Muscle Models

1 Linear Model

A third order linear differential equation model was of optimal order for fitting the muscle force over a range of pulse inputs [6]. Therefore, a linear model of third order is used.

$$\theta_3 \ddot{F}(t) + \theta_2 \dot{F}(t) + \theta_1 F(t) = \theta_0 u(t)$$

Where $F(t)$ is the muscle force as a function of time and θ , $j = 1, ..., 4$, are the model parameters. The model has 4 parameters θ_3, θ_2, θ_1 and θ_0 with units of s^3, s^2, s and dimensionless, respectively, for the case where the force is normalised.

The starting values for the model parameters are: $\theta_3 = 1$, $\theta_2 = 25$, $\theta_1 = -1$ and $\theta_0 = 14$.

2 Wiener Model

A second-order linear differential equation is used to describe the linear dynamics, and the static nonlinearity is assumed to have a similar form to that of existing models [3, 4].

$$\theta_3 \ddot{q}(t) + \theta_2 \dot{q}(t) + \theta_1 q(t) = \theta_0 u(t)$$
$$f(t) = \frac{q(t)^m}{q(t)^m + k^m}$$

This model has 5 parameters θ_2, θ_1, θ_0, m and k with units of s^2, s, dimensionless, dimensionless and dimensionless, respectively, for the case where the force is normalised. A dummy variable $q(t)$ is used to couple the nonlinear to the linear model.

The starting values for the model parameters are: $\theta_3 = 1$, $\theta_2 = 25$, $\theta_1 = -1$, $\theta_0 = 14$ and $A = 100$.

3 Adapted Model

The model of [2] was adapted to give a model of similar, yet simpler form to that of [1]. This new, Adapted model is presented in [5].

$$\dot{C}_N(t) + \frac{C_N(t)}{\tau_c} = u(t)$$

$$\dot{F} + \frac{F(t)}{\tau_1} = AC_N(t)$$

This model has 3 parameters, τ_c, τ_1 and A. For the case where $F(t)$ is normalised, the time constants τ_c and τ_1 have units of s, A units of s^{-1}.

The starting values for the model parameters are: $\tau_c = \tau_1 = 1$ and $A = 100$.

References

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