Safety	If a process delivers message $M_2$ , it must have already delivered every message $M_1$ such that $M_1 \rightarrow M_2$ .
<b>All Properties from Uniform Reliable Broadcast</b>		



## No Wait Implementation

One implementation of this spec is a *causal reliable broadcast* that never waits. This is done by dropping any message that precedes the delivered message that has not already been delivered.

- Each message has a list of past messages `m_past`
- The `m_past` contains all causally preceding messages as a bundle.
- Hence whenever *URB* delivering a message all preceding messages are already available to *CRB* deliver first.

**Causal Delivery** Ensured as each message contains all of its past messages which are *CRB* delivered prior to the message.

**No Creation, No Duplication and Validity** from *Uniform Reliable Broadcast*

Past will grow large over time as the set of preceding messages grows.

- Large past uses up memory and network bandwidth
- Can selectively purge/garbage collect past messages (e.g when it is known a message recipient has already received some past messages)

```

defmodule Causal_Reliable_Broadcast_No_Wait do
  def start do
    receive do
      { :bind, client, urb } -> next(client, urb, [ ], MapSet.new)
    end
  end

  # past      -> messages that have been crb_broadcast or crb_delivered
  #           (the list of messages that are causally precede)
  # delivered -> messages that have been crb-delivered
  #
  # Message Formats:
  # { :crb_broadcast, msg }
  #
  # Note: m_past are the preceding messages
  # { :urb_deliver, from, { :crb_data, m_past, msg } }
  defp next(client, urb, past, delivered) do
    receive do
      { :crb_broadcast, msg } ->
        send urb, { :urb_broadcast, { :crb_data, past, msg } }

        # Add this message to the delivered messages
        new_past = past ++ [{ self(), msg }]

        next(client, urb, new_past, delivered)

      { :urb_deliver, from, { :crb_data, m_past, msg } } ->
        if msg in delivered do
          next(client, urb, past, delivered)
        else

```

```

# specify all preceding messages as delivered (even if they have not yet been urb_delivered - m
old_msgs =
  for { past_sender, past_msg } = past_data <- m_past,
                                past_msg not in delivered
    into: MapSet.new
    # syntax error here
  do send c, { :crb_deliver, past_sender, past_msg }
    past_data
  end

# crb deliver this message
send c, { :crb_deliver, from, msg }

# old messages marked as delivered
new_delivered = MapSet.put(MapSet.union(delivered, old_msgs), msg)
new_past = past ++ old_msgs ++ [{from, msg}]

next(client, urb, new_past, new_delivered)
end
end
end
end

```

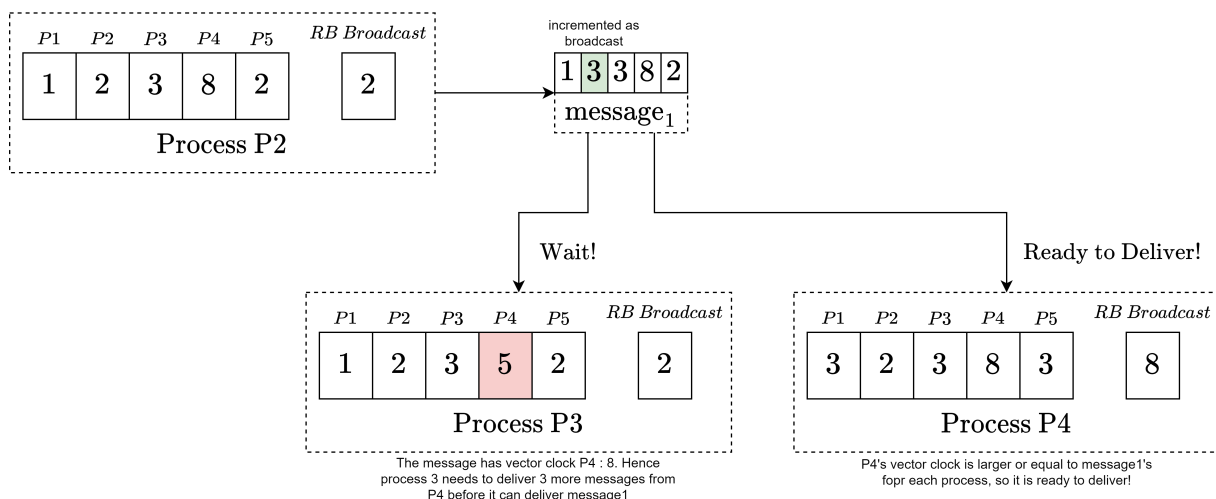
## Vector Clock Implementation

### Dynamic Deadlock Detection

*Extra Fun! 3.5.1*

Vector clocks can also be used in dynamically detecting data races in programs, as discussed in 60007 - Theory and practice of Concurrent Programming.

- Each process maintains a vector clock of (processes  $\rightarrow$  messages *CRB delivered*) and a count of messages that it has *RB broadcast*.
- When sending a message, the the vector clock and the *RB Broadcasts* count are sent.
- A message is only delivered if the sender's vector clock is  $\leq$  the receiver's vector clock (the current process has seen all the messages the sender had seen, when it sent this message)



```

defmodule Causal_Reliable_Broadcast_Vector_Clock do
  def start () do
    receive do
      { :bind, client, rb } -> next(client, rb, 0, Map.new, [ ])
    end
  end
end

```



```

# client -> The client to deliver messages to
# rb      -> Reliable broadcast (used by crb to broadcast)
# vc      -> Vector Clock: a map (pid -> number of messages crb delivered)
# pnum    -> This process's unique number
defp next(client, rb, rb_broadcasts, vc, pending) do
  receive do
    { :crb_broadcast, msg } ->
      # Create a new vector clock with this broadcast included and send
      send_vc = Map.put(vc, self(), rb_broadcasts)
      send rb, { :rb_broadcast, { :crb_data, send_vc, msg }}

      # continue
      next(client, rb, rb_broadcasts + 1, vc, pending)

    { :rb_deliver, sender, { :crb_data, s_vc, s_msg }} ->
      # Add delivered messages to pending and determine which can now be delivered.
      { new_vc, new_pending } = deliver(client, vc, pending ++ [{ sender, s_vc, s_msg }])

      next(client, rb, rb_broadcasts, new_vc, new_pending)
  end
end

defp deliver(client, vc, pending) do
  for pending_tuple <- pending, reduce: {vc, []} do
    {vc, still_pending} ->
      { sender, s_vc, s_msg } = pending_tuple
      # <= is true if s_vc[p] <= vc[p] for every entry p
      if s_vc <= vc do
        # Deliver the message
        send c, { :crb_deliver, sender, s_msg }

        # Update the sender's entry in vector clock
        new_vc = Map.put(vc, sender, Map.get(vc, sender, 0) + 1)

        {new_vc, still_pending}
      else
        {vc, still_pending ++ [pending_tuple]}
      end
  end
end
end
end

```

### 3.5.3 Total Order Message Delivery

Total Order/TO Message Delivery	Definition 3.5.4
<p>All correct messages deliver the same global order of messages.</p> <ul style="list-style-type: none"> <li>• Impossible in an asynchronous system as there is no shared clock, so no way to determine a shared ordering.</li> <li>• Does not need to be <i>FIFO</i> but is usually implemented so.</li> <li>• Sometimes called <i>atomic broadcast</i>.</li> </ul> <p><b>Uniform Total Order</b>    Safety    If a correct or crashed process delivers <math>M_1 \prec M_2</math>, then no correct process delivers <math>M_2 \prec M_1</math>.</p> <p style="text-align: center;"><b>All Properties from Uniform Reliable Broadcast</b></p> <p>In order to have a total order, processes must reach a consensus on the global order.</p>	

# Chapter 4

## Consensus

### 4.1 Motivation

Many algorithms require a set of processes running in a distributed system to agree on values (e.g order of messages, program state).

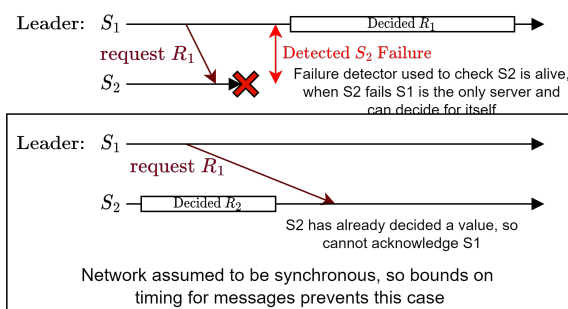
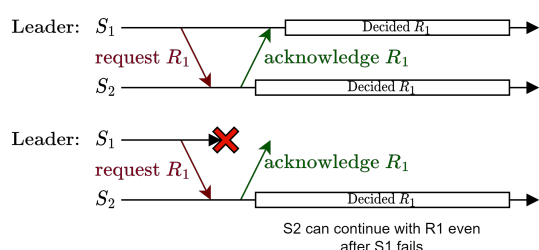
- Processes each propose a value, some agreement algorithm occurs, and then all decide on the same value.
- Required for all processes to get a consistent view, even if a single leader decided on a value there would then be a consensus required on which process is the leader to start, and when leaders fail.
- Often a *replicated server/replica* stores the state replicated over all processes (e.g the sequence of transactions for a database, the current player count in a game).

Uniform Consensus		Definition 4.1.1
<b>Validity</b>	Safety	If a process decides on a value, then this value was proposed by some process.
<b>Integrity</b>	Safety	A process can only decide on one value at most.
<b>Termination</b>	Liveness	Each correct process eventually decides.
<b>Uniform Agreement</b>	Safety	Processes cannot decide on different values.

Regular Consensus	Definition 4.1.2
A strengthening of <i>Uniform Consensus</i> to replace <b>Uniform Agreement</b> .	
<b>Validity, Integrity and Termination Properties from Uniform Consensus</b> <b>Uniform Agreement</b> Safety <b>Correct</b> Processes cannot decide on different values.	

### 4.2 Primary Backup

A simple consensus algorithm between two servers.



- One server is the leader, a failure detector is used by the leader to check the other server.
- Only works in a synchronous system (time bound on all messages), violations on order of requests, and timing will violate consensus.

## 4.3 FLP Impossibility Result

Fisher Lynch & Paterson

Extra Fun! 4.3.1

From the paper Impossibility of Distributed Consensus with One Faulty Process:

*"The consensus problem involves an asynchronous system of processes, some of which may be unreliable. The problem is for the reliable processes to agree on a binary value. In this paper, it is shown that every protocol for the problem has the possibility of non-termination, even with only one faulty process."*

Michael Fischer, Nancy Lynch, Mike Paterson

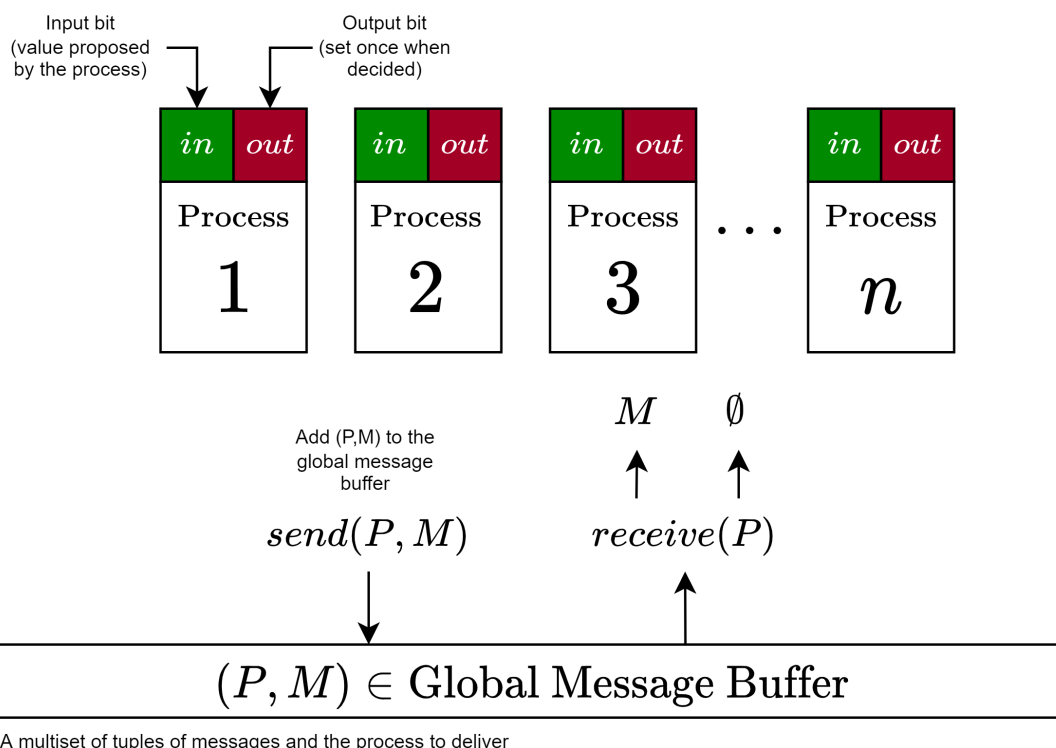
### FLP Impossibility Result

### Definition 4.3.1

In a purely asynchronous system we cannot use message timings to determine if a process has crashed (no guarantee on timings), this even applies when:

- Agreeing on a single bit
- Reliable message passing is used
- Only one process crashes

### 4.3.1 FLP Model



- *receive* can return empty even if messages are present for  $P$ .
- Messages are delivered non-deterministically and can be received in any order with any arbitrary delay.
- If *receive* is called infinitely many times, then every message will eventually be delivered.
- A message takes finite (but unbounded) time.
- Message buffer is a multiset, so can contain duplicates.

**Configuration**  $([P_1 : S_1, \dots], \{(P, M), \dots\})$  All process states and the global message buffer.

**Initial Configuration** Input bit of each process is set, message buffer is empty.

$$C_1 \rightarrow C_2$$

A step occur when a single process  $P$ :

- Performs  $receive(P)$  to get a message  $M$  or  $\emptyset$
- Executes some code and changes its internal state
- Sends a finite number of messages to the global message buffer with  $send$ .

$E = (P, M)$

Recepit of message  $M$  by process  $P$  is an event  $E$ .

$C_2 = E(C_1)$

Applying event  $E$  to configuration  $C_1$  to get new configuration  $C_2$ .

$E_1 \circ E_2 \circ \dots \circ E_n \triangleq \sigma$

A *schedule* is a series of events composed.

$\sigma(C)$

A *schedule* is an *execution* if applied to the initial configuration.

$\sigma(C) = C \rightarrow C' \rightarrow \dots$

A sequence of steps corresponding to a schedule is called a *run*.

$\sigma(C) = C'$

$C'$  is reachable from  $C$ , and accessible if  $C$  is the initial configuration.

A process can take infinitely many steps to run. *Runs* can be categorised as:

**Deciding Run**

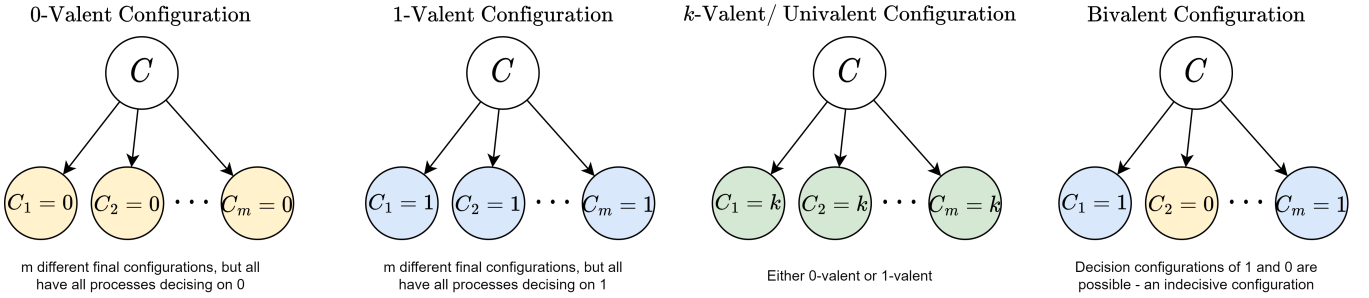
A *run* resulting in some process making a decision (writing to output bit).

**Admissable Run**

A *run* where at least one process is faulty and every message is eventually *received* (every process can *receive* infinitely many times).

A consensus protocol is *totally correct* if every *admissable run* is a *deciding run*.

### 4.3.2 Valent Configurations



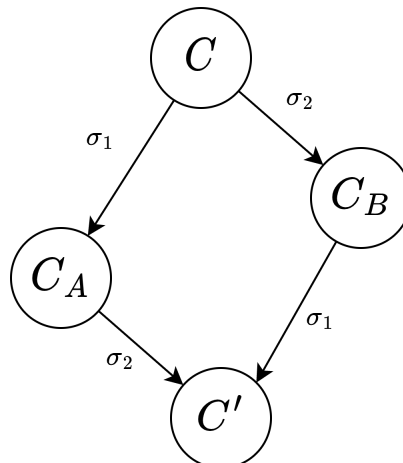
Proof is done by contradiction.

- Assume there is an algorithm  $\mathcal{A}$  that solves consensus.
- Construct an *execution* in which  $\mathcal{A}$  never reaches a decision (indecisive forever).
- Hence  $\mathcal{A}$  cannot solve consensus, so by contradiction there can be no  $\mathcal{A}$ .

By showing it is possible to start in a *bivalent configuration* and continue doing steps without reaching a *decisive configuration* (*univalent*) we demonstrate it is impossible to certainly reach consensus.

### 4.3.3 Lemmas

**Confluence**



Given configuration  $C$  and *schedules*  $\sigma_1$  and  $\sigma_2$  such that set of processes with steps in  $\sigma_1$  and  $\sigma_2$  are disjoint:

$$\sigma_1(\sigma_2(C)) \equiv \sigma_2(\sigma_1(C))$$

Initial Bivalent Configuration

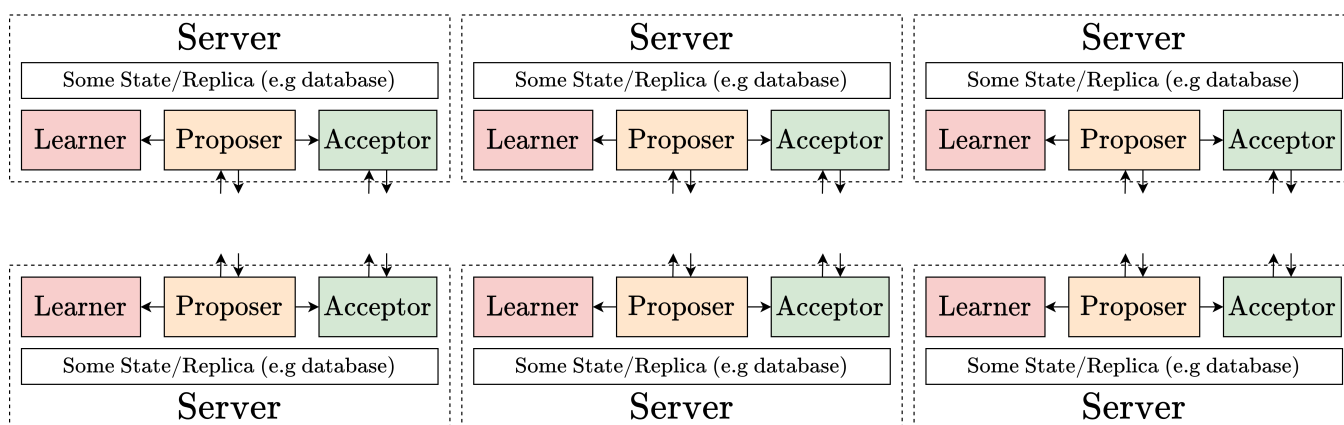
**UNFINISHED!!!**

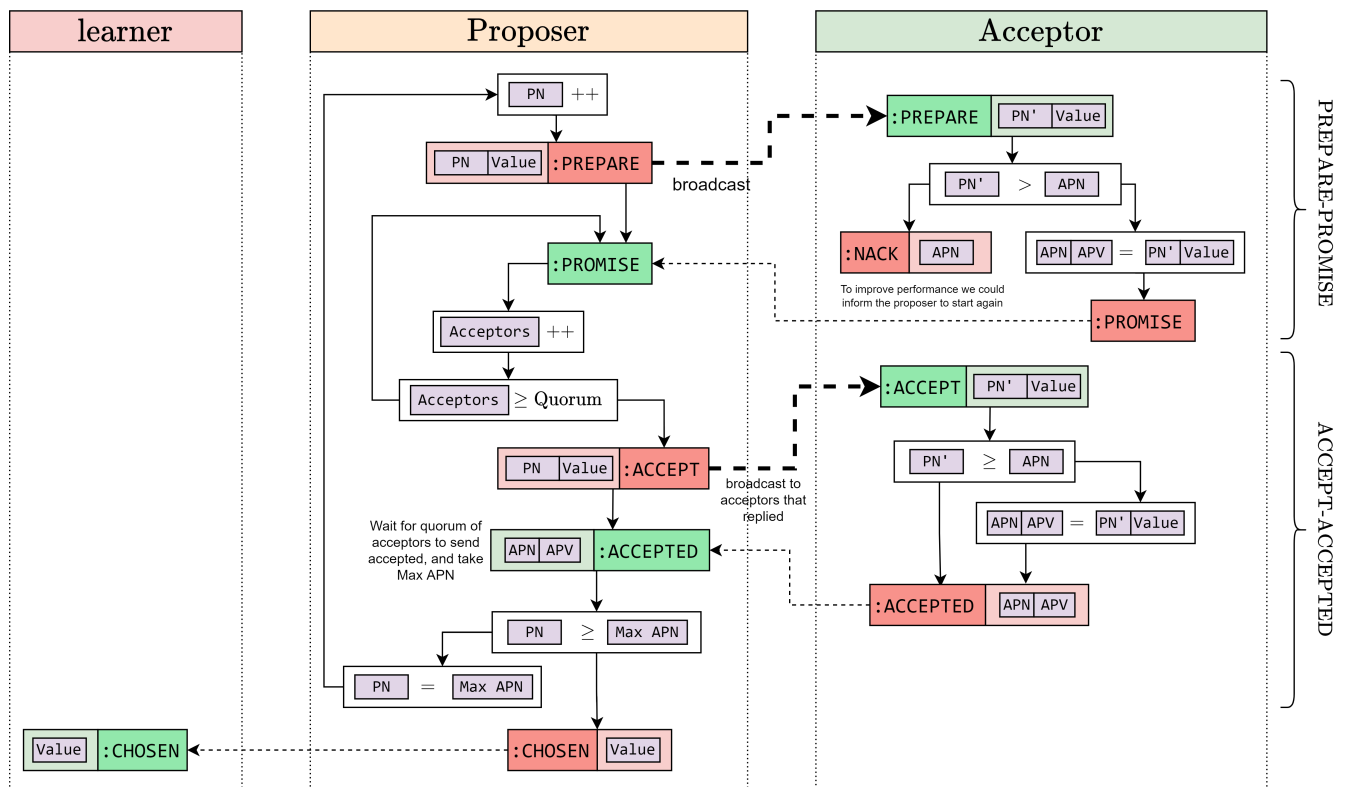
## 4.4 Common Consensus Algorithms

<b>Multipaxos</b>	Most popular algorithm, variants are used across industry; Google chubby (a distributed lock manager), BigTable (a Google DBMS), AWS, Azure Fabric, Neo4j (a graph DBMS), Apache Mesos (a distributed systems kernel).
<b>Raft</b>	( <b>R</b> eliable, <b>R</b> eplicated, <b>R</b> edundant <b>A</b> nd <b>F</b> ault <b>T</b> olerant) A newer algorithm (formally verified, and easier to understand) used in Meta's Hydrabase, Kubernetes and Docker Swarm.
<b>PBFT</b>	( <b>P</b> ractical <b>B</b> yzantine <b>F</b> ault <b>T</b> olerance) and proof of work/proof of stake are used for many blockchains backing cryptocurrencies such as Bitcoin.
<b>Viewstamped Replication</b>	An early consensus algorithm designed to be easily added to non-distributed programs, it has been improved upon with VSR Revisited.
<b>Atomic Broadcast</b>	Implemented in Apache Zookeeper (ZAB protocol) for building coordination services and is used for services such as Apache Hadoop (similar to MapReduce).
<b>CRDTs</b>	( <b>C</b> onflict- <b>F</b> ree <b>R</b> eplicated <b>D</b> atatypes) A data structure that can be updated independently & across a distributed system and can resolve any inconsistencies itself, with all eventually converging to the same value.

## 4.5 Paxos

Paxos	Definition 4.5.1
A consensus algorithm wherein each server has:	
<b>Learner</b>	Receives decisions, alters the state based on agreed values.
<b>Proposer</b>	Proposes values to <b>Acceptors</b> , associated with its proposal number. Receives accepted values.
<b>Acceptor</b>	Accepts values with increasing ballot numbers.





UNFINISHED!!!

#### 4.5.1 leadership Based Paxos

The algorithm is split into rounds, in each round there is a **leader**.

- The leader requests the last accepted value from each acceptor
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# Chapter 5

## Temporal Logic of Actions

### 5.1 Introduction

- A summary of TLA
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### 5.2 Terminology

Stuttering Step	Definition 5.2.1
A transition where all state variables stay the same. Represented in TLA+ using the actions:	
$[A]_v$	Action $A$ occurs, or $v$ is unchanged in successor
$[A]_{\langle v_1, v_2, v_3 \rangle}$	Same as above but with many variables

Actions	Definition 5.2.2
Change the state of a module (primed variables $\rightarrow$ non-primed)	

#### 5.2.1 TLA+ Constructs

Based on an excellent cheat sheet created by professor Narankar Dulay, based on Model Based Testing Informal Systems's own.

Booleans Functions Integers Sets Tuples \& Sequences

#### Logic

BOOLEAN	BOOLEAN	Set of boolean values $\{true, false\}$
TRUE	TRUE	
FALSE	FALSE	
$\neg$	$\sim e$	Logical negation
$a \wedge b$	$a \ /\ \ b$	Logical and
$a \vee b$	$a \ \vee \ b$	Logical or
$a = b$	$a = b$	Equality
$a \neq b$	$a \ \#\# \ b$	Not equal
$a \Rightarrow b$	$a \Rightarrow b$	Logical Implication ( $b \vee \neg a$ ) or IF a THEN b ELSE TRUE
$a \equiv b$	$a \Leftrightarrow b$	Equivalence

#### Quantifiers

$\forall var \in S : e$	$\forall A \ var \ \text{in} \ S : e$	Expression $e$ is <i>true</i> for all elements of set $S$
$\exists var \in S : e$	$\exists E \ var \ \text{in} \ S : e$	Expression $e$ is <i>true</i> for some element of set $S$
CHOOSE $var \in S : e$	CHOOSE $var \ \text{in} \ S : e$	Always picks the same element $e$ from set $S$ (undefined for empty sets)

## Integers

<i>Int</i>	<b>Int</b>	Set of all integers
<i>Nat</i>	<b>Nat</b>	Set of all natural numbers (not including 0)
1, -2, 12542355	1, -2, 12542355	Integer literals
$a \dots b$	<b>a..b</b>	Integer range as a set (inclusive and empty is $a > b$ )
$a + b, a - b, a * b$	<b>a + b, a - b, a * b</b>	Integer arithmetic
$a^b, a \% b$	<b>a ^ b, a \% b</b>	
$a > b, a \geq b, a < b, a \leq b$	<b>a &gt; b, a &gt;= b, a &lt; b, a &lt;= b</b>	Comparison operations

## Strings

<b>STRING</b>	<b>STRING</b>	The set of all finite strings
"", "hello world"	"", "hello world"	String literals

## Finite Sets

$\{a, b, c\}$	<b>{a,b,c}</b>	A set constructed of $a, b$ and $c$ (all the same type)
$\text{Cardinality}(S)$	<b>Cardinality(S)</b>	Get the size/cardinality of set $S$
$e \in S, e \notin S$	<b>e \in S, e \notin S</b>	Checking set membership
$S1 \subseteq S2$	<b>S1 \subseteq S2</b>	Checking a $S1$ is a subset (can be equal)
$S1 \cup S2$	<b>S1 \cup S2 or S1 \cup S2</b>	Set union operation
$S1 \cap S2$	<b>S1 \cap S2 or S1 \cap S2</b>	Set intersection
$S1 \setminus S2$	<b>S1 \setminus S2</b>	Set difference ( $S1 - S2$ )
$\{var \in S : P(var)\}$	<b>{var \in S: P(m)}</b>	Filter elements of $S$ using predicate $P$
$\{e : k \in KeyS\}$	<b>{e: k \in KeyS}</b>	Map all keys from $Keys$ with expression $e$

## Functions & Maps

$k \in keys \mapsto e$	<b>[k \in KeyS  -&gt; e]</b>	[Function Construction] map all keys $k$ to expression $e$ (which potentially uses $k$ )
$fn[k]$	<b>fn[k]</b>	[Function Application] get value associated to key $k$ by function $fn$
$[fn \text{ EXCEPT } ![k1] = e1, \dots]$	<b>[fn EXCEPT ![k1] = e1, ...]</b>	Remap the key $k1$ for function $fn$ (can use $@$ to reference the original $fn[k1]$ ) with other remappings (the ...)
$[Keys \rightarrow Values]$	<b>Keys -&gt; Values</b>	The set of all functions mapping the set of $Keys$ to the set of $Values$ , (e.g $STRING \rightarrow Nat$ )

## Records

$[f1 \mapsto e1, f2 \mapsto e2, \dots]$	<b>[f1  -&gt; e1, f2  -&gt; e2, ...]</b>	Construct a record of fields $fs$ containing expressions $es$
$myRec.f$	<b>myRec.f</b>	Access field $f$ from a record $myRec$
$[myRec \text{ EXCEPT } !.f1 = e1, \dots]$	<b>[rec EXCEPT !.f1 = e1, ...]</b>	Rebinding fields (similar to rebinding keys for functions)
$[f1 : S1, f2 : S2, \dots]$	<b>[f1: S1, f2: S2, ...]</b>	The set of all records with field names $fs$ in sets $Ss$

## Sequences

$\langle e1, e2, e3 \rangle$	<b>&lt;&lt;e1, e2, e3&gt;&gt;</b>	Construct a sequence (list) from expressions (all the same type)
$mySeq[i]$	<b>mySeq[i]</b>	Get index $i$ of sequence $mySeq$
$seq1 \circ seq2$	<b>seq1 \circ seq2</b>	Concatenation of sequences
$Len(mySeq)$	<b>Len(mySeq)</b>	Length of a given sequence
$Append(mySeq, e)$	<b>Append(mySeq, e)</b>	Add to end of a sequence
$Head(mySeq)$	<b>head(mySeq)</b>	Get first element of $mySeq$
$Seq(S)$	<b>Seq(S)</b>	The set of all finite sequences over set $S$



## Tuples

$\langle a, b, c \rangle$	<code>&lt;&lt;a, b, c&gt;&gt;</code>	Construct a tuple (types of elements can be different)
$myTup[i]$	<code>myTup[i]</code>	Index a tuple
$S1 \times S2 \times \dots \times S_n$	<code>S1 \X S2 \X ... \X S_n</code>	Set of the cartesian product of the sets of tuples (each tuple of form $\langle s1, s2, \dots sn \rangle$ )

## Miscellaneous

$LET\ var \triangleq e1 \in e2$	<code>LET var == e1 \in e2</code>	A let statement (e.g same as in Haskell)
$IF\ e\ THEN\ e1\ ELSE\ e2$	<code>IF e THEN e1 ELSE e2</code>	If statements (statement is an expression itself - e.g like Elixir, Haskell, Rust)

## Actions

$var'$	<code>var'</code>	[Primed variable] denotes the non-primed $var$ in the next state
$UNCHANGED\ \langle v1, v2, \dots \rangle$	<code>UNCHANGED &lt;&lt;v1, v2, ...&gt;&gt;</code>	Shorthand for $v1 = v1' \wedge v2 = v2' \wedge \dots$
$[A]_v, [A]_{\langle v1, v2, \dots \rangle}$	<code>[A]_v, [A]_&lt;&lt;v1, v2, ...&gt;&gt;</code>	Stuttering action (can apply action or variables $v, v1, v2, \dots$ are unchanged)
$\langle A \rangle_v, \langle A \rangle_{\langle v1, v2, \dots \rangle}$	<code>&lt;&lt;A&gt;&gt;_v, &lt;A&gt;_&lt;&lt;v1, v2, v3&gt;&gt;</code>	Non-stuttering action, the variables must $v, v1, v2, \dots$ change
$ENABLED\ A$	<code>ENABLED A</code>	<i>true</i> if action $A$ is enabled

## Temporal Logic

$\Box F$	<code>[] F</code>	$F$ is always <i>true</i>
$\Diamond F$	<code>&lt;&gt; F</code>	$F$ is eventually <i>true</i>
$F1 \leadsto F2$	<code>F1 ~&gt; F2</code>	$F1$ leads to $F2$
$WF_v(A), SF_v(A)$	<code>WF_v(A), SF_v(A)</code>	Strong and weak fairness for action $A$

## 5.3 Examples

### 5.3.1 One Bit Clock

<pre> MODULE OneBitClock VARIABLE b Type <math>\triangleq b \in \{0, 1\}</math>  Init <math>\triangleq b = 0 \vee b = 1</math> Next <math>\triangleq ((b = 0) \wedge (b' = 1)) \vee ((b = 1) \wedge (b' = 0))</math> Spec <math>\triangleq Init \wedge \Box[Next]_b</math>  Typed <math>\triangleq \Box Type</math> </pre>	<pre> ---- MODULE OneBitClock ---- VARIABLE b Type == b \in {0,1}  ----- Init == b=0 /\ b=1 Next == ((b=0) /\ (b'=1)) /\ ((b=1) /\ (b'=0)) Spec == Init /\ [] [Next]_b ----- Typed == [] Type ===== </pre>
--	--

A basic counter with states  $\dots \rightarrow 0 \rightarrow 1 \rightarrow 0 \rightarrow 1 \rightarrow \dots$

- Contains a single variable  $b$  ( $b'$  is the value of  $b$  in the next state).
- Starts as 0 or 1, and is always 0 or 1 (by the theorem *Typed* which states *Type* is always true)
- $b$  is always updated in the next

The use of  $Init \wedge \Box[Next]_b$  is equivalent to  $Init \wedge \Box(Next \vee (b = b'))$  and allows for a *stuttering step*.

### 5.3.2 12 Hour Clock

<pre> MODULE TwelveHourClock EXTENDS Naturals VARIABLE hour  Init <math>\triangleq</math> hour <math>\in</math> 0 .. 11 Next <math>\triangleq</math> hour' = (hour + 1)%12 Spec <math>\triangleq</math> Init <math>\wedge</math> <math>\square</math>[Next]<sub>hour</sub>  Typed <math>\triangleq</math> <math>\square</math>Init </pre>	<pre> ---- MODULE TwelveHourClock ---- EXTENDS Naturals VARIABLE hour  Init == hour \in 0..11 Next == hour' = (hour + 1) % 12 Spec == Init /\ [] [Next]_hour  Typed == [] Init ==== </pre>
---	--

The *Init* predicate is always true (from *Typed*  $\triangleq$   $\square$ *Init*) hence TLC can check the correctness of our *Next* implementation.

### 5.3.3 24 Hour Clock

We can make use of TLC provided functions such as *Print* and *PrintT*.

<pre> MODULE 24HourClock EXTENDS Naturals, TLC VARIABLE hour  Init <math>\triangleq</math> hour <math>\in</math> 0 .. 23 Next <math>\triangleq</math> hour' = (hour + 1)%24 <math>\wedge</math> (   (hour <math>\leq</math> 12 <math>\wedge</math> PrintT(<math>\langle</math>"[Morning] time:", hour<math>\rangle</math>)) <math>\vee</math> (hour &gt; 12 <math>\wedge</math> hour &lt; 18 <math>\wedge</math> PrintT(<math>\langle</math>"[Afternoon] time:", hour<math>\rangle</math>)) <math>\vee</math> (hour <math>\geq</math> 18 <math>\wedge</math> PrintT(<math>\langle</math>"[Evening] time:", hour<math>\rangle</math>)) ) Spec <math>\triangleq</math> Init <math>\wedge</math> <math>\langle</math>Next<math>\rangle</math><sub>hour</sub>  Typed <math>\triangleq</math> <math>\square</math>Init </pre>	<pre> ---- MODULE 24HourClock ---- EXTENDS Naturals, TLC VARIABLE hour  Init == hour \in 0..23 Next == hour' = (hour + 1) % 24 /\ (   (hour &lt;= 12 /\ PrintT(&lt;&lt;"[Morning] time:", hour&gt;&gt;))   /\ (hour &gt; 12 /\ hour &lt; 18 /\ PrintT(&lt;&lt;"[Afternoon] time:", hour&gt;&gt;))   /\ (hour &gt;= 18 /\ PrintT(&lt;&lt;"[Evening] time:", hour&gt;&gt;)) ) Spec == Init /\ &lt;&lt;Next&gt;&gt;_hour  Typed == [] Init ==== </pre>
--	---

We can see the short-circuiting of  $\vee$  resulting in messages being printed, *PrintT* always returns *true*:

<<"[Morning] time:", 0>>	<<"[Morning] time:", 8>>	<<"[Afternoon] time:", 16>>
<<"[Morning] time:", 1>>	<<"[Morning] time:", 9>>	<<"[Afternoon] time:", 17>>
<<"[Morning] time:", 2>>	<<"[Morning] time:", 10>>	<<"[Evening] time:", 18>>
<<"[Morning] time:", 3>>	<<"[Morning] time:", 11>>	<<"[Evening] time:", 19>>
<<"[Morning] time:", 4>>	<<"[Morning] time:", 12>>	<<"[Evening] time:", 20>>
<<"[Morning] time:", 5>>	<<"[Afternoon] time:", 13>>	<<"[Evening] time:", 21>>
<<"[Morning] time:", 6>>	<<"[Afternoon] time:", 14>>	<<"[Evening] time:", 22>>
<<"[Morning] time:", 7>>	<<"[Afternoon] time:", 15>>	<<"[Evening] time:", 23>>

# Chapter 6

# Credit

## Image Credit

## Content

Based on the distributed algorithms course taught by Prof Narankar Dulay.

These notes were written by Oliver Killane.