Team Name: P.L.S.

Team Members: Savannah Blasi, Lindsay Sanford, Peter Tikalsky
State: Nebraska

Executive Summary Letter

The purpose of this report is to present an Integer Programming (IP) model that creates one or more valid congressional redistricting plans for the state of Nebraska. Our plan satisfies traditional redistricting principles as well as state and federal criteria. The model we chose for our project was that of Hess' model for moment of inertia compactness with added constraints suggested from other authors (see References) to optimize compactness and minimum deviation while also maintaining contiguity. Our initial objective was to simply minimize deviation, but found the result lacked compactness and contiguity, rendering the solution infeasible and suboptimal. After making adjustments to our program to match the modified Hess model, we were able to produce a solution that satisfied contiguity, compactness, and deviation constraints.

The Federal government requires population balance and prohibits plans that discriminate against minorities. Nebraska has state criteria that require districts to be contiguous, compact, preserve political subdivisions, and preserve the cores of prior districts. Nebraska also prohibits protecting incumbents and the use of partisan data. After fully understanding these constraints, we created an integer program model using Gurobi optimization software and generated map results using geopandas.

After comparing our model with the official 2021 maps, we have come to the understanding that our solutions can be considered optimal for the provided constraints, and any differences can be explained by our choice to keep counties whole to abide with the National Conference of State Legislatures' criterion of preserving political boundaries, whereas the ratified districts apportion with voting precincts as the smallest unit of order.

Introduction

For this project, we developed a program to redraw the congressional districts for the state of Nebraska. Congressional districts must be redrawn every 10 years based on the United States Census. The purpose of congressional districts is to ensure that all citizens have equal representation in Congress. According to American Civil Liberties Union, "When it's conducted fairly, it accurately reflects population changes and racial diversity, and is used by legislators to equitably allocate representation in Congress and state legislatures"("What is Redistricting and Why Should We Care?" 2021). The process of developing new congressional boundaries varies from each state based on state criteria. According to the Congressional Research Service, "Congressional redistricting involves creating geographic boundaries for U.S. House districts within a state" (Congressional Research Service 2021).

Criteria

Federal Criteria

There are two major criteria set by the federal government that all states must follow. The first criterion is population equality, which comes from the Apportionment Clause of Article I, Section 2, of the U.S. Constitution. The clause requires that "all districts be as nearly equal in population as practicable, which essentially means exactly equal"(National Conference of State Legislatures 2021). The second criterion is racial and language minority protection, found in Section 2 of the Voting Rights Act of 1965 which "prohibits plans that intentionally or inadvertently discriminate on the basis of race, which could dilute the minority vote" (National Conference of State Legislatures 2021). This states from drawing congressional districts that "have the effect of reducing, or diluting, minority voting strength" (National Conference of State Legislatures 2021).

State Criteria

Required: According to the Nebraska State Constitution, Article III-5...

- Compact and contiguous: For a compact district, the congressional district will have smooth borders and resemble a geometrical shape. For a contiguous district, a person should be capable of traveling across the district without crossing into another district.

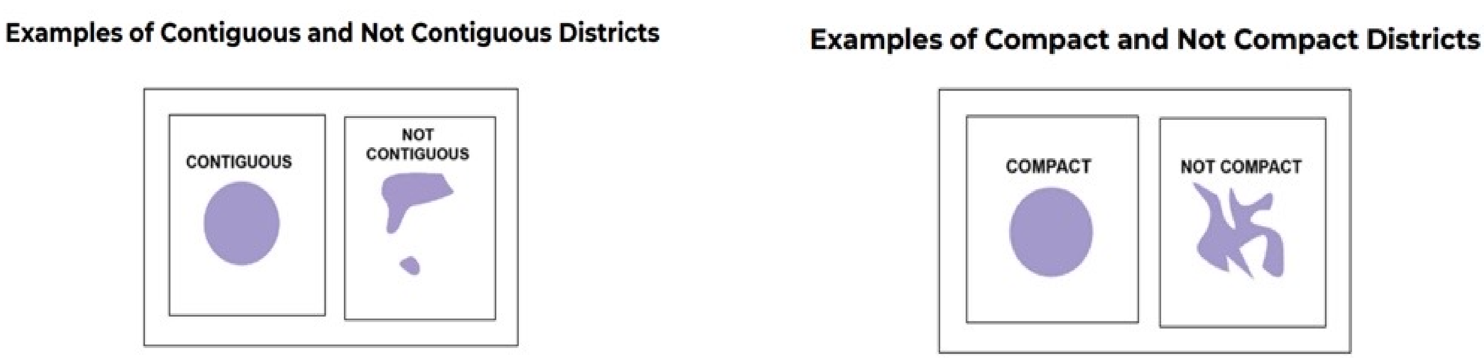

Figure 1: Examples of district compactness and contiguity

- Preserve Political Subdivisions: This requires not crossing county, city, or town boundaries when drawing the districts.

- Preserve Cores of Prior Districts: This requires that the state maintains districts as previously drawn which leads to continuity of representation.

Prohibited: according to the Legislative Resolution 102...

- Protecting Incumbencies: Nebraska prohibits redistricting in a way that favors and protects the party currently in power.
- Use of Partisan Data: Line drawers are prohibited from using incumbent residences, election results, party registration, or other socio-economic data as an input when redrawing districts.

Problem Statement

Create models that develop one or more congressional redistricting plans for Nebraska using an integer program and Gurobi optimization software. The program must include constraints accounting for contiguous and compact districts, keep counties whole, and abide by the accepted population deviations along with all other state and federal criteria.

Integer Program Model

Mathematical Model

$$\min \sum_{i \in V} \sum_{j \in V} w_{ij} x_{ij} \tag{1a}$$

$$\sum_{j \in V} x_{ij} = 1 \qquad \forall i \in V \tag{1b}$$

$$\sum_{j \in V} x_{jj} = k \tag{1c}$$

$$L x_{jj} \leq \sum_{i \in V} p_i x_{ij} \leq U x_{jj} \qquad \forall j \in V \tag{1d}$$

$$x_{ij} \leq x_{jj} \qquad \forall i, j \in V \tag{1e}$$

$$x_{ij} \in \{0, 1\} \qquad \forall i, j \in V. \tag{1f}$$

Fig. 2: Mathematical description of Hess' moment-of-inertia compactness model (Source: Buchanan 2022)

$$\sum_{u \in N(i)} (f^j_{ui} - f^j_{iu}) = x_{ij} \qquad \forall i \in V \setminus \{j\},\ \forall j \in V \qquad (2a)$$

$$\sum_{u \in N(i)} f^j_{ui} \leq (n-1)x_{ij} \qquad \forall i \in V \setminus \{j\},\ \forall j \in V \qquad (2b)$$

$$\sum_{u \in N(i)} f^j_{uj} = 0 \qquad \forall j \in V \qquad (2c)$$

$$f^v_{ij},\ f^v_{ji} \geq 0 \qquad \forall \{i,j\} \in E,\ \forall v \in V. \qquad (2d)$$

Fig. 3: Additional constraints defined by the below-mentioned authors to maintain contiguity (Source: Buchanan 2022)

Verbal Explanation

The model we chose for our project was a combination of Hess' model for moment of inertia compactness (Hess et al. 1965), seen in fig. 2, and additional constraints for maintaining contigiuity from Shirabe (Shirabe 2004) (Shirabe 2009), Oherlein and Haunert (Oerhlein and Haunert 2017), and Validi et al (Validi, Buchanan, and Lykhovyd 2022). Hess' model considers each county *i* the vertex of a graph with "bodies"' representing the number of districts in a state. Each district has a centroid (center of mass) j at a county that is determined throughout the solve of the problem, as each centroid is set in relation to the county populations around it.

The objective of Hess' model is to minimize the geographical size of the body *k* in relation to its centroid, that is, the "center of mass" of a given congressional district *k*'s location w.r.t. the nearest adjacent counties' populations (1a). In order to do this, the population of a county p_i is multiplied by the distance squared from a given county to its centroid d_{ij}^2 . This value (the "penalty") serves as a weight to a binary variable x_{ij} , which is equal to 1 if a county *i* is assigned to the centroid *j* in question and 0 otherwise.

Hess' model also includes several constraints to ensure each state is modeled accurately. All counties must be used in the program (1b) and belong to one of the given districts (1c). The total population of each district must also conform to the lower and upper population bounds determined valid for each state (1d).

Additional constraints are added to the model as shown in fig. 3 that have been suggested by the aforementioned group of authors to force contiguity into the program. To accomplish this, an additional variable f_{ij}^v is created to represent the artificial concept of "flow" within a given district. For a given district center v, one unit of flow is sent from v to county i along the path $\{i, j\}$. If the county has a path $\{i, j\}$ to center v, the county consumes one flow unit (2a). In constraint (2b), it is set that county i may only receive a flow unit from a center j iff the county has a path $\{i, j\}$ tied to said center.

Constraints (1e), (1f), and (2d) are added for program stability. Constraint (2c) is added to prevent flow from circulating through the district (flow may only pass between a county and its district center along its given path) (Buchanan 2022).

Experiments

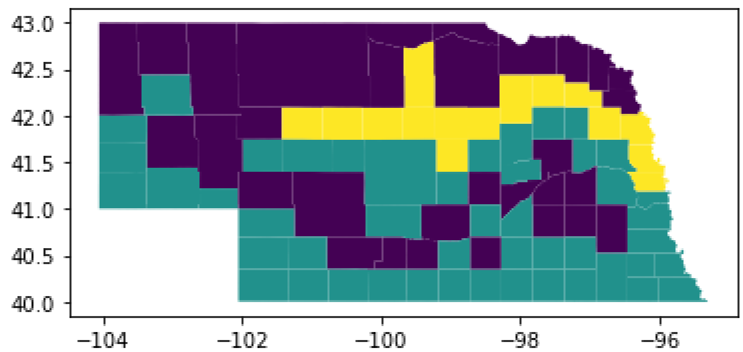

Fig. 4: First iteration. Minimum deviation, no compactness/contiguity constraints

Our first objective during experimentation was to simply minimize deviation between the smallest and largest district populations without taking compactness and contiguity into account. This first solution took almost two minutes, and as shown in fig. 4, while the map finds a solution with regard to minimum deviation, there is poor compactness and almost no contiguity, rendering this iteration unsatisfactory. After adjusting our model to the Hess model with contiguity constraint alterations, we produced figs. 6 & 7, which became the basis of our solution.

While the originally supplied code by Dr. Buchanan suggested printing the objective value, we found it to be ineffective in conveying the information needed from the model. Instead, we chose to print the counties in each district. Each solution was found to be optimal with the provided population and district-number constraints.

Plans & Maps

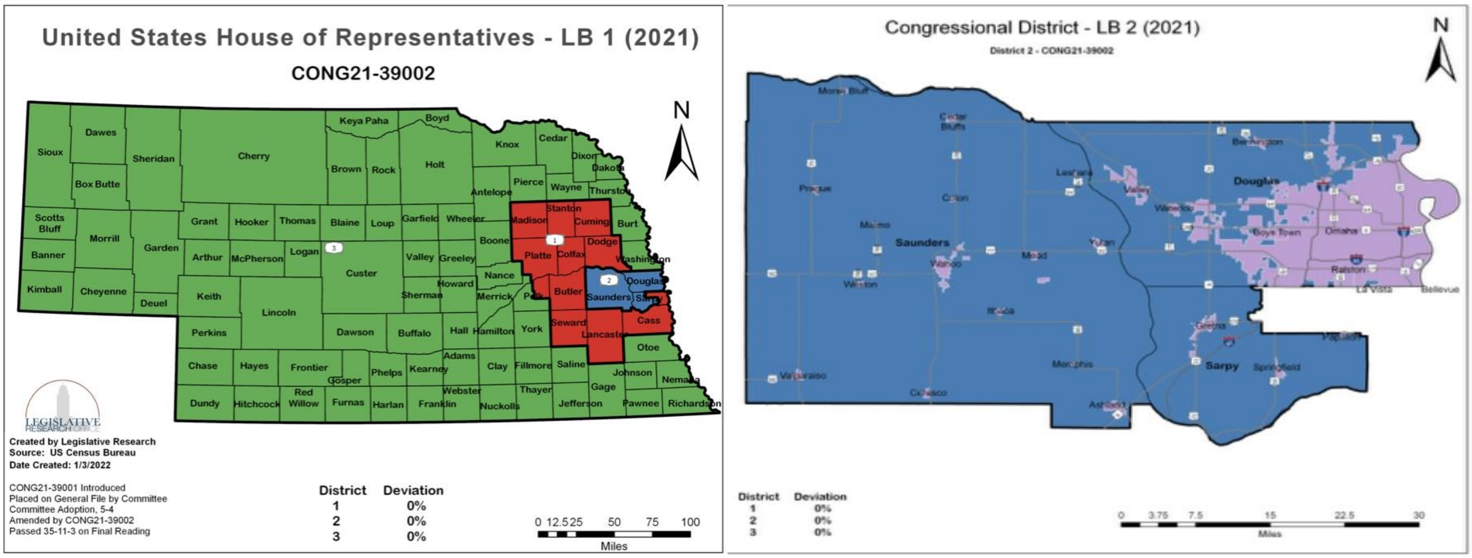

Fig. 5: 2021 approved congressional districts for Nebraska (left), with Sarpy county's district split highlighted (right)

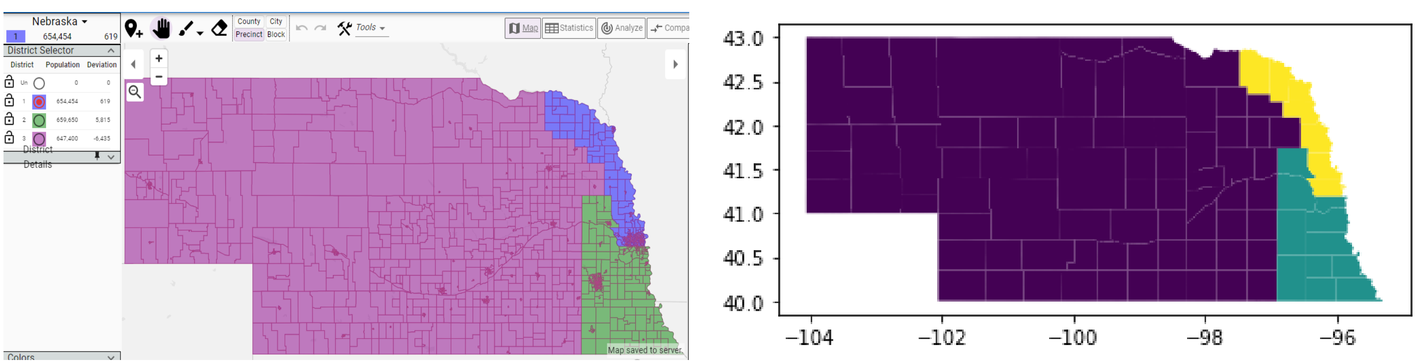

Fig. 6: PLS Solution 1: Estimated result drawn in DRA 2020 (left) compared to IP solution (right)

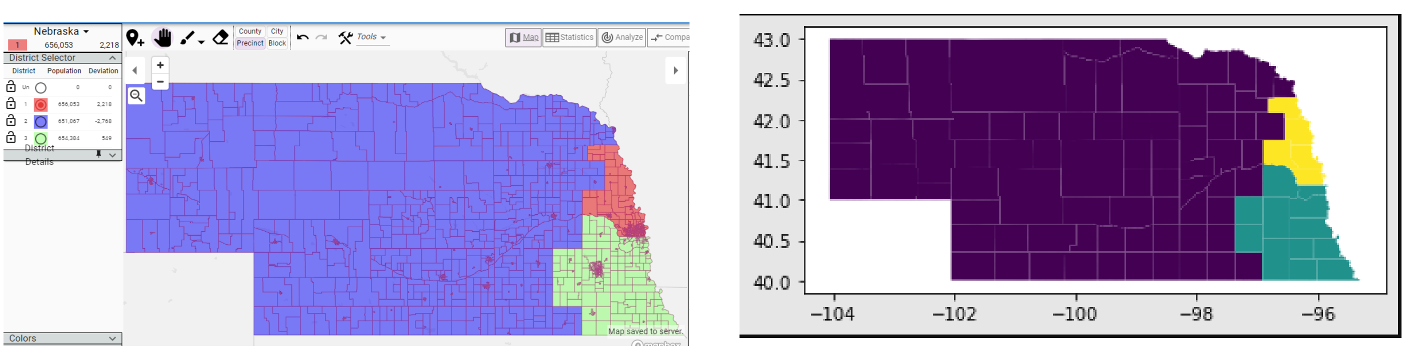

Fig. 7: PLS Solution 2: Estimated result drawn in DRA 2020 (left) comparted to IP solution (right)

Evaluation of Solutions

On September 30, 2021, elected officials finalized the new Congressional District map after the 2020 apportionment (Ballotpedia, n.d.). This map became effective for the 2022 election. The 2021 map followed all criteria except Preserving Political Subdivisions, as some counties were divided into different districts (fig. 5). The current deviation based on the 2020 apportionment is 0%, following the state issued guidelines of +/-0.5%, but legally-acceptable district population may vary from 650,566 to 657,104.

When running the model, we wanted to see the difference between a 1% and 2% deviation. Figure 6 was run by the model based on 2% deviation, and fig. 7 was run with 1% deviation. Fig. 7 is our final model that falls within the state of Nebraska's +/- 0.5 deviation criterion, with a total deviation of 4,986 compared to fig. 6's 12,250. Comparing the two sets of maps, one can clearly see that the smallest amount of alterations to the accepted deviation noticeably alters the congressional districts.

Comparing our final model to the official 2021 maps from the 2020 apportionment, any differences can be attributed to our choice to keep counties whole, fulfilling the criterion of Preserving Political Subdivision. We kept the counties whole while the official map allowed the districts to be based on precincts and not counties. Based on the legally-accepted constraints, the official map can be considered a more optimal solution since selecting voting precincts over counties as graph vertices creates a smaller deviation.

Conclusion

In conclusion, after adopting the modified Hess model, we were able to formulate a program that successfully redistricted the state of Nebraska into three congressional districts. Our model followed the criteria set by the state to ensure the congressional districts were equal by population, compact, contiguous and used partisan data while obtaining the minimal deviation constraints of +/- 0.5% of the ideal district population. We discovered by using only minimum deviation as our objective without the added constraints, the data did not formulate a map where the congressional districts followed the needed requirements. Even though our final model followed the constraints and was still within the given criteria, the official 2021 map created from the 2020 apportionment would most likely be chosen over ours due to a smaller deviation since they constrained by precincts and not counties.

References

Buchanan, Austin. 2022. "IEM 4013: Two MIPs for redistricting," supplied document outlining two different IP models for re-apportioning congressional districts. GitHub Repo: Districting-Examples-2020. https://github.com/AustinLBuchanan/Districting-Examples-2020 .

Hess, S.W., J.B. Weaver, H.J. Siegfeldt, J.N. Whelan, and P.A. Zitlau. 1965. "Nonpartisan Political Redistricting by Computer." *Operations Research* 13, no. 6 (November-December): 879-1060. https://doi.org/10.1287/opre.13.6.998 .

Oerhlein, Johannes, and Jan-Henrik Haunert. 2017. "A cutting-plane method for contiguity-constrained spatial aggregation." *Journal of Spatial Information Science*, no. 15 (12), 89-120. 10.5311. http://dx.doi.org/10.5311/JOSIS.2017.15.379 .

Shirabe, Takeshi. 2004. "A Model of Contiguity for Spatial Unit Allocation." *Geographical Analysis* 37, no. 1 (December): 2-16. https://doi.org/10.1111/j.1538-4632.2005.00605 .

Shirabe, Takeshi. 2009. "Districting Modeling with Exact Contiguity Constraints." *Environment and Planning B: Planning and Design* 36, no. 6 (December): 951-1134. https://doi.org/10.1068/b34104 .

Validi, Hamidreza, Austin Buchanan, and Eugene Lykhovyd. 2022. "Imposing Contiguity Constraints in Political Districting Models." *Operations Research* 70, no. 2 (December): iii-viii, 641-1291, C2-C3. https://doi.org/10.1287/opre.2021.2141 .

"What is Redistricting and Why Should We Care?" 2021. ACLU.
https://www.aclu.org/news/voting-rights/what-is-redistricting-and-why-should-we-care.