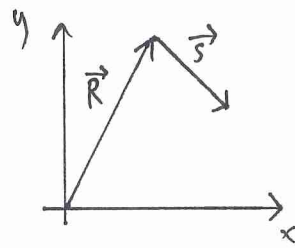


$$\vec{r}(\theta, t) = \vec{R}(t) + \vec{s}(\theta)$$

$$\dot{\vec{r}}(\theta, t) = \dot{\vec{R}}(t) + \dot{\theta} \vec{s}'(\theta)$$



$$\dot{\vec{r}}^2(\theta, t) = |\dot{\vec{R}}(t)|^2 + |\vec{s}'(\theta)|^2 \dot{\theta}^2 + 2 \dot{\theta} \dot{\vec{R}}^+(t) \vec{s}'(\theta)$$

$$T(\theta, \dot{\theta}, t) = \frac{1}{2} m V^2(t) + \frac{1}{2} m |\vec{s}'(\theta)|^2 \dot{\theta}^2 + m \dot{\theta} \vec{V}^+(t) \vec{s}'(\theta)$$

$$V(\theta, t) = mg \vec{R}(t) \cdot \hat{k} + mg \vec{s}(\theta) \cdot \hat{k}$$

$$\mathcal{L}(\theta, \dot{\theta}, t) = \frac{1}{2} m |\vec{V}(t)|^2 + \frac{1}{2} m |\vec{s}'(\theta)|^2 \dot{\theta}^2 + m \dot{\theta} \vec{V}^+(t) \vec{s}'(\theta) - mg \vec{R}(t) \cdot \hat{k} - mg \vec{s}(\theta) \cdot \hat{k}$$

$$\frac{\partial \mathcal{L}}{\partial \dot{\theta}} = m |\vec{s}'(\theta)|^2 \dot{\theta} + m \vec{V}^+(t) \vec{s}'(\theta)$$

$$\frac{d}{dt} \left(\frac{\partial \mathcal{L}}{\partial \dot{\theta}} \right) = 2m \vec{s}'^+(t) \vec{s}''(\theta) \dot{\theta} + m |\vec{s}'(\theta)|^2 \ddot{\theta} + m \vec{A}^+(t) \vec{s}'(\theta) + m \vec{V}^+(t) \vec{s}''(\theta) \dot{\theta}$$

$$\frac{\partial \mathcal{L}}{\partial \theta} = m \vec{s}'^+(t) \vec{s}''(\theta) \dot{\theta} + m \vec{V}^+(t) \vec{s}''(\theta) \dot{\theta} - mg \vec{s}'(\theta) \cdot \hat{k}$$

$$\frac{d}{dt} \left(\frac{\partial \mathcal{L}}{\partial \dot{\theta}} \right) - \frac{\partial \mathcal{L}}{\partial \theta} = 0 \Rightarrow$$

$$\Rightarrow m |\vec{s}'(\theta)|^2 \ddot{\theta} + m \vec{s}'^+(t) \vec{s}''(\theta) \dot{\theta} - mg \vec{s}'(\theta) \cdot \hat{k} = -m \vec{A}^+(t) \vec{s}'(\theta)$$

$$\text{If } \vec{s}(\theta) = l \begin{bmatrix} \sin \theta \\ -\cos \theta \end{bmatrix} :$$

$$\ddot{\theta} - \frac{g}{l} \sin \theta = -\frac{A_x(t)}{l} \cos \theta - \frac{A_y(t)}{l} \sin \theta$$

$$\frac{d}{dt} |\vec{s}(\theta)|^2 = 2 \vec{s}^+(t) \vec{s}'(\theta) \dot{\theta}$$