Chapter 18

Accelerated Test Models

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Chapter 18
Accelerated Test Models
Objectives

- Describe motivation and applications of accelerated reliability testing.

- Explain the connections between degradation, physical failure, and acceleration of reliability tests.

- Examine the basis for temperature and humidity acceleration.

- Examine the basis for voltage and pressure stress acceleration.

- Show how to compute time-acceleration factors.

- Review other accelerated test models and assumptions.

Accelerated Tests Increasingly Important

Today's manufactures need to develop newer, higher technology products in record time while improving productivity, reliability, and quality.

Important issues:

- Rapid product development.

- Rapidly changing technologies.

- More complicated products with more components.

- Higher customer expectations for better reliability.

Need for Accelerated Tests

Need timely information on high reliability products.

• Modern products designed to last for years or decades.

• Accelerated Tests (ATs) used for timely assessment of reliability of product components and materials.

• Tests at high levels of use rate, temperature, voltage, pressure, humidity, etc.

• Estimate life at **use conditions**.

Note: Estimation/prediction from ATs involves **extrapolation.**

Applications of Accelerated Tests

Applications of Accelerated Tests include:

- Evaluation the effect of stress on life.

- Assessing component reliability.

- Demonstrating component reliability.

- Detecting failure modes.

- Comparing two or more competing products.
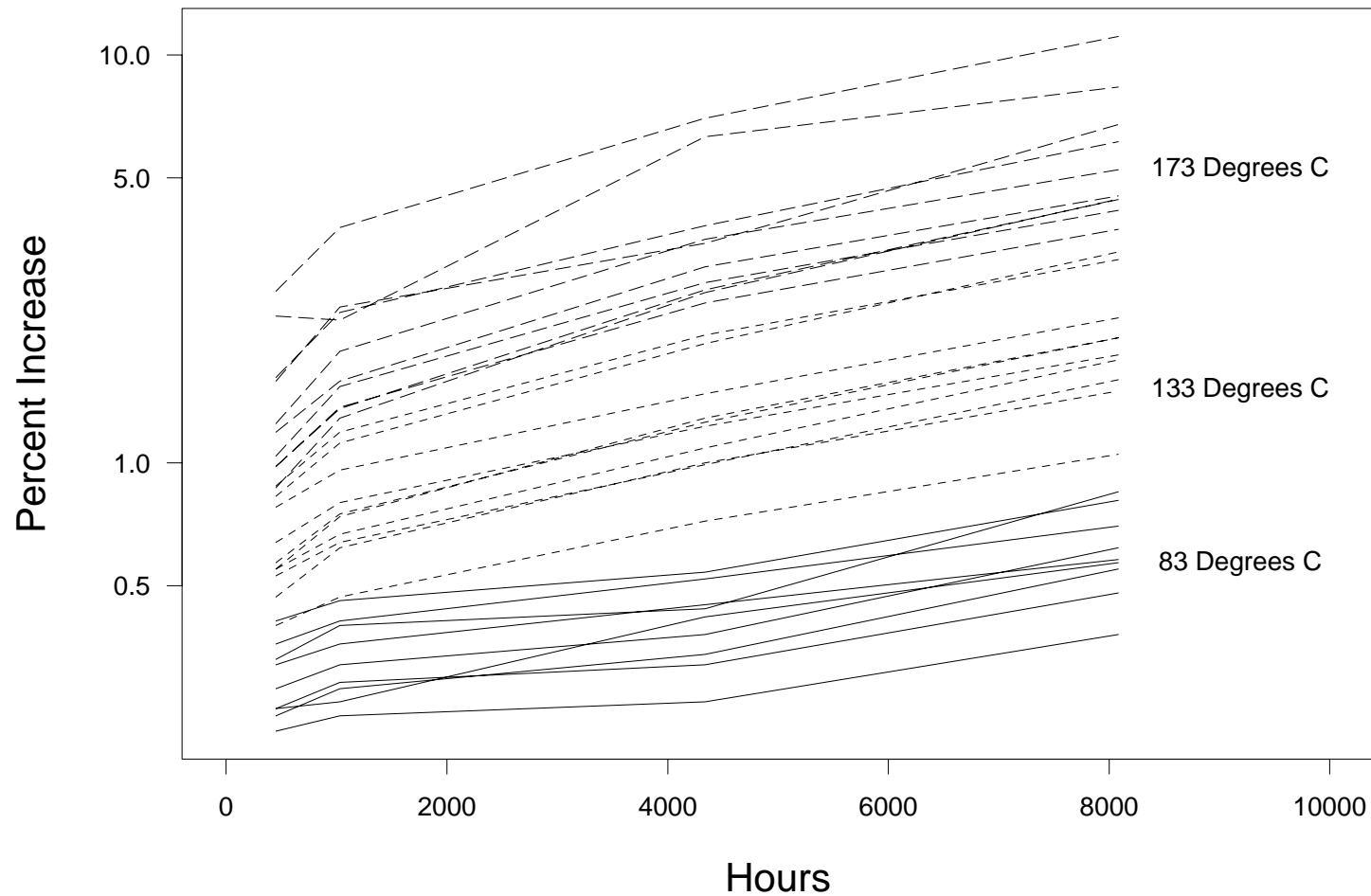
- Establishing safe warranty times.

Methods of Acceleration

Three fundamentally different methods of accelerating a reliability test:

- Increase the use-rate of the product (e.g., test a toaster 200 times/day). Higher use rate reduces test time.

- Use elevated temperature or humidity to increase rate of failure-causing chemical/physical process.

- Increase stress (e.g., voltage or pressure) to make degrading units fail more quickly.

  Use a **physical/chemical** (preferable) or **empirical** model relating degradation or lifetime at **use** conditions.

Change in Resistance Over Time
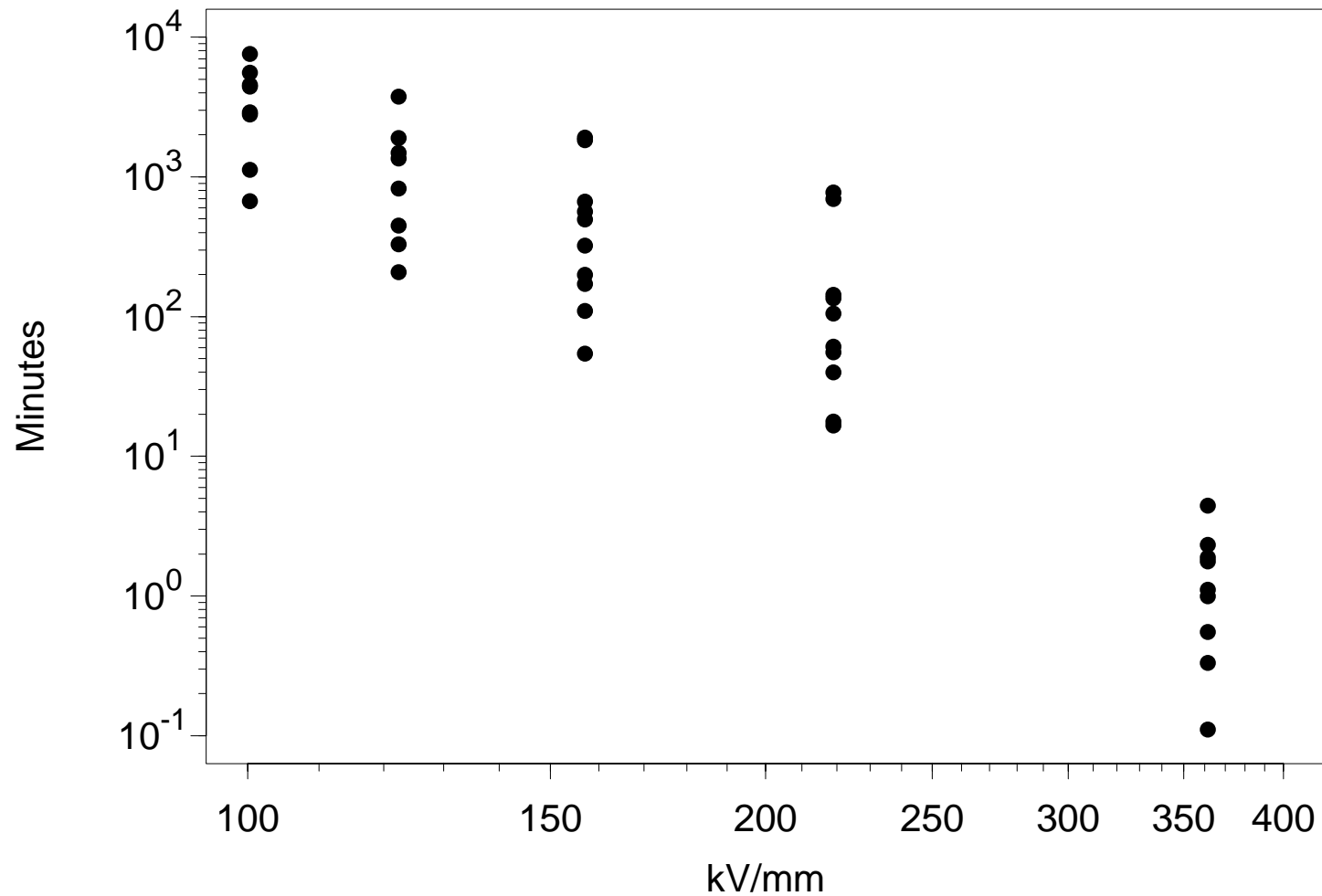# of Carbon-Film Resistors
(Shiomi and Yanagisawa 1979)

173 Degrees C

133 Degrees C

83 Degrees C

Percent Increase

Hours

Accelerated Degradation Tests (ADTs)

Response: Amount of degradation at points in time.

Model components:

- Model for degradation over time.

- A definition of **failure** as a function of degradation variable.

- Relationship(s) between degradation model parameters (e.g., chemical process reaction rates) and acceleration variables (e.g., temperature or humidity).

Breakdown Times in Minutes of a Mylar-Polyurethane Insulating Structure (from Kalkanis and Rosso 1989)

Accelerated Life Tests (ALTs)

Response:

- Failure time (or interval) for units that fail.

- Censoring time for units that do not fail.

Model Components:

- Constant-stress time-to-failure distribution.

- Relationship(s) between one (or more) of the constant-stress model parameters and the accelerating variables.

Use-Rate Acceleration

Basic idea: Increase use-rate to accelerate failure-causing wear or degradation.

Examples:

- Running automobile engines or appliances continuously.

- Rapid cycling of relays and switches.

- Cycles to failure in fatigue testing.

Simple assumption: Useful if life adequately modeled by cycles of operation. Reasonable if cycling simulates actual use and if test units return to steady state after each cycle.

More complicated situations: Wear rate or degradation rate depends on cycling frequency or product deteriorates in stand-by as well as during actual use.

Elevated Temperature Acceleration of Chemical Reaction Rates

- The **Arrhenius** model **Reaction Rate**, $\mathcal{R}(\mathtt{temp})$, is

$$\mathcal{R}(\mathtt{temp}) = \gamma_0 \exp\left(\frac{-E_a}{k_{\mathsf{B}}(\mathtt{temp}\,^\circ\mathsf{C} + 273.15)}\right) = \gamma_0 \exp\left(\frac{-E_a \times 11605}{\mathtt{temp}\,\mathsf{K}}\right)$$

 where $\mathtt{temp}\,\mathsf{K} = \mathtt{temp}\,^\circ\mathsf{C} + 273.15$ is temperature in degrees Kelvin and $k_{\mathsf{B}} = 1/11605$ is Boltzmann's constant in units of electron volts per K. The reaction activation energy, E_a, and γ_0 are characteristics of the product or material being tested.

- The reaction rate **Acceleration Factor** is

$$
\begin{aligned}
\mathcal{AF}(\mathtt{temp}, \mathtt{temp}_U, E_a) &= \frac{\mathcal{R}(\mathtt{temp})}{\mathcal{R}(\mathtt{temp}_U)} \\
&= \exp\left[E_a\left(\frac{11605}{\mathtt{temp}_U\,\mathsf{K}} - \frac{11605}{\mathtt{temp}\,\mathsf{K}}\right)\right]
\end{aligned}
$$

- When $\mathtt{temp} > \mathtt{temp}_U$, $\mathcal{AF}(\mathtt{temp}, \mathtt{temp}_U, E_a) > 1$.

Acceleration Factors for the SAFT Arrhenius Model

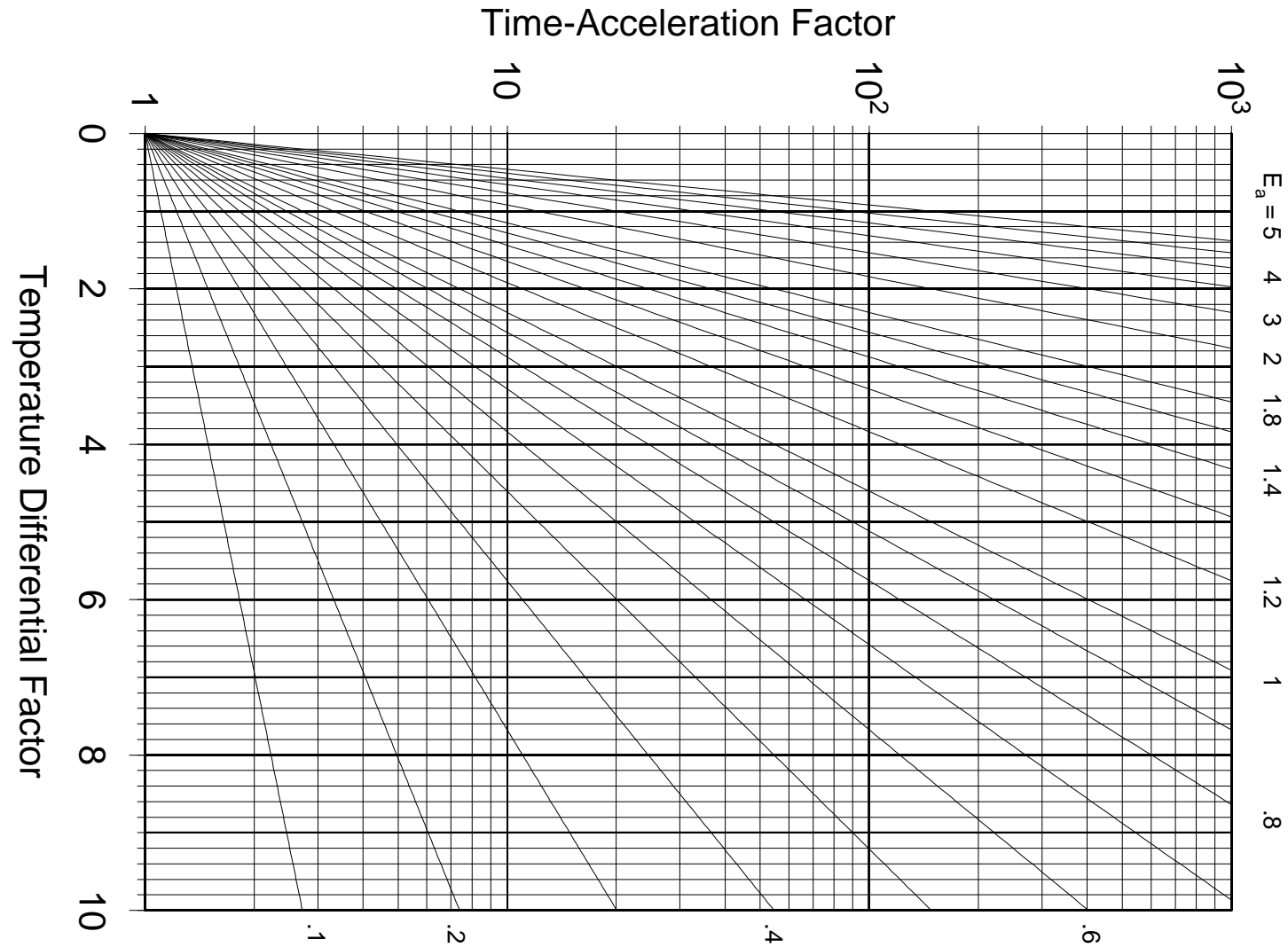- Table 18.2 gives the **Temperature Differential Factors** (TDF)

$$\mathsf{TDF} = \left(\frac{11605}{\mathtt{temp_{Low}}\ \mathsf{K}} - \frac{11605}{\mathtt{temp_{High}}\ \mathsf{K}} \right).$$

- Figure 18.3 gives

$$\mathcal{AF}(\mathtt{temp_{High}}, \mathtt{temp_{Low}}, E_a) = \exp\left(E_a \times \mathsf{TDF} \right)$$

- We use $\mathcal{AF}(\mathtt{temp}) = \mathcal{AF}(\mathtt{temp}, \mathtt{temp}_U, E_a)$ when $\mathtt{temp}_U$ and E_a are understood to be, respectively, product use temperature and reaction-specific activation energy.

Time-Acceleration Factor as a Function of Temperature Differential Factor
(Figure 18.3)

Nonlinear Degradation Reaction-Rate Acceleration

- Consider the simple chemical degradation path model

$$\mathcal{D}(t; \mathtt{temp}) = \mathcal{D}_\infty \times \{1 - \exp\left[-\mathcal{R}_U \times \mathcal{AF}(\mathtt{temp}) \times t\right]\}$$

where $\mathcal{R}_U$ is the rate reaction at use temperature ($\mathtt{temp}_U$) and for $\mathtt{temp} > \mathtt{temp}_U$, $\mathcal{AF}(\mathtt{temp}) > 1$.

- For $\mathcal{D}_\infty > 0$, failure occurs when $\mathcal{D}(T; \mathtt{temp}) > \mathcal{D}_\mathsf{f}$. Equating $\mathcal{D}(T; \mathtt{temp})$ to $\mathcal{D}_\mathsf{f}$ and solving for failure time, $T(\mathtt{temp})$, gives
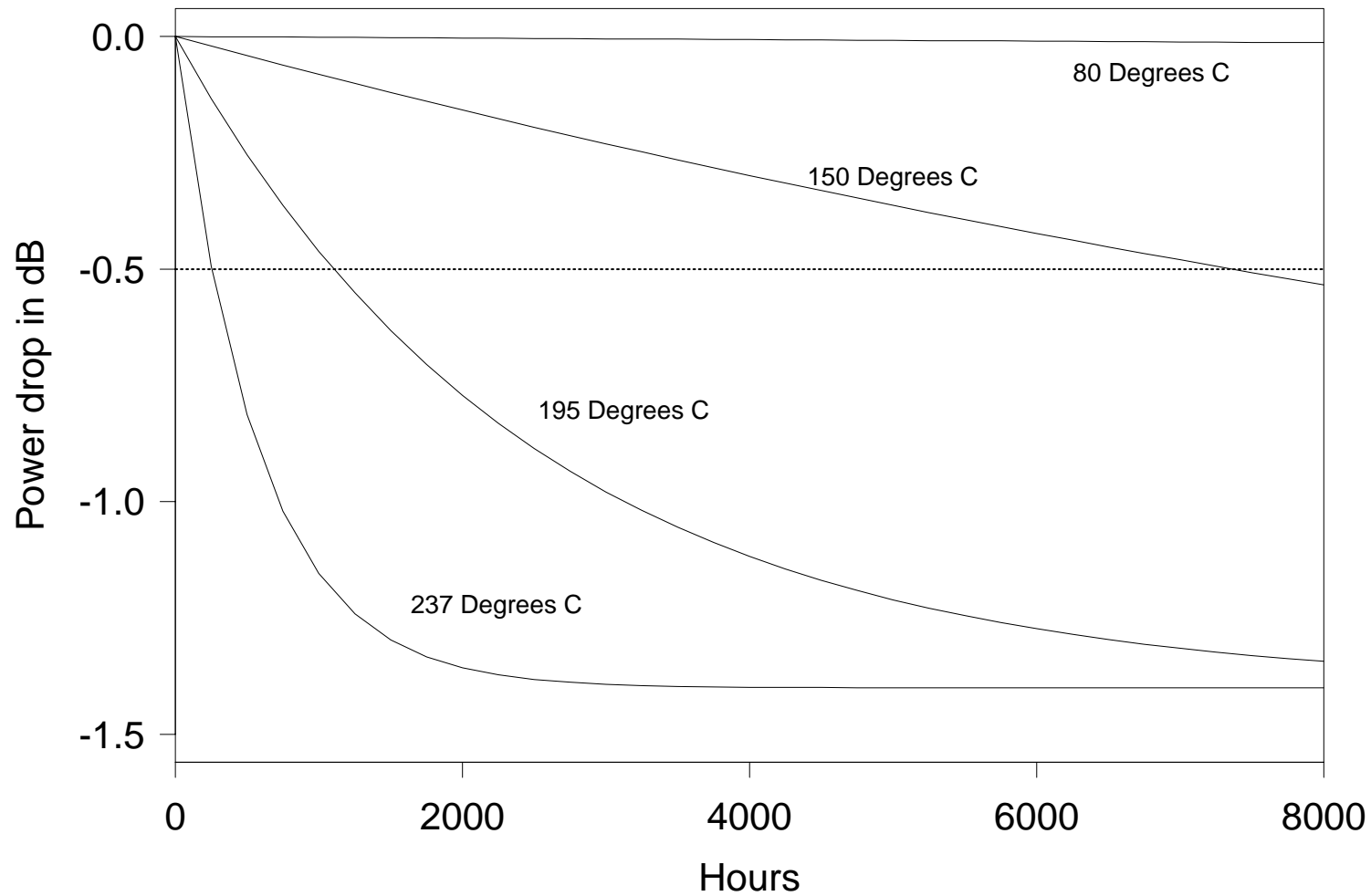
$$T(\mathtt{temp}) = \frac{T(\mathtt{temp}_U)}{\mathcal{AF}(\mathtt{temp})} = \frac{\left[-\frac{1}{\mathcal{R}_U}\log\left(1 - \frac{\mathcal{D}_\mathsf{f}}{\mathcal{D}_\infty}\right)\right]}{\mathcal{AF}(\mathtt{temp})}$$

where $T(\mathtt{temp}_U)$ is failure time at use conditions.

- This is an SAFT model.

SAFT Model from Nonlinear Degradation Paths

$$\mathcal{D}(t; \mathtt{temp}) = \mathcal{D}_\infty \times \{1 - \exp\left[-\mathcal{R}_U \times \mathcal{AF}(\mathtt{temp}) \times t\right]\}$$

The Arrhenius-Lognormal Regression Model

The Arrhenius-lognormal regression model is

$$\Pr[T \leq t; \mathtt{temp}] = \Phi_{\mathsf{nor}} \left[\frac{\log(t) - \mu}{\sigma} \right]$$
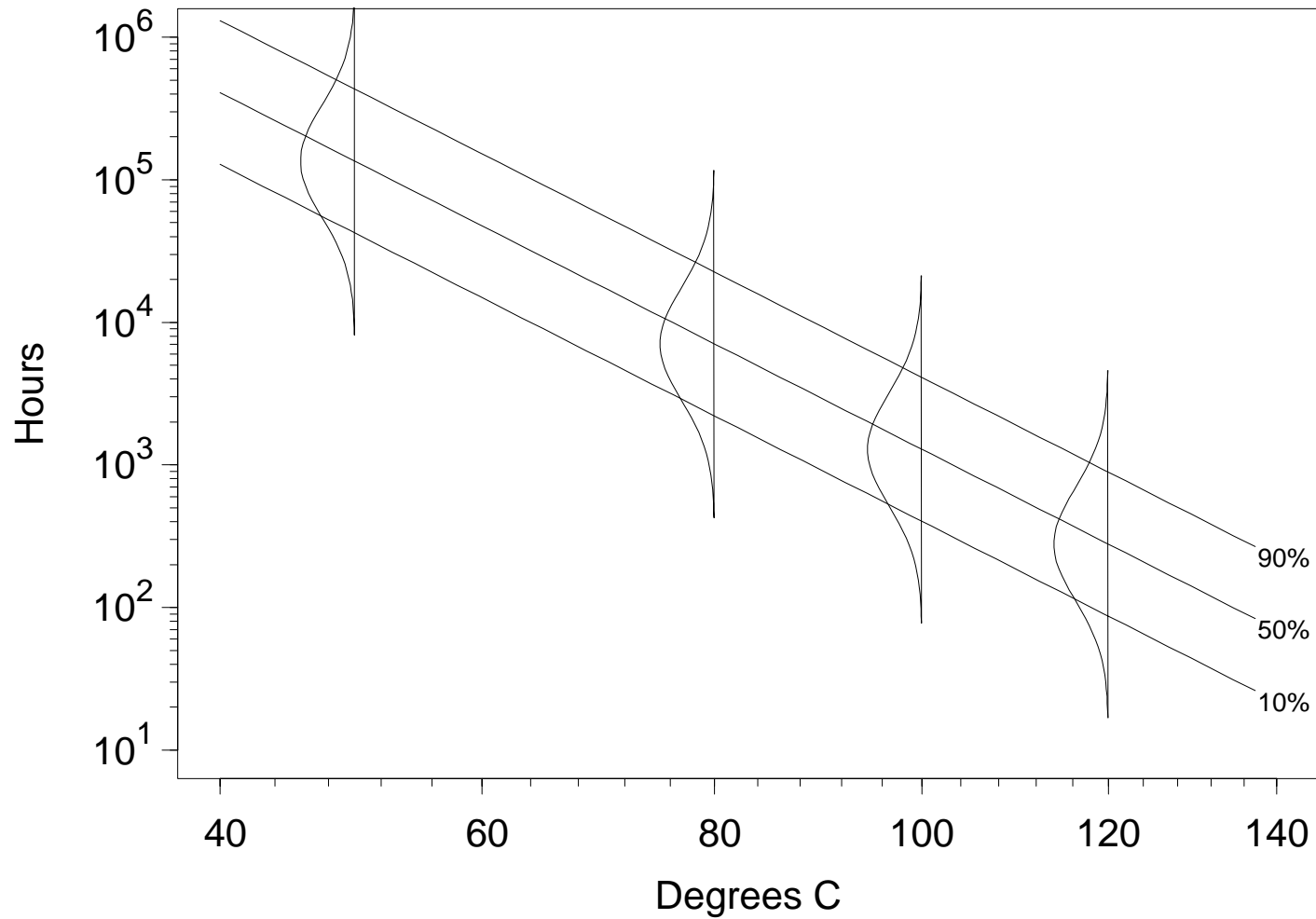
where

- $\mu = \beta_0 + \beta_1 x,$

- $x = 11605/(\mathtt{temp}\,\mathsf{K}) = 11605/(\mathtt{temp}\,^{\circ}\mathsf{C} + 273.15)$

- and $\beta_1 = E_a$ is the effective activation energy
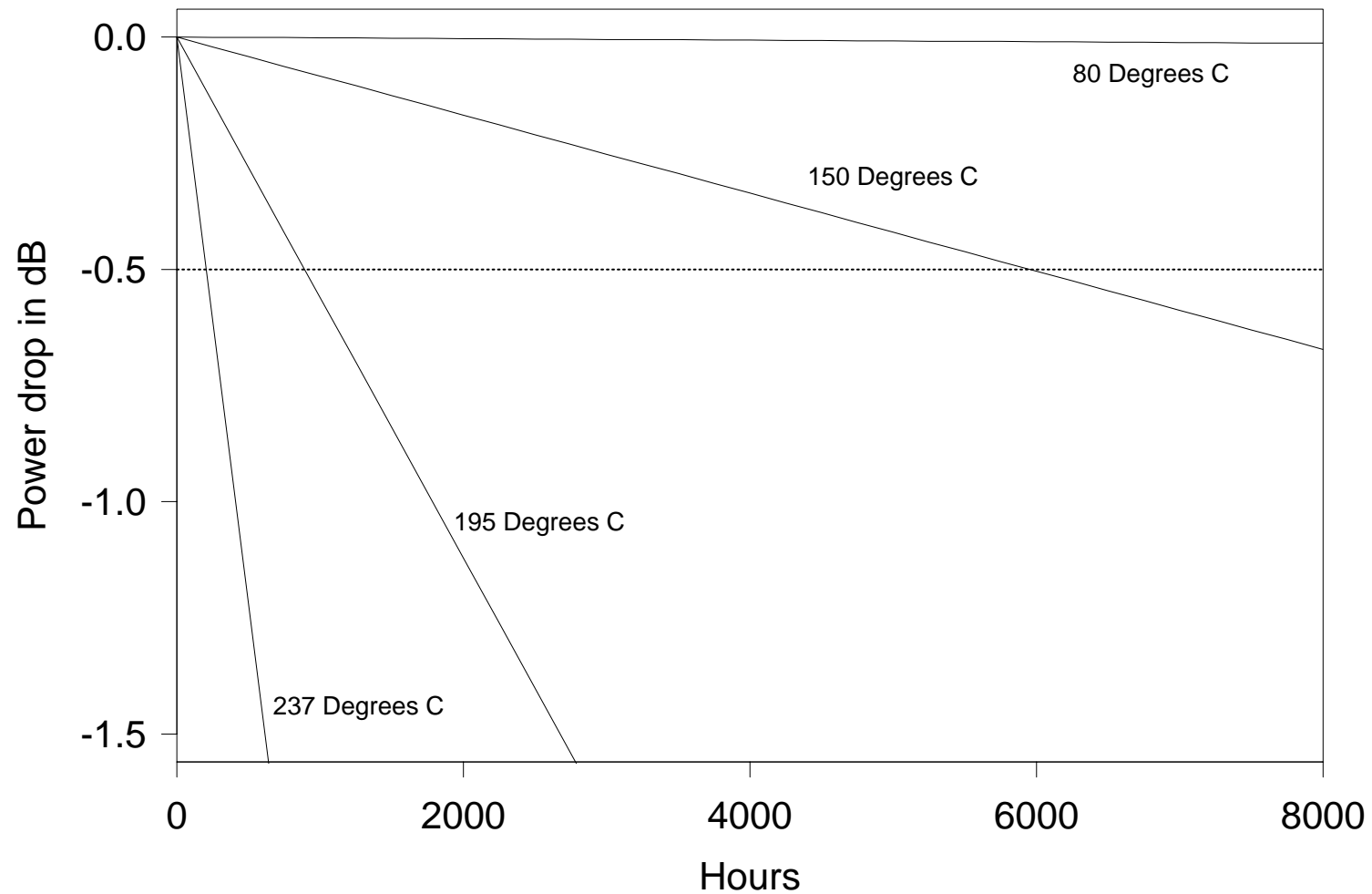
- σ is constant

- This implies that

$$t_p(\mathtt{temp}_U) = t_p(\mathtt{temp}) \times \mathcal{AF}(\mathtt{temp})$$

Example Arrhenius-Lognormal Life Model

$$\log[t_p(\texttt{temp})] = \beta_0 + \beta_1 x + \Phi_{\text{nor}}^{-1}(p)\sigma$$

SAFT Model from Linear Degradation Paths

80 Degrees C

150 Degrees C

195 Degrees C

237 Degrees C

Linear Degradation Reaction-Rate Acceleration

If $\mathcal{R}_U \times \mathcal{AF}(\text{temp}) \times t$ is small so that $\mathcal{D}(t)$ is small relative to $\mathcal{D}_\infty$, then

$$
\begin{aligned}
\mathcal{D}(t; \text{temp}) \;&=\; \mathcal{D}_\infty \times \{1 - \exp\left[-\mathcal{R}_U \times \mathcal{AF}(\text{temp}) \times t\right]\} \\
&\approx\; \mathcal{D}_\infty \times \mathcal{R}_U \times \mathcal{AF}(\text{temp}) \times t = \mathcal{R}_U^+ \times \mathcal{AF}(\text{temp}) \times t
\end{aligned}
$$

is approximately linear in t.

- Also some degradation processes are linear in time:

$$
\mathcal{D}(t; \text{temp}) = \mathcal{R}_U \times \mathcal{AF}(\text{temp}) \times t.
$$

- Failure occurs when $\mathcal{D}(T; \text{temp}) > \mathcal{D}_{\mathsf{f}}$. Equating $\mathcal{D}(T; \text{temp})$ to $\mathcal{D}_{\mathsf{f}}$ and solving for failure time, $T(\text{temp})$,

$$
T(\text{temp}) \;=\; \frac{T(\text{temp}_U)}{\mathcal{AF}(\text{temp})}
$$

where $T(\text{temp}_U) = \mathcal{D}_{\mathsf{f}}/\mathcal{R}_{2U}$ is failure time at use conditions.

- This is an SAFT model and, for example, $T(\text{temp}_U) \sim$ WEIB(μ, σ) implies $T(\text{temp}) \sim$ WEIB $\left[\mu - \log(\mathcal{AF}(\text{temp})), \sigma)\right]$.

Non-SAFT Degradation Reaction-Rate Acceleration

Consider the more complicated chemical degradation path

$$
\begin{aligned}
\mathcal{D}(t; \texttt{temp}) \;=\;\; & \mathcal{D}_{1\infty} \times \{1 - \exp\left[-\mathcal{R}_{1U} \times \mathcal{AF}_1(\texttt{temp}) \times t\right]\} \\
& + \mathcal{D}_{2\infty} \times \{1 - \exp\left[-\mathcal{R}_{2U} \times \mathcal{AF}_2(\texttt{temp}) \times t\right]\}
\end{aligned}
$$

$\mathcal{R}_{1U}, \mathcal{R}_{2U}$ are the rates of the reactions contributing to failure.

This is **not** an SAFT model. Temperature affects the two degradation processes differently, inducing a nonlinearity into the acceleration function relating times at two different temperatures.

Voltage Acceleration and Voltage Stress
Inverse Power Relationship

- Depending on the failure mode, voltage can be raised to:

  ▶ Increase the strength of electric fields. This can accelerate some failure-causing reactions.

  ▶ Increase the stress level (e.g., voltage stress relative to declining **voltage strength**).

- An **empirical** model for life at volt relative to use conditions $\mathtt{volt}_U$ is

$$T(\mathtt{volt}) = \frac{T(\mathtt{volt}_U)}{\mathcal{AF}(\mathtt{volt})} = \left(\frac{\mathtt{volt}}{\mathtt{volt}_U}\right)^{\beta_1} T(\mathtt{volt}_U)$$

where $\mathcal{AF}(\mathtt{volt}) = \mathcal{AF}(\mathtt{volt}, \mathtt{volt}_U, \beta_1)$,

$$\mathcal{AF}(\mathtt{volt}) = \mathcal{AF}(\mathtt{volt}, \mathtt{volt}_U, \beta_1) = \frac{T(\mathtt{volt}_U)}{T(\mathtt{volt})} = \left(\frac{\mathtt{volt}}{\mathtt{volt}_U}\right)^{-\beta_1}$$

and β_1 is a material characteristic. $T(\mathtt{volt}), T(\mathtt{volt}_U)$ are the failure times at increased voltage and use conditions.

Inverse Power Relationship-Weibull Model

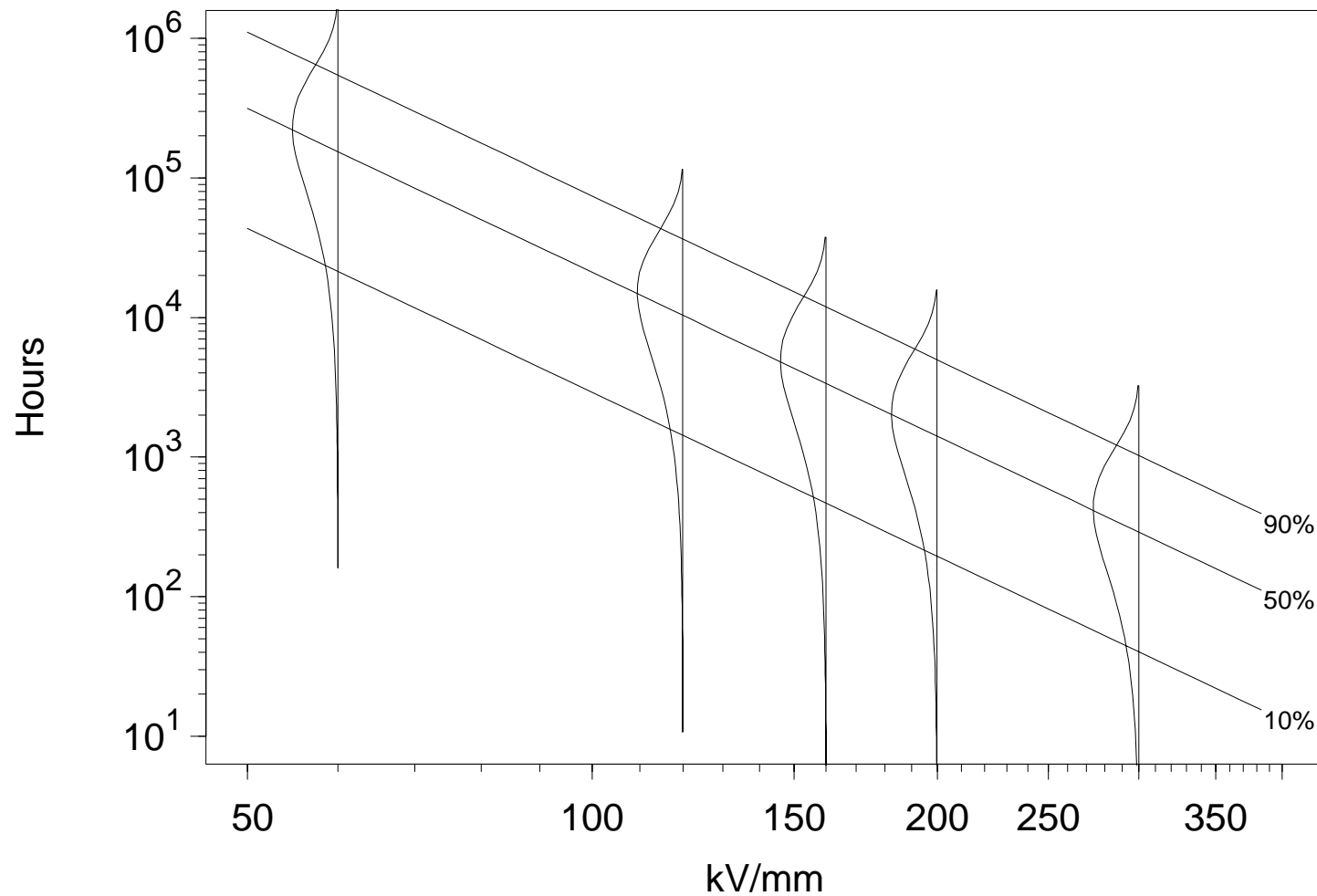The inverse power relationship-Weibull model is

$$\Pr[T \leq t; \mathtt{volt}] = \Phi_{\mathsf{sev}} \left[\frac{\log(t) - \mu}{\sigma} \right]$$

where

- $\mu = \beta_0 + \beta_1 x$, and

- $x = \log(\text{Voltage Stress})$.

- σ assumed to be constant.

Example Weibull Inverse Power Relationship Between Life and Voltage Stress

$$\log[t_p(\mathtt{volt})] = \beta_0 + \beta_1 x + \Phi_{\mathtt{sev}}^{-1}(p)\sigma$$

Other Commonly Used Life-Stress Relationships

Other commonly used SAFT models have the simple form:

$$T(x) \; = \; \frac{T(x_U)}{\mathcal{AF}(x)}$$

where $\mathcal{AF}(x) = \mathcal{AF}(x, x_U, \beta_1) = \exp\left[\beta_1(x - x_U)\right]$. β_1 is a material characteristic.

Examples include:

- Cycling rate: $x = \log(\text{frequency})$.

- Current density: $x = \log(\text{current})$.

- Size: $x = \log(\text{thickness})$.

- Humidity 1: $x = \log(\text{RH})$, RH is % relative humidity/100.

- Humidity 2: $x = \log[\text{RH}/(1 - \text{RH})]$.

Some of these models are empirical. For a location-scale time-to-failure distribution $\mu = \beta_0 + \beta_1 x$.

Eyring Temperature Relationship

- Arrhenius relationship obtained from **empirical** observation.

- Eyring developed **physical theory** describing the effect that temperature has on a reaction rate:

$$\mathcal{R}(\texttt{temp}) = \gamma_0 \times A(\texttt{temp}) \times \exp\left(\frac{-E_a}{k_{\mathsf{B}} \times \texttt{temp}\,\mathsf{K}}\right)$$

- $A(\texttt{temp})$ is a function of temperature depending on the specifics of the reaction dynamics; γ_0 and E_a are constants.

- Applications in the literature have used $A(\texttt{temp}) = (\texttt{temp}\,\mathsf{K})^m$ with a fixed value of m ranging between $m = 0$ (Boccaletti et al. 1989), $m = .5$ (Klinger 1991a), to $m = 1$ (Nelson 1990a and Mann Schafer and Singpurwalla 1974). Difficult to identify m from limited data.

- Eyring showed how to include other accelerating variables.

The Eyring Regression Model
(e.g., for Weibull or Lognormal Distributions)

The Eyring temperature-acceleration regression model is

$$\Pr(T \leq t; \texttt{temp}) = \Phi\left[\frac{\log(t) - \mu}{\sigma}\right]$$

where

- $\mu = -m\log(\texttt{temp}\,^{\circ}\mathrm{C} + 273.15) + \beta_0 + \beta_1 x$.

- $x = 11605/(\texttt{temp}\,^{\circ}\mathrm{C} + 273.15)$.

- $\beta_1 = E_a$ is the activation energy.

- m is usually given; σ is constant, but usually unknown.

- With $m > 0$, Arrhenius provides a useful first order approximation to the Eyring model, with conservative extrapolation to lower temperatures.

Humidity Acceleration Models

- Useful for accelerating failure mechanisms involving **corrosion** and certain other kinds of **chemical degradation**.

- Often used in conjunction with elevated temperature.

- Most humidity models have been developed empirically.

- Empirical and limited theoretical results for corrosion on thin films (Gillen and Mead 1980, Peck 1986, and Klinger 1991b) suggest the use of RH instead of P_v (vapor pressure) as the independent (or experimental) variable in humidity relationships when temperature is also to be varied.

- RH is the preferred variable because the change in life, as a function of RH, does not depend on temperature. That is,

$$\frac{\partial^2 \text{Life}}{\partial \text{RH} \, \partial \texttt{temp}} = 0$$

or no **statistical interaction**.

Humidity Regression Relationships

- Consider the Weibull/lognormal lifetime regression model

$$\Pr(T \le t; \texttt{humidity}) = \Phi_{\textsf{sev}} \left[\frac{\log(t) - \mu}{\sigma} \right]$$

  where $\mu = \beta_0 + \beta_1 x_1$ and σ is constant. Letting $0 < \texttt{RH} < 1$ denote **relative humidity**, possible humidity relationships are:

  ▶ $x_1 = \texttt{RH}$ [Intel, empirical].

  ▶ $x_1 = \log(\texttt{RH})$ [Peck, empirical].

  ▶ $x_1 = \log[\texttt{RH}/(1 - \texttt{RH})]$ [Klinger, corrosion on thin films].

- For temperature and humidity acceleration, possible relationships include

$$
\begin{aligned}
\mu &= \beta_0 + \beta_1 x_1 + \beta_2 x_2 \\
\text{or} \quad \mu &= \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2 \\
\text{or} \quad \mu &= \beta_0 + \beta_2 x_2 + \beta_3 x_1 x_2
\end{aligned}
$$

  where $x_2 = 11605/(\texttt{temp}\,^\circ\text{C} + 273.15)$.

Temperature/Humidity Acceleration Factors with RH versus `temp` K **No Interaction**

- Peck's relationship

$$\mathcal{AF}(\texttt{temp}, \texttt{RH}) \;=\; \frac{\mathcal{R}(\texttt{temp}, \texttt{RH})}{\mathcal{R}(\texttt{temp}_U, \texttt{RH}_U)}$$

$$= \; \left(\frac{\texttt{RH}_U}{\texttt{RH}}\right)^{\beta_1} \exp\left[E_a\left(\frac{11605}{\texttt{temp}_U \, \mathsf{K}} - \frac{11605}{\texttt{temp}\,\mathsf{K}}\right)\right].$$

- Klinger's relationship

$$\mathcal{AF}(\texttt{temp}, \texttt{RH}) \;=\; \frac{\mathcal{R}(\texttt{temp}, \texttt{RH})}{\mathcal{R}(\texttt{temp}_U, \texttt{RH}_U)}$$

$$= \; \left[\left(\frac{\texttt{RH}_U}{1-\texttt{RH}_U}\right)\left(\frac{1-\texttt{RH}}{\texttt{RH}}\right)\right]^{\beta_1} \times$$

$$\exp\left[E_a\left(\frac{11605}{\texttt{temp}_U\,\mathsf{K}} - \frac{11605}{\texttt{temp}\,\mathsf{K}}\right)\right].$$

Thermal Cycling

- Fatigue is an important failure mechanism for many products and materials.

- Mechanical expansion and contraction from thermal cycling can lead to fatigue cracking and failure.

- Applications include:

  ▶ Power-on/power-off cycling of electronic equipment and effect on component encapsulement and solder joints.

  ▶ Take-off power-thrust in jet engines and its effect on crack initiation and growth in fan disks.

  ▶ Power-up/power-down of nuclear power plants and effect on the growth of cracks in heat generator tubes.

  ▶ Thermal inkjet printhead delamination could be caused by temperature cycling during normal use.

Coffin-Manson Relationship

- The **Coffin-Manson** relationship says that the **typical** number of cycles to failure is

$$N = \frac{\delta}{(\Delta\texttt{temp})^{\beta_1}}$$

  where $\Delta\texttt{temp}$ is the temperature range and δ and β_1 are properties of the material and test setup. This power-rule relationship explains the effect that temperature range has on thermal-fatigue life. For some metals, $\beta_1 \approx 2$.

- Letting T be the random number of cycles to failure (e.g., $T = N \times \epsilon$ where ϵ is a random variable), the acceleration factor when $\Delta\texttt{temp}$, relative to the number of cycles when $\Delta\texttt{temp}_U$, is

$$\mathcal{AF}(\Delta\texttt{temp}) = \frac{T(\Delta\texttt{temp}_U)}{T(\Delta\texttt{temp})} = \left(\frac{\Delta\texttt{temp}}{\Delta\texttt{temp}_U}\right)^{\beta_1}$$

- There may be a $\Delta\texttt{temp}$ threshold below which little or no fatigue damage is done during thermal cycling.

Generalized Coffin-Manson Relationship

- Empirical evidence has shown that effect of temperature cycling can depend importantly on $\texttt{temp}_{\textsf{max}}\,\textsf{K}$, the maximum temperature in the cycling (e.g., if $\texttt{temp}_{\textsf{max}}\,\textsf{K}$ is more than .2 or .3 times a metal's melting point).

- The effect of temperature cycling can also depend on the cycling rate (e.g., due to heat buildup).

- An **empirical** extension of the Coffin-Manson relationship is

$$N = \frac{\delta}{(\Delta\texttt{temp})^{\beta_1}} \times \frac{1}{(\texttt{freq})^{\beta_2}} \times \exp\left(\frac{E_a \times 11605}{\texttt{temp}_{\textsf{max}}\,\textsf{K}}\right)$$

where $\texttt{freq}$ is the cycling frequency, and E_a is an activation energy.

- Caution must be used when using such a model outside the range of available data and past experience.

Other Topics in Chapter 18

- Other accelerated degradation models and relationships to accelerated time models.

- Discussion of stress-cycling models.

- Other models for two or more experimental factors.