CSE100 Midterm 2 Review

This review doc aims to provide a concise summary of the concepts, algorithms and lecture slides covered in CSE110. *Created by M.*, feel free to collaborate.

List of Topics:

Midterm 1:

Time and space complexity
C++ (compare to java, classes, reference, const, vector, I/O, template)
Tree
Binary search Tree (Average-case time complexity computation)
Iterators
KD-Tree
Multi-way Tree
Ternary Search Tree

Midterm 2:

- Hashing, properties of hash functions, hash table, optimization based on probability theory, collision resolution strategies(linear, double, random, separate chaining, cuckoo), hash map(map abstract data type)
- Markov text generation
- AVL tree; single/double/edge case rotation, runtime complexity proof;
- Red-black Tree; general structure, coloring, insertion in different cases, insertion, removal
- Graph; types of graphs, representation(adjacency matrix, adjacency list)
- Encoding with graph: huffman algorithm
- Graph Search; BFS, BST's queue ADT implementation; DFS, DFS's stack implementation; runtime: O(|V| + |E|).
- Shortest path; Dijkstra's algorithm on weighted and unweighted, runtime O(|V| + |E| log |E|), A* graph search (modification on priority function: f(n) = g(n) + h(n), best estimate cost)

- **Multiway Trie**
- **Ternary Search Tree (TST)**

- Find algorithm
    - **If** *letter* < *node*'s label:
        - **If** left, traverse down to *left*. Otherwise, **failed**
    - **If** *letter* > *node*'s label:
        - **If** *right*, traverse down to *right*. Otherwise, **failed.**
    - **If** *letter* = *node*'s label:
        - **If** last letter of *key* and if *node* is labeled as a "word", **found.**
        - **If not, failed.**
        - **Otherwise**, if *middle*, traverse down to *node*'s middle child and set *letter* to the next character of *key*; if not, we have failed (*key* does not exist in this **Ternary Search Tree**)
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- **Hashing**
    - Faster find array[i] at **O(1);** Size is **Prime**
    - Hash function

      Key + hash function = hash value;

      Necessary: **if k = l, then h(k) = h(l)**; Nice to have: if k != l, then h(k) != h(l)
    - Hash Table: array of size M

      Average complexity = O(1), independent of # elements it stored.

      Impossible to iterator in order: **unordered set**.
    - Optimization

      Reasonable capacity (load factor), good indexing (hash function).

      Probability theory: P(no collisions) = 1 * (m-1)/m * (m-2)/m …. (m-n-1)/m; expected collisions: **M ~= 1.3N**.

      Load factor: performance depend on load factor, not number of elements; insertion is bad when load factor > 70%.
    - **Resize** - double to the nearest prime

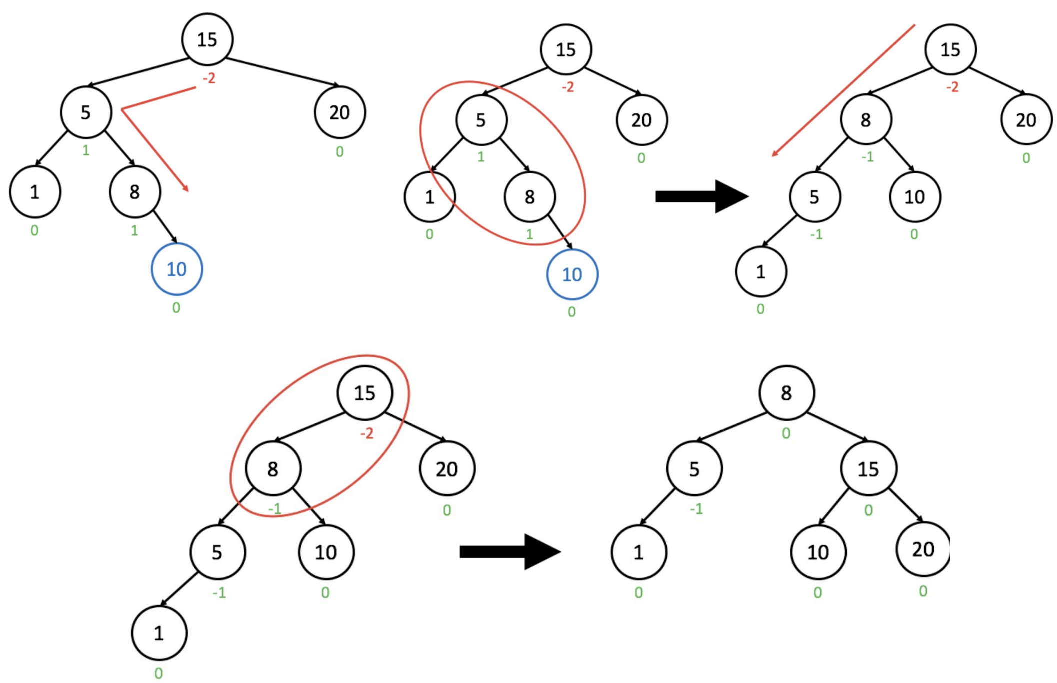
- **Resolution of collision**
    - Linear probing: Best O(1), Worst O(n)
        - If occupied, go to next slot: index = (index + 1) % size
        - **Open addressing** (the keys are open to move to an address other than the address to which they initially hashed.)
        - Remove
            - **delete** flag needed, cannot actually remove element
        - Pros - when not full, average performance **O(1)**
        - Cons

        - **clumps**
        - Unequal probability of getting filled
- Double Hashing
    - H_1 to calculate the index, H_2 to calculate the offset
    - Next Index = (Index + offset) % size
- Random Hashing
    - Key as seed, generate a sequence of index
- Separate Chaining: Best O(1); Worst O(n)
    - Keep pointers as linked lists - new key to the **front**
    - **Closed addressing** (the key *must* be located in the original address)
    - Pros:
        - In general, the average-case performance is considered much better than **Linear Probing** and **Double Hashing** as the amount of keys approaches, and even *exceeds*, *M*, the capacity of the **Hash Table**
    - Cons:
        - Requires extra space for pointers
        - All the data in our **Hash Table** is no longer huddled near 1 memory location (since pointers can point to memory anywhere), and as a result, this poor locality causes poor cache performance (i.e., it often takes the computer longer to find data that isn't located near previously accessed data)
- Cuckoo Hashing: Best O(1); Worst O(1)
    - **Two hash functions, both return a position in Hash table**
    - Two hash tables **(d ≥ 2) (size = M / d)**
        - If collision, kick out item in T_1, insert the kicked item into T_2
        - **Infinite cycle - kicking out the same element**
    - find & delete
        - Need to check two tables
- Bloom Filter
    - If words are real, give true; if words are not real, might be wrong
    - Fill the bit
        - Multiple hash functions, fill every bit at all hash values
    - When checking if in the table
        - Check corresponding indice
        - **Might be wrong** - slot filled by other words
- Count-min - sacrificing exact solution to solve heavy hitters
    - Heavy Hitters: Given a set of elements of size n, some much smaller value k, determine which elements occur at least **n/k** times.
    - Multiple hash functions
    - Find the **min** occurrence among the values at corresponding indice

- **Markov Text Generation**

- Train: build model based on data, for each new word keep track of next words, calculate probability
- Generate: find the current word, generate next word based on probability

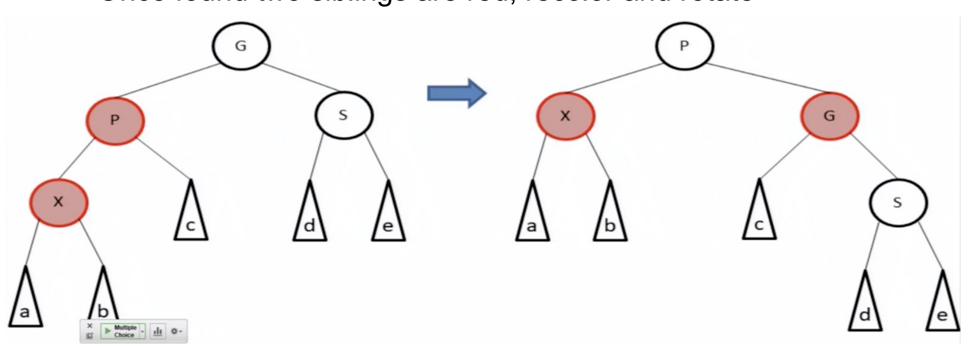
- **AVL Tree**
  - a **Binary Search Tree** in which, for all nodes in the tree, the *heights* of the two child subtrees of the node differ by at most one
  - **balance factor -**
    - **right_height - left_height**
    - Is etheri 1, 0, -1
  - Time complexity
    - **O(log(n))**
  - To keep balanced → **Rotation**
    - **Single rotation on the node out of balance**
      - Left || right
      - Check if has two child
        - If yes, connect children
    - **Double rotation**



Complicated cases… 感觉挺难的 不是太懂 有什么方法性的东西吗？

- **Red-Black Tree**
  - Properties
    - All nodes must be "colored" either **red** or **black**
    - **root** must be **black**

    - If a node is red, all of its children must be **black**
    - For any given node *u*, every possible path from *u* to a "null reference" (i.e., an empty left or right child) must contain the same number of **black** nodes
- Null reference is also colored **black**
- *some* **Red-Black Trees** are *also* **AVL Trees**, *not all* **Red-Black Trees** are **AVL Trees**.
- Height
    - At most 2 times of the height of AVL Tree with the same amount of elements. (consider remove all red nodes, then all black nodes same height)
    - **O(log(n))**
- To keep balanced - insertion
    - **Recoloring**
        - Newly inserted node marked as **red**
        - If newly inserted node is a child of **black node, we are done!**
        - If newly inserted node is a child of **red node**
            - **If nodes are straight line**
                - **single rotation**
                - **recoloring**
            - **Else**
                - **Double rotation**
                - **Recoloring**
    - **Parent's sibling is Red**
        - As going down the tree, check nodes' two children
        - Recoloring before insertion
            - Once found two siblings are red, recolor and rotate



If X's Parent (P) is red, P is a left child of G, X is a left child of P, (and P's sibling (S) is black), then Rotate P right, flip colors of P and G

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        -

- **AVL vs Red-Black**
    - AVL Trees perform better with find operation
    - RB Trees perform better with insert and remove operations.
    - **Red-Black Trees** are the **Binary Search Tree** of choice for ordered data structures in many programming languages (e.g. the C++ map or set)

- Red-Black Tree
    - out of balance, take roughly the same amount of time to **remove or insert** an element in comparison to the corresponding AVL Tree (around 2 log *n* vs. around 2 log *n* operations).
    - pretty balanced, take roughly ***half*** the time to **remove or insert** an element in comparison to the corresponding AVL Tree (around log *n* vs. around 2 log *n* operations).

- **Worst-Case Time Complexity (AVL Tree)**
    - Find: **O(log *n*)** — AVL Trees must be balanced by definition
    - Insert: **O(log *n*)** — AVL Trees must be balanced by definition, and the rebalancing is O(log *n*)
    - Remove: **O(log *n*)** — AVL Trees must be balanced by definition, and the rebalancing is O(log *n*)
- **Average-Case Time Complexity (AVL Tree)**
    - Find: **O(log *n*)**
    - Insert: **O(log *n*)**
    - Remove: **O(log *n*)**
- **Best-Case Time Complexity (AVL Tree)**
    - Find: **O(1)** — If the query is the root
    - Insert: **O(log *n*)** — AVL Trees must be balanced, so we have to go down the entire O(log *n*) height of the tree
    - Remove: **O(log *n*)**
- **Space Complexity (AVL Tree)**
    - **O(*n*)** — Each node contains either 3 pointers (parent, left child, and right child) and the data, so O(1) space for each node, and we have exactly *n* nodes

- **Worst-Case Time Complexity (Red-Black Tree)**
    - Find: **O(log *n*)** — Red-Black Trees must be balanced
    - Insert: **O(log *n*)** — Red-Black Trees must be balanced, so we have to go down the entire O(log *n*) height of the tree, and the rebalancing occurs with O(1) cost during the insertion
    - Remove: **O(log *n*)** — Red-Black Trees must be balanced, and the rebalancing occurs with O(1) cost during the removal
- **Average-Case Time Complexity (Red-Black Tree)**
    - Find: **O(log *n*)**
    - Insert: **O(log *n*)**
    - Remove: **O(log *n*)**
- **Best-Case Time Complexity (Red-Black Tree)**

- **Find: O(1)** — If the query is the root
- **Insert: O(log *n*)** — Red-Black Trees must be balanced, so we have to go down the entire O(log *n*) height of the tree, and the rebalancing occurs with O(1) cost during the insertion
- **Remove: O(log *n*)**

- **Space Complexity (Red-Black Tree)**
  - **O(*n*)** — Each node contains either 3 pointers (parent, left child, and right child) and the data, so O(1) space for each node, and we have exactly *n* nodes

- **Huffman Encoding** - **Tree Construction**
  - Encoding depends on the **frequencies** of symbols
  - Construction:
    - Have a bunch of nodes (some subtrees that only have **root**)
    - Pick the least frequent nodes and make a **Tree**
      - **Mark left edge 1, right 0**
    - Repeat **step-2** till no subtrees exists
  - once have a tree, we can start encoding
  - Decryption
    - Traverse down the tree by bits
  - **Runtime analysis**
    - **File - O(n)**
    - **Use priority queue - O(clogc)**

- **Graph**
  - Nodes, edges
  - Unstructured, sequential, hierarchical, structured/connected/disconnected
  - Directed, undirected, weighted
  - Representation
    - **Adjacency Matrix**: coming from row i going to column j. **O(V^2)**

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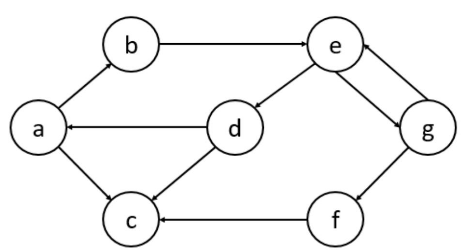
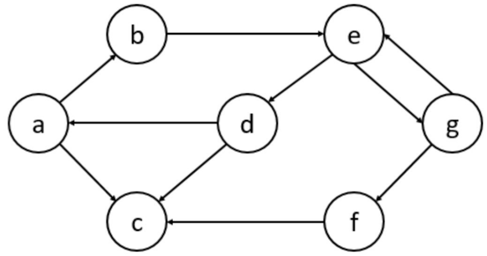
	a	b	c	d	e	f	g
a	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0
b	0	0	0	0	1	0	0
c	0	0	0	0	0	0	0
d	1	0	1	0	0	0	0
e	0	0	0	1	0	0	1
f	0	0	1	0	0	0	0
g	0	0	0	0	1	1	0



- **Adjacency List:** smaller storage, O(V+E). More dense: O(V+V^2)

```
a: {b, c}
b: {e}
c: {}
d: {a, c}
e: {d, g}
f: {c}
g: {e, f}
```



  - Storage - |V| + |E|

Graph Search

- BFS: Search all nodes on the same layer before continuing to next layer, **queue ADT(first in, first out)**
  - **Application: shortest path search (unweighted)**

```
BFS(s):
    initially, give all vertices in the graph a distance of INFINITY   O(|V|)
    start at s; give s distance = 0   O(1)
    enqueue s into a queue   O(1)
    while the queue is not empty:   O(|E|)
        dequeue the vertex v from the head of the queue   O(1)
        for each of v's adjacent nodes that have not yet been visited:   O(|E|)
            mark its distance as as 1 + distance to v   O(1)
            enqueue it in the queue   O(1)
```

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Unweighted Shortest Path: C++ code

```
/** Traverse the graph using BFS */
void BFSTraverse( vector<Vertex*> theGraph, int from )
{
  // assume code to initialize each Vertex's dist to INFINITY
  queue<Vertex*> toExplore;          //set prev to -1
  Vertex* start = theGraph[from];
  start->dist = 0;
  toExplore.push(start);
  while ( !toExplore.empty() ) {

    Vertex* next = toExplore.front();
    toExplore.pop();
    vector<int>::iterator it = next->adj.begin();
    for ( ; it != next->adj.end(); ++it ){
      Vertex* neighbor = theGraph[*it];
      if (next->dist+1 < neighbor->dist){
        neighbor->dist = next->dist + 1;
        neighbor->prev = next->index;
        toExplore.push(neighbor);
      }
    }
  }
}
```

- DFS: Search from root to leaf, go all the way down. **Stack(first in, last out)**
    - **Runtime: O(V + E).**

```
DFS(u,v):
    s = an empty stack
    push (0,u) to s // (0,u) -> (length from u, current vertex)
    while s is not empty:
        (length,curr) = s.pop()
        if curr == v: // if we have reached the vertex we are searching for
            return length
        for all outgoing edges (curr,w) from curr: // otherwise explore all neighbors
            if w has not yet been visited:
                add (length+1,w) to s
    return "FAIL" // if we reach this point, then no path exists from u to v
```

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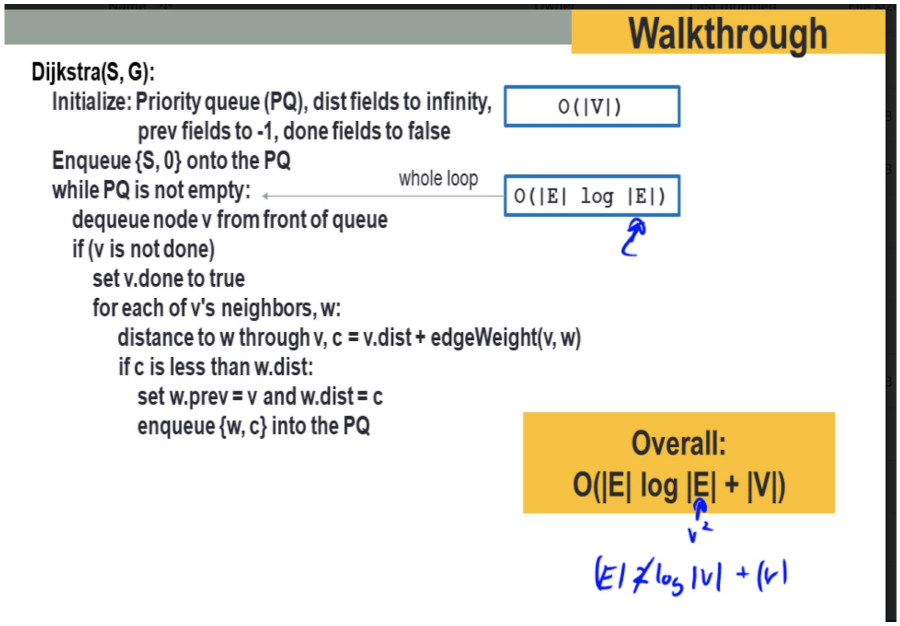
Shortest Path

- Dijkstra's Algorithm: (**BFS based**)
    - **no negative weight edges can exist in the graph**
    - lowest overall path weight. Search for next shortest and see which vertex it discovers.
    - Implementation: **Priority Queue** ADT, store one immediate vertex at the end of path
    - Fields of node: Distance, previous, done
    - Each vertex: first time removed, **shortest path found**
    - Runtime: O(V + **E log(E)**)
        - Insertion of priority queue - **O(ElogE)**



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- **A star search**
    - **Priority function: f(n) = g(n) {distance from start to vertex n} + h(n) {heuristic estimated cost from vertex n to goal vertex}**

Minimum Spanning Tree Creation (Not dis time)

- **Prim's Algorithm**
    - Starting at a vertex
    - Enqueue every adjacent vertex
    - Dequeue the very first vertex and connect with its previous
    - Done when queue is empty

- **Kruskal's Algorithm**
    - Create a forest of vertices
    - Create a priority queue containing all the edges, ordered by edge weight
    - While fewer than |*V*| - 1 edges have been added back to the forest:
        - Deque the smallest-weight edge (*v*,*w*, cost), from the priority queue
        - **If:** *v* and *w* already belong to the same tree in the forest, go to deque
        - **Else:** Join those vertices with that edge and continue
- Kruskal's Algorithm and Prim's Algorithm can easily work with negative edge