Asymmetric Distributed Trust

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Abstract

Quorum systems are a key abstraction in distributed fault-tolerant computing for capturing trust assumptions. They can be found at the core of many algorithms for implementing reliable broadcasts, shared memory, consensus and other problems. This paper introduces *asymmetric Byzantine quorum systems* that model subjective trust. Every process is free to choose which combinations of other processes it trusts and which ones it considers faulty. Asymmetric quorum systems strictly generalize standard Byzantine quorum systems, which have only one global trust assumption for all processes. This work also presents protocols that implement abstractions of shared memory and broadcast primitives with processes prone to Byzantine faults and asymmetric trust. The model and protocols pave the way for realizing more elaborate algorithms with asymmetric trust.

1 Introduction

Byzantine quorum systems [19] are a fundamental primitive for building resilient distributed systems from untrusted components. Given a set of nodes, a quorum system captures a trust assumption on the nodes in terms of potentially malicious protocol participants and colluding groups of nodes. Based on quorum systems, many well-known algorithms for *reliable broadcast*, *shared memory*, *consensus* and more have been implemented; these are the main abstractions to synchronize the correct nodes with each other and to achieve consistency despite the actions of the faulty, so-called *Byzantine* nodes.

Traditionally, trust in a Byzantine quorum system for a set of processes $\mathcal{P}$ has been *symmetric*. In other words, a global assumption specifies which processes may fail, such as the simple and prominent *threshold quorum* assumption, in which any subset of $\mathcal{P}$ of a given maximum size may collude and act against the protocol. The most basic threshold Byzantine quorum system, for example, allows all subsets of up to $f < n/3$ processes to fail. Some classic works also model arbitrary, non-threshold symmetric quorum systems [19, 15], but these have not actually been used in practice.

However, trust is inherently subjective. *De gustibus non est disputandum.* Estimating which processes will function correctly and which ones will misbehave may depend on personal taste. A myriad of local choices influences one process' trust in others, especially because there are so many forms of "malicious" behavior. Some processes might not even be aware of all others, yet a process should not depend on unknown third parties in a distributed collaboration. How can one model asymmetric trust in distributed protocols? Can traditional Byzantine quorum systems be extended to subjective failure assumptions? How do the standard protocols generalize to this model?

In this paper, we answer these questions and introduce models and protocols for asymmetric distributed trust. We formalize *asymmetric quorum systems* for asynchronous protocols, in which every process can make its own assumptions about Byzantine faults of others. We introduce several protocols with asymmetric trust that strictly generalize the existing algorithms, which require common trust.

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Our formalization takes up earlier work by Damgård et al. [10] and starts out with the notion of a fail-prone system that forms the basis of a symmetric Byzantine quorum system. A global fail-prone system for a process set $\mathcal{P}$ contains all maximal subsets of $\mathcal{P}$ that might jointly fail during an execution. In an asymmetric quorum system, every process specifies its *own* fail-prone system and a corresponding set of local quorums. These local quorum systems satisfy a *consistency condition* that ranges across all processes and a local *availability condition*, and generalize symmetric Byzantine quorum system according to Malkhi and Reiter [19].

Interest in consensus protocols based on Byzantine quorum systems has surged recently because of their application to permissioned blockchain networks [6, 2]. Typically run by a consortium, such distributed ledgers often use *Byzantine-fault tolerant (BFT)* protocols like PBFT [7] for consensus that rely on symmetric threshold quorum systems. The Bitcoin blockchain and many other cryptocurrencies, which triggered this development, started from different assumptions and use so-called permissionless protocols, in which everyone may participate. Those algorithms capture the relative influence of the participants on consensus decisions by an external factor, such as "proof-of-work" or "proof-of-stake."

A middle ground between permissionless blockchains and BFT-based ones has been introduced by the blockchain networks of Ripple (`https://ripple.com`) and Stellar (`https://stellar.org`). Their stated model for achieving network-level consensus uses subjective trust in the sense that each process declares a local list of processes that it "trusts" in the protocol.

Consensus in the *Ripple* blockchain (and for the *XRP* cryptocurrency on the *XRP Ledger*) is executed by its validator nodes. Each validator declares a *Unique Node List (UNL)*, which is a "list of transaction validators a given participant believes will not conspire to defraud them;" but on the other hand, "Ripple provides a default and recommended list which we expand based on watching the history of validators operated by Ripple and third parties" [23]. Many questions have therefore been raised about the kind of decentralization offered by the Ripple protocol. This debate has not yet been resolved.

Stellar was created as an evolution of Ripple that shares much of the same design philosophy. The Stellar consensus protocol [20] powers the *Stellar Lumen (XLM)* cryptocurrency and introduces *federated Byzantine quorum systems (FBQS)*; these bear superficial resemblance with our asymmetric quorum systems but differ technically. Stellar's consensus protocol uses *quorum slices*, which are "the subset of a quorum that can convince one particular node of agreement." In an FBQS, "each node chooses its own quorum slices" and "the system-wide quorums result from these decisions by individual nodes" [26]. However, standard Byzantine quorum systems and FBQS are *not* comparable because (1) an FBQS when instantiated with the same trust assumption for all processes does not reduce to a symmetric quorum system and (2) existing protocols do not generalize to FBQS.

Understanding how such ideas of subjective trust, as manifested in the Ripple and Stellar blockchains, relate to traditional quorum systems is the main motivation for this work. Our protocols for asymmetric trust generalize the well-known, classic algorithms in the literature and therefore look superficially similar. This should be seen as a feature, actually, because simplicity and modularity are important guiding principles in science.

Our contributions are as follows:

- We introduce asymmetric Byzantine quorum systems formally in Section 4 as an extension of standard Byzantine quorum systems and discuss some of their properties.

- In Section 5, we show two implementations of a shared register, with single-writer, multi-reader regular semantics, using asymmetric Byzantine quorum systems.

- We examine broadcast primitives in the Byzantine model with asymmetric trust in Section 6. In particular, we define and implement Byzantine consistent and reliable broadcast protocols. The latter primitive is related to a "federated voting" protocol used by Stellar consensus [20].

Before presenting the technical contributions, we discuss related work in Section 2 and state our system model in Section 3.

2 Related work

Damgård et al. [10] introduce asymmetric trust in the context of synchronous protocols for secure distributed computation by modeling process-specific fail-prone systems. They state the consistency property of asymmetric Byzantine quorums and claim (without proof) that the B^3 property is required for implementing a synchronous broadcast protocol in this setting (cf. Section 4.2). However, they do not formalize quorum systems nor discuss asynchronous protocols.

The *Ripple* consensus protocol is run by an open set of validator nodes. The protocol uses votes, similar to standard consensus protocols, whereby each validator only communicates with the validators in its UNL. Each validator chooses its own UNL, which makes it possible for anyone to participate, in principle, similar to proof-of-work blockchains. Early literature suggested that the intersection of the UNLs for every two validators should be at least 20% of each list [24], assuming that also less than one fifth of the validators in the UNL of every node might be faulty. An independent analysis by Armknecht et al. [3] later argued that this bound must be more than 40%. A recent technical paper of Chase and MacBrough [9, Thm. 8] concludes, under the same assumption of $f < n/5$ faulty nodes in every UNL of size n, that the UNL overlap should actually be at least 90%.

However, the same paper also derives a counterexample to the liveness of the Ripple consensus protocol [9, Sec. 4.2] as soon as two validators don't have "99% UNL overlap." By generalizing the example, this essentially means that the protocol can get stuck *unless all nodes have the same UNL*. According to the widely shared understanding in the field of distributed systems, though, a protocol needs to satisfy safety *and* liveness because achieving only one of these properties is trivial. Chase and MacBrough therefore present a devastating verdict on the merit of Ripple's protocol.

The *Stellar consensus protocol (SCP)* also features open membership and lets every node express its own set of trusted nodes [20]. Generalizing from Ripple's flat lists of unique nodes, every node declares a collection of trusted sets called *quorum slices*, whereby a slice is "the subset of a quorum convincing one particular node of agreement." A *quorum* in Stellar is a set of nodes "sufficient to reach agreement," defined as a set of nodes that contains one slice for each member node. The quorum choices of all nodes together yield a *federated Byzantine quorum systems (FBQS)*. The Stellar white paper states properties of FBQS and protocols that build on them. However, these protocols do not map to known protocol primitives in distributed computing.

García-Pérez and Gotsman [12] build a link from FBQS to existing quorum-system concepts by investigating a Byzantine reliable broadcast abstraction in an FBQS. They show that the *federated voting protocol* of Stellar [20] is similar to Bracha's reliable broadcast [4] and that it implements a variation of Byzantine reliable broadcast on an FBQS for executions that contain, additionally, a set of so-called intact nodes.

The paper [12], however, uses the FBQS concept of Mazières [20] that is at odds with the usual notion of a Byzantine quorum system in the sense that it does not reduce to a symmetric quorum system for symmetric trust choices. In contrast, we show in this paper that a natural extension of a symmetric Byzantine quorum system suffices for implementing various protocol primitives with asymmetric trust. We explain, in particular, how to implement the prominent register abstraction with asymmetric trust and investigate multiple broadcast primitives. In particular, Stellar's federated voting protocol and the Byzantine reliable broadcast over an FBQS as described by García-Pérez and Gotsman [12], can be shown as straightforward generalizations of Byzantine reliable broadcast with symmetric trust. Section 5.4, furthermore, discusses why FBQS are not adequate for emulating a shared register.

3 System model

Processes. We consider a system of n *processes* $\mathcal{P} = \{p_1, \ldots, p_n\}$ that communicate with each other. The processes interact asynchronously with each other through exchanging messages. The system itself is asynchronous, i.e., the delivery of messages among processes may be delayed arbitrarily and the processes have no synchronized clocks. Every process is identified by a name, but such identifiers are not

made explicit. A protocol for $\mathcal{P}$ consists of a collection of programs with instructions for all processes. Protocols are presented in a modular way using the event-based notation of Cachin et al. [5].

Functionalities. A *functionality* is an abstraction of a distributed computation, either a primitive that may be used by the processes or a service that they will provide. Every functionality in the system is specified through its *interface*, containing the events that it exposes to protocol implementations that may call it, and its *properties*, which define its behavior. A process may react to a received event by changing their state and triggering further events.

There are two kinds of events in an interface: *input events* that the functionality receives from other abstractions, typically to invoke its services, and *output events*, through which the functionality delivers information or signals a condition a process. The behavior of a functionality is usually stated through a number of properties or through a sequential implementation.

We assume there is a low-level functionality for sending messages over point-to-point links between each pair of processes. In a protocol, this functionality is accessed through the events of "sending a message" and "receiving a message". Point-to-point messages are authenticated, delivered reliably, and output in FIFO order among processes [14, 5].

Executions and faults. An *execution* starts with all processes in a special initial state; subsequently the processes repeatedly trigger events, react to events, and change their state through computation steps. Every execution is *fair* in the sense that, informally, processes do not halt prematurely when there are still steps to be taken or events to be delivered (see the standard literature for a formal definition [18]).

A process that follows its protocol during an execution is called *correct*. On the other hand, a *faulty* process may crash or even deviate arbitrarily from its specification, e.g., when *corrupted* by an adversary; such processes are also called *Byzantine*. We consider only Byzantine faults here and assume for simplicity that the faulty processes fail right at the start of an execution.

Idealized digital signatures. A *digital signature scheme* provides two operations, $sign_i$ and $verify_i$. The invocation of $sign_i$ specifies a process p_i and takes a bit string $m \in \{0,1\}^*$ as input and returns a signature $\sigma \in \{0,1\}^*$ with the response. Only p_i may invoke $sign_i$. The operation $verify_i$ takes a putative signature σ and a bit string m as parameters and returns a Boolean value with the response. Its implementation satisfies that $verify_i(\sigma, m)$ returns TRUE for any $i \in \{1, \ldots, n\}$ and $m \in \{0,1\}^*$ if and only if p_i has executed $sign_i(m)$ and obtained σ before; otherwise, $verify_i(\sigma, m)$ returns FALSE. Every process may invoke *verify*.

4 Asymmetric Byzantine quorum systems

4.1 Symmetric trust

Quorum systems are well-known in settings with symmetric trust. As demonstrated by many applications to distributed systems, ordinary quorum systems [21] and Byzantine quorum systems [19] play a crucial role in formulating resilient protocols that tolerate faults through replication [8]. A quorum system typically ensures a consistency property among the processes in an execution, despite the presence of some faulty processes.

For the model with Byzantine faults, *Byzantine quorum systems* have been introduced by Malkhi and Reiter [19]. This notion is defined with respect to a *fail-prone system* $\mathcal{F} \subseteq 2^{\mathcal{P}}$, a collection of subsets of $\mathcal{P}$, none of which is contained in another, such that some $F \in \mathcal{F}$ with $F \subseteq \mathcal{P}$ is called a *fail-prone set* and contains all processes that may at most fail together in some execution [19]. A fail-prone system is the same as the *basis* of an *adversary structure*, which was introduced independently by Hirt and Maurer [15].

A fail-prone system captures an assumption on the possible failure patterns that may occur. It specifies all maximal sets of faulty processes that a protocol should tolerate in an execution; this means that a

protocol designed for $\mathcal{F}$ achieves its properties as long as the set F of actually faulty processes satisfies $F \in \mathcal{F}^*$. Here and from now on, the notation $\mathcal{A}^*$ for a system $\mathcal{A} \subseteq 2^{\mathcal{P}}$, denotes the collection of all subsets of the sets in $\mathcal{A}$, that is, $\mathcal{A}^* = \{A' | A' \subseteq A, A \in \mathcal{A}\}$.

Definition 1 (Byzantine quorum system [19]). A *Byzantine quorum system* for $\mathcal{F}$ is a collection of sets of processes $\mathcal{Q} \subseteq 2^{\mathcal{P}}$, where each $Q \in \mathcal{Q}$ is called a *quorum*, such that the following properties hold:

Consistency: The intersection of any two quorums contains at least one process that is not faulty, i.e.,

$$\forall Q_1, Q_2 \in \mathcal{Q}, \forall F \in \mathcal{F} : Q_1 \cap Q_2 \nsubseteq F.$$

Availability: For any set of processes that may fail together, there exists a disjoint quorum in $\mathcal{Q}$, i.e.,

$$\forall F \in \mathcal{F} : \exists Q \in \mathcal{Q} : F \cap Q = \emptyset.$$

The above notion is also known as a *Byzantine dissemination quorum system* [19] and allows a protocol to be designed despite arbitrary behavior of the potentially faulty processes. The notion generalizes the usual threshold failure assumption for Byzantine faults [22], which considers that any set of f processes are equally likely to fail.

We say that a set system $\mathcal{T}$ *dominates* another set system $\mathcal{S}$ if for each $S \in \mathcal{S}$ there is some $T \in \mathcal{T}$ such that $S \subseteq T$ [11]. In this sense, a quorum system for $\mathcal{F}$ is *minimal* whenever it does not dominate any other quorum system for $\mathcal{F}$.

Similarly to the threshold case, where $n > 3f$ processes overall are needed to tolerate f faulty ones in many Byzantine protocols, Byzantine quorum systems can only exist if not "too many" processes fail.

Definition 2 (Q^3-condition [19, 15]). A fail-prone system $\mathcal{F}$ satisfies the Q^3-*condition*, abbreviated as $Q^3(\mathcal{F})$, whenever it holds
$$\forall F_1, F_2, F_3 \in \mathcal{F} : \mathcal{P} \nsubseteq F_1 \cup F_2 \cup F_3.$$

In other words, $Q^3(\mathcal{F})$ means that no *three* fail-prone sets together cover the whole system of processes. A Q^k-condition can be defined like this for any $k \geq 2$ [15].

The following lemma considers the *bijective complement* of a process set $\mathcal{S} \subseteq 2^{\mathcal{P}}$, which is defined as $\overline{\mathcal{S}} = \{\mathcal{P} \setminus S | S \in \mathcal{S}\}$, and turns $\mathcal{F}$ into a Byzantine quorum system.

Lemma 1 ([19, Theorem 5.4]). *Given a fail-prone system $\mathcal{F}$, a Byzantine quorum system for $\mathcal{F}$ exists if and only if $Q^3(\mathcal{F})$. In particular, if $Q^3(\mathcal{F})$ holds, then $\overline{\mathcal{F}}$, the bijective complement of $\mathcal{F}$, is a Byzantine quorum system.*

The quorum system $\mathcal{Q} = \overline{\mathcal{F}}$ is called the *canonical quorum system* of $\mathcal{F}$. According to the duality between $\mathcal{Q}$ and $\mathcal{F}$, properties of $\mathcal{F}$ are often ascribed to $\mathcal{Q}$ as well; for instance, we say $Q^3(\mathcal{Q})$ holds if and only if $Q^3(\mathcal{F})$. However, note that the canonical quorum system is not always minimal. For instance, if $\mathcal{F}$ consists of all sets of $f \ll n/3$ processes, then each quorum in the canonical quorum system has $n - f$ members, but also the family of all subsets of $\mathcal{P}$ with $\lceil \frac{n+f+1}{2} \rceil < n - f$ processes forms a quorum system.

Core sets. A *core set* C for $\mathcal{F}$ is a minimal set of processes that contains at least one correct process in every execution. More precisely, $C \subseteq \mathcal{P}$ is a core set whenever (1) for all $F \in \mathcal{F}$, it holds $\mathcal{P} \setminus F \cap C \neq \emptyset$ (and, equivalently, $C \nsubseteq F$) and (2) for all $C' \subsetneq C$, there exists $F \in \mathcal{F}$ such that $\mathcal{P} \setminus F \cap C' = \emptyset$ (and, equivalently, $C' \subseteq F$). With the threshold failure assumption, every set of $f + 1$ processes is a core set. A *core set system* $\mathcal{C}$ is the minimal collection of all core sets, in the sense that no set in $\mathcal{C}$ is contained in another. Core sets can be complemented by *survivor sets*, as shown by Junqueira et al. [16]. This yields a dual characterization of resilient distributed protocols, which parallels ours using fail-prone sets and quorums.

4.2 Asymmetric trust

In our model with asymmetric trust, every process is free to make its own trust assumption and to express this with a fail-prone system. Hence, an *asymmetric fail-prone system* $\mathbb{F} = [\mathcal{F}_1, \ldots, \mathcal{F}_n]$ consists of an array of fail-prone systems, where $\mathcal{F}_i$ denotes the trust assumption of p_i. One often assumes $p_i \notin F_i$ for practical reasons, but this is not necessary. This notion has earlier been formalized by Damgård et al. [10].

Definition 3 (Asymmetric Byzantine quorum system). An *asymmetric Byzantine quorum system* for $\mathbb{F}$ is an array of collections of sets $\mathbb{Q} = [\mathcal{Q}_1, \ldots, \mathcal{Q}_n]$, where $\mathcal{Q}_i \subseteq 2^{\mathcal{P}}$ for $i \in [1, n]$. The set $\mathcal{Q}_i \subseteq 2^{\mathcal{P}}$ is called the *quorum system of p_i* and any set $Q_i \in \mathcal{Q}_i$ is called a *quorum (set) for p_i*. It satisfies:

Consistency: The intersection of two quorums for any two processes contains at least one process for which both processes assume that it is not faulty, i.e.,

$$\forall i, j \in [1, n], \forall Q_i \in \mathcal{Q}_i, \forall Q_j \in \mathcal{Q}_j, \forall F_{ij} \in \mathcal{F}_i^* \cap \mathcal{F}_j^* : Q_i \cap Q_j \nsubseteq F_{ij}.$$

Availability: For any process p_i and any set of processes that may fail together according to p_i, there exists a disjoint quorum for p_i in $\mathcal{Q}_i$, i.e.,

$$\forall i \in [1, n], \forall F_i \in \mathcal{F}_i : \exists Q_i \in \mathcal{Q}_i : F_i \cap Q_i = \emptyset.$$

The existence of asymmetric quorum systems can be characterized with a property that generalizes the Q^3-condition for the underlying asymmetric fail-prone systems as follows.

Definition 4 (B^3-condition). An asymmetric fail-prone system $\mathbb{F}$ satisfies the B^3-*condition*, abbreviated as $B^3(\mathbb{F})$, whenever it holds that

$$\forall i, j \in [1, n], \forall F_i \in \mathcal{F}_i, \forall F_j \in \mathcal{F}_j, \forall F_{ij} \in \mathcal{F}_i^* \cap \mathcal{F}_j^* : \mathcal{P} \nsubseteq F_i \cup F_j \cup F_{ij}$$

The following result is the generalization of Lemma 1 for asymmetric quorum systems; it was stated by Damgård et al. [10] without proof. As for symmetric quorum systems, we use this result and say that $B^3(\mathbb{Q})$ holds whenever the asymmetric $\mathbb{Q}$ consists of the canonical quorum systems for $\mathbb{F}$ and $B^3(\mathbb{F})$ holds.

Theorem 2. *An asymmetric fail-prone system $\mathbb{F}$ satisfies $B^3(\mathbb{F})$ if and only if there exists an asymmetric quorum system for $\mathbb{F}$.*

Proof. Suppose that $B^3(\mathbb{F})$. We let $\mathbb{Q} = [\mathcal{Q}_1, \ldots, \mathcal{Q}_n]$, where $\mathcal{Q}_i = \overline{\mathcal{F}_i}$ is the canonical quorum system of $\mathcal{F}_i$, and show that $\mathbb{Q}$ is an asymmetric quorum system. Indeed, let $Q_i \in \mathcal{Q}_i$, $Q_j \in \mathcal{Q}_j$, and $F_{ij} \in \mathcal{F}_i^* \cap \mathcal{F}_j^*$ for any i and j. Then $F_i = \mathcal{P} \setminus Q_i \in \mathcal{F}_i$ and $F_j = \mathcal{P} \setminus Q_j \in \mathcal{F}_j$ by construction, and therefore, $F_i \cup F_j \cup F_{ij} \neq \mathcal{P}$. This means there is some $p_k \in \mathcal{P} \setminus (F_i \cup F_j \cup F_{ij})$. This implies in turn that $p_k \in Q_i \cap Q_j$ but $p_k \notin F_{ij}$ and proves the consistency condition. The availability property holds by construction of the canonical quorum systems.

To show the reverse direction, let $\mathbb{Q}$ be a candidate asymmetric Byzantine quorum system for $\mathbb{F}$ that satisfies availability and assume towards a contradiction that $B^3(\mathbb{F})$ does not hold. We show that consistency cannot be fulfilled for $\mathbb{Q}$. By our assumption there are sets F_i, F_j, F_{ij} in $\mathbb{F}$ such that $F_i \cup F_j \cup F_{ij} = \mathcal{P}$, which is the same as $\mathcal{P} \setminus (F_i \cup F_j) \subseteq F_{ij}$. The availability condition for $\mathbb{Q}$ then implies that there are sets $Q_i \in \mathcal{Q}_i$ and $Q_j \in \mathcal{Q}_j$ with $F_i \cap Q_i = \emptyset$ and $F_j \cap Q_j = \emptyset$. Now for every $p_k \in Q_i \cap Q_j$ it holds that $p_k \notin F_i \cup F_j$ by availability and therefore $p_k \in \mathcal{P} \setminus (F_i \cup F_j)$. Taken together this means that $Q_i \cap Q_j \subseteq \mathcal{P} \setminus (F_i \cup F_j) \subseteq F_{ij}$. Hence, $\mathbb{Q}$ does not satisfy the consistency condition and the statement follows. $\square$

Kernels. Given a symmetric Byzantine quorum system $\mathcal{Q}$, we define a *kernel K* as a set of processes that overlaps with every quorum and that is minimal in this respect. Formally, $K \subseteq \mathcal{P}$ is a *kernel of $\mathcal{Q}$* if and only if
$$\forall Q \in \mathcal{Q} : K \cap Q \neq \emptyset$$
and
$$\forall K' \subsetneq K : \exists Q \in \mathcal{Q} : K \cap Q = \emptyset.$$
The *kernel system $\mathcal{K}$* of $\mathcal{Q}$ is the set of all kernels of $\mathcal{Q}$.

For example, under a threshold failure assumption where any f processes may fail and the quorums are all sets of $\left\lceil \frac{n+f+1}{2} \right\rceil$ processes, every set of $\left\lfloor \frac{n-f+1}{2} \right\rfloor$ processes is a kernel.

The definition of a kernel is related to that of a core set in the sense that for a given maximal fail-prone system $\mathcal{F}$, in the sense that its canonical quorum system $\mathcal{Q} = \overline{\mathcal{F}}$ is minimal, the kernel of $\mathcal{Q}$ is the same as the core-set system for $\mathcal{F}$.

Asymmetric core sets and kernels. Let $\mathbb{F} = [\mathcal{F}_1, \ldots, \mathcal{F}_n]$ be an asymmetric fail-prone system. An *asymmetric core set system* $\mathbb{C}$ is an array of collections of sets $[\mathcal{C}_1, \ldots, \mathcal{C}_n]$ such that each $\mathcal{C}_i$ is a core set system for the fail-prone system $\mathcal{F}_i$. We call a set $C_i \in \mathcal{C}_i$ a *core set for p_i*.

Given an asymmetric quorum system $\mathbb{Q}$ for $\mathbb{F}$, an *asymmetric kernel system* for $\mathbb{Q}$ is defined analogously as the array $\mathbb{K} = [\mathcal{K}_1, \ldots, \mathcal{K}_n]$ that consists of the kernel systems for all processes in $\mathcal{P}$ with respect to $\mathbb{Q}$; a set $K_i \in \mathcal{K}_i$ is called a *kernel for p_i*.

Naïve and wise processes. The faults or corruptions occurring in a protocol execution with an underlying quorum system imply a set F of actually *faulty processes*. However, no process knows F and this information is only available to an observer outside the system. With a traditional quorum system $\mathcal{Q}$ designed for a fail-prone set $\mathcal{F}$, the guarantees of a protocol usually hold as long as $F \in \mathcal{F}$. Recall that such protocol properties apply to *correct* processes only but not to faulty ones.

With asymmetric quorums, we further distinguish between two kinds of correct processes, depending on whether they considered F in their trust assumption or not. Given a protocol execution, the processes are therefore partitioned into three types:

Faulty: A process $p_i \in F$ is *faulty*.

Naïve: A correct process p_i for which $F \notin \mathcal{F}_i^*$ is called *naïve*.

Wise: A correct process p_i for which $F \in \mathcal{F}_i^*$ is called *wise*.

The naïve processes are new for the asymmetric case, as all processes are wise under a symmetric trust assumption. Protocols for asymmetric quorums cannot guarantee the same properties for naïve processes as for wise ones, since the naïve processes may have the "wrong friends."

Guilds. If too many processes are naïve or even fail during a protocol run with asymmetric quorums, then protocol properties cannot be ensured. A *guild* is a set of wise processes that contains at least one quorum for each member; its existence ensures liveness and consistency for typical protocols. This generalizes from protocols for symmetric quorum systems, where the correct processes in every execution form a quorum by definition. (A guild represents a group of influential and well-connected wise processes, like in the real world.)

Definition 5 (Guild). Given a fail-prone system $\mathbb{F}$, an asymmetric quorum system $\mathbb{Q}$ for $\mathbb{F}$, and a protocol execution with faulty processes F, a *guild $\mathcal{G}$ for F* satisfies two properties:

Wisdom: $\mathcal{G}$ is a set of wise processes:
$$\forall p_i \in \mathcal{G} : F \in \mathcal{F}_i^*.$$

Closure: $\mathcal{G}$ contains a quorum for each of its members:
$$\forall p_i \in \mathcal{G} : \exists Q_i \in \mathcal{Q}_i : Q_i \subseteq \mathcal{G}.$$

Superficially a guild seems similar to a "quorum" in the Stellar consensus protocol [20], but the two notions actually differ because a guild contains only wise processes and Stellar's quorums do not distinguish between naïve and wise processes.

Observe that for a specific execution, the union of two guilds is again a guild, since the union consists only of wise processes and contains again a quorum for each member. Hence, every execution with a guild contains a unique *maximal guild* $\mathcal{G}_{\max}$.

Example. We define an example asymmetric fail-prone system $\mathbb{F}_A$ on $\mathcal{P} = \{p_1, p_2, p_3, p_4, p_5\}$. The notation $\Theta^n_k(\mathcal{S})$ for a set $\mathcal{S}$ with n elements denotes the "threshold" combination operator and enumerates all subsets of $\mathcal{S}$ of cardinality k. W.l.o.g. every process trusts itself. The diagram below shows fail-prone sets as shaded areas and the notation ^n_k in front of a fail-prone set stands for k out of the n processes in the set.

$$\mathbb{F}_A: \quad \begin{aligned} \mathcal{F}_1 &= \Theta^4_1(\{p_2, p_3, p_4, p_5\}) \\ \mathcal{F}_2 &= \Theta^4_1(\{p_1, p_3, p_4, p_5\}) \\ \mathcal{F}_3 &= \Theta^2_1(\{p_1, p_2\}) * \Theta^2_1(\{p_4, p_5\}) \\ \mathcal{F}_4 &= \Theta^4_1(\{p_1, p_2, p_3, p_5\}) \\ \mathcal{F}_5 &= \{\{p_2, p_4\}\} \end{aligned}$$

The operator $*$ for two sets satisfies $\mathcal{A} * \mathcal{B} = \{A \cup B | A \in \mathcal{A}, B \in \mathcal{B}\}$.

As one can verify in a straightforward way, $B^3(\mathbb{F}_A)$ holds. Let $\mathbb{Q}_A$ be the canonical asymmetric quorum system for $\mathbb{F}_A$. Note that since $\mathbb{F}_A$ contains the fail-prone systems of p_3 and p_5 that permit two faulty processes each, this fail-prone system cannot be obtained as a special case of $\Theta^5_1(\{p_1, p_2, p_3, p_4, p_5\})$. When $F = \{p_2, p_4\}$, for example, then processes p_3 and p_5 are wise and p_1 is naïve.

5 Shared memory

This section illustrates a first application of asymmetric quorum systems: how to emulate shared memory, represented by a *register*. Maintaining a shared register reliably in a distributed system subject to faults is perhaps the most fundamental task for which ordinary, symmetric quorum systems have been introduced, in the models with crashes [13] and with Byzantine faults [19]. After presenting definitions and two protocols to implement a register with asymmetric quorums, this section also explains why federated Byzantine quorum systems according to Stellar [20] fail to directly emulate shared memory.

5.1 Definitions

Operations and precedence. For the particular *shared-object* functionalities considered here, the processes interact with an object Λ through *operations* provided by Λ. Operations on objects take time and are represented by two events occurring at a process, an *invocation* and a *response*. The *history* of an execution σ consists of the sequence of invocations and responses of Λ occurring in σ. An operation is *complete* in a history if it has a matching response.

An operation o *precedes* another operation o' in a sequence of events σ, denoted $o <_\sigma o'$, whenever o completes before o' is invoked in σ. A sequence of events π *preserves the real-time order* of a history σ if for every two operations o and o' in π, if $o <_\sigma o'$ then $o <_\pi o'$. Two operations are *concurrent* if neither one of them precedes the other. A sequence of events is *sequential* if it does not contain concurrent operations. An execution on a shared object is *well-formed* if the events at each process are alternating invocations and matching responses, starting with an invocation.

Semantics. A *register* with domain $\mathcal{X}$ provides two operations: *write(x)*, which is parameterized by a value $x \in \mathcal{X}$ and outputs a token ACK when it completes; and *read*, which takes no parameter for

invocation but outputs a value $x \in \mathcal{X}$ upon completion.

We consider a *single-writer* (or *SW*) register, where only a designated process p_w may invoke *write*, and permit *multiple readers* (or *MR*), that is, every process may execute a *read* operation. The register is initialized with a special value x_0, which is written by an imaginary *write* operation that occurs before any process invokes operations. We consider *regular* semantics under concurrent access [17]; the extension to other forms of concurrent memory, including an atomic register, proceeds analogously.

It is customary in the literature to assume that the writer and reader processes are correct; with asymmetric quorums we assume explicitly that readers and writers are *wise*. We illustrate below why one cannot extend the guarantees of the register to naïve processes.

Definition 6 (Asymmetric Byzantine SWMR regular register). A protocol emulating an *asymmetric SWMR regular register* satisfies:

Liveness: If a wise process p invokes an operation on the register, p eventually completes the operation.

Safety: Every *read* operation of a wise process that is not concurrent with a *write* returns the value written by the most recent, preceding *write* of a wise process; furthermore, a *read* operation of a wise process concurrent with a *write* of a wise process may also return the value that is written concurrently.

5.2 Protocol with authenticated data

In Algorithm 1, we describe a protocol for emulating a regular SWMR register with an asymmetric Byzantine quorum system, for a designated writer p_w and a reader $p_r \in \mathcal{P}$. The protocol uses *data authentication* implemented with digital signatures. This protocol is the same as the classic one of Malkhi and Reiter [19] that uses a Byzantine dissemination quorum system and where processes send messages to each other over point-to-point links. The difference lies in the individual choices of quorums by the processes and that it ensures safety and liveness for wise processes.

In the register emulation, the writer p_w obtains ACK messages from all processes in a quorum $Q_w \in \mathcal{Q}_w$; likewise, the reader p_r waits for a VALUE message carrying a value/timestamp pair from every process in a quorum $Q_r \in \mathcal{Q}_r$ of the reader.

The function *highestval*(S) takes a set of timestamp/value pairs S as input and outputs the value in the pair with the largest timestamp, i.e., v such that $(ts, v) \in S$ and $\forall (ts', v') \in S : ts' < ts \vee (ts', v') = (ts, v)$. Note that this v is unique in Algorithm 1 because p_w is correct. The protocol uses digital signatures, modeled by operations $sign_i$ and $verify_i$, as introduced earlier.

Theorem 3. *Algorithm 1 emulates an asymmetric Byzantine SWMR regular register.*

Proof. First we show liveness for wise writer p_w and reader p_r, respectively. Since p_w is wise by assumption, $F \in \mathcal{F}_w{}^*$, and by the availability condition of the quorum system there is $Q_w \in \mathcal{Q}_w$ with $F \cap Q_w = \emptyset$. Therefore, the writer will receive sufficiently many [ACK] messages and the *write* will return. As p_r is wise, $F \in \mathcal{F}_r{}^*$, and by the analogous condition, there is $Q_r \in \mathcal{Q}_r$ with $F \cap Q_r = \emptyset$. Because p_w is correct and by the properties of the signature scheme, all responses from processes $p_j \in Q_r$ satisfy the checks and *read* returns.

Regarding safety, it is easy to observe that any value output by *read* has been written in some preceding or concurrent *write* operation, and this even holds for naïve readers and writers. This follows from the properties of the signature scheme; *read* verifies the signature and outputs only values with a valid signature produced by p_w.

We now argue that when both the writer and the reader are wise, then *read* outputs a value of either the last preceding *write* or a concurrent *write* and the protocol satisfies safety for a regular register. On a high level, note that $F \in \mathcal{F}_w{}^* \cap \mathcal{F}_r{}^*$ since both are wise. So if p_w writes to a quorum $Q_w \in \mathcal{Q}_w$ and p_r reads from a quorum $Q_r \in \mathcal{Q}_r$, then by consistency of the quorum system $Q_w \cap Q_r \not\subseteq F$ because p_w and p_r are wise. Hence, there is some correct $p_i \in Q_w \cap Q_r$ that received the most recently written value from p_w and returns it to p_r. $\square$

Algorithm 1 Emulation of an asymmetric SWMR regular register (process p_i).

State
> wts: sequence number of write operations, stored only by writer p_w
> rid: identifier of read operations, used only by reader
> ts, v, σ: current state stored by p_i: timestamp, value, signature

upon invocation *write(v)* **do**　　　　　　　　　　　　　　　　　　　　// only if p_i is writer p_w
> $wts \leftarrow wts + 1$
> $\sigma \leftarrow_\$ sign_w(\text{WRITE}\|w\|wts\|v)$
> send message $[\text{WRITE}, wts, v, \sigma]$ to all $p_j \in \mathcal{P}$
> **wait for** receiving a message $[\text{ACK}]$ from all processes in some quorum $Q_w \in \mathcal{Q}_w$

upon invocation *read* **do**　　　　　　　　　　　　　　　　　　　　　// only if p_i is reader p_r
> $rid \leftarrow rid + 1$
> send message $[\text{READ}, rid]$ to all $p_j \in \mathcal{P}$
> **wait for** receiving messages $[\text{VALUE}, r_j, ts_j, v_j, \sigma_j]$ from all processes in some $Q_r \in \mathcal{Q}_r$ **such that**
> 　　$r_j = rid$ **and** $verify_w(\sigma_j, \text{WRITE}\|w\|ts\|v_j)$
> **return** $highestval(\{(ts_j, v_j)|j \in Q_r\}$

upon receiving a message $[\text{WRITE}, ts', v', \sigma']$ from p_w **do**　　　　　　　　　// every process
> **if** $ts' > ts$ **then**
> 　　$(ts, v, \sigma) \leftarrow (ts', v', \sigma')$
> send message $[\text{ACK}]$ to p_w

upon receiving a message $[\text{READ}, r]$ from p_r **do**　　　　　　　　　　　// every process
> send message $[\text{VALUE}, r, ts, v, \sigma]$ to p_r

Example.　We show why the guarantees of this protocol with asymmetric quorums hold only for wise readers and writers. Consider $\mathbb{Q}_A$ from the last section and an execution in which p_2 and p_4 are faulty, and therefore p_1 is naïve and p_3 and p_5 are wise. A quorum for p_1 consists of p_1 and three processes in $\{p_2, \ldots, p_5\}$; moreover, every process set that contains p_3, one of $\{p_1, p_2\}$ and one of $\{p_4, p_5\}$ is a quorum for p_3.

We illustrate that if naïve p_1 writes, then a wise reader p_3 may violate safety. Suppose that all correct processes, especially p_3, store timestamp/value/signature triples from an operation that has terminated and that wrote x. When p_1 invokes *write(u)*, it obtains $[\text{ACK}]$ messages from all processes except p_3. This is a quorum for p_1. Then p_3 runs a *read* operation and receives the outdated values representing x from itself (p_3 is correct but has not been involved in writing u) and also from the faulty p_2 and p_4. Hence, p_3 outputs x instead of u.

Analogously, with the same setup of every process initially storing a representation of x but with wise p_3 as writer, suppose p_3 executes *write(u)*. It obtains $[\text{ACK}]$ messages from p_2, p_3, and p_4 and terminates. When p_1 subsequently invokes *read* and receives values representing x, from correct p_1 and p_5 and from faulty p_2 and p_4, then p_1 outputs x instead of y and violates safety as a naïve reader.

Since the sample operations are not concurrent, the implication actually holds also for registers with only safe semantics.

5.3　Double-write protocol without data authentication

This section describes a second protocol emulating an asymmetric Byzantine SWMR regular register. In contrast to the previous protocol, it does not use digital signatures for authenticating the data to the reader. Our algorithm generalizes the construction of Abraham et al. [1] and also assumes that only a finite number of write operations occur (*FW-termination*). Furthermore, this algorithm illustrates the use of asymmetric core set systems in the context of an asymmetric-trust protocol.

Algorithm 2 Double-write emulation of an asymmetric SWMR regular register (process p_i).
___

State

    wts: sequence number of write operations, stored only by writer p_w

    rid: identifier of read operations, used only by reader

    pts, *pv*, *ts*, *v*: current state stored by p_i: pre-written timestamp and value, written timestamp and value

upon invocation *write(v)* **do**   // only if p_i is writer p_w

    $wts \leftarrow wts + 1$

    send message $[\text{PREWRITE}, wts, v]$ to all $p_j \in \mathcal{P}$

    wait for receiving a message $[\text{PREACK}]$ from all processes in some quorum $Q_w \in \mathcal{Q}_w$

    send message $[\text{WRITE}, wts, v]$ to all $p_j \in \mathcal{P}$

    wait for receiving a message $[\text{ACK}]$ from all processes in some quorum $Q_w \in \mathcal{Q}_w$

upon invocation *read* **do**   // only if p_i is reader p_r

    $rid \leftarrow rid + 1$

    send message $[\text{READ}, rid]$ to all $p_j \in \mathcal{P}$

upon receiving a message $[\text{VALUE}, r_j, pts_j, pv_j, ts_j, v_j]$ from p_j **such that**   // only if p_i is reader p_r

        $r_j = rid \wedge \big(pts_p = ts_p + 1 \vee (pts, pv) = (ts, v)\big)$ **do**

    $readlist[j] \leftarrow (pts_j, pv_j, ts_j, v_j)$

    if there exist ts^*, v^*, a core set $C_r \in \mathcal{C}_r$ for p_r, and a quorum $Q_r \in \mathcal{Q}_r$ for p_r **such that**

        $C_r \subseteq \big\{p_k | readlist[k] = (pts_k, pv_k, ts_k, v_k)\big\} \wedge \big((pts_k, pv_k) = (ts^*, v^*) \vee (ts_k, v_k) = (ts^*, v^*)\big)\big\}$ **and**

        $Q_r = \big\{p_k | readlist[k] = (pts_k, pv_k, ts_k, v_k)$

                $\wedge \big((ts_k < ts^*) \vee (pts_k, pv_k) = (ts^*, v^*) \vee (ts_k, v_k) = (ts^*, v^*)\big)\big\}$ **then**

        return v^*

    else

        send message $[\text{READ}, rid]$ to all $p_j \in \mathcal{P}$

upon receiving a message $[\text{PREWRITE}, ts', v']$ from p_w **such that** $ts' = pts + 1 \wedge pts = ts$ **do**

    $(pts, pv) \leftarrow (ts', v')$

    send message $[\text{PREACK}]$ to p_w

upon receiving a message $[\text{WRITE}, ts', v']$ from p_w **such that** $ts' = pts \wedge v' = pv$ **do**

    $(ts, v) \leftarrow (ts', v')$

    send message $[\text{ACK}]$ to p_w

upon receiving a message $[\text{READ}, r]$ from p_r **do**

    send message $[\text{VALUE}, r, pts, pv, ts, v]$ to p_r
___

Theorem 4. *Algorithm 2 emulates an asymmetric Byzantine SWMR regular register, provided there are only finitely many write operations.*

Proof. We first establish safety when the writer p_w and the reader p_r are wise. In that case, $F \in \mathcal{F}_w^* \cap \mathcal{F}_r^*$. During in a *write* operation, p_w has received PREACK and ACK messages from $Q_w \in \mathcal{Q}_i$ and $Q_w' \in \mathcal{Q}_i$, respectively, and for all $Q_r \in \mathcal{Q}_r$ it holds that $Q_w \cap Q_r \not\subseteq F$ and $Q_w' \cap Q_r \not\subseteq F$.

We now argue that any pair (ts^*, v^*) returned by p_r was written by p_w either in a preceding or a concurrent *write*. From the condition on the core set C_r and (ts^*, v^*) it follows that at least one correct process exists in C_r that stores (ts^*, v^*) as a pre-written or as a written value. Thus, the pair was written by p_w before.

Next we argue that for every completed *write*(v^*) operation, in which p_w has sent $[\text{WRITE}, wts, v^*]$, and for any subsequent *read* operation that selects (ts^*, v^*) and returns v^*, it must hold $wts \leq ts^*$. Namely, the condition on Q_r implies that $ts^* \geq ts_k$ for all $p_k \in Q_r$. By the consistency of the quorum system, it holds that $Q_w' \cap Q_r \not\subseteq F$, so there is a correct process $p_\ell \in Q_w' \cap Q_r$ that has sent ts_ℓ to p_r. Then $ts^* \geq ts_\ell \geq wts$ follows because the timestamp variable of p_ℓ only increases.

The combination of the above two paragraphs implies that for *read* operations that are not concurrent with any *write*, the pair (ts^*, v^*) chosen by *read* was actually written in the immediately preceding *write*. If the *read* operation occurs concurrently with a *write*, then the pair (ts^*, v^*) chosen by *read* may also originate from the concurrent *write*. This establishes the safety property of the SWMR regular register.

We now show liveness. First, if p_w is wise, then there exists a quorum $Q_w \in \mathcal{Q}_w$ such that $Q_w \cap F = \emptyset$. Second, any correct process will eventually receive all $[\text{PREWRITE}, wts, v]$ and $[\text{WRITE}, wts, v]$ messages sent by p_w and process them in the correct order by the assumption of FIFO links. This means that p_w will receive $[\text{PREACK}]$ and $[\text{ACK}]$ messages, respectively, from all processes in one of its quorums, since at least the processes in Q_w will eventually send those.

Liveness for the reader p_r is shown under the condition that p_r is wise and that the *read* operation is concurrent with only finitely many *write* operations. The latter condition implies that there is one last *write* operation that is initiated, but does not necessarily terminate, while *read* is active.

By the assumption that p_w is correct and because messages are received in FIFO order, all messages of that last *write* operation will eventually arrive at the correct processes. Notice also that p_r simply repeats its steps until it succeeds and returns a value that fulfills the condition. Hence, there is a time after which all correct processes reply with VALUE messages that contain pre-written and written timestamp/value pairs from that last operation. It is easy to see that there exist a core set and a quorum for p_r that satisfy the condition and the reader returns. In conclusion, the algorithm emulates an asymmetric regular SWMR register, where liveness holds only for finitely many write operations. $\square$

5.4 SWMR register emulation with a federated Byzantine quorum system

As stated in Section 2, Stellar's consensus protocol introduces federated Byzantine quorum systems (FBQS) for expressing flexible trust. Each process specifies a set of *quorum slices*, which are subsets of $\mathcal{P}$. When process "hears such a quorum slice of processes assert a statement, it assumes no functioning process will ever contradict that statement" [20].

Consider then the example of a tiered quorum system that serves to explain FBQS [20, Fig. 3]: It consists of three tiers:

- A *top* tier with processes p_1, p_2, p_3, p_4. The slices for p_i (with $i = 1, \ldots, 4$) are all sets of three out of the four processes that contain p_i as well.

- At a *middle* tier, the slices for processes p_5, p_6, p_7, p_8 consist of the process itself plus any two processes of the top tier.

- The *bottom* tier with processes p_9, p_{10}. A slice for $p_i \in \{p_9, p_{10}\}$ contains p_i and two nodes of the middle tier.

Suppose one uses this FBQS in a protocol like Algorithm 1, by interpreting a quorum for p_r and p_w, respectively, as one of their respective quorum slices. The resulting protocol would violate the satisfy

of the register because p_5, say, acting as the writer of a value u might have received ACK messages from $\{p_1, p_2, p_5\}$ only and terminate. Processes p_3 and p_4 store a previously written value x and would respond to a reader p_8 with $x \neq u$. As these processes form a quorum slice $\{p_3, p_4, p_8\}$ for p_8, it would read x. This implies either that new, different protocols are needed for a register emulation with FBQS, or that the FBQS concept is not appropriate for generalizing the established notions.

6 Broadcast

This section shows how to implement two *broadcast primitives* tolerating Byzantine faults with asymmetric quorums. Recall from the standard literature [14, 8, 5] that reliable broadcasts offer basic forms of reliable message delivery and consistency, but they do not impose a total order on delivered messages (as this is equivalent to consensus). The Byzantine broadcast primitives described here, *consistent broadcast* and *reliable broadcast*, are prominent building blocks for many more advanced protocols.

With both primitives, the sender process may broadcast a message m by invoking *broadcast(m)*; the broadcast abstraction outputs m to the local application on the process through a *deliver(m)* event. Moreover, the notions of broadcast considered in this section are intended to deliver only one message per instance. Every instance has a distinct (implicit) label and a designated sender p_s. With standard multiplexing techniques one can extend this to a protocol in which all processes may broadcast messages repeatedly [5].

Byzantine consistent broadcast.   The simplest such primitive, which has been called *(Byzantine) consistent broadcast* [5], ensures only that those correct processes which deliver a message agree on the content of the message, but they may not agree on termination. In other words, the primitive does not enforce "reliability" such that a correct process outputs a message if and only if all other correct processes produce an output. The events in its interface are denoted by *c-broadcast* and *c-deliver*.

The change of the definition towards asymmetric quorums affects most of its guarantees, which hold only for wise processes but not for all correct ones. This is similar to the definition of a register in Section 5.

Definition 7 (Asymmetric Byzantine consistent broadcast). A protocol for *asymmetric (Byzantine) consistent broadcast* satisfies:

Validity: If a correct process p_s *c-broadcasts* a message m, then all wise processes eventually *c-deliver* m.

Consistency: If some wise process *c-delivers* m and another wise process *c-delivers* m', then $m = m'$.

Integrity: For any message m, every correct process *c-delivers* m at most once. Moreover, if the sender p_s is correct and the receiver is wise, then m was previously *c-broadcast* by p_s.

The following protocol is an extension of "authenticated echo broadcast" [5], which goes back to Srikanth and Toueg [25]. It is a building block found in many Byzantine fault-tolerant protocols with greater complexity. The adaptation for asymmetric quorums is straightforward: Every process considers its own quorums before *c-delivering* the message.

Theorem 5. *Algorithm 3 implements asymmetric Byzantine consistent broadcast.*

Proof. For the *validity* property, it is straightforward to see that every correct process sends [ECHO, m]. According to the availability condition for the quorum system $\mathcal{Q}_i$ of every wise process p_i and because $F \subseteq F_i$ for some $F_i \in \mathcal{F}_i$, there exists some quorum Q_i for p_i of correct processes that echo m to p_i. Hence, p_i *c-delivers* m.

To show *consistency*, suppose that some wise process p_i has *c-delivered* m_i because of [ECHO, m_i] messages from a quorum Q_i and another wise p_j has received [ECHO, m_j] from all processes in $Q_j \in \mathcal{Q}_j$. By the consistency property of $\mathbb{Q}$ it holds $Q_i \cap Q_j \not\subseteq F$; let p_k be this process in $Q_i \cap Q_j$ that is not in F. Because p_k is correct, p_i and p_j received the same message from p_k and $m_i = m_j$.

Algorithm 3 Asymmetric Byzantine consistent broadcast protocol with sender p_s (process p_i)

State
    sentecho ← FALSE: indicates whether p_i has sent ECHO
    echos ← $[\bot]^N$: collects the received ECHO messages from other processes
    delivered ← FALSE: indicates whether p_i has delivered a message

upon invocation *c-broadcast*(m) **do**
    send message $[\text{SEND}, m]$ to all $p_j \in \mathcal{P}$

upon receiving a message $[\text{SEND}, m]$ from p_s **such that** ¬*sentecho* **do**
    sentecho ← TRUE
    send message $[\text{ECHO}, m]$ to all $p_j \in \mathcal{P}$

upon receiving a message $[\text{ECHO}, m]$ from p_j **do**
    if *echos*$[j] = \bot$ **then**
        echos$[j] \leftarrow m$

upon exists $m \neq \bot$ **such that** $\{p_j \in \mathcal{P} | echos[j] = m\} \in \mathcal{Q}_i$ **and** ¬*delivered* **do**
    delivered ← TRUE
    output *c-deliver*(m)

The first condition of *integrity* is guaranteed by using the *delivered* flag; the second condition holds because because the receiver is wise, and therefore the quorum that it uses for the decision contains some correct processes that have sent $[\text{ECHO}, m]$ with the message m they obtained from p_s according to the protocol. $\qquad\square$

Example. We illustrate the broadcast protocols using a six-process asymmetric quorum system $\mathbb{Q}_B$, defined through its fail-prone system $\mathbb{F}_B$. In $\mathbb{F}_B$, as shown below, for p_1, p_2, and p_3, each process always trusts itself, some other process of $\{p_1, p_2, p_3\}$ and one further process in $\{p_1, \ldots, p_5\}$. Process p_4 and p_5 each assumes that at most one other process of $\{p_1, \ldots, p_5\}$ may fail (excluding itself). Moreover, none of the processes p_1, …, p_5 ever trusts p_6. For p_6 itself, the fail-prone sets consist of p_1 and one process of $\{p_2, p_3, p_4, p_5\}$.

$$
\mathbb{F}_B: \quad
\begin{aligned}
\mathcal{F}_1 &= \Theta_2^3(\{p_2, p_4, p_5\}) * \{\{p_6\}\} \\
\mathcal{F}_2 &= \Theta_2^3(\{p_3, p_4, p_5\}) * \{\{p_6\}\} \\
\mathcal{F}_3 &= \Theta_2^3(\{p_1, p_4, p_5\}) * \{\{p_6\}\} \\
\mathcal{F}_4 &= \Theta_1^4(\{p_1, p_2, p_3, p_5\}) * \{\{p_6\}\} \\
\mathcal{F}_5 &= \Theta_1^4(\{p_1, p_2, p_3, p_4\}) * \{\{p_6\}\} \\
\mathcal{F}_6 &= \{\{p_1\}\} * \Theta_1^4(\{p_2, p_3, p_4, p_5\})
\end{aligned}
$$

It is easy to verify that $B^3(\mathbb{F}_B)$ holds; hence, let $\mathbb{Q}_B$ be the canonical quorum system of $\mathbb{F}_B$. Again, there is no reliable process that could be trusted by all and $\mathbb{Q}_B$ is not a special case of a symmetric threshold Byzantine quorum system. With $F = \{p_1, p_5\}$, for instance, processes p_3 and p_6 are wise, p_2 and p_4 are naïve, and there is no guild.

Consider an execution of Algorithm 3 with sender p_4^* and $F = \{p_4^*, p_5^*\}$ (we write p_4^* and p_5^* to denote that they are faulty). This means processes p_1, p_2, p_3 are wise and form a guild because $\{p_1, p_2, p_3\}$ is a quorum for all three; furthermore, p_6 is naïve.

$$
\begin{array}{ll}
p_1: \ [\text{ECHO}, x] \to \mathcal{P} & p_1: \textit{c-deliver}(x) \\
p_2: \ [\text{ECHO}, u] \to \mathcal{P} & p_2: \text{no quorum of } [\text{ECHO}] \text{ in } \mathcal{Q}_2
\end{array}
$$

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$$p_3: \quad [\text{ECHO}, x] \to \mathcal{P} \qquad\qquad p_3: \text{no quorum of } [\text{ECHO}] \text{ in } \mathcal{Q}_3$$

$$p_4^*: \begin{cases} [\text{SEND}, x] \to p_1, p_3 \\ [\text{SEND}, u] \to p_2, p_6 \end{cases} \qquad p_4^*: \begin{cases} [\text{ECHO}, x] \to p_1 \\ [\text{ECHO}, u] \to p_6 \end{cases}$$

$$p_5^*: \begin{cases} [\text{ECHO}, x] \to p_1 \\ [\text{ECHO}, u] \to p_6 \end{cases}$$

$$p_6: \quad [\text{ECHO}, u] \to \mathcal{P} \qquad\qquad p_6: \textit{c-deliver}(u)$$

Hence, p_1 receives $[\text{ECHO}, x]$ from $\{p_1, p_3, p_4^*, p_5^*\} \in \mathcal{Q}_1$ and *c-delivers* x, but the other wise processes do not terminate. The naïve p_6 gets $[\text{ECHO}, u]$ from $\{p_2, p_4^*, p_5^*, p_6\} \in \mathcal{Q}_6$ and *c-delivers* $u \neq x$.

Byzantine reliable broadcast. In the symmetric setting, consistent broadcast has been extended to *(Byzantine) reliable broadcast* in a well-known way to address the disagreement about termination among the correct processes [5]. This primitive has the same interface as consistent broadcast, except that its events are called *r-broadcast* and *r-deliver* instead of *c-broadcast* and *c-deliver*, respectively.

A reliable broadcast protocol also has all properties of consistent broadcast, but satisfies the additional *totality* property stated next. Taken together, *consistency* and *totality* imply a notion of *agreement*, similar to what is also ensured by many crash-tolerant broadcast primitives. Analogously to the earlier primitives with asymmetric trust, our notion of an *asymmetric reliable broadcast*, defined next, ensures agreement on termination only for the wise processes, and moreover only for executions with a guild. Also the *validity* of Definition 7 is extended by the assumption of a guild. Intuitively, one needs a guild because the wise processes that make up the guild are self-sufficient, in the sense that the guild contains a quorum of wise processes for each of its members; without that, there may not be enough wise processes.

Definition 8 (Asymmetric Byzantine reliable broadcast). A protocol for *asymmetric (Byzantine) reliable broadcast* is a protocol for asymmetric Byzantine consistent broadcast with the revised *validity* condition and the additional *totality* condition stated next:

Validity: In all executions with a guild, if a correct process p_s *c-broadcasts* a message m, then all processes in the maximal guild eventually *c-deliver* m.

Totality: In all executions with a guild, if a wise process *r-delivers* some message, then all processes in the maximal guild eventually *r-deliver* a message.

The protocol of Bracha [4] implements reliable broadcast subject to Byzantine faults with symmetric trust. It augments the authenticated echo broadcast from Algorithm 3 with a second all-to-all exchange, where each process is supposed to send READY with the payload message that will be *r-delivered*. When a process receives the same m in $2f + 1$ READY messages, in the symmetric model with a threshold Byzantine quorum system, then it *r-delivers* m. Also, a process that receives $[\text{READY}, m]$ from $f + 1$ distinct processes and has not yet sent a READY chimes in and also sends $[\text{READY}, m]$. These two steps ensure totality.

For asymmetric quorums, the conditions of a process p_i receiving $f + 1$ and $2f + 1$ equal READY messages, respectively, generalize to receiving the same message from a kernel for p_i and from a quorum for p_i. Intuitively, the change in the first condition ensures that when a wise process p_i receives the same $[\text{READY}, m]$ message from a kernel for itself, then this kernel intersects with some quorum of wise processes. Therefore, at least one wise process has sent $[\text{READY}, m]$ and p_i can safely adopt m. Furthermore, the change in the second condition relies on the properties of asymmetric quorums to guarantee that whenever some wise process has *r-delivered* m, then enough correct processes have sent a $[\text{READY}, m]$ message such that all wise processes eventually receive a kernel of $[\text{READY}, m]$ messages and also send $[\text{READY}, m]$.

Applying these changes to Bracha's protocol results in the asymmetric reliable broadcast protocol shown in Algorithm 4. Note that it strictly extends Algorithm 3 by the additional round of READY messages, in the same way as for symmetric trust. For instance, when instantiated with the symmetric

Algorithm 4 Asymmetric Byzantine reliable broadcast protocol with sender p_s (process p_i)

State
    sentecho ← FALSE: indicates whether p_i has sent ECHO
    echos ← $[\bot]^N$: collects the received ECHO messages from other processes
    sentready ← FALSE: indicates whether p_i has sent READY
    readys ← $[\bot]^N$: collects the received READY messages from other processes
    delivered ← FALSE: indicates whether p_i has delivered a message

upon invocation *r-broadcast*(m) **do**
    send message $[\text{SEND}, m]$ to all $p_j \in \mathcal{P}$

upon receiving a message $[\text{SEND}, m]$ from p_s **such that** ¬*sentecho* **do**
    sentecho ← TRUE
    send message $[\text{ECHO}, m]$ to all $p_j \in \mathcal{P}$

upon receiving a message $[\text{ECHO}, m]$ from p_j **do**
    if *echos*$[j] = \bot$ **then**
        echos$[j] \leftarrow m$

upon exists $m \neq \bot$ **such that** $\{p_j \in \mathcal{P}|echos[j] = m\} \in \mathcal{Q}_i$ **and** ¬*sentready* **do**   // a quorum for p_i
    sentready ← TRUE
    send message $[\text{READY}, m]$ to all $p_j \in \mathcal{P}$

upon exists $m \neq \bot$ **such that** $\{p_j \in \mathcal{P}|readys[j] = m\} \in \mathcal{K}_i$ **and** ¬*sentready* **do**   // a kernel for p_i
    sentready ← TRUE
    send message $[\text{READY}, m]$ to all $p_j \in \mathcal{P}$

upon receiving a message $[\text{READY}, m]$ from p_j **do**
    if *readys*$[j] = \bot$ **then**
        readys$[j] \leftarrow m$

upon exists $m \neq \bot$ **such that** $\{p_j \in \mathcal{P}|readys[j] = m\} \in \mathcal{Q}_i$ **and** ¬*delivered* **do**
    delivered ← TRUE
    output *r-deliver(m)*

---

threshold quorum system of $n = 3f + 1$ processes, of which f may fail, then every set of $f + 1$ processes is a kernel.

In Algorithm 4, there are two conditions that let a correct p_i send $[\text{READY}, m]$: either when receiving a quorum of $[\text{ECHO}, m]$ messages for itself or after obtaining a kernel for itself of $[\text{READY}, m]$. For the first case, we say p_i *sends* READY *after* ECHO; for the second case, we say p_i *sends* READY *after* READY.

Lemma 6. *In any execution with a guild, there exists a unique m such that whenever a wise process sends a* READY *message, it contains m.*

Proof. Consider first all READY messages sent by wise processes after ECHO. The fact that Algorithm 4 extends Algorithm 3 achieving consistent broadcast, combined with the consistency property in Definition 7 implies immediately that the lemma holds for READY messages sent by wise processes after ECHO.

For the second case, consider the first wise process p_i which sends $[\text{READY}, m']$ after READY. From the protocol it follows that all processes in some kernel $K_i \in \mathcal{K}_i$, which triggered p_i to send $[\text{READY}, m']$, have sent $[\text{READY}, m']$ to p_i. Moreover, according to the definition of a kernel, K_i overlaps with all quorums for p_i. Since there exists a guild in the execution, at least one of the quorums for p_i consists exclusively of wise processes. Hence, some wise process p_j has sent $[\text{READY}, m']$ to p_i. But since p_i is the first wise process to send READY after READY, it follows that p_j sent $[\text{READY}, m']$ after ECHO; therefore, $m' = m$ from the proof in the first case. Continuing this argument inductively over all READY

messages sent after READY by wise processes, in the order these were sent, shows that all those messages contain m and establishes the lemma. $\qquad\square$

Theorem 7. *Algorithm 4 implements asymmetric Byzantine reliable broadcast.*

Proof. Recall that the *validity* property assumes there exists a guild $\mathcal{G}$. Since the sender p_s is correct and according to asymmetric quorum availability, every wise process p_i in $\mathcal{G}$ eventually receives a quorum of $[\text{ECHO}, m]$ messages for itself, containing the message m from p_s. According to the protocol, p_i therefore sends $[\text{READY}, m]$ after ECHO unless *sentready* = TRUE; if this is the case, however, p_i has already sent $[\text{READY}, m]$ after READY as ensured by Lemma 6. Hence, every process in $\mathcal{G}$ eventually sends $[\text{READY}, m]$. Then every process in $\mathcal{G}$ receives a quorum for itself of $[\text{READY}, m]$ and *r-delivers* m, as ensured by the properties of a guild and by the protocol.

To establish the *totality* condition, suppose that some wise process p_i has *r-delivered* a message m. Then it has obtained $[\text{READY}, m]$ messages from the processes in some quorum $Q_i \in \mathcal{Q}_i$. Consider any other wise process p_j. Since p_i and p_j are both wise, it holds $F \in \mathcal{F}_i^*$ and $F \in \mathcal{F}_j^*$, which implies $F \in \mathcal{F}_i^* \cap \mathcal{F}_j^*$. Then, the set $K = Q_i \setminus F$ intersects every quorum of p_j by quorum consistency and is a kernel for p_j by definition. Since K consists only of correct processes, all of them have sent $[\text{READY}, m]$ also to p_j and p_j eventually sends $[\text{READY}, m]$ as well. This implies that all wise processes eventually send $[\text{READY}, m]$ to all processes. Every process in $\mathcal{G}_{\max}$ therefore receives a quorum for itself of $[\text{READY}, m]$ and *r-delivers* m, as required for totality.

The *consistency* property follows immediately from the preceding argument and from Lemma 6, which implies that all wise processes deliver the same message.

Finally, *integrity* holds because of the *delivered* flag in the protocol and because of the argument showing validity together with Lemma 6. $\qquad\square$

Example. Consider again the protocol execution with $\mathbb{Q}_B$ introduced earlier for illustrating asymmetric consistent broadcast. Recall that $F = \{p_4^*, p_5^*\}$, the set $\{p_1, p_2, p_3\}$ is a guild, and p_6 is naïve. The start of the execution is the same as shown previously and omitted. Instead of *c-delivering* x and u, respectively, p_1 and p_6 send $[\text{READY}, x]$ and $[\text{READY}, u]$ to all processes:

...	$p_1 : [\text{READY}, x] \to \mathcal{P}$		$p_1 : \textit{r-deliver}(x)$
...	$p_2 : \text{no quorum}$	$p_2 : [\text{READY}, x] \to \mathcal{P}$	$p_2 : \textit{r-deliver}(x)$
...	$p_3 : \text{no quorum}$	$p_3 : [\text{READY}, x] \to \mathcal{P}$	$p_3 : \textit{r-deliver}(x)$
...	$p_4^* : -$		
...	$p_5^* : -$		
...	$p_6 : [\text{READY}, u] \to \mathcal{P}$	$p_6 : \text{no quorum}$	

Note that the kernel systems of processes p_1, p_2, and p_3 are $\mathcal{K}_1 = \{\{p_1\}, \{p_3\}\}$, $\mathcal{K}_2 = \{\{p_1\}, \{p_2\}\}$, and $\mathcal{K}_3 = \{\{p_2\}, \{p_3\}\}$. Hence, when p_2 receives $[\text{READY}, x]$ from p_1, it sends $[\text{READY}, x]$ in turn because $\{p_1\}$ is a kernel for p_2, and when p_3 receives this message, then it sends $[\text{READY}, x]$ because $\{p_2\}$ is a kernel for p_3.

Furthermore, since $\{p_1, p_2, p_3\}$ is the maximal guild and contains a quorum for each of its members, all three wise processes *r-deliver* x as implied by *consistency* and *totality*. The naïve p_6 does not *r-deliver* anything, however.

Remarks. Asymmetric reliable broadcast (Definition 8) ensures validity and totality only for processes in the maximal guild. On the other hand, an asymmetric consistent broadcast (Definition 7) ensures validity also for all *wise* processes. We leave it as an open problem to determine whether these guarantees can also be extended to wise processes for asymmetric reliable broadcast and the Bracha protocol. This question is equivalent to determining whether there exist any wise processes outside the maximal guild.

Another open problem concerns the conditions for reacting to READY messages in the asymmetric reliable broadcast protocol. Already in Bracha's protocol for the threshold model [4], a process (1)

sends its own READY message upon receiving $f + 1$ READY messages and (2) *r-delivers* an output upon receiving $2f + 1$ READY messages. These conditions generalize for arbitrary, non-threshold quorum systems to receiving messages (1) from any set that is guaranteed to contain at least one correct process and (2) from any set that still contains at least one process even when any two fail-prone process sets are subtracted. In Algorithm 4, in contrast, a process delivers the payload only after receiving READY messages from one of its quorums. But such a quorum (e.g., $\lceil \frac{n+f+1}{2} \rceil$ processes) may be larger than a set in the second case (e.g., $2f + 1$ processes). It remains interesting to find out whether this discrepancy is necessary.

Acknowledgments

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