

Fourier's law: $\underline{H} = -K \underline{\nabla} T(\underline{x}, t)$

1st law: $\Delta E = \Delta W + \Delta Q$
work heat

$$\Delta E = \Delta Q$$

$$\Delta E = C \rho \Delta T \quad \text{density}$$

↑
heat cap.

$$E(t) = \int d^3x \, (c_p T(x, t))$$

$$\frac{\partial E}{\partial t} = \frac{\partial}{\partial t} \int d^3x \mathcal{L}_S T(x,t) = - \oint \underline{H} \cdot \underline{dA}$$

CHANGE IN ENERGY INSIDE = HEAT FLOW ACROSS BOUNDARY

$$\frac{\partial E}{\partial t} = - \oint \underline{H} \cdot d\underline{A}$$

$$\int d^3x \frac{\partial}{\partial t} C_{ST}(x,t) = - \oint (-\kappa \underline{\nabla} T) \cdot d\underline{A} = + \int d^3x \kappa \underline{\nabla} \cdot \underline{\nabla} T$$

Gauss
divergence : $\int d^3x \nabla \cdot \underline{E} = \oint \underline{E} \cdot d\underline{A}$ } $\oint \nabla \phi \cdot d\underline{A} = \int d^3x \nabla \cdot \nabla \phi$

$$\int d^3x \frac{\partial}{\partial t} C_S T(x, t) = + \int d^3x \kappa \nabla^2 T(x, t)$$

$$\left[\frac{\partial}{\partial t} C_S T(x, t) = \kappa \nabla^2 T(x, t) \right]$$

$$\left[\frac{\partial T}{\partial t} = \frac{\kappa}{C_S} \nabla^2 T(x, t) \right]$$

heat equation

General heat/diffusion equation

$\kappa(x, t)$ ← diffusion coefficient

Fick's law

$$\frac{\partial u}{\partial t} = \nabla \cdot D(x) \nabla u$$

$$\vec{j} = D \nabla u$$

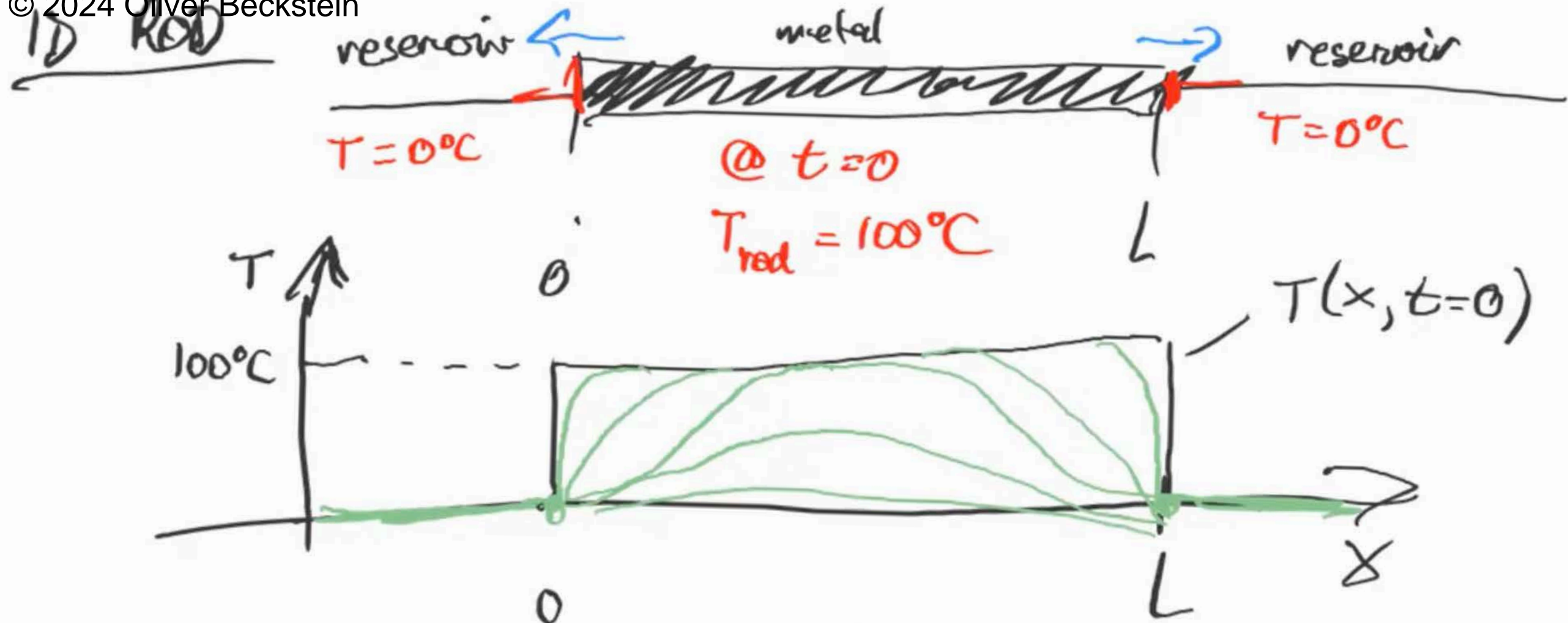
$D = \text{const}$ + mass consv.

$$\frac{\partial u}{\partial t} = D \nabla^2 u(x, t)$$

• Black-Scholes

• Schrödinger's eq.

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \psi + V\psi$$



$$T(x, t > 0) = ?$$

$$T(x, t)$$

$$T(x, t \rightarrow \infty)$$

- 1) analytically
- 2) numerically

PHY202

PHY201 (Math Methods)