ECE3522 Practicum 2: Error Rates in Digital Communications

PART I:

σ and Variance calculation

Based on the input $SNR = 5\ dB$, the standard deviation of $x = s + w$ can be calculated by finding σ, which represents the standard deviation.

$SNRdB$ is represented by the following equations:

$$SNR_{dB} = 10log_{10}(SNR)$$

$$SNR_{dB} = -20log_{10}(\sigma)$$

Therefore, σ can be calculated as

$$\sigma = 10^{-\frac{\alpha}{20}}$$

Since $SNR = 5\ dB$, $\alpha = 5$, so

$$\sigma = 10^{-\frac{5}{20}}$$

$$\sigma = 0.5623$$

Furthermore, the Variance can be stated as σ^2 and is found as $0.5623^2 = 0.316227$

Expected Value calculation

The practicum states that the signal can be received as $x = s + w$. To find the expected value based upon Gaussian distribution (μ, σ). By taking the integral of the PDF function $f_x(x) = \frac{1}{\sqrt{2\pi\sigma}} exp(-\frac{(x-\mu)^2}{2\sigma^2})$ from -∞ to ∞,

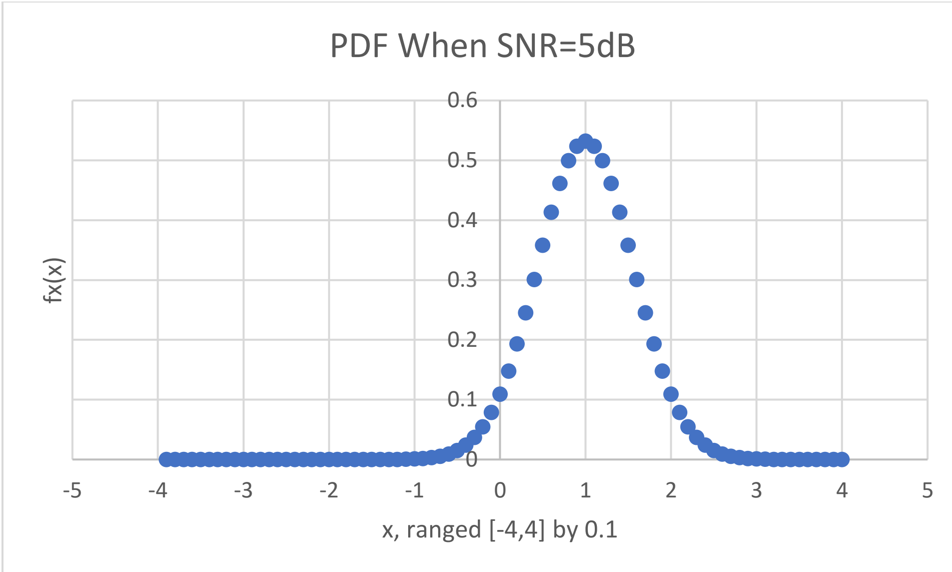
$$E[x] = \frac{1}{\sqrt{2\pi\sigma}} \int_{-\infty}^{\infty} xexp(-\frac{(x-\mu)^2}{2\sigma^2})\,dx$$

$E[x]$ simplifies out to $E[x] = \mu$. Based upon the equation $E[x] = s + E[w]$, where s is the signal and w is the additive noise distributed from 0 to σ. The expected value of the noise "w" is $\mu = 0$, therefore finding $E[x] = s + E[w] => E[x] = s + 0$, and s=1 in this case, therefore $E[x] = 1 = \mu$ for the received signal.

Analytically Sketching the PDF

To calculate the PDF of the Normal Gaussian Distribution, a range of -4 to 4, incrementing by 0.1 was given by the practicum. To obtain an accurate sketch of the PDF, I created a table within Excel where I used a simple function to start from -4 and add 0.1 until it reached 4 to define my range. From that, I inserted the PDF equation into excel, $f_x(x) = \frac{1}{\sqrt{2\pi\sigma}} exp(-\frac{(x-\mu)^2}{2\sigma^2})$ where $\mu = 1$, since this is the expected value of just the signal, $\sigma = 0.5623$, and x were the values of my range.

The values for each column can be seen within the appendix. From that, I inserted a scatter plot which yielded the following graph:



Calculating the BER

To compute the probability $P[x < 0]$, representing the Bit Error Rate (BER), first calculate the $-Z$ cutoff point from using standard gaussian distribution, which would be when $x = 0$ when the pdf yields 0.109432222, based upon my excel sheet. Finding Z, $Z = \frac{x-\mu}{\sigma} = \frac{0-1}{0.5623} = -1.7857$, since this is a negative value, finding $\Phi(Z)$ from the lookup table, which can be found within the appendix taken from Quiz 6, yields 0.9625. Finally, subtracting $1 - \Phi(Z) = 0.0377$. Confirming this in MATLAB, using $normcdf$, I entered the parameters $X = 0$, $\mu = 1$, and my calculated $\sigma = 0.5623$. MATLAB returned a value of 0.0377 which is also the BER.

PART II

By using the sample code provided within the manual, I experimentally found $\sigma = 0.5620$ yielding a 0.0003% difference, the Expected value for X as $E[x] = 1$ with 0% difference, and an experimental BER of 0.0378, which is only 0.0001% difference. This was done by the following code:

```
%Experimentally finding sigma
sigma_EX=std(received);


%Experimentally finding the Experimental Signal Value E[X]
E_X=mean((signal*2-1)+mu_w);


%Experimental BER Analysis
detect = (received > 0);                    % detected result
N_err = sum(abs(detect-signal));      % # of erroneously detected bits
BER=N_err/N;                               % Ratio of Error to total bits
```

Figure 1. Finding Experimental STD, E[X], and BER when SNR=5 dB

The resulting graph displays the relative frequency, which experimentally is the PDF:

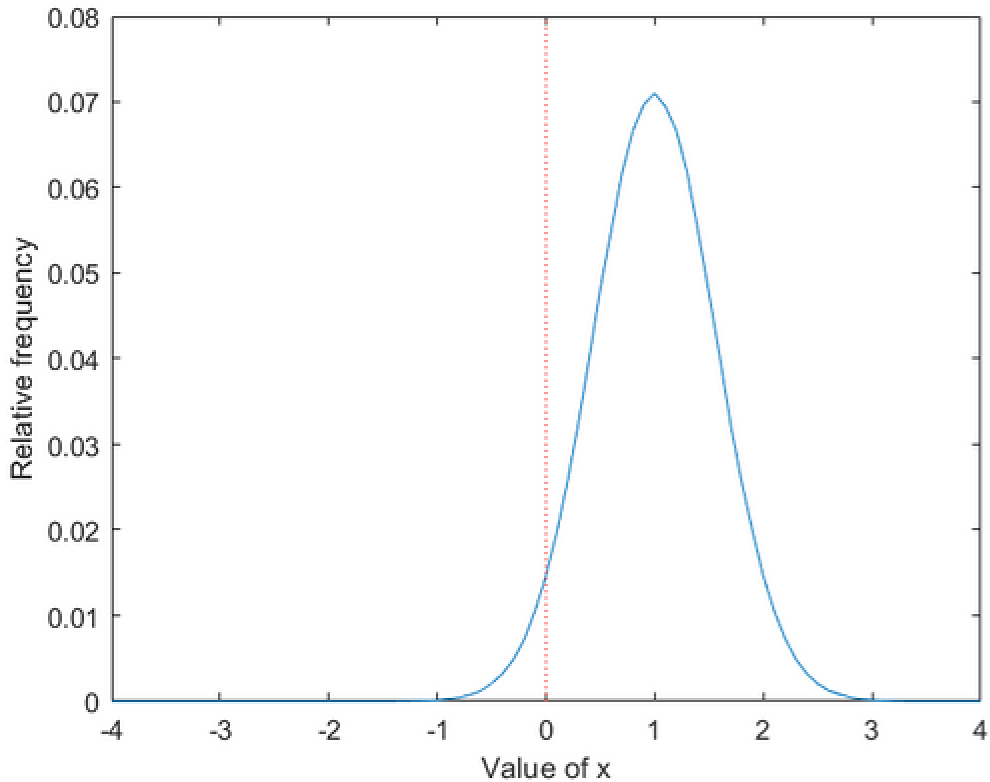


Figure 2. Relative Frequency Plot when SNR=5dB

MATLAB prints the following output for values:

```
STD(sigma) = 5.619102e-01, E[x] = 1
N = 1000000 bits transmitted; N_err = 37635 error bits received
 BER = 3.763500e-02
```

Figure 3. Experimental Values

PART III:

σ and Variance calculation

Changing the SNR input to be $10\ dB$, the calculations for σ and variance can be done similarly to Part I:

$$SNR_{dB} = 10log_{10}(SNR)$$

$$SNR_{dB} = -20log_{10}(\sigma)$$

Therefore,

$$\sigma = 10^{-\frac{\alpha}{20}}$$

$$\sigma = 10^{-\frac{10}{20}}$$

$$\sigma = 0.3162278$$

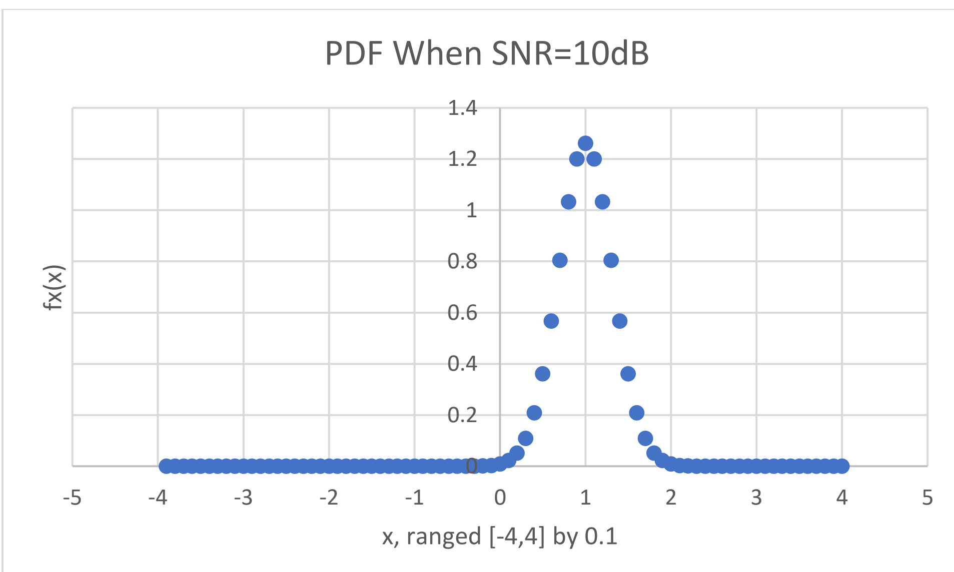
$$Var[x] = \sigma^2 = 3.162278^2 = 0.1$$

Expected Value calculation

Despite changing the SNR input, the expected value will remain at 0, since $\mu = 0$, therefore the Expected value of the signal, $E[X]$ should remain 1.

Analytically Sketching the PDF

Inserting another column into the Excel sheet, I used the same range from Part I and updated my value for σ to be 0.3162278, yielding the following graph:



Calculating the BER

Computing the probability $P[x < 0]$, I used the same method as I did in part 1.

Therefore, $Z = \frac{0-1}{0.3162278} = -3.16227$, finding the positive $\Phi(3.16)$ from the table within the lecture slides and textbook, $\Phi(3.16) = 0.9992$, then taking 1-0.9992=0.0008. Using $normcdf$ in MATLAB to further calculate this, yields 0.00078270. The difference is probably my calculator simply rounding up.

By recalculating the values for when $SNR = 10\ dB$, the σ proved to be smaller, due to the formula now raising 10 to power of $-10/20$, rather than $-5/20$. The expected value remained at 1 since this is not dependent on the SNR input. The PDF's peak values became larger compared to $SNR = 5\ dB$, which is due to the equation $f_x(x) = \frac{1}{\sqrt{2\pi}\sigma} exp(-\frac{(x-\mu)^2}{2\sigma^2})$, where σ is now 0.3162278, changing the shape of the curve to appear more narrower in shape. By changing SNR input to $10\ dB$, the BER calculation is now smaller due to the smaller denominator when finding Z.

PART IV:

By changing $SNR = 10$ and rerunning my program, I experimentally found $\sigma = 0.3159$ yielding a 0.0003% difference, the Expected value stayed the same as X as $E[x] = 1$ with 0% difference since it does not depend on the SNR input, and I obtained an experimental BER of 0.000805, which is yields a 0.000005% difference.

The resulting graph displays the relative frequency when $SNR = 10\ dB$:

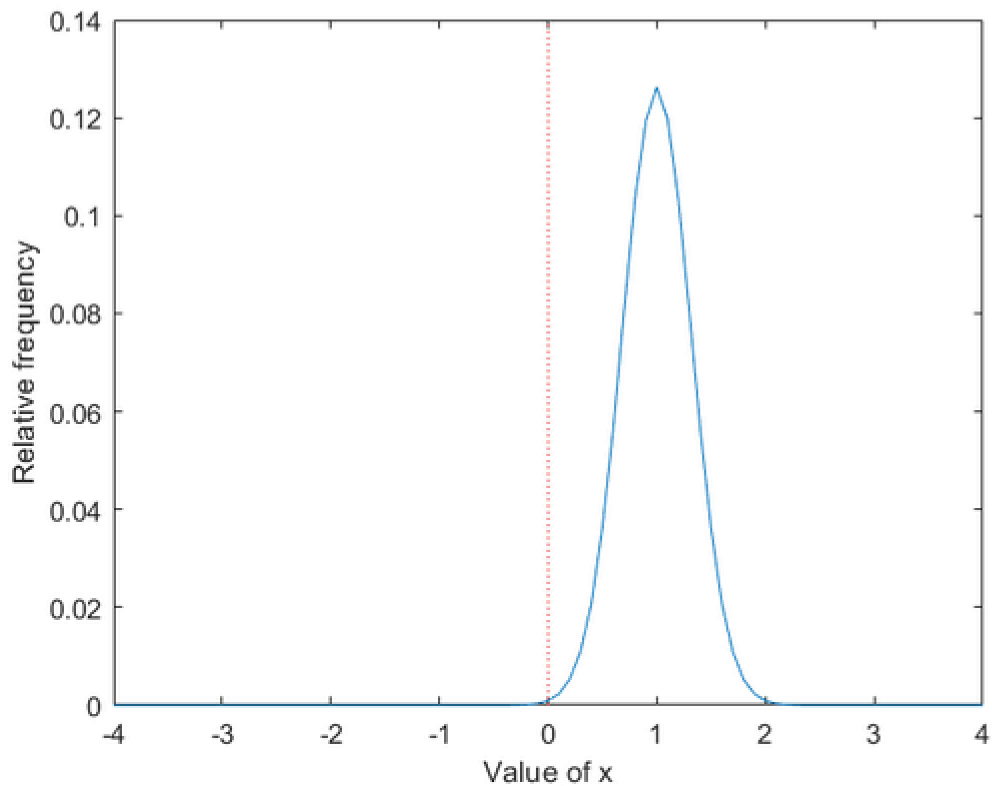


Figure 4. Relative Frequency Plot when SNR=10dB

MATLAB prints the following output for values:

```
STD(sigma) = 3.159102e-01, E[x] = 1
N = 1000000 bits transmitted; N_err = 805 error bits received
 BER = 8.050000e-04
```

Figure 5. Experimental Values when SNR=10 dB

Upon obtaining the experimental values for when $SNR = 10\ dB$, the same relationships apply as when I found the values analytically, with the σ decreasing, E[X] remaining the same, the relative frequency producing an increase in values/narrower curve, and the BER as a smaller error ratio compared to when $\sigma = 0.5623$.

Appendix:

MATLAB LiveScript code:

```
%% ========== Matlab code sample for Practicum 2 ===========
% Simulation: Error Rates in Digital Communications
% Robert Bara
%
clear
N = 1e6;                                 % number of bits transmitted


%Make 5 for Parts 1 and 2, make 10 for Parts 3 and 4
SNR_dB = 10;                               % signal-to-noise ratio in dB
sigma = 10^(-SNR_dB/20);                % noise standard deviation
mu_w=0;
mu_s=1;




BER_P=normcdf(0,mu_s,sigma);          % Analytically Calculating P(x<0) based on
sigma and Mu

signal = ones(N,1);                    % bit stream with only 1's
noise = randn(N,1)* sigma;             % additive Gaussian noise
received = (signal*2-1) + noise;           % received noisy signal


x_range = [-4:0.1:4];


% Plotting Relative Frequency
hist_x = hist(received, [-4:0.1:4]);
plot(x_range, hist_x/N);
xlabel('Value of x')
ylabel('Relative frequency')
vline([0])


%Experimentally finding sigma
sigma_EX=std(received);


%Experimentally finding the Experimental Signal Value E[X]
E_X=mean((signal*2-1)+mu_w);


%Experimental BER Analysis
```

```
detect = (received > 0);                        % detected result
N_err = sum(abs(detect-signal));      % # of erroneously detected bits
BER=N_err/N;                               % Ratio of Error to total bits
fprintf("STD(sigma) = %d, E[x] = %d\n", sigma_EX, E_X)
fprintf('N = %d bits transmitted; N_err = %d error bits received\n BER = %d \n',
N, N_err,BER)
```

Standard Gaussian Distribution Table

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

Figure 6. Look up Table, taken from Quiz 6

EXCEL Tables:

x	PDF SNR=5dB	PDF SNR=10dB
-4	3.60086E-18	6.51796E-55
-3.9	1.72313E-17	9.20171E-53
-3.8	7.98907E-17	1.17543E-50
-3.7	3.58871E-16	1.35861E-48
-3.6	1.56187E-15	1.4209E-46
-3.5	6.58589E-15	1.34463E-44
-3.4	2.6906E-14	1.15137E-42
-3.3	1.065E-13	8.92059E-41
-3.2	4.08425E-13	6.25381E-39
-3.1	1.51754E-12	3.96704E-37
-3	5.46302E-12	2.27698E-35
-2.9	1.90542E-11	1.18256E-33
-2.8	6.43889E-11	5.55719E-32
-2.7	2.10813E-10	2.36297E-30
-2.6	6.68724E-10	9.09146E-29
-2.5	2.05523E-09	3.16504E-27
-2.4	6.11984E-09	9.96998E-26
-2.3	1.76556E-08	2.84172E-24
-2.2	4.93504E-08	7.32888E-23
-2.1	1.33648E-07	1.71027E-21
-2	3.5067E-07	3.61129E-20
-1.9	8.91455E-07	6.89971E-19
-1.8	2.19566E-06	1.19281E-17
-1.7	5.23955E-06	1.86586E-16
-1.6	1.2114E-05	2.64095E-15
-1.5	2.7136E-05	3.38229E-14
-1.4	5.88937E-05	3.91951E-13
-1.3	0.000123838	4.10983E-12
-1.2	0.000252294	3.89929E-11
-1.1	0.000497992	3.34749E-10
-1	0.000952364	2.60029E-09
-0.9	0.001764604	1.82766E-08
-0.8	0.003167789	1.16236E-07
-0.7	0.00550972	6.68892E-07
-0.6	0.009284685	3.48291E-06
-0.5	0.015158953	1.64096E-05
-0.4	0.023979253	6.9956E-05
-0.3	0.036750772	0.00026985

-0.2	0.054570969	0.000941869
-0.1	0.078509326	0.002974603
0	0.109432222	0.008500375
0.1	0.147786094	0.021979497
0.2	0.193368758	0.051424251
0.3	0.245133931	0.108865123
0.4	0.301082098	0.208535559
0.5	0.358286812	0.361444844
0.6	0.413086589	0.566858298
0.7	0.461440545	0.804410155
0.8	0.499407191	1.032883028
0.9	0.523670629	1.200038832
1	0.53201763	1.261566125
1.1	0.523670629	1.200038832
1.2	0.499407191	1.032883028
1.3	0.461440545	0.804410155
1.4	0.413086589	0.566858298
1.5	0.358286812	0.361444844
1.6	0.301082098	0.208535559
1.7	0.245133931	0.108865123
1.8	0.193368758	0.051424251
1.9	0.147786094	0.021979497
2	0.109432222	0.008500375
2.1	0.078509326	0.002974603
2.2	0.054570969	0.000941869
2.3	0.036750772	0.00026985
2.4	0.023979253	6.9956E-05
2.5	0.015158953	1.64096E-05
2.6	0.009284685	3.48291E-06
2.7	0.00550972	6.68892E-07
2.8	0.003167789	1.16236E-07
2.9	0.001764604	1.82766E-08
3	0.000952364	2.60029E-09
3.1	0.000497992	3.34749E-10
3.2	0.000252294	3.89929E-11
3.3	0.000123838	4.10983E-12
3.4	5.88937E-05	3.91951E-13
3.5	2.7136E-05	3.38229E-14
3.6	1.2114E-05	2.64095E-15
3.7	5.23955E-06	1.86586E-16
3.8	2.19566E-06	1.19281E-17
3.9	8.91455E-07	6.89971E-19
4	3.5067E-07	3.61129E-20