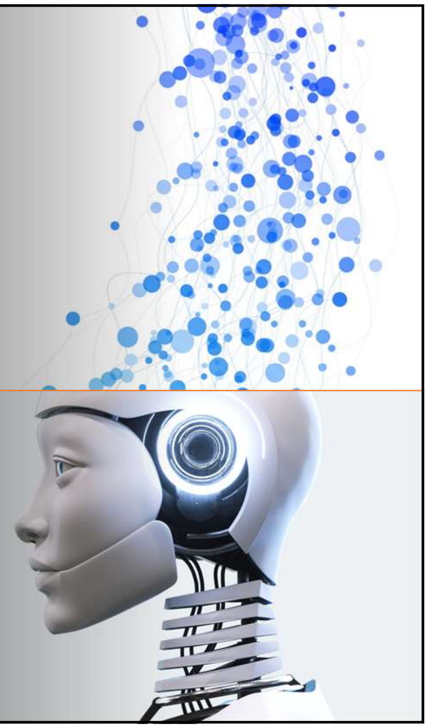
Machine Learning Foundations

An Introduction to your AI Journey

Sumudu Tennakoon, PhD



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Course Objective

Provide comprehensive understanding of the core principles of Machine Learning with hands-on training on applying machine learning to solve real-world problems.

A learner who completes this course should be able to define a machine learning problem, understand the solution path, and display the ability to carry out the end-to-end process of building a machine learning application.

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Machine Learning Career Prospectus

- Data Scientist
- AI Scientist
- ML/AI Engineer
- Data Engineer
- Data Analyst
- AI/ML Developer
- IoT Developer
- Solutions Architect
- Freelancer
- …

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Schedule and Format

Duration: 60 hours

Schedule: 3-month program/12 weeks, two sessions per week.

Format: Live/Recorded Lectures, Demonstrations, Hands-on Exercises/Labs.

Evaluation: Quizzes (2), Project (1)

Additional Practice: Students must spend extra time on exercises and the capstone project.

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Prerequisites

- Basics of computer programming, mathematics, and statistics.
- Basic knowledge in computer applications:
  - Spreadsheet
  - word processor
  - presentation authoring

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Platform and Data for Hands-on Exercises and Project

Programing Language: Python 3 will be used as the primary programming language in teaching, practice examples and assignments.

Python Libraries: Scikit-learn, TensorFlow, Pandas, NumPy, Matplotlib, Seaborn, Flask.

Applications/Tools: Jupyter Notebook/Lab, IDE (Spyder/VS Code/Atom/PyCharm), Spreadsheet (MS Excel/LibreOffice Calc).

Data: Data for exercises, case studies, and projects will be obtained from open data repositories.

Computing Environment: Cloud platform (will be decided on class consensus and service availability) or locally installed Python distribution in student's PC.

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Session Topics

#	Topic Name	Training Week #
1	Introduction to Machine Learning (ML), History, and Applications	1
2	Setting up a Computing Environment, Python and Required Libraries.	2
3	Knowledge Foundations to ML (Computing, Statistics, and Mathematics) *	2-3
4	Exploratory Data Analysis (EDA) and Feature Engineering *	4-5
5	Supervised and Unsupervised Learning (concepts)	6
6	Machine Learning Algorithms *	6-7
7	Explaining ML Models and Predictions (introduction) *	7
8	Deep Learning and Neural Networks (introduction) *	8
9	Design and Develop and Deploy ML Solutions *	9-10
10	Capstone Project *	11-12

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Evaluations and Grading

- **Completion Requirement:**
  - 80 % Attendance (at least 19 out of 24 sessions)
  - Final Grade > 70 %
- **Completion with Distinction:**
  - Final Grade > 90 %

	Topic #	%
Quiz1 (Basic Concepts)	1-6	20
Quiz 2 (Advanced Concepts, Deep Learning and Application Building)	7-9	20
Deliverable and Project Report	10	50
Presentation (video narration)	10	10
		100

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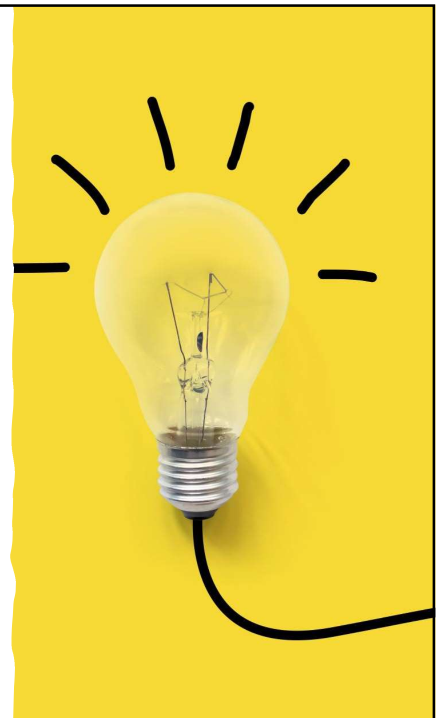
Introduction to Machine Learning

History and core concepts of ML to navigate the future lessons.

Applications of ML.



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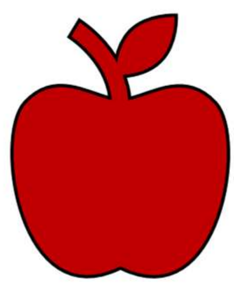
Example: Identify Objects

What facts you consider to identify these object?

Pineapple



Apple

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How we Learn ?

- Memorize Facts
  - Declarative Knowledge
  - Limited by memory and time to observe

- Infer (deduce new information from previously known facts)
  - Imperative Knowledge
  - Limited by accuracy of predictions and drifts (present is not behaving the same way as past)

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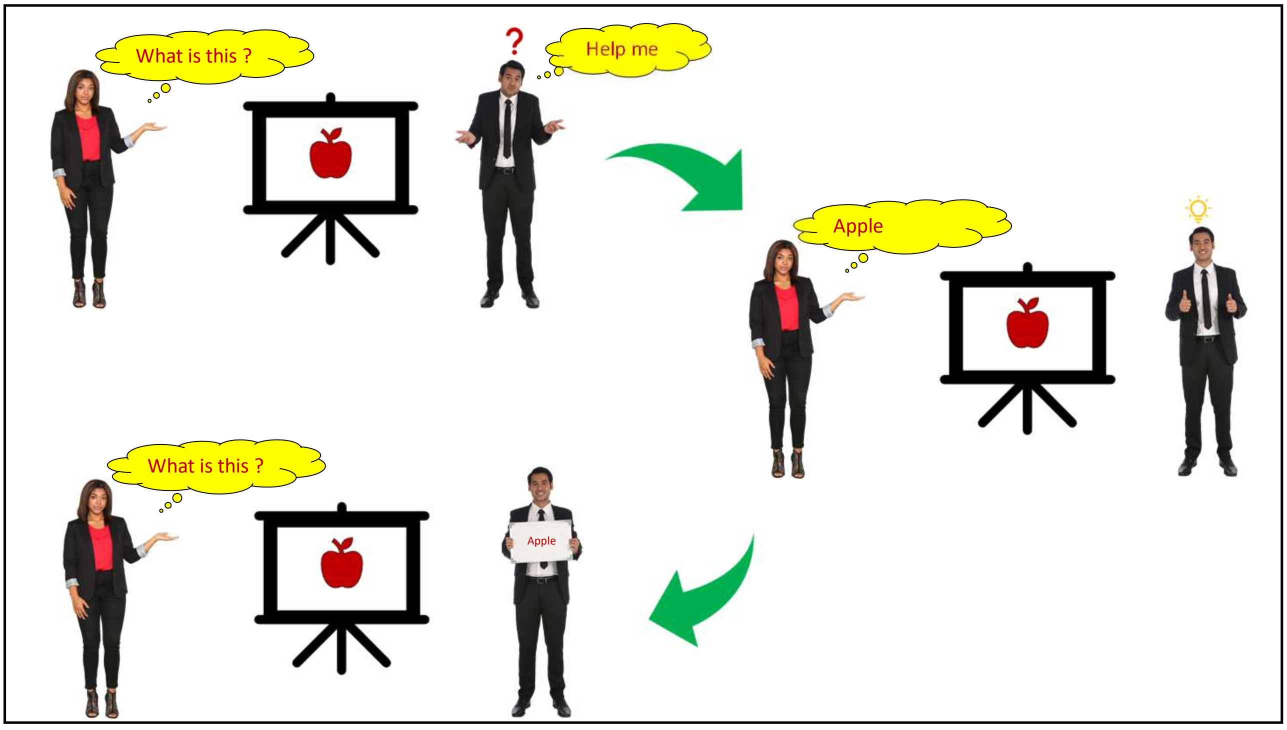

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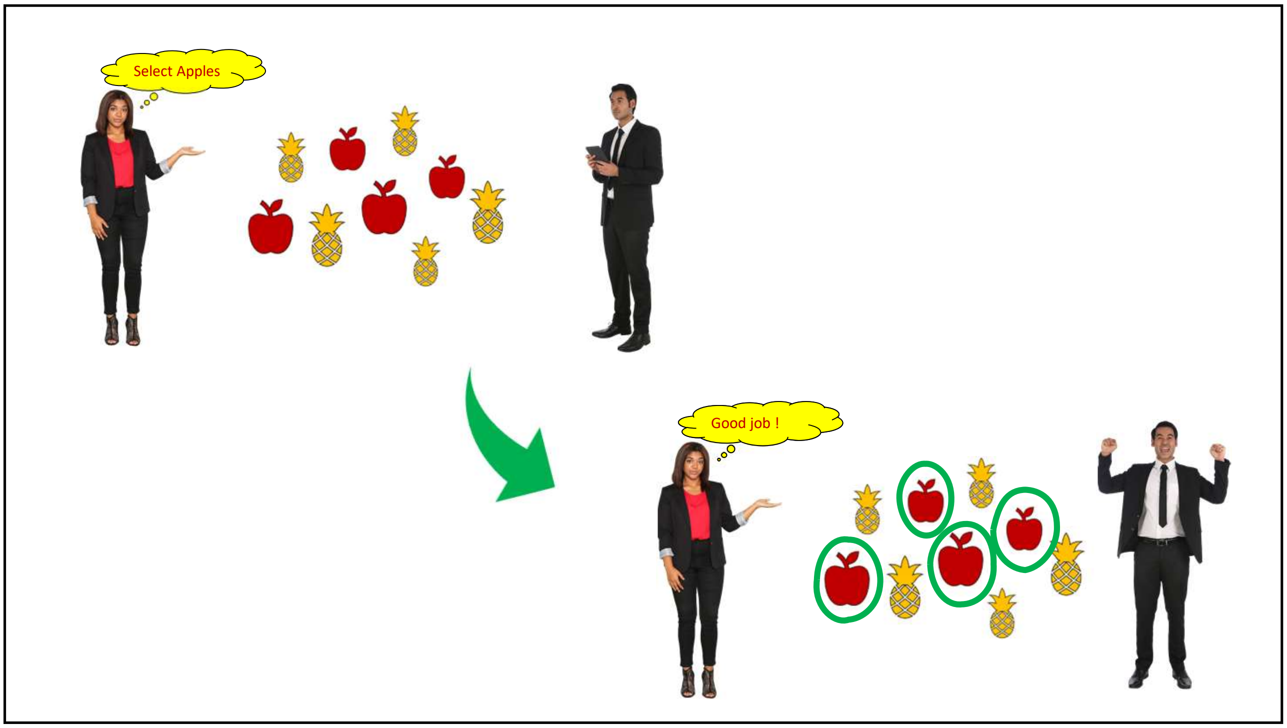

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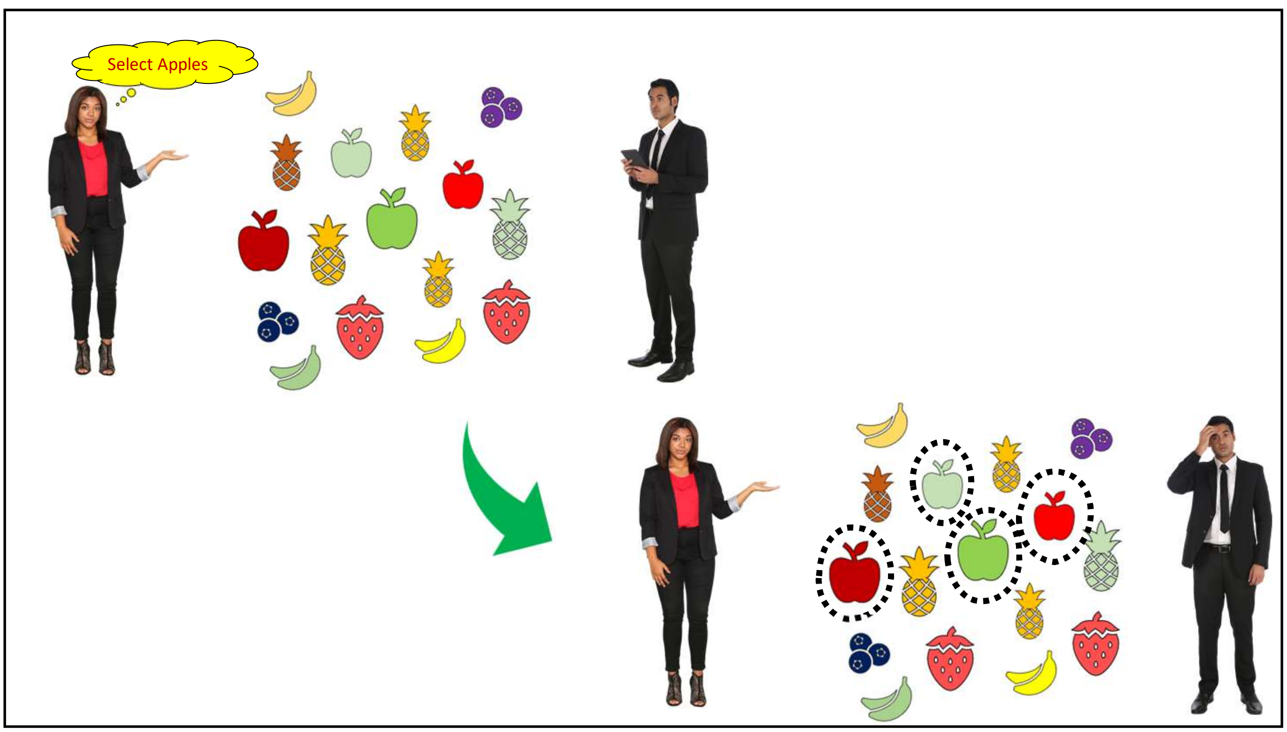

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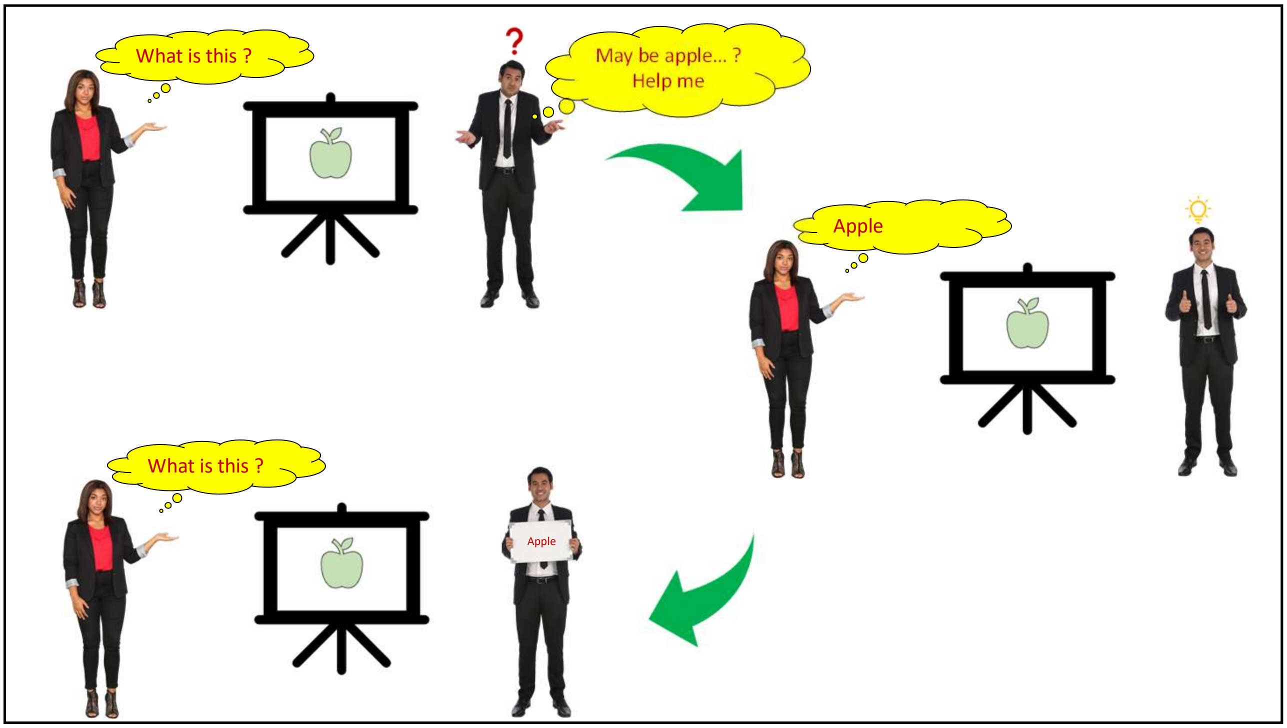

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Exercise

- What are the Observations/Measurements can be used to make a determination.
- Design a simple classifier logic.
- Is it easy tor difficult to converting this logic to a computer program (code)?
- What are the considerations when converting this logic to a computer program (code)?
- What are the points of failures in this approach?

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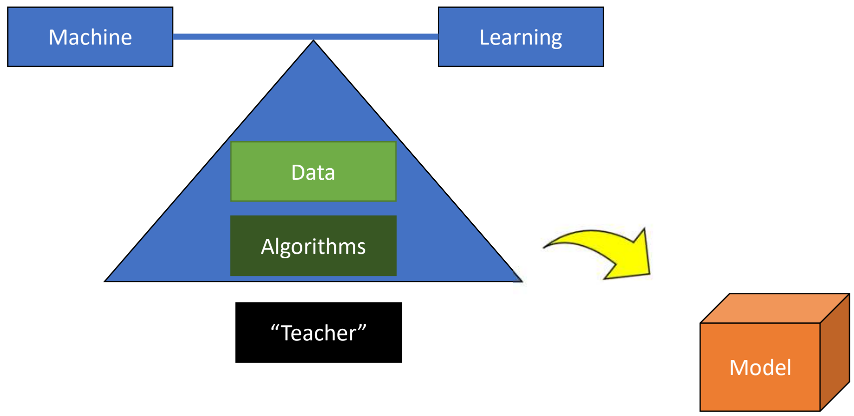
What is Machine Learning ?

- Learn from Data



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Machine learning model

“Machine learning models are built on mathematical algorithms and are trained using data and human expertise to help us accurately predict outcomes based on input data such as images, text, or language.”

https://developer.nvidia.com/ai-models

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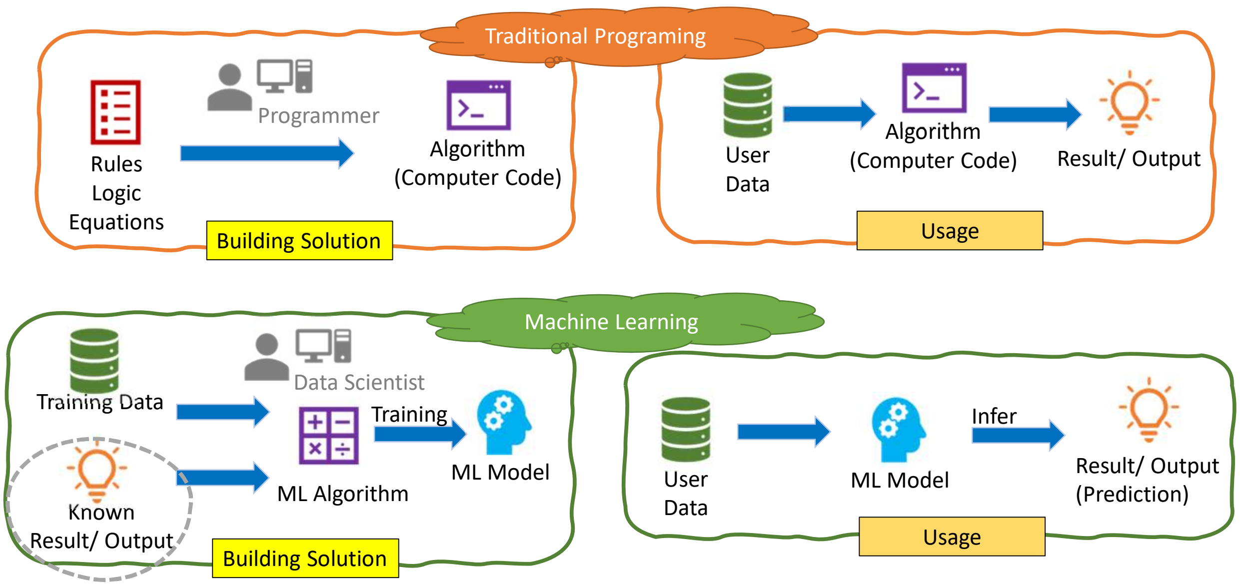
Computer Program vs. Machine Learning



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What is Machine Learning?

"The field of study that gives computers the ability to learn without being explicitly programmed."

~ Arthur Samuel (1959)

Author of first self-learning program to learn how to play checkers by learning from experience (past movements and results)

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What is Machine Learning?

" A computer program is said to learn from experience E with respect to some class of **tasks T** and **performance measure P**, if its performance at tasks in T, as measured by P, improves with **experience E**."

~ *Tom Mitchell (1997)*

Example: playing checkers.

- E = the experience of playing many games of checkers
- T = the task of playing checkers.
- P = the probability that the program will win the next game.

Mitchell, T. (1997). *Machine Learning*. McGraw Hill. p. 2. ISBN 978-0-07-042807-2.

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AI and Machine Learning

- AI (Mimic Cognitive Functions of Human)
  - Computer Vision
  - Speech Recognition and Synthesis
  - Language Processing and Understanding
  - Motion
  - Decision Making
  - Prescribe or Predict
  - Reasoning
- Machine Learning (ML)
  - Machines learn on Data/Prior Knowledge
  - Statistical Modeling/Algorithms
  - Backbone of AI is Machine Learning
  - Algorithms to Find meanings of data
  - Find Relationships
  - Making Predictions
  - Problem-Solution Types
    - Classification
    - Regression
    - Clustering

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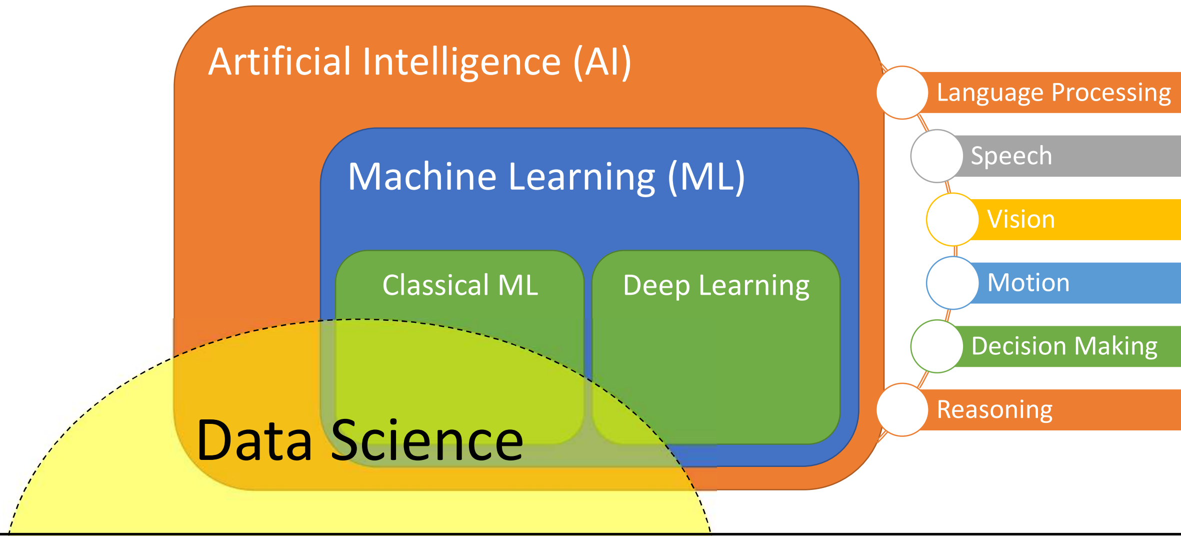
Machine Learning/Deep Learning/AI



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Levels of AI

Artificial General Intelligence (AGI) known as "Strong AI"

- AGI is the ability to solve *any* problem rather than finding a solution to a particular problem.
- Machine can understand or learn any intellectual task that a human being can.
- The machine can think and perform tasks on its own, just like a human being.
- In the Movies! We are not there yet.

Weak Artificial Intelligence (Weak AI),

- Implements a limited part of human cognitive abilities.
- **Narrow AI** is a special case of Weak AI focused on a specific problem or task.
- Currently, existing AI systems are likely operating as a narrow AI.
- devices cannot follow these tasks independently but are made to look intelligent (simulate human behavior).

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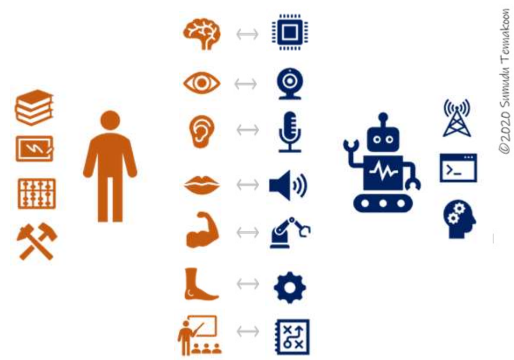
Building Blocks of an AI System

- Image recognition (computer vision)
- Signal processing (sound, sensor data feed, etc.)
- Speech Recognition (Speech to text/STT)
- Natural language processing (NLP)
- Visual Synthesis (Computer Graphics)
- Sound Synthesis (Text to Speech/TTS)
- Software/Algorithms
- Applications (Anomaly Detection, Classification, Prediction, Pattern Recognition)
- Memory (Storage, RAM, Cache)
- Processor (GPU, CPU, TPU)
- Connectivity (Wi-Fi, Satellite, 5G, ethernet, etc.)
- Hardware (Computer, Mechanical Components, etc.)



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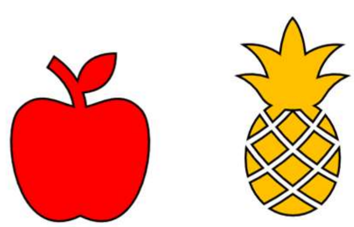
Exercise



- What are Observations/Measurements can be used to make a determination.
- Design a simple classifier logic.
- Is it easy or difficult to converting this logic to a computer program (code)?
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- What are the points of failure in this approach?

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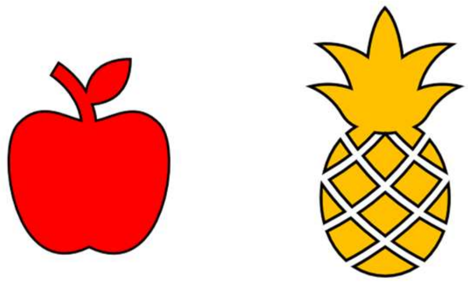
Input and Output ?



Input → (Features/Attributes)

Color	red	yellow
Weight	50 g	200 g
Diameter	10 cm	20 cm

Output → Apple (A) / Pineapple (P)

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$$y = f(X)$$

Independent Variables

$$X = [x1, x2, x3, \dots.]$$

$\{Color, Diameter, Weight\}$

Dependent Variable

$$y$$

$\{Apple, Pineapple\}$

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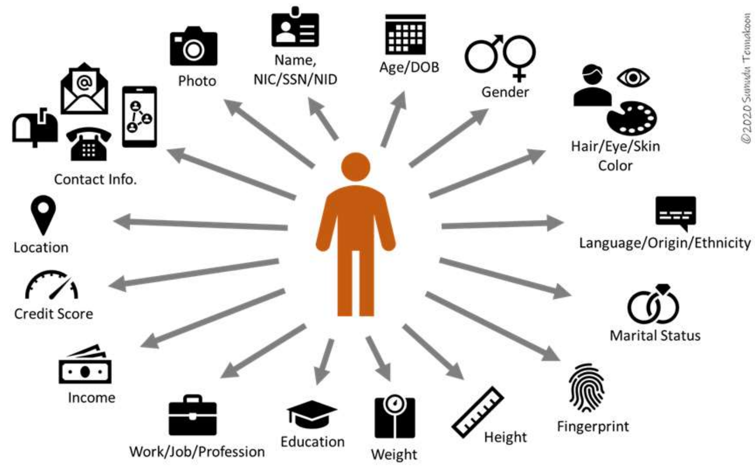
Features/Attributes



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Tabular Data

Column 1 Column 2 Column 3

	ID	Name	DOB
Row 1	10001	John Doe	1988-01-01
Row 2	10002	Jane Doe	1990-12-31
Row 3	...	...	...

← Column Names

Row (Record)

Column (Data Field)

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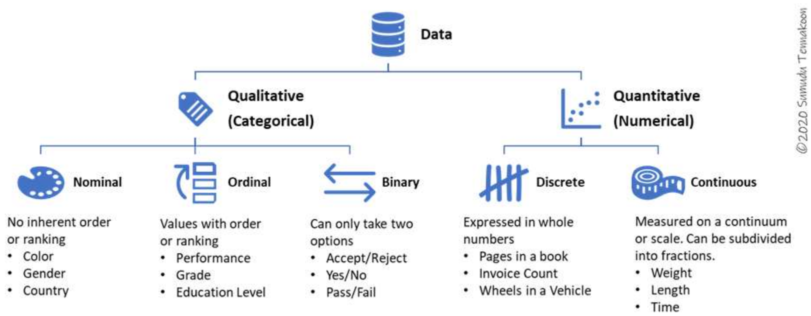
Understanding Data



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Machine Learning Approaches

- Supervised
- Unsupervised
- Reinforcement learning

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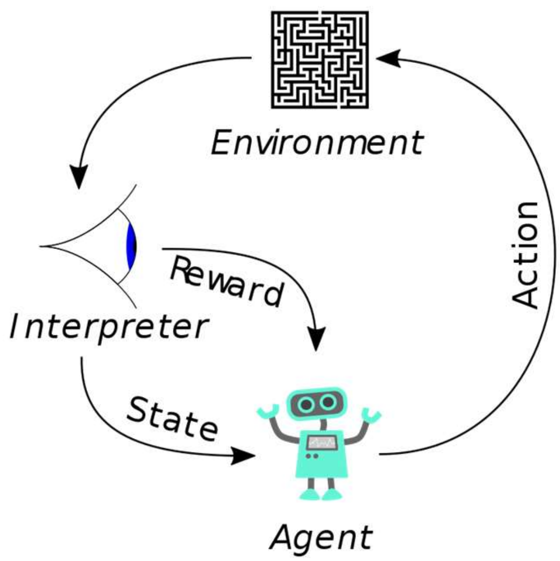

https://en.wikipedia.org/wiki/Reinforcement_learning

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Types of ML Algorithms

Regression

Linear

Polynomial

Classification

Tree Classifiers

Logistic Regression

Support Vector Machines (SVM)

Clustering

K-Means

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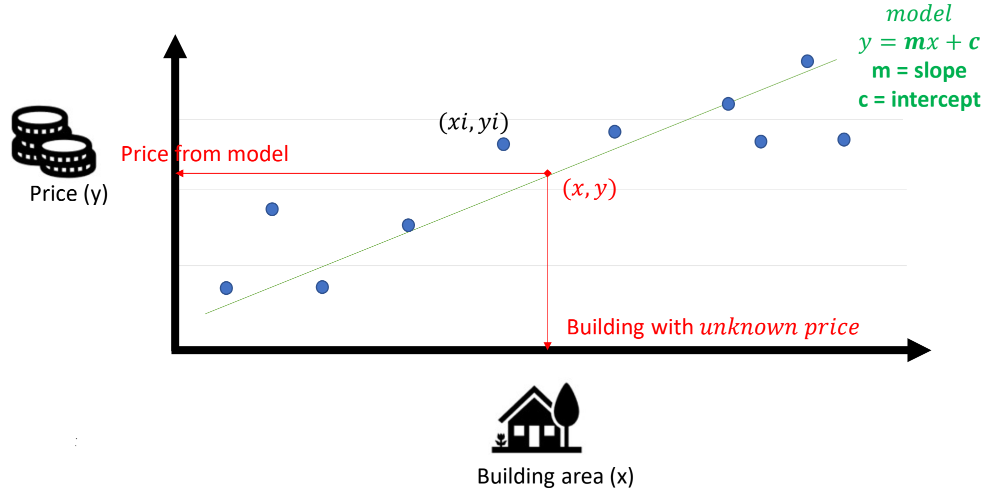
Regression



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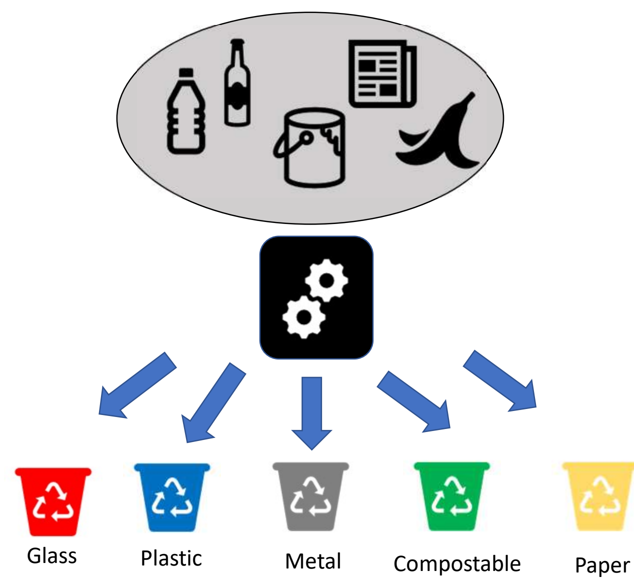
Classification



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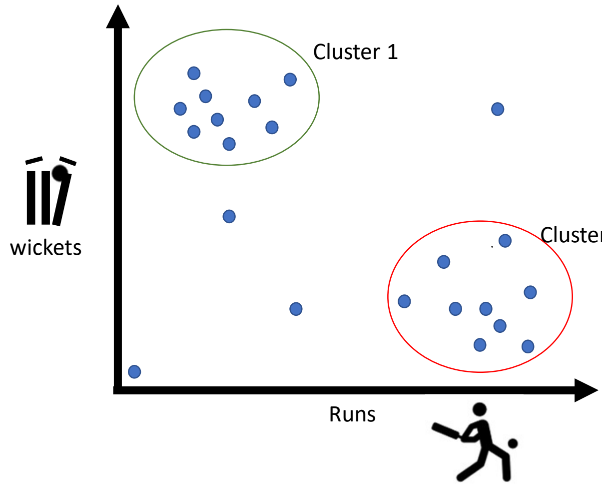
Clustering



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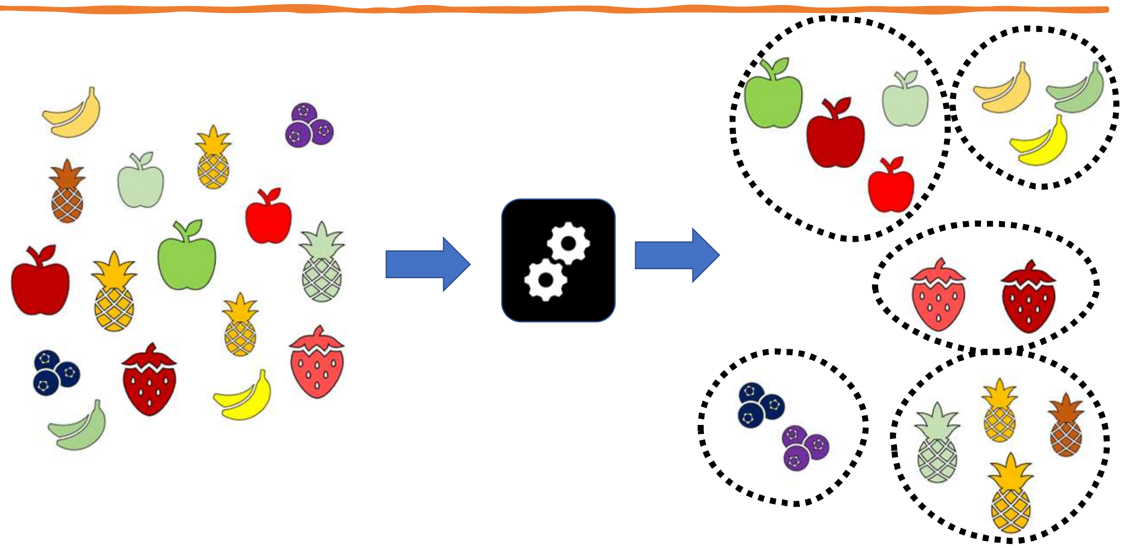
Clustering



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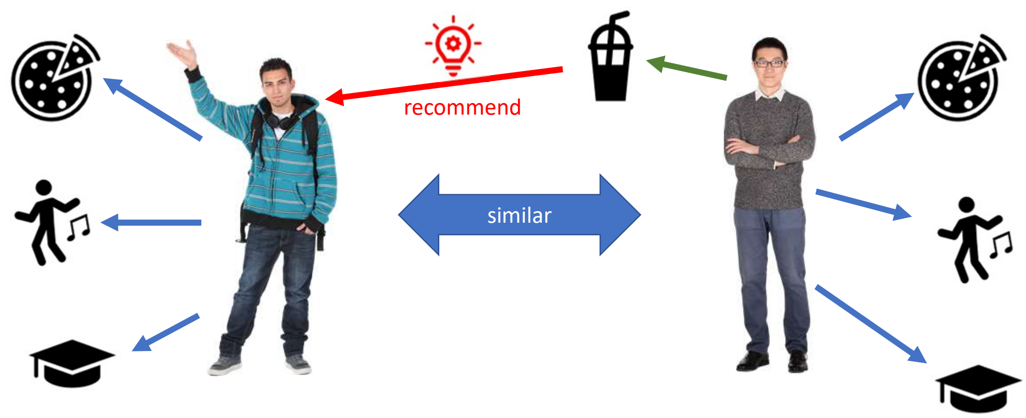
Recommender Systems



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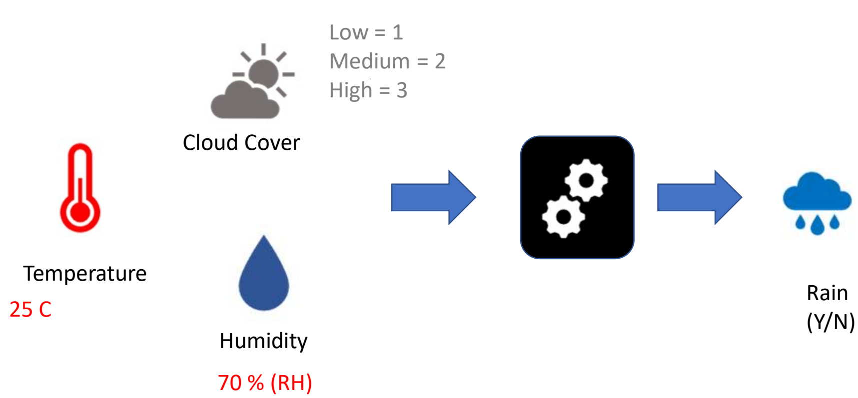
Prediction



$$y = f(X)\ ?$$

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Steps of Building ML Model

Prepare Data → Identifying Solution Approach (ML Model) → Train → Score (Predict on Test Sample)

→ Model Evaluation → Tuning the Model → Deploy

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Why we need Machine Learning?

- Simulate human intelligence
- Automation
- Help humans with informed decision making
- Solve multidimensional problems
- Predict future outcome based on historical observations

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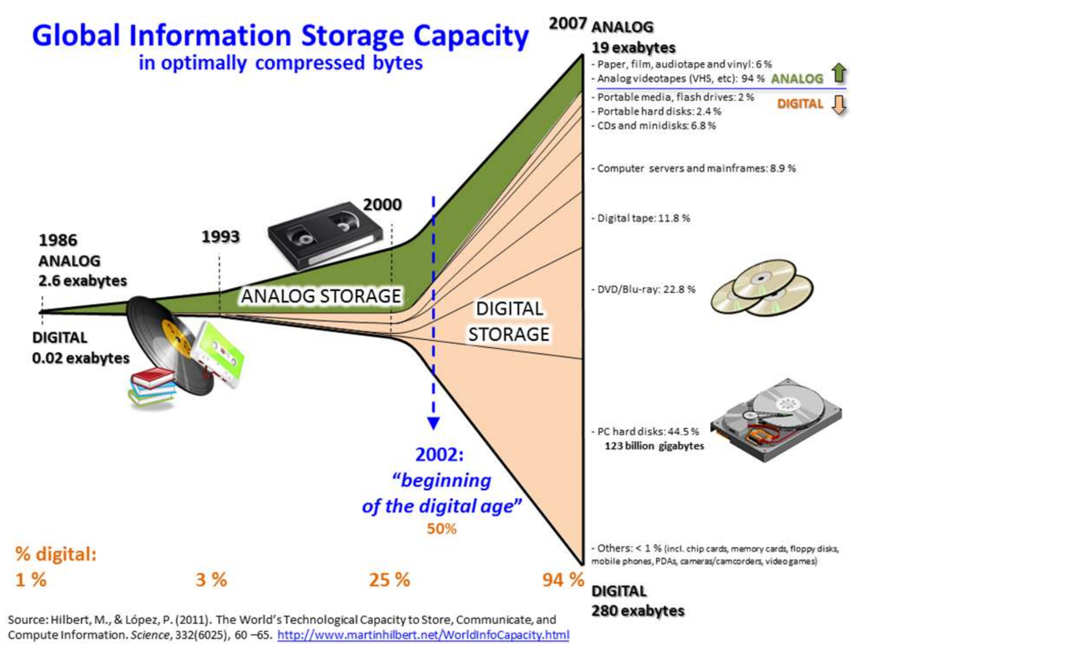

Scholz, R.. "Sustainable Digital Environments: What Major Challenges Is Humankind Facing?" Sustainability 8 (2016): 1-31.

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Machine Learning Applications

- Spam Email Filtering
- Approve or Reject Loan Application
- Predicting Stock Price
- Credit Card Fraud Detection
- Recommending Items to Purchase (Advertising)

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Application Areas

- Finance
- Marketing
- Information Technology
- Cyber Security
- Agriculture
- Government
- Automobile
- Manufacturing
- Retail
- Entertainment
- …

Everywhere!

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Why should everyone get familiar with ML?

- Applications of machine learning are all around us.
- ML is used in many industries and domains.
- Job opportunities.
- It is fun to learn and helps train your brain.

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Machine Learning Career Prospectus

- Data Scientist
- AI Scientist
- ML/AI Engineer
- Data Engineer
- Data Analyst
- AI/ML Developer
- IoT Developer
- Solutions Architect
- …

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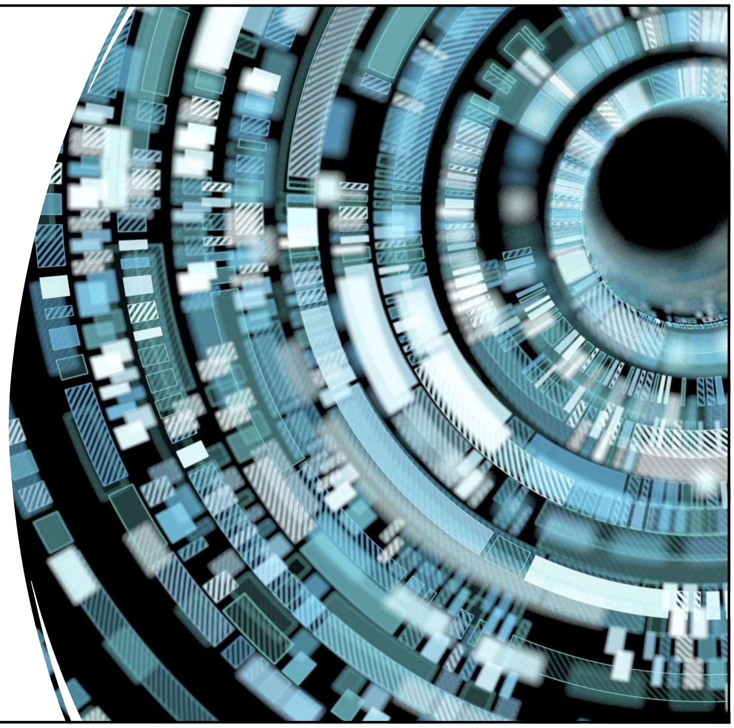
Setting up Computing Environment

Cloud Computing Platform (Google Colab)

Python (Install, Libraries)



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Install Python in Local Computer

- Python: https://www.python.org/downloads/
- Anaconda Python: https://www.anaconda.com/products/individual
- Python: Libraries: https://www.anaconda.com/open-source

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Python Libraries

- Data Handling
  - **Pandas**
  - Dask (distributed)
- Machine Learning
  - **Scikit-learn**
  - **TensorFlow**
  - PyTorch
- Visualizing
  - **Matplotlib**
  - Seaborn
- Numerical and Scientific Computing
  - **SciPy**
  - **NumPy**
- Machine Learning Model Interpretation
  - **LIME**
  - **SHAP**
- Web Services/API
  - **Flask**
  - Django

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Install Python Libraries

- **Using pip package manager for Python**
  - https://packaging.python.org/tutorials/installing-packages
  - **`pip install some-package-name`**
  - NumPy: `pip install numpy`
  - Pandas: `pip install pandas`
  - Scikit-Learn: `pip install scikit-learn`
  - Matplotlib: `pip install matplotlib`
- **Using Conda package manager**
  - https://docs.conda.io/projects/conda/en/latest/commands/install.html
  - **`conda install -c conda-forge some-package-name`**
  - NumPy: `conda install -c conda-forge numpy`
  - Pandas: `conda install -c conda-forge pandas`
  - Scikit-Learn: `conda install -c conda-forge scikit-learn`
  - Matplotlib: `conda install -c conda-forge matplotlib`

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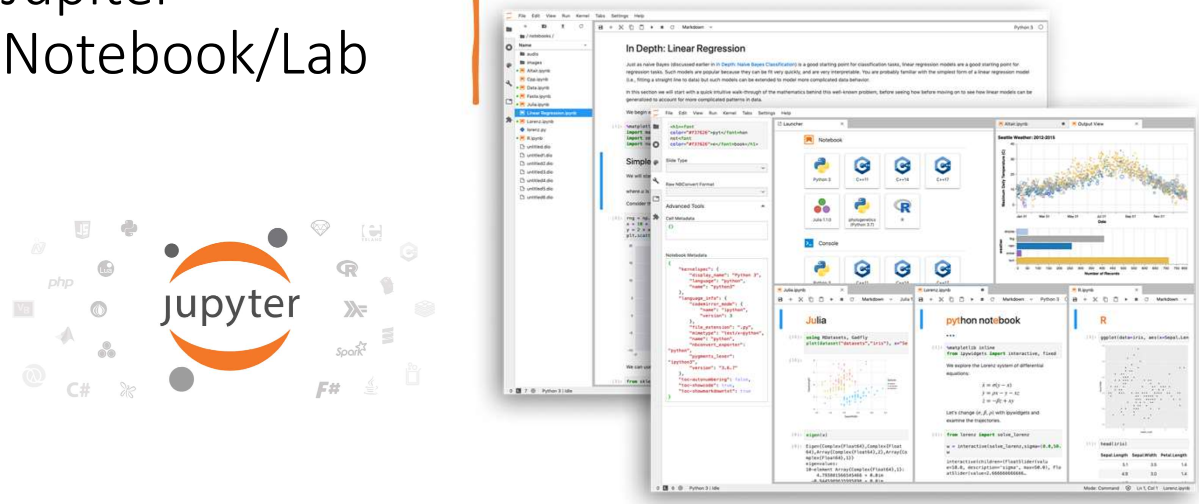
Jupiter Notebook/Lab



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Google Colab

https://colab.research.google.com

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Getting Started

- Creating New Notebook
- Opening Notebook from GitHub
- Opening Notebook from file

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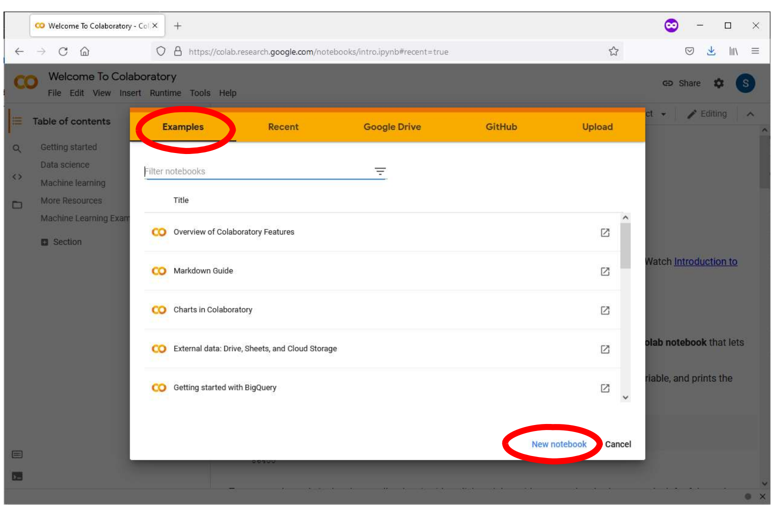
Creating New Notebook



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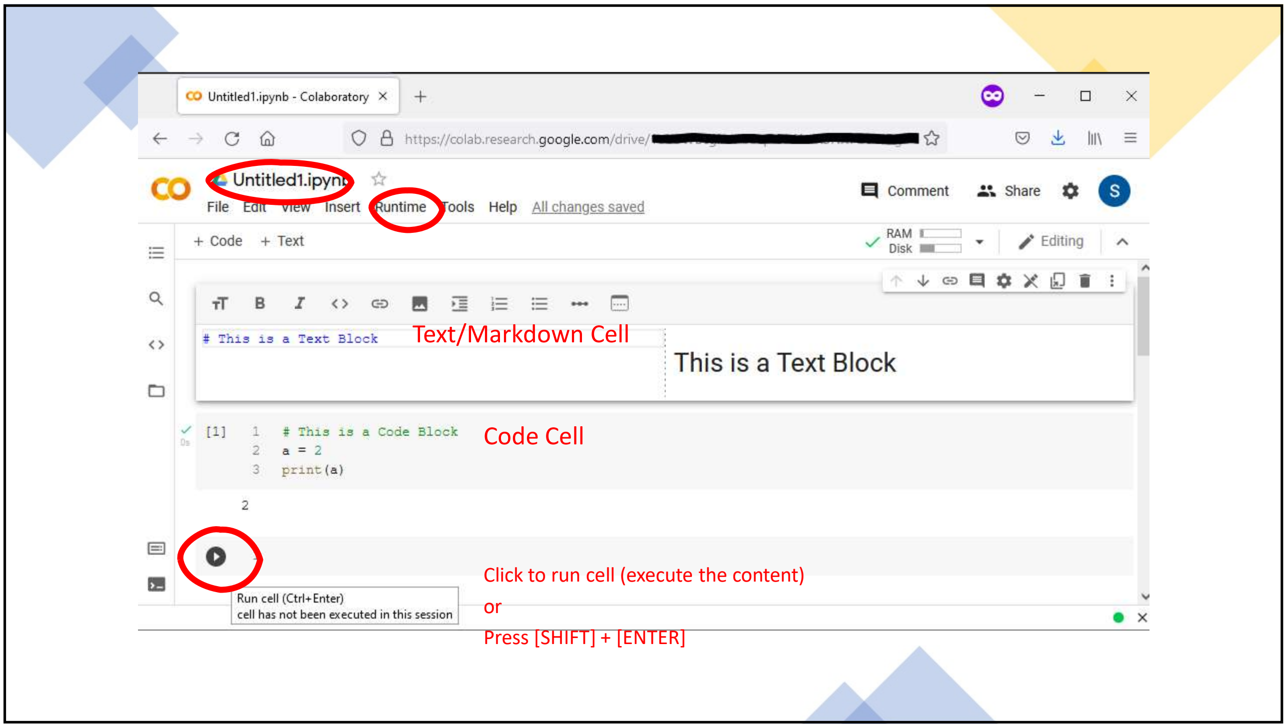

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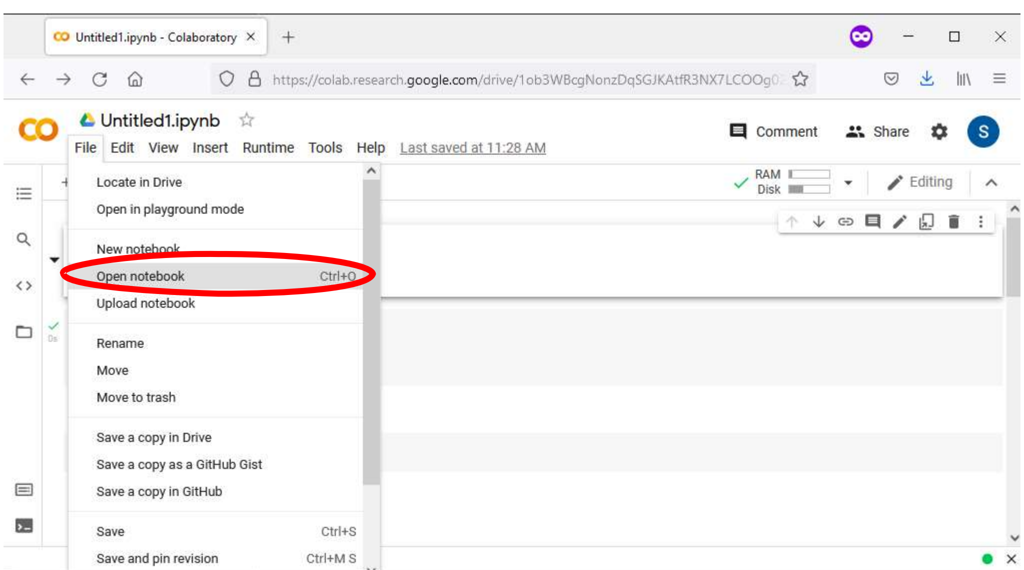
Open Notebook



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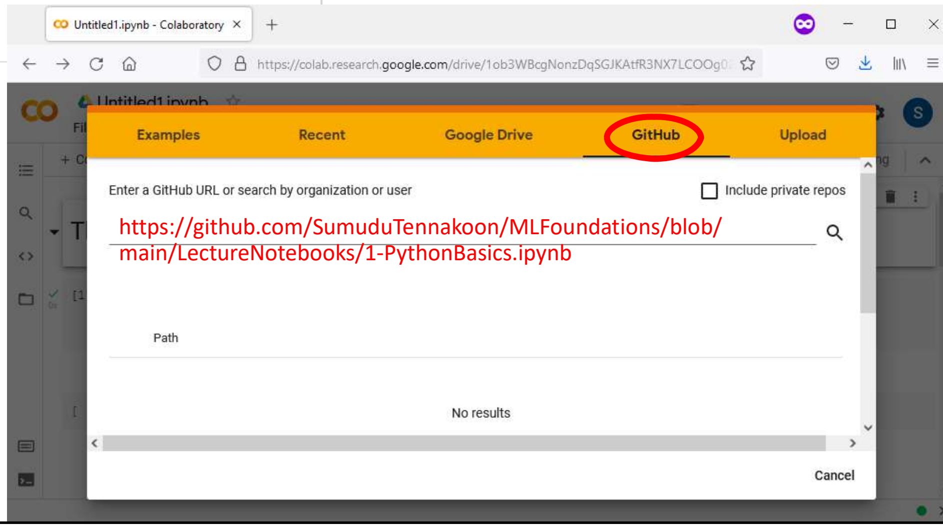
Open Notebook from GitHub



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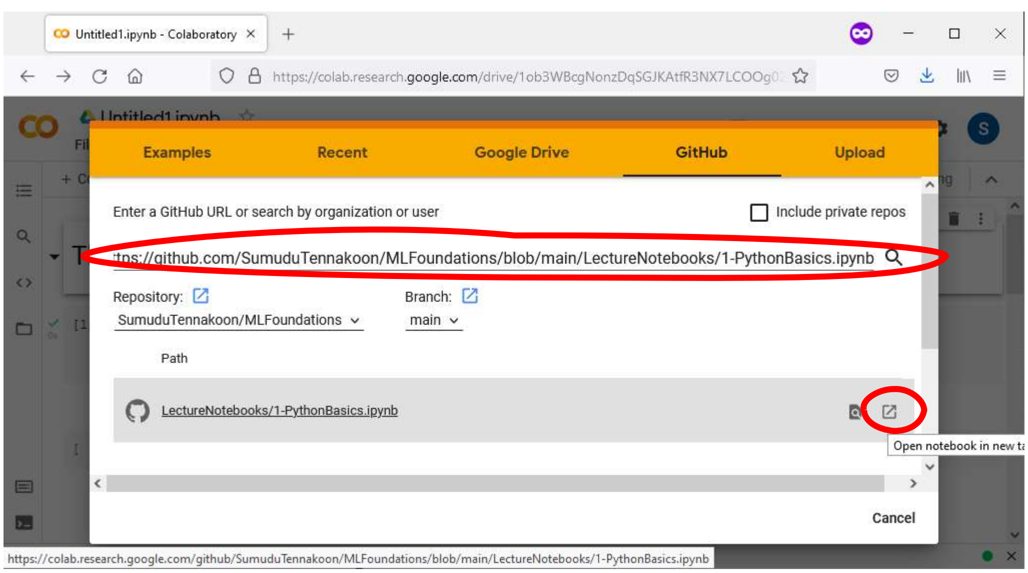

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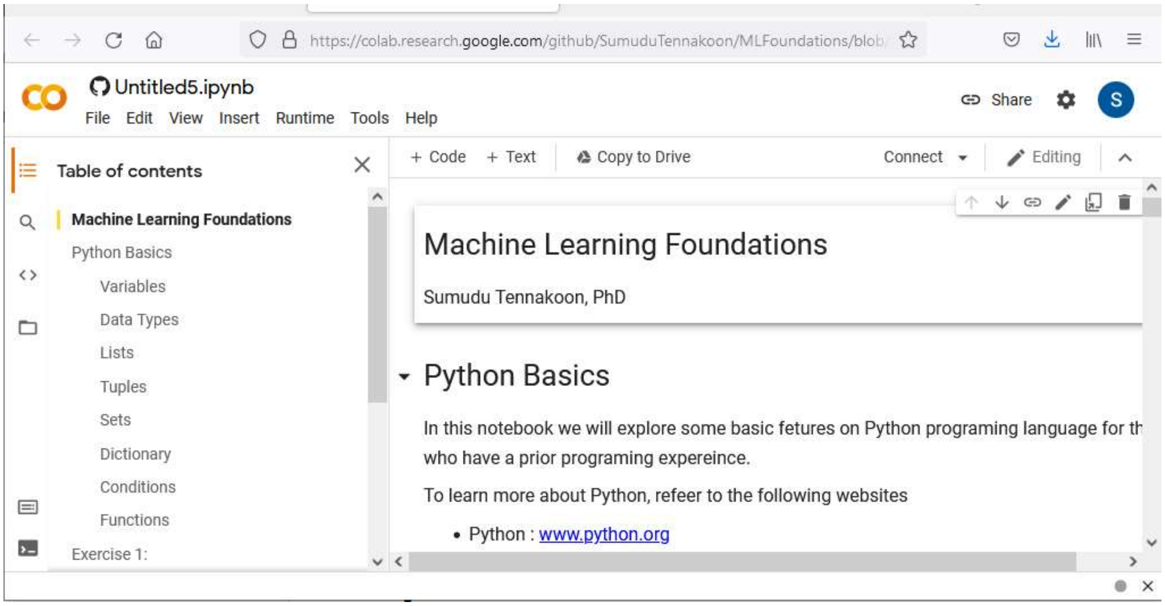

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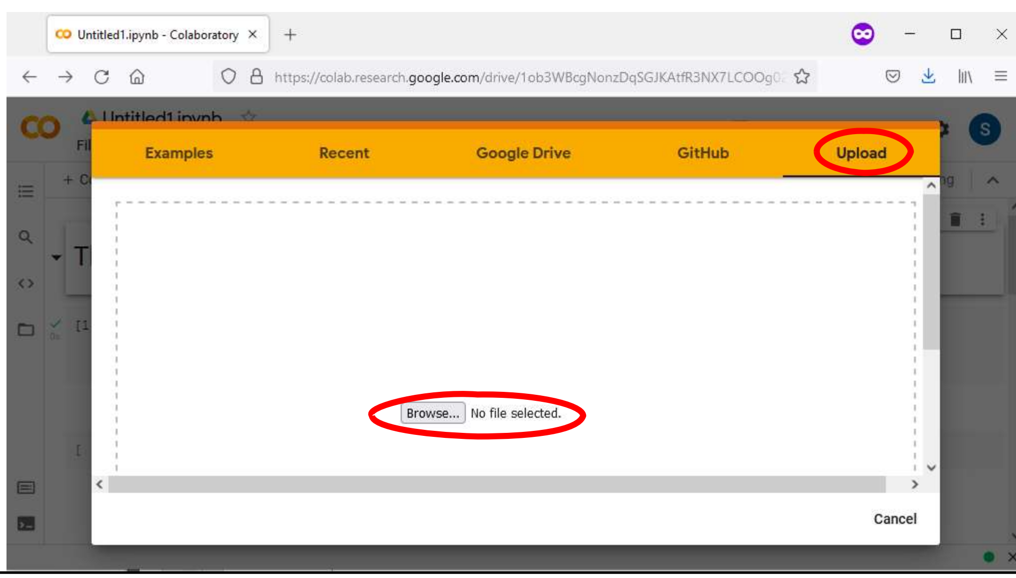
Upload Notebook (.ipynb file)



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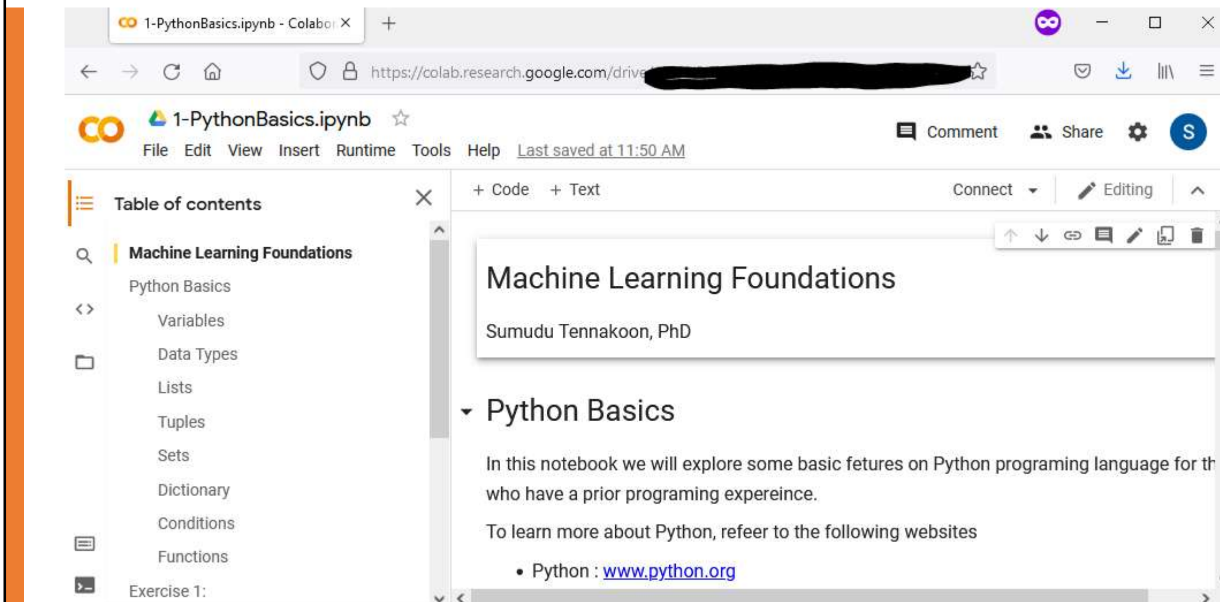

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GitHub Link to Lecture Notebooks

Folder:
https://github.com/sumudutennakoon/mlfoundations/tree/main/lecturenotebooks

Python basics notebook:
https://github.com/sumudutennakoon/mlfoundations/blob/main/lecturenotebooks/1-pythonbasics.ipynb

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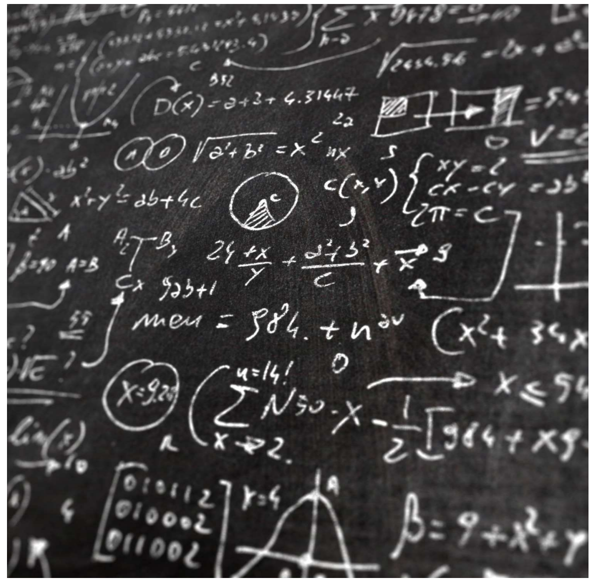
Mathematics for Machine Learning



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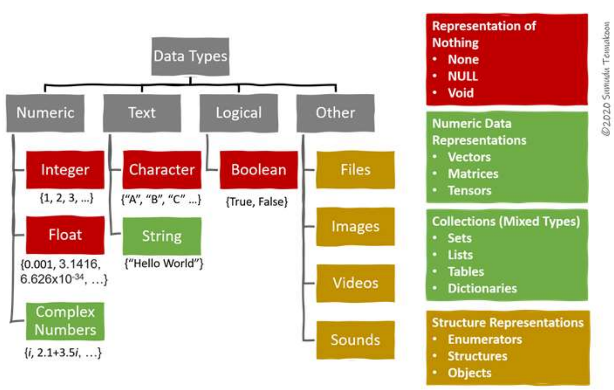
Data Types and Representations



https://pub.towardsai.net/data-science-for-everyone-getting-to-know-your-data-part-1-bb8b6d7782b1

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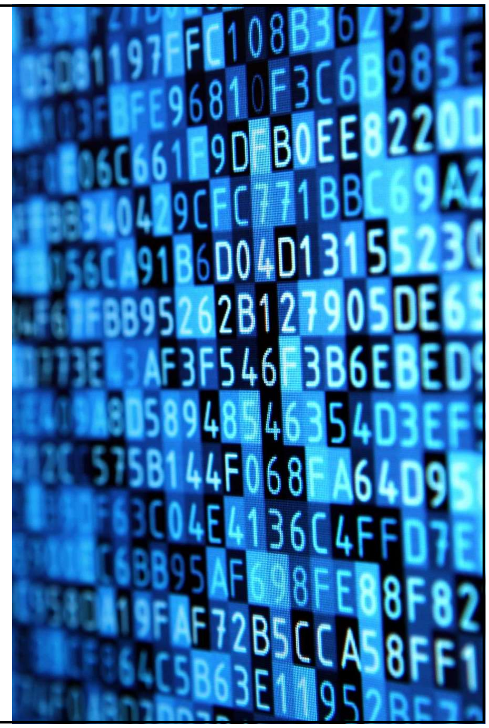
Data Representations

- In computing everything must convert into numbers !
- Numeric Data Structures:
  - Scalars: 3.14
  - Vectors: [1,2,3]
  - Matrices: [[1,2], [3,4]]
  - Tensors: [[[1,2], [3,4]] , [[5,6], [7,8]]]



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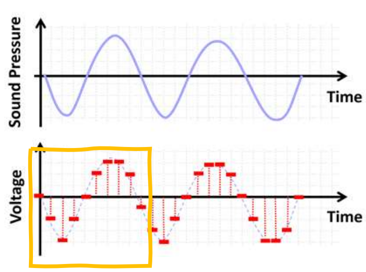
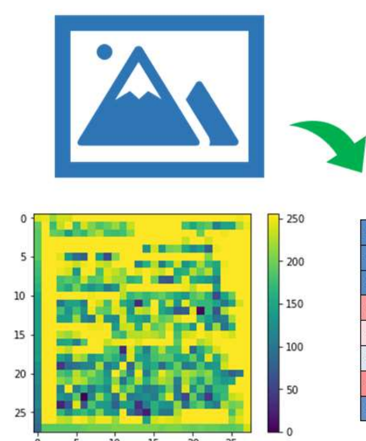
Why we need Vectors, Matrices and Tensors in Machine Learning?



$$\begin{aligned} signal &= [0 \quad -2 \quad -4 \quad -2 \quad 0 \quad 2 \quad 3 \quad 3 \quad 2 \quad -1 \quad -3] \\ time &= [0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \quad 9 \quad 10] \end{aligned}$$



1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
1.0	0.9	0.7	0.9	0.6	0.8	0.7	0.8
1.0	0.9	0.8	0.9	0.8	0.9	0.9	0.9
0.7	0.7	0.7	0.8	1.0	0.6	0.8	0.8
0.9	0.9	0.9	0.9	1.0	0.9	0.9	0.9
0.9	1.0	1.0	0.9	1.0	1.0	1.0	1.0
0.7	0.6	0.8	0.8	0.9	0.6	1.0	0.8
1.0	0.9	1.0	1.0	1.0	0.9	1.0	1.0

https://pub.towardsai.net/data-science-for-everyone-getting-to-know-your-data-part-1-bb8b6d7782b1

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Vectors and Matrices

Scalar

1

- *Temperature*
- *Mass*
- Speed

Vector

columns

rows

Column Vector

$$\begin{bmatrix}1\\2\\3\end{bmatrix}_{3\times1}$$

Row Vector

$$\begin{bmatrix}1 & 2 & 3\end{bmatrix}_{1\times3}$$

- *Distance [x, y, z]*
- *Velocity* $[Vx, Vy, Vz]$

Matrix

$$\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\end{bmatrix}_{2\times3}$$

rows x columns

$$\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}_{2\times2}$$

Tensor

$$\begin{bmatrix}[1 & 2] & [3 & 4]\\ [5 & 6] & [7 & 8]\end{bmatrix}_{2\times2\times2}$$

[What's a Tensor? - YouTube]

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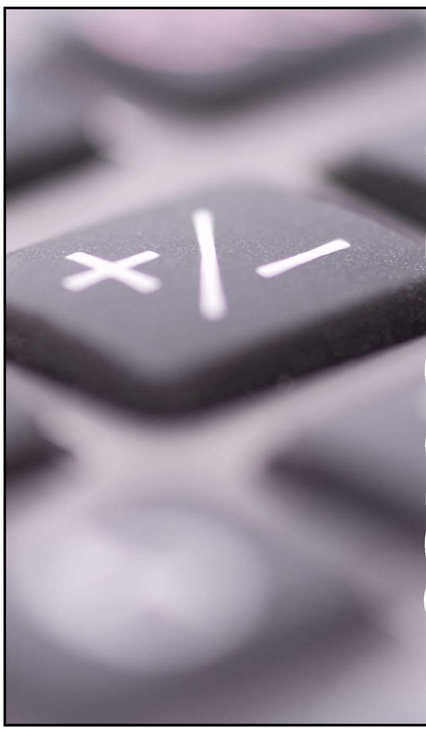

Matrix Algebra

- Transpose
- Sum
- Diagonal
- Determinant
- Adjugate
- Inverse

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Matrix Notation

- Matrix Notation
- $A = \begin{bmatrix} a_{11} & a_{12} & .. & a_{1n} \\ a_{21} & a_{22} & .. & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & . & a_{mn} \end{bmatrix}_{m \times n}$
- Representation of elements in python index numbers
  - $A = \begin{bmatrix} A[0][0] & A[0][1] & .. & A[0][n-1] \\ A[1][0] & A[1][1] & .. & A[1][n-1] \\ \vdots & \vdots & \vdots & \vdots \\ A[m-1][0] & A[m-1][1] & .. & A[m-1][n-1] \end{bmatrix}$

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Transpose, Sum, Diagonal

Columns (j)

Rows (i)

- $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} = [a_{ij}]$

Diagonal

Rows ↔Columns

- $A^T = \begin{bmatrix} a_{11} & a_{21} & a_{31} & a_{41} \\ a_{12} & a_{22} & a_{32} & a_{42} \\ a_{13} & a_{23} & a_{33} & a_{43} \\ a_{14} & a_{24} & a_{34} & a_{44} \end{bmatrix} = [a_{ji}]$
- $Sum(A) = \sum_{ij} a_{ij}$
- $Diagonal(A) = [a_{11} \quad a_{22} \quad a_{33} \quad a_{44}]$
- $Trace(A) = a_{11} + \ a_{22} + \ a_{33} + \ a_{44}$

E.g.,

- $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$
- $A^T = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$
- $Sum(A) = 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 = 45$
- $Diagonal(A) = [1 \quad 5 \quad 9]$
- $Trace(A) = 1 + 5 + 9 = 15$

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Determinant

2 × 2 Martix

$$\det\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = \mathrm{ad} - \mathrm{bc}$$

$$\det\left(\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\right) = 1 \times 4 - 2 \times 3 = -2$$

3 × 3 Martix

$$\det\left(\begin{array}{c} \begin{array}{ccc} + & - & + \end{array} \\ \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \end{array}\right) = a\boxed{\begin{vmatrix} e & f \\ h & i \end{vmatrix}} - \mathrm{b}\begin{vmatrix} d & f \\ g & i \end{vmatrix} + \mathrm{c}\begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

$$\det\left(\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}\right) = 1\begin{vmatrix} 5 & 6 \\ 8 & 9 \end{vmatrix} - 2\begin{vmatrix} 4 & 6 \\ 7 & 9 \end{vmatrix} + 3\begin{vmatrix} 4 & 5 \\ 7 & 8 \end{vmatrix} = 0$$

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Inverse Matrix

- $A^{-1} = \frac{1}{|A|} adj(A)$
- $A^{-1}A = AA^{-1} = \mathrm{I}$
- $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$
- $|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = \mathrm{ad} - \mathrm{bc}$
- $adj(A) = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$
- $A^{-1} = \frac{1}{\mathrm{ad-bc}} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

https://en.wikipedia.org/wiki/Invertible_matrix
https://en.wikipedia.org/wiki/Adjugate_matrix

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Properties of Matrix

$$A + B = B + A$$
$$A + 0 = A$$
$$AB \neq BA$$

$$A(BC) = (AB)C$$
$$A(B+C)=AB+AC$$
$$(A+B)C=AC+BC$$

$$\alpha\,(A + B) = \alpha A + \alpha B$$
$$(\alpha + \beta)A = kA + pA$$

$$0.A = 0$$
$$A.0 = 0$$

$$(A^T)^T = A$$
$$(A + B)^T = A^T + B^T$$
$$(AB)^T = B^T A^T$$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$$

$$(A^{-1})^{-1} = A$$
$$A^{-1}A = AA^{-1} = \mathrm{I}$$

$$(A^T)^{-1} = (A^{-1})^T$$
$$(AB)^{-1} = B^{-1}A^{-1}$$

$$|A^{-1}| = \frac{1}{|A|}$$
$$|AB| = |A||B|$$

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Special Vectors/Matrices

- Square Matrix: $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2\times 2}$
- Symmetric Matrix: $S = \begin{bmatrix} 1 & 4 & 5 \\ 4 & 2 & 6 \\ 5 & 6 & 3 \end{bmatrix}$
- Zero Matrix: $0 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$
- Unit Vector: $\hat{x} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

- Diagonal Matrix: $D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$
- Identity Matrix: $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
- Scalar Diagonal Matrix: $D = \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} = \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \lambda I$
- Upper Triangular matrix: $U = \begin{bmatrix} 1 & 4 & 5 \\ 0 & 2 & 6 \\ 0 & 0 & 3 \end{bmatrix}$
- Lower Triangular Matrix: $L = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 2 & 0 \\ 5 & 6 & 3 \end{bmatrix}$

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Matrix Operations and Applications

- Addition/Subtraction
- Scalar Multiplication
- Matrix Multiplication (Dot Product)
- Matrix-Vector Multiplication
- Row operations
- Solving Linear Equations
- Linear transformations
- Decomposition

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Matrix Multiplication

- $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$
- $B = \begin{bmatrix} p & q \\ r & s \end{bmatrix}$
- $A \cdot B = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} p & q \\ r & s \end{bmatrix}$
- $A \cdot B = \begin{bmatrix} ap + br & aq + bs \\ cp + dr & cq + ds \end{bmatrix}$

https://en.wikipedia.org/wiki/Matrix_multiplication

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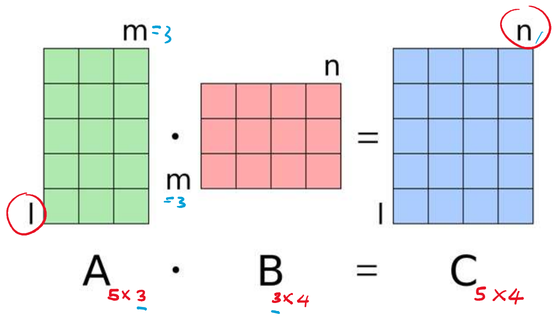
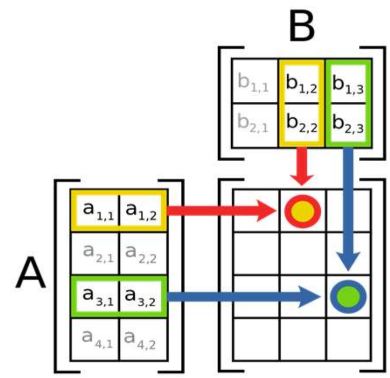
Matrix Multiplication (Dot Product)





https://en.wikipedia.org/wiki/Matrix_multiplication

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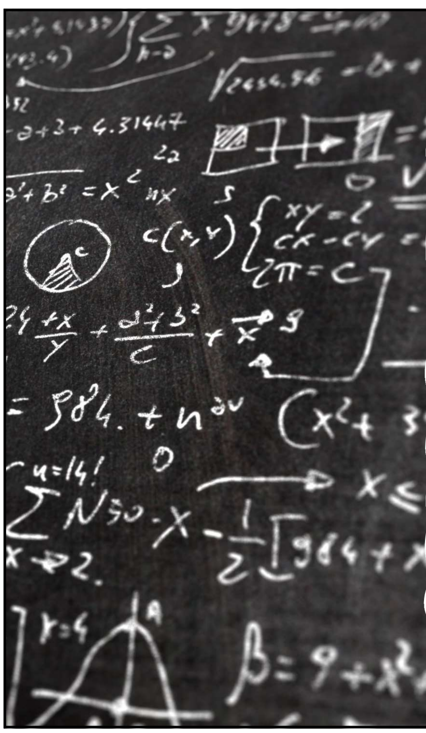

Solving Linear Equations

Problem:

$$\mathbf{3x + 2y = 12 \rightarrow (1)}$$
$$\mathbf{4x + 5y = 23 \rightarrow (2)}$$

$$\boxed{\begin{aligned} \boldsymbol{x} &= ? \\ \boldsymbol{y} &= ? \end{aligned}}$$

Solution Approach:

$$x = \frac{12 - 2y}{3} = 4 - \frac{2}{3}y$$
$$4\left(4 - \frac{2}{3}y\right) + 5y = 23$$
$$16 - \frac{8}{3}y + 5y = 16 + \frac{15 - 8}{3}y = 16 + \frac{7}{3}y = 23$$
$$y = \frac{3(23 - 16)}{7} = \frac{3(7)}{7} = 3$$
$$x = 4 - \frac{2}{3}(3) = 4 - 2 = \underline{2}$$

$$\boxed{\begin{aligned} \boldsymbol{x} &= \mathbf{2} \\ \boldsymbol{y} &= \mathbf{3} \end{aligned}}$$

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Formulating the Problem in Matrix Form

$$\boxed{\begin{matrix} \boldsymbol{x_1 = x} \\ \boldsymbol{x_2 = y} \end{matrix}}$$

$$\boldsymbol{3x_1 + 2x_2 = 12 \rightarrow (1)}$$

$$\boldsymbol{4x_1 + 5x_2 = 23 \rightarrow (2)}$$

Solve using Matrices:

$$\begin{bmatrix}3\\4\end{bmatrix}x_1 + \begin{bmatrix}2\\5\end{bmatrix}x_2 = \begin{bmatrix}12\\23\end{bmatrix}$$

$$\begin{bmatrix}3 & 2\\4 & 5\end{bmatrix} \cdot \begin{bmatrix}x_1\\x_2\end{bmatrix} = \begin{bmatrix}12\\23\end{bmatrix}$$

$$\begin{bmatrix}\boldsymbol{a_{11}} & \boldsymbol{a_{11}}\\\boldsymbol{a_{11}} & \boldsymbol{a_{11}}\end{bmatrix} \cdot \begin{bmatrix}x_1\\x_2\end{bmatrix} = \begin{bmatrix}\boldsymbol{b_1}\\\boldsymbol{b_2}\end{bmatrix}$$

$$\boxed{Ax = b}$$

$$A = \begin{bmatrix}3 & 2\\4 & 5\end{bmatrix} = \begin{bmatrix}a_{11} & a_{11}\\a_{11} & a_{11}\end{bmatrix}$$

$$x = \begin{bmatrix}x_1\\x_2\end{bmatrix}$$

$$b = \begin{bmatrix}b_1\\b_2\end{bmatrix} = \begin{bmatrix}12\\23\end{bmatrix}$$

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Obtaining the Solution in Matrix Form

$$\boxed{\begin{matrix} Ax = b \\ A^{-1}Ax = A^{-1}b \\ Ix = A^{-1}b \\ \boldsymbol{x = A^{-1}b} \end{matrix}}$$

$$\boxed{\begin{matrix} x = A^{-1}b \\ x = \left(\dfrac{1}{|A|} \times adj(A)\right)\mathrm{b} \end{matrix}}$$

Example $A = \begin{bmatrix}3 & 2\\4 & 5\end{bmatrix}$

$$b = \begin{bmatrix}12\\23\end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}adj(A)$$

$$|A| = 3 \times 5 - 4 \times 2 = 7$$

$$A^{-1} = \frac{1}{7}\begin{bmatrix}5 & -2\\-4 & 3\end{bmatrix}$$

$$x = \frac{1}{7}\begin{bmatrix}5 & -2\\-4 & 3\end{bmatrix} \cdot \begin{bmatrix}12\\23\end{bmatrix} = \begin{bmatrix}\boldsymbol{2}\\\boldsymbol{3}\end{bmatrix} = \begin{bmatrix}\boldsymbol{x_1}\\\boldsymbol{x_2}\end{bmatrix}$$

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Solving Linear Equations (General Form)

$$\boxed{\boldsymbol{Ax = b}}$$

$$\begin{bmatrix} a_{11} & a_{12} & .. & a_{1n} \\ a_{21} & a_{22} & .. & a_{2n} \\ : & : & : & : \\ a_{m1} & a_{m2} & . & a_{mn} \end{bmatrix}_{m\times n} \cdot \begin{bmatrix} x_1 \\ x_2 \\ : \\ x_n \end{bmatrix}_{n\times 1} = \begin{bmatrix} b_1 \\ b_2 \\ : \\ b_m \end{bmatrix}_{m\times 1}$$

$m = number\ of\ linearly\ independent\ equations$
$n = number\ of\ unknowns\ (x_i)$

$m = \mathrm{n} \rightarrow unique\ solution$
$m < \mathrm{n} \rightarrow infintely\ many\ solutions$

$$\boxed{\boldsymbol{x = A^{-1}b}}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ : \\ x_n \end{bmatrix}_{n\times 1} = \underbrace{\begin{bmatrix} a'_{11} & a_{,12} & .. & a'_{1m} \\ a'_{21} & a'_{22} & .. & a'_{2m} \\ : & : & : & : \\ a'_{n1} & a'_{n2} & . & a'_{nm} \end{bmatrix}}_{\boldsymbol{A^{-1}}}{}_{n\times m} \cdot \begin{bmatrix} b_1 \\ b_2 \\ : \\ b_m \end{bmatrix}_{m\times 1}$$

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Linear Transformations

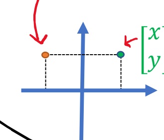
$$p = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$p' = T \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x' \\ y' \end{bmatrix}$$

- Stretching $= \begin{bmatrix} k & 0 \\ 0 & 1 \end{bmatrix}$
- Squeezing $= \begin{bmatrix} k & 0 \\ 0 & 1/k \end{bmatrix}$
- Rotation $= \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$

counterclockwise

$$\begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$



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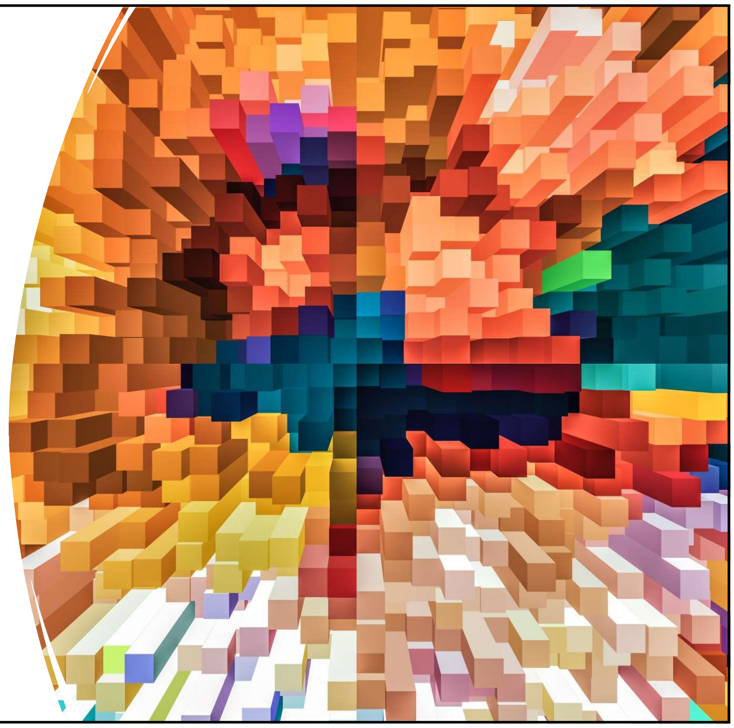
Sets

- Basics of Set Theory
- Set Operators
  - Union
  - Intersection
  - Complement
  - Difference



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Basics of Set Theory

- A set is a collection of things (elements).
  - $A = \{♣, ♦, ♥, ♠\}$
  - $B = \{\times, \div, +, -\}$
  - $C = \{ apple\ ,\ orange, mango, banana \}$
  - $D = \{x | x\ satisfies\ some\ property\}$
  - $\mathbb{N} = \{ 1, 2, 3, 4, \ldots \}$ *is set of natural numbers*
  - $\mathbb{Z} = \{ \ldots, -3, -2, -1, 0, 1, 2, 3, 4, \ldots \}$ *is set of integers*
  - $E = \{x | x \in \mathbb{Z}, -2 \leq x < 10\}$
  - $\emptyset = \{\quad\}$ is Null set
- Items belongs to a set (element of)
  - $♥ \in A$
  - $\div \in B$
  - $apple \in C$
- Items not belongs to a set (not an element of)
  - $⧗ \notin A$
  - $* \notin B$
  - $stawberry\ \notin$ C
- Subset
  - $\{♦, ♥\} \subset A$
  - $\mathbb{N} \subset \mathbb{Z}$

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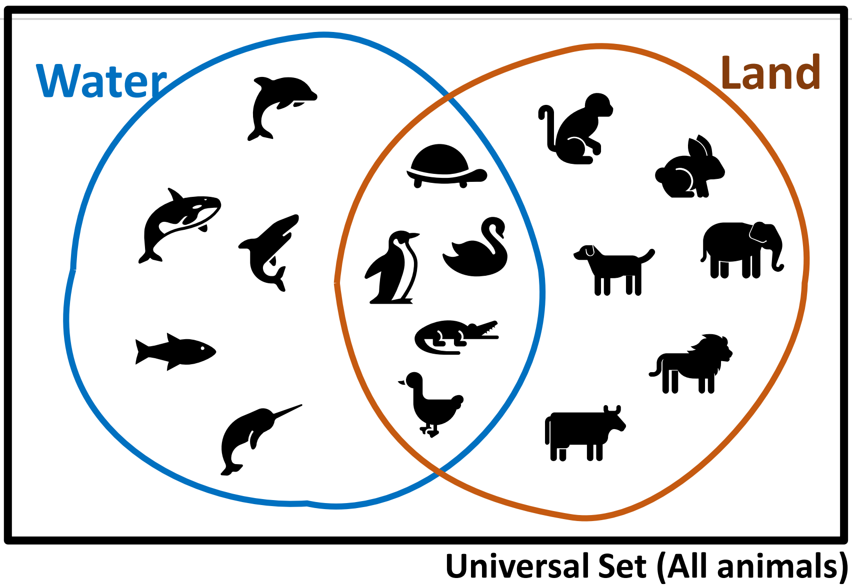
Venn Diagrams



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Cardinality (size of the set)

$$A = \{♣, ♦, ♥, ♠\}$$
$$|A| = 4$$

$$B = \{1,2,3,4,5,6\}$$
$$|B| = 6$$

https://en.wikipedia.org/wiki/Cardinality

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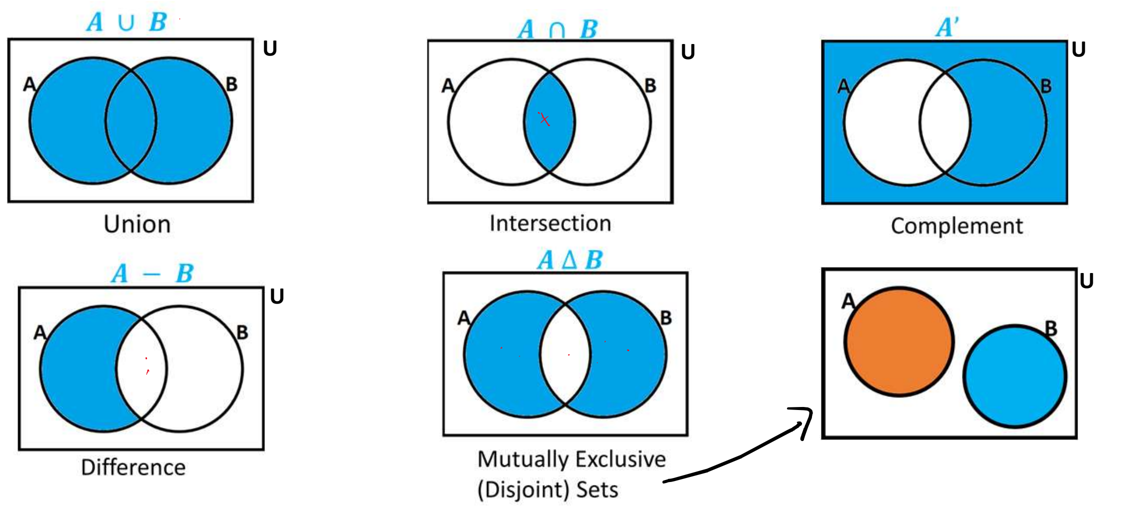
Set Operations



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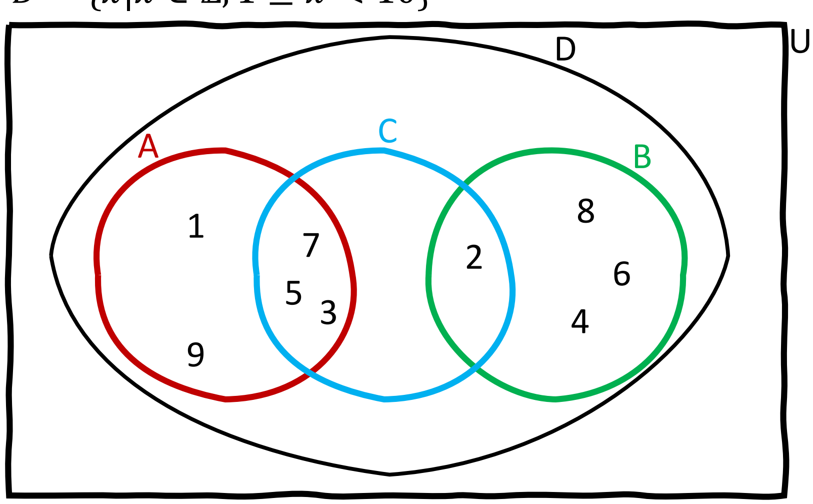
$$A = \{x | x \in \mathbb{Z}, 1 \leq x < 10, x \text{ is an odd number}\}$$
$$B = \{x | x \in \mathbb{Z}, 1 \leq x < 10, x \text{ is an even number}\}$$
$$C = \{x | x \in \mathbb{Z}, 1 \leq x < 10, x \text{ is a prime number}\}$$
$$D = \{x | x \in \mathbb{Z}, 1 \leq x < 10\}$$



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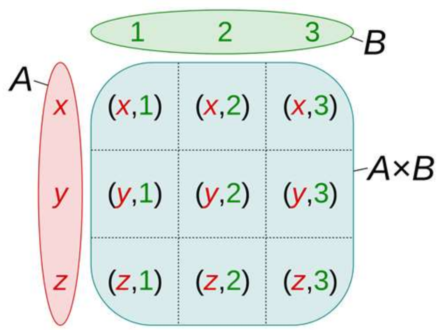
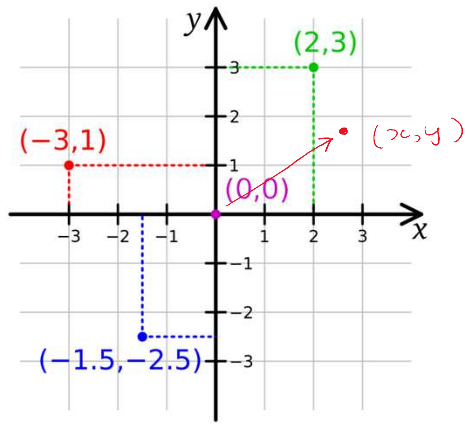
Cartesian product

- $A \times B = \{(a, b)|a \in A \text{ and } b \in B)$
- $A \times B \neq \mathrm{B} \times A$





https://en.wikipedia.org/wiki/Cartesian_product

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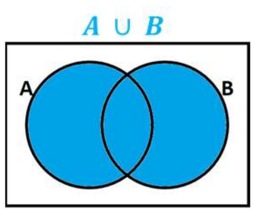
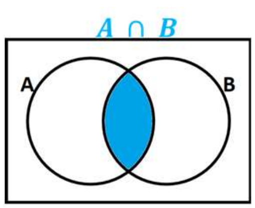
Laws of Set Theory

- Identity
  - $A \cap U = A$
  - $A \cup \emptyset = A$
- Dominance
  - $A \cap \emptyset = \emptyset$
  - $A \cup U = U$
- Idempotence
  - $A \cap A = A$
  - $A \cup A = A$



Union



Intersection

- Complement
  - $A \cap A' = \emptyset$
  - $A \cup A' = U$
- Double Compliment
  - $(A')' = A$

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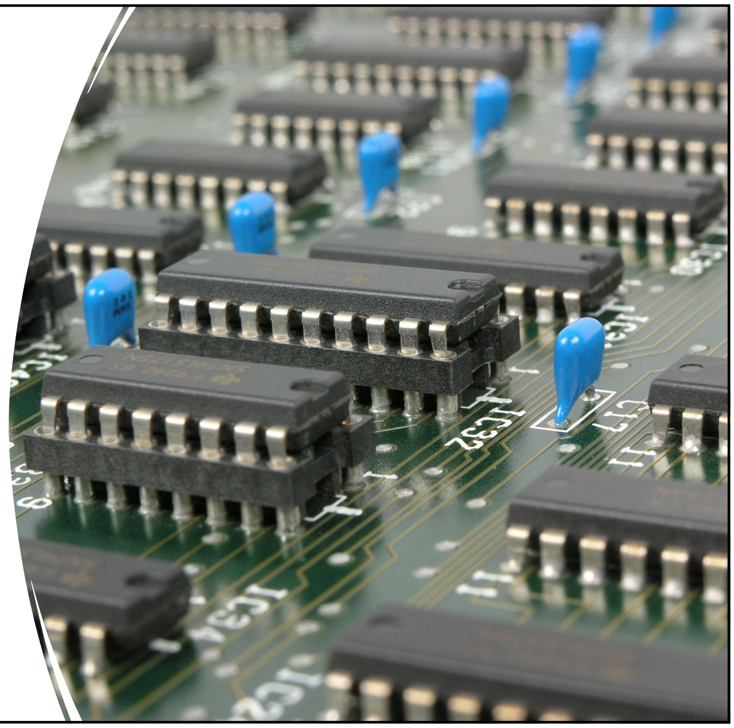
Logic Gates

- Logical Statements
- Logical Operators
  - AND
  - OR
  - NOT



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Logical Statements and Binary Logic

- TRUE = 1
- FALSE = 0

- "5 is an odd number" is TRUE
- "4 is a prime number" is FALSE
- "Kandy is the Capital City of Sri Lanka" is FALSE
- "A triangle has three sides" is TRUE

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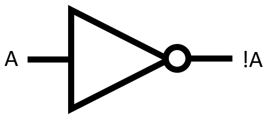
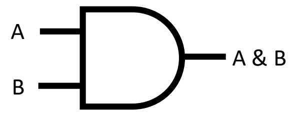
Logic Gates

NOT



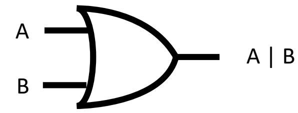
A	!A
1	0
0	1

AND

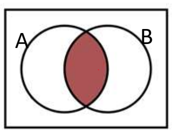


A	B	A & B
1	1	1
1	0	0
0	1	0
0	0	0

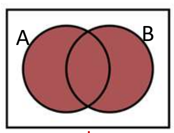
OR



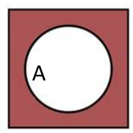
A	B	A | B
1	1	1
1	0	1
0	1	1
0	0	0



A & B



A | B



!A

https://en.wikipedia.org/wiki/Boolean_algebra

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Laws of Boolean Algebra

- A is a Boolean variable
- A can take either 1 or 0

- Identity
  - A & 1 = A
  - A | 0 = A
- Annihilator
  - A & 0 = 0
  - A | 1 = 1
- Idempotence
  - A & A = A
  - A | A = A
- Associativity
  - A & (B&C) = (A & B) & C
  - A | (B | C) = (A | B) | C
- Commutativity
  - A & B = B & A
  - A | B = B | A
- Distributivity
  - A & (B | C) = (A & B) | (A & C)
  - A | (B & C) = (A | B) & (A | C)

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Laws of Boolean Algebra

- Complement
  - A & !A = 0
  - A | !A = 1
- Double Negation
  - !(!A) = A
- De Morgan's laws:
  - !A & !B = !(A | B)
  - !A | !B = !(A & B)

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Exponents and Logarithms

	Exponent Representation	Logarithmic representation
Base n	$n^x = y$	$\log_n y = x$
Base 2	$2^3 = 8$	$\log_2 8 = 3$
Base 10	$10^3 = 1000$	$\log_{10} 1000 = 3$
Base e	$e^{6.907755} \approx 1000$	$\log_e 1000 \approx 6.907755$

$\log_e(1000) = \ln(1000)$

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Functions

- $y = f(x)$
- $f(x) = \frac{1}{x}$
- $f(x) = a_0 + a_1 x + a_2 x^2$
- $f(x) = a_0 + a_1 x_1 + a_2 x_2$
- $f(x) = a_0 + a_1 x_1 + a_2 x_2$

- *Step Functions*
  - $f(x) = \begin{cases} 0, & x < 0 \\ x, & x \geq 0 \end{cases}$
- Logistic Function
  - $f(x) = \frac{L}{1+e^{-k(x-x_0)}}$

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Series

- $\boldsymbol{S_n = \sum_{i=1}^{n} a_i = a_1 + a_2 + \ldots + a_n}$
- Example
  - Let $a_i = 2(\mathrm{i} - 1) + 1$
  - $a_1 = 1$
  - $a_2 = 2 + 1 = 3$
  - $a_3 = 2 \times 2 + 1 = 5$
  - $a_4 = 2 \times 3 + 1 = 7$
  - $S_4 = 1 + 3 + 5 + 7 = 16$

https://en.wikipedia.org/wiki/Series_(mathematics)

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Sequence

- $(a_i)_{i=1}^{n} = [a_0, a_1, a_2, \dots, a_n]$
- Example
  - Let $a_i = i^2$
  - $a_1 = 1$
  - $a_2 = 2^2 = 4$
  - $a_3 = 3^2 = 9$
  - $a_4 = 4^2 = 16$
  - $(i^2)_{i=1}^{4} = 1, 4, 9, 16$

https://en.wikipedia.org/wiki/Series_(mathematics)

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Trigonometric Functions

- $sin(x)$
- $cos(x)$
- $tan(x)$
- Sound Signal:
  - $amplitude = \sin(\omega t) = sin(2\pi f t)$

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Take-home Exercise

- Mention one machine learning problem (specify whether it is a classification, regression or clustering problem) and the training approach you would take (supervised or unsupervised).
- Your answer must be 240 characters or fewer.

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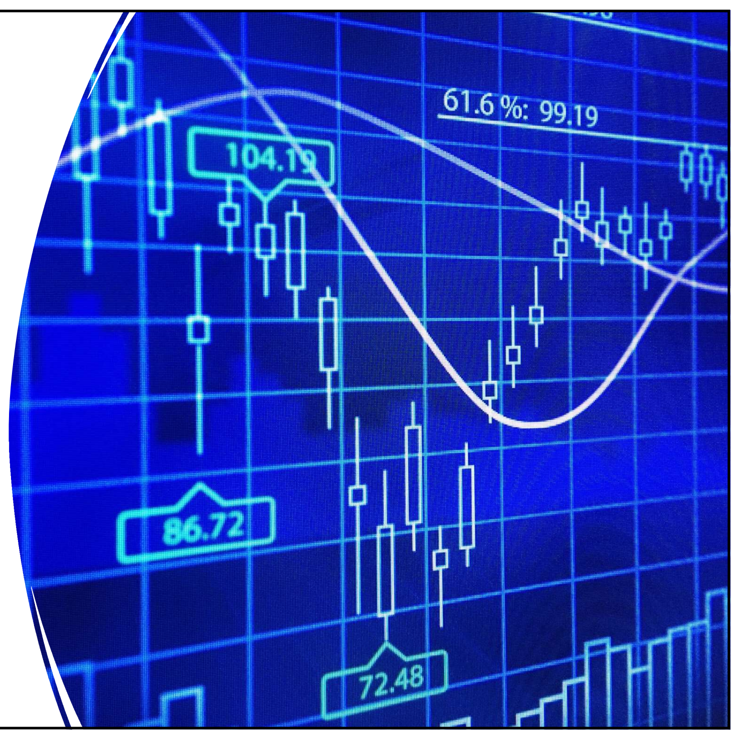
Probability and Statistics Concepts for Machine Learning

- Probability
- Statistical Distributions
- Descriptive Statistics



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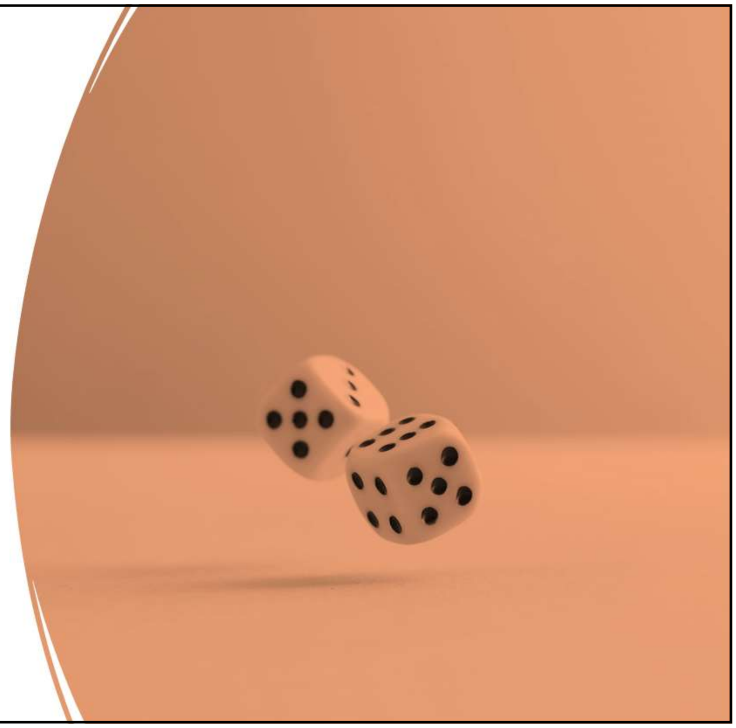
Probability



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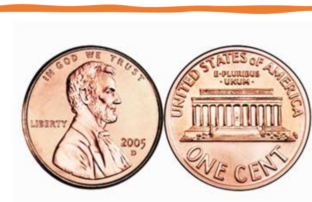
Probability Example

- Coin (single toss)
  - Possible outcomes: {Head, Tail}



- Dice (single roll)
  - Possible outcomes: {1, 2, 3, 4, 5, 6l}

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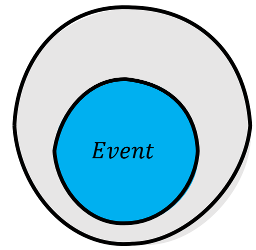
Term Definitions

- **Experiment**: A procedure that can be repeated and has a well-defined set of possible outcomes.
  - **Random experiment**: outcome is unknown before the experiment. Results different outcomes when repeating the experiment in the same manner.
  - **Deterministic experiment**: outcome may be predicted with certainty beforehand. Has a definite outcome. Outcome is predictable.
- **Sample Space** ($\boldsymbol{\Omega}$)**:** Set of all possible outcomes or results of an experiment.
- **Outcome:** Possible result of an experiment. An element of Ω.
- **Event:** A set of selected outcomes of an experiment. Subset of the sample space.
  - Independent Event: probabilities of occurrence do not depend on one another
- **Random Variable:** A variable that can take different values of the sample space (describes the outcomes of a random experiment).



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Probability

- Let $\Omega = \{\omega_1, \omega_2, \dots, \omega_n\}$ be a sample space of an experiment.
- Let the event $A \subset \Omega$
- Probability (How likely the event A can occur) : $P(A) = \frac{|A|}{|\Omega|}$
- $0 \le P(A) \le 1$
- Random Variable (X): $\{\omega \in \Omega | u < X(\omega) \le v\}$
- Cumulative Probability: $\sum_x P(X = x) = 1$

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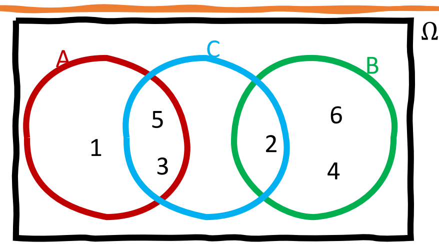
Probability Example

- Coin (single toss)
  - Two possible outcomes
  - $\Omega = \{H, T\}$
  - $|\Omega| = 2$
  - $P(A = \{H\}) = \frac{1}{2}$
  - $P(A = \{T\}) = \frac{1}{2}$

- Dice (single roll)
  - Six possible outcomes
  - $\Omega = \{1, 2, 3, 4, 5, 6\}$
  - $|\Omega| = 6$
  - $P(A = \{1\}) = \frac{1}{6}$
  - $P(A = \{6\}) = \frac{1}{6}$
  - $P(A = \{1,3,5\}) = \frac{3}{6} = \frac{1}{2}$ (get odd number as outcome)



Odd number: $P(A) = \frac{|A|}{|\Omega|} = \frac{1}{6} = \frac{1}{2}$

Even number: $P(B) = \frac{|B|}{|\Omega|} = \frac{1}{2}$

Prime number: $P(C) = \frac{|c|}{|\Omega|} = \frac{1}{2}$

$P(A \cup C) = \frac{|A \cup C|}{|\Omega|} = \frac{4}{6} = \frac{2}{3}$

$P(A \cap C) = \frac{|A \cap C|}{|\Omega|} = \frac{2}{6} = \frac{1}{3}$

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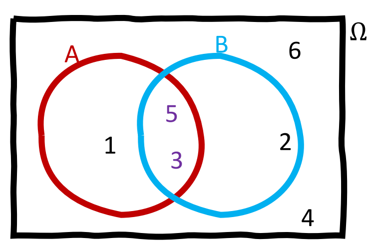
Conditional Probability

- Probability of A given B : $P(A|B) = \frac{P(A \cap B)}{P(B)}, if\ P(B) \neq 0$
- Example
  - $A = \{2,3,5\}$, prime numbers
  - $B = \{1,3,5\}$, odd numbers

- If the outcome is an odd number, probability of that Number being a prime.
  - $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{1}{3}\Big/\frac{1}{2} = \frac{2}{3}$
  - $P(A \cap B) = \frac{|A \cap B|}{|\Omega|} = \frac{|\{3,5\}|}{|\{1,2,3,4,5,6\}|} = \frac{2}{6} = \frac{1}{3}$
  - $P(B) = \frac{|B|}{|\Omega|} = \frac{|\{1,3,5\}|}{|\{1,2,3,4,5,6\}|} = \frac{3}{6} = \frac{1}{2}$



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Conditional Probability: Example

- $A: Selected\ person\ is\ vaccinated$
- B: $Age\ of\ the\ selected\ person >= 30$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

- If a person with age >=30 is selected, probability that the person is vaccinated?
  - $P(A) = \frac{|A|}{|\Omega|} = \frac{55}{100}$
  - $P(A \cap B) = \frac{|A \cap B|}{|\Omega|} = \frac{45}{100}$
  - $P(B) = \frac{|B|}{|\Omega|} = \frac{50}{100}$
  - $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{45}{100} \Big/ \frac{50}{100} = \frac{45}{50} = 0.9$

90 %

	Vaccinated (YES)	Vaccinated (NO)	Total
Age < 30	10	40	50
Age >= 30	45	5	50
Total	55	45	100

A

B

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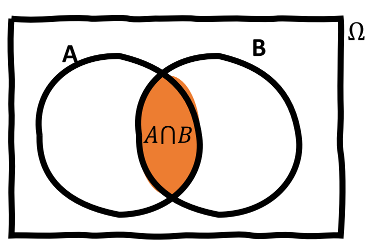
Bayes Rule

- Conditional Probability of A given B
  - $P(A|B) = \frac{P(A \cap B)}{P(B)}, if\ P(B) \neq 0 \longrightarrow (1)$
- Commutativity in Set theory
  - $P(A \cap B) = P(B \cap A) \longrightarrow (2)$
- $(2)\ in\ (1)$
  - $P(B \cap A) = P(A|B) \cdot P(B) \longrightarrow (3)$
- Conditional Probability of B given A
  - $P(B|A) = \frac{P(B \cap A)}{P(A)} \longrightarrow (4)$
- $(3)\ in\ (4)$
  - $\boldsymbol{P(B|A) = \frac{P(A|B) \cdot P(B)}{P(A)}, if\ P(A) \neq 0}$



https://en.wikipedia.org/wiki/Bayes%27_theorem

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Conditional Probability: Example

Events

- I : $Infected$
- N: $Not\ infected$
- T_P: Positive Test Result
- T_N: Negative Test Result

Measurements/Probabilities

known
- Infected population ratio: $P(I) = 0.04 = 4\ \%$
- Positive Test Result if infected: $P(T_P|I) = 0.98 = 98\%$
- Positive Test Result if not infected: $P(T_P|N) = 0.01 = 1\%$
- A person get positive result regardless of the infection status: $P(T_P)$
  - $P(N) = 1 - P(I)$
  - $P(T_P) = \sum_{x=\{I,N\}} P(x)P(T_P|x)$
  - $\boldsymbol{P(T_P) = P(I)P(T_P|I) + P(N)P(T_P|N)}$
  - $P(T_P) = 0.04 \times 0.98 + 0.96 \times 0.01 = 0.0488$

John got a positive test result. What is probability that John is infected.

$$\boldsymbol{P(I|T_P)} = \frac{\mathbf{P(T_P|I) \cdot P(I)}}{\boldsymbol{P(T_P)}} = \frac{0.98 \times 0.04}{0.0488} = 0.803$$

80 %

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Probability Distributions

Discrete

- Probability distribution of a random variable that can take only a finite set of values.
- Probability mass function (PMF)
  - $\sum_{x \in X} P(x) = 1$

Continuous

- Probability distribution of a random variable that can take an infinite number of values.
- Probability density function (PDF)
  - $\int P(x)\, dx = 1$

https://en.wikipedia.org/wiki/Probability_distribution

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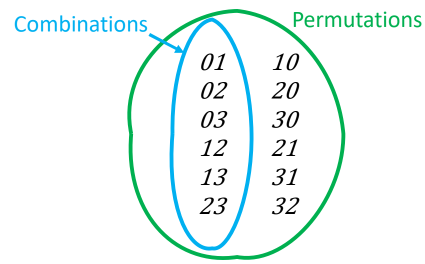
Combinatorics

- Permutations
  - Order matters
  - Without repetition: $P_x^n = \frac{n!}{(n-x)!}$
  - With repetitions: n^x
- Combinations
  - Order does not matter
  - Can take with or without reptations
  - Without repetitions: $C_x^n = \frac{n!}{x!\,(n-x)!}$
  - With repetitions: $C_x^{n+x-1} = \frac{(n+x-1)!}{x!\,(n-x)!}$

Example

Set of Digits
- $S = \{0, 1, 2, 3\}$
- $\mathrm{n} = |S| = 4$

Pick 2 elements (digits at a time) without repetition.
- $\mathrm{x} = 2$



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Permutations and Combinations

Set of Digits
- $S = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$
- $|S| = 10$

Pick 4 elements (digits at a time)
E.g.,
- $A = 1234$
- $B = 2341$

How many permutations can be made?

How many combinations can be made?

$$P_x^n = \frac{n!}{(n-x)!}$$

$$C_x^n = \frac{n!}{x!\ (n-x)!}$$

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Statistics

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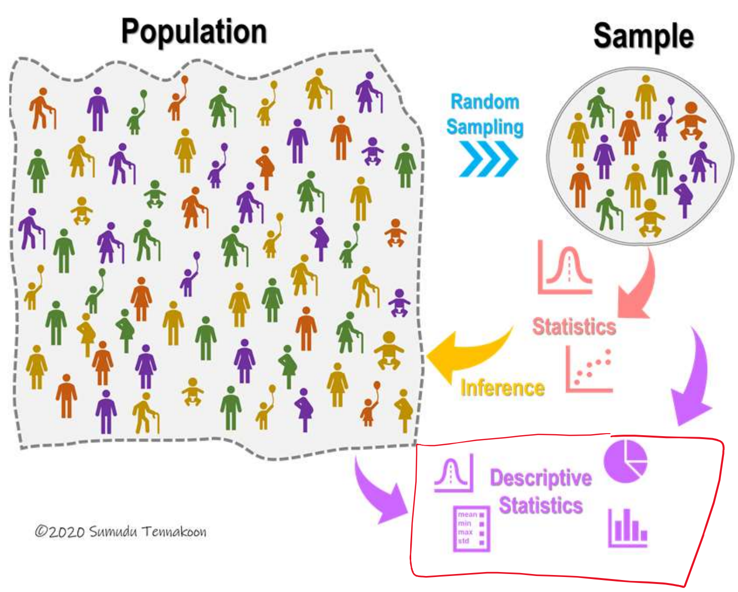
Sampling



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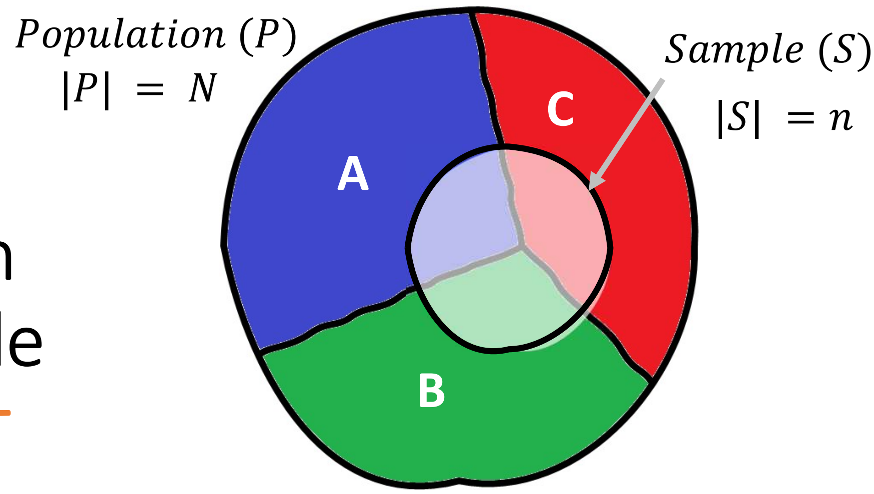
Population and Sample



- $P = A \cup B \cup C$
- $S \subset P$
- $S = (A \cap S) \cup (B \cap S) \cup (C \cap S)$
- $|S| = \mathrm{n} = |A \cap S| + |B \cap S| + |C \cap S| = \mathrm{a} + \mathrm{b} + \mathrm{c}$

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Sampling Method depends on

- Nature of population
- Regulatory Restrictions
- Resources available
  - Time
  - Money
  - Data Collectors/Probes

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Sampling Considerations

Unbiased: each element is equally likely to be chosen

Representative of the Population

Minimize the **Sampling Error**

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Probabilistic Sampling Techniques

Simple Random Sampling

Stratified Sampling

Systematic sampling

Cluster Sampling

…

https://en.wikipedia.org/wiki/Sampling_(statistics)
https://en.wikipedia.org/wiki/Stratified_sampling

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Non-Probabilistic Sampling Techniques

- Convenience Sampling
- Judgmental Sampling
- Quota sampling
- Snowball Sampling

...

https://en.wikipedia.org/wiki/Sampling_(statistics)
https://en.wikipedia.org/wiki/Stratified_sampling

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Types of Statistics

Descriptive Statistics: Organizing and summarizing data. Use data from the population or sample.

Mean, Median, Mode, Range, Variance

Inferential Statistics: Use the sample data to make an inference or draw the population's conclusion. It uses probability to find out the confidence of the predictions we make.

Distributions, Regression, …

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Types of Analytics

Descriptive: What do we have now/ had

Predictive: What can happen if the conditions are set to

Prescriptive: Define rules/ constrains

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Descriptive Statistics

- Range
- Mean
- Median
- Mode
- Variance
- Standard Deviation
- Coefficient of Variation
- Co-variance
- Co-relation Coefficient

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Range, Mean, Median and Mode

- Range
  - Max Value - Min value
- Mean
  - Population Mean: $\mu = \frac{\sum_{i=1}^{n} x_i}{N}$
  - Sample Mean: $\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$
  - Has influence of outliers
- Weighted Mean (w_is are weights)
  - $\overline{x_w} = \frac{\sum_i^n w_i x_i}{\sum_{i=1}^{n} w_i}$
- Geometric Mean:
  - $\overline{x_g} = \sqrt[n]{\prod_{i=1}^{n} x_i}$
- Median:
  - Mid value when ordered
  - Removes influence of outliers
- Mode
  - Value that occur most often

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Variance , Standard Deviation and Coefficient of variance

- Variance
  - How data spread around mean
  - Population : $\sigma^2 = \frac{\sum_{i=1}^{n}(x_i-\mu)^2}{N}$
  - Sample : $S^2 = \frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n-1}$
- Standard Deviation (STD)
  - Population :$\sigma = \sqrt{\frac{\sum_{i=1}^{n}(x_i-\mu)^2}{N}}$
  - Sample : $S = \sqrt{\frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n-1}}$
- Coefficient of variance (CV)
  - Also known as relative standard deviation (RSD)
  - Population : $\mathrm{CV} = \frac{\sigma}{\mu}$
  - Sample : $\mathrm{CV} = \frac{S}{\bar{x}}$

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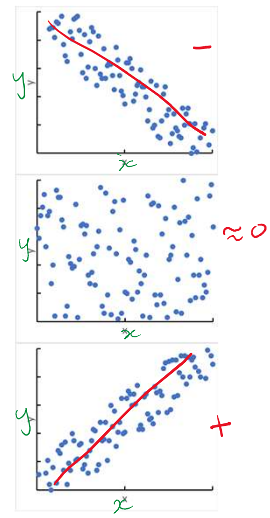
Co-variance and Co-relation Coefficient

- Co-variance:
  - Check whether 2 variables moving together
  - $\text{COV}(x, y) = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{n-1}$
  - $\text{COV} \begin{cases} < 0, & \textit{if variables moving opposite direction} \\ = 0, & \textit{if independent variables} \\ > 0, & \textit{if variables moving together} \end{cases}$
- Co-relation Coefficient
  - $r = \frac{\text{COV}(x,y)}{S(x) \cdot S(y)}$
  - $-1 \leq r \leq 1$
  - $r = 1 \text{ and } r = -1\text{: } perfect\ corelation$
  - $\text{r} \begin{cases} < 0, & \textit{if negative corelation} \\ = 0, & \textit{if independent variables} \\ > 0, & \textit{if positive corelation} \end{cases}$



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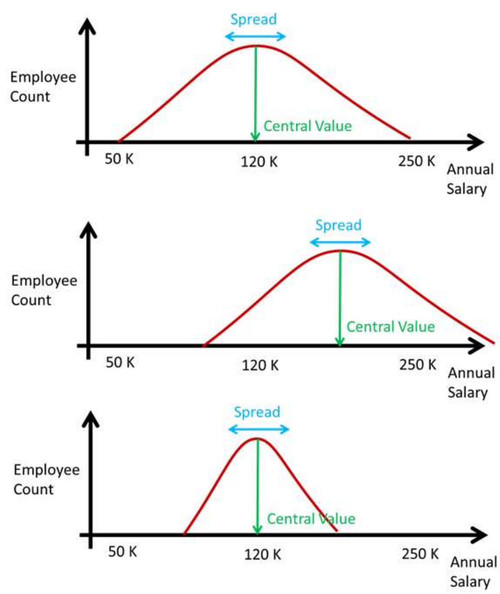
Distributions

Central value and Spread



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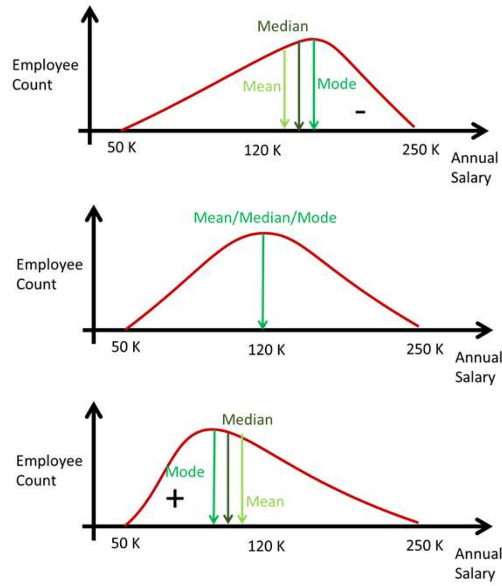
Distributions
Skewness

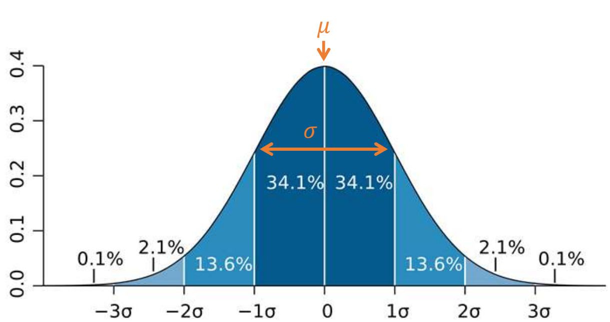


https://en.wikipedia.org/wiki/Skewness
https://docs.scipy.org/doc/scipy/reference/generated/scipy.stats.skew.html
https://mathworld.wolfram.com/PearsonsSkewnessCoefficients.html

131

Standard Normal Distribution

- $Standard\ Deviation\ (\sigma) = 1$
- $Mean\ (\mu) = 0 = \text{Median} = \text{Mode}$



https://en.wikipedia.org/wiki/Probability_distribution

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Figure 6A.15: Distributional Choices

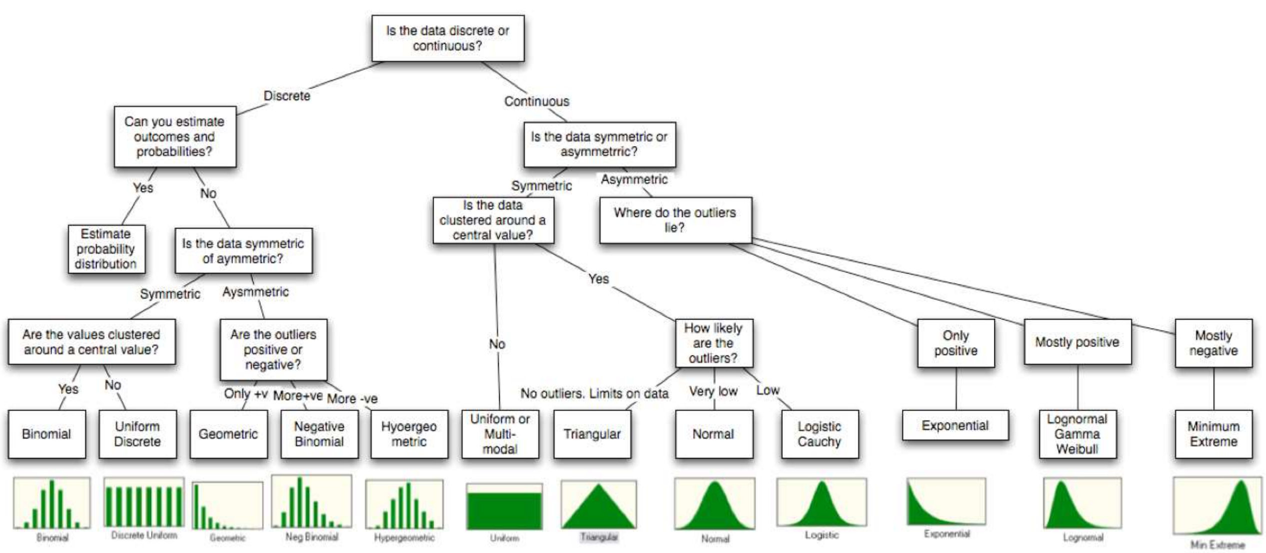

Image: [Probabilistic approaches to risk](#) by [Aswath Damodaran](#)

133

Exploratory Data Analysis and Feature Engineering

- Descriptive Statistics
- Univariate Analysis
- Multivariate Analysis

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Range, Mean, Median and Mode

- Range
  - Max Value - Min value
- Mean
  - Population Mean: $\mu = \frac{\sum_{i=1}^{n} x_i}{N}$
  - Sample Mean: $\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$
  - Has influence of outliers
- Weighted Mean (w_is are weights)
  - $\overline{x_w} = \frac{\sum_{i}^{n} w_i x_i}{\sum_{i=1}^{n} w_i}$
- Geometric Mean:
  - $\overline{x_g} = \sqrt[n]{\prod_{i=1}^{n} x_i}$
- Median:
  - Mid value when ordered
  - Removes influence of outliers
- Mod
  - Value that occur most often

135

Variance , Standard Deviation and Coefficient of variance

- Variance
  - How data spread around mean
  - Population : $\sigma^2 = \frac{\sum_{i=1}^{n}(x_i-\mu)^2}{N}$
  - Sample : $S^2 = \frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n-1}$
- Standard Deviation (STD)
  - Population :$\sigma = \sqrt{\frac{\sum_{i=1}^{n}(x_i-\mu)^2}{N}}$
  - Sample : $S = \sqrt{\frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n-1}}$
- Coefficient of variance (CV)
  - Also known as relative standard deviation (RSD)
  - Population : $\mathrm{CV} = \frac{\sigma}{\mu}$
  - Sample : $\mathrm{CV} = \frac{S}{\bar{x}}$

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Data Pre-Processing

Treat Missing Values

Treat Outliers

Scaling Variables

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Treat Missing Values

- Drop
  - Drop rows/columns with missing values
- Impute
  - Impute (replacing with value)
    - Interpolate
    - Descriptive statistics: Median, Mean, Mode
    - Use machine learning model
    - Derive using other column (e.g., gender by title, name)
    - Replace with a dummy value (e.g., NA, N/A, null, -9999)
- Revisit
  - Revisit data collection
- Request
  - Request missing information from data provider

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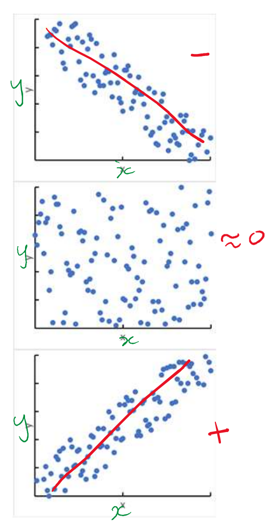
Treating Outliers

- Detecting Outliers
  - Box Plot
  - Percentiles
- Remove Outliers
  - Drop rows/columns
  - Replace outlier values
    - Set cutoff value/clip (piecewise function)
    - Convert to ranges

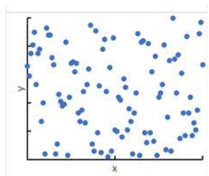
139

Co-variance and Co-relation Coefficient

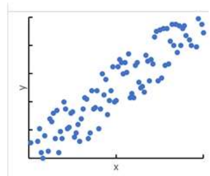
- Co-variance:
  - Check whether 2 variables moving together
  - $\mathrm{COV}(x, y) = \frac{\sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y})}{n-1}$
  - $\mathrm{COV}\begin{cases} < 0, & if\ variables\ moving\ opposite\ direction \\ = 0, & if\ independent\ variables \\ > 0, & if\ variables\ moving\ together \end{cases}$
- Co-relation Coefficient
  - $r = \frac{\mathrm{COV(x,y)}}{S(x) \cdot S(y)}$
  - $-1 \leq r \leq 1$
  - $r = 1\ and\ r = -1:\ perfect\ corelation$
  - $\mathrm{r}\begin{cases} < 0, & if\ negative\ corelation \\ = 0, & if\ independent\ variables \\ > 0, & if\ positive\ corelation \end{cases}$



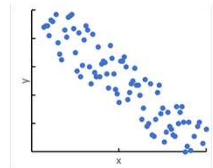
140



A



B



C

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Univariate Analysis

- Single Variable
- Techniques
  - Descriptive Statistics
  - Frequency Table
  - Count Plot/Bar plot (categorical)
  - Histogram (numerical)
  - Box Plot

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Bivariate Analysis

- Two variables
- Techniques
  - Cross tables (two-way tables)
  - Scatter Plot
  - Correlation coefficient
  - Stacked plot
  - Heatmaps
  - Pair Plot
  - Marginal Probability/Conditional Probability/Joint Probability
  - Regression

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Preparing Dataset for Machine Learning

- Dimensionality Reduction
- Feature Selection
  - Select best features based on highest predictive power

- Feature Engineering
  - Derive new features from existing features
  - Transform features
  - Combine multiple features (create interactions)

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Dimensionality Reduction

- Transform data from higher dimension space to lower dimension space (e.g., 4D to 3D, 3D to 2D)
- Retains characteristics of original data.
- Useful for
  - visualization purpose
  - Observe patterns more clearly
- Methods
  - Principle Component Analysis (PCA)
  - Linear discriminant analysis (LDA)
  - Kernel PCA
  - Autoencoder

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Feature Scaling

- Normalization (value between 0 and 1)
  - Min-max
  - Mean
- Standardization (scale based on standard normal distribution)
  - Z-score
- Other Scaling Techniques
  - Log transform
  - Power transform
- Not necessary in Tree based models

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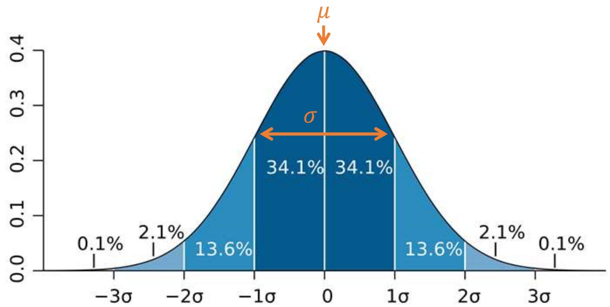
Standard Normal Distribution

- $Standard\ Deviation\ (\sigma) = 1$
- $Mean\ (\mu) = 0 = \text{Median} = \text{Mode}$



https://en.wikipedia.org/wiki/Probability_distribution

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Encoding

(1)

- Numerical Variables -> Categorical Variables
- Categorical Variables
  - One hot encoding (1/0, create dummy variables)
  - Mean Encoding (mean of each category)
  - Label Encoding (ordinal) Education → Edu... num
  - Target guided ordinal encoding (rank of mean)

col.	A	B	C	NA
A	1	0	0	0
B	0	1	0	0
C	0	0	1	0
A	1	0	0	0
~~B~~	~~0~~	~~0~~	~~0~~	0
Null	0	0	0	

1.
2
3
4
⋮

Binary

#	Gender	M	F	y
1	M	0.4552	0	1
2	F	0.550	1	0
3	M	1	0	1
4	F	0	1	1
5	M	1	0	0
6	M	1	0	0

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Feature Selection

- Based on Missing Values
  - Remove features with high % or missing values (define a threshold)
  - Create new feature to indicate missingness (binary column)
- Based on Variance
  - Remove features with zero variation
  - Remove features with low variation (define a threshold)
- Based on correlation
  - Analyze correlation between features (x variables) and those with the target (y) variable
  - Keep from a set of highly correlated variables
    - Keep one having highest correlation coeffect with the target (y) variable.
  - Drop features with low correlation with the target (y)
    - This could miss a useful feature, therefore use with cation

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Feature Selection

- Forward Selection
  - Start with best feature (feature with most predictive power) to build model
  - Evaluate the model performance
  - Add next feature, build model and evaluate
  - If the mode performance increase, keep feature if not, drop
- Backward Elimination
  - Start with all features, build model, evaluate
  - Drop least useful feature in each iteration

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Evaluating Machine Learning Models

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	Positive	Negative	Total
Infected	48	2	50
Not Infected	1	49	50
Total	49	51	100

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Confusion Matrix: Terms

- condition positive (P)
  - the number of real positive cases in the data
- condition negative (N)
  - the number of real negative cases in the data

- true positive (TP)
- true negative (TN)
- false positive (FP)
- false negative (FN)

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Confusion Matrix

		Predicted condition		Sources: [13][14][15][16][17][18][19][20] view · talk · edit	
	Total population $= P + N$	Positive (PP)	Negative (PN)	Informedness, bookmaker informedness (BM) $= TPR + TNR - 1$	Prevalence threshold (PT) $= \frac{\sqrt{TPR \times FPR} - FPR}{TPR - FPR}$
Actual condition	Positive (P)	**True positive (TP),** hit	**False negative (FN),** type II error, miss, underestimation	True positive rate (TPR), recall, sensitivity (SEN), probability of detection, hit rate, power $= \frac{TP}{P} = 1 - FNR$	False negative rate (FNR), miss rate $= \frac{FN}{P} = 1 - TPR$
	Negative (N)	**False positive (FP),** type I error, false alarm, overestimation	**True negative (TN),** correct rejection	False positive rate (FPR), probability of false alarm, fall-out $= \frac{FP}{N} = 1 - TNR$	True negative rate (TNR), specificity (SPC), selectivity $= \frac{TN}{N} = 1 - FPR$
	Prevalence $= \frac{P}{P+N}$	Positive predictive value (PPV), precision $= \frac{TP}{PP} = 1 - FDR$	False omission rate (FOR) $= \frac{FN}{PN} = 1 - NPV$	Positive likelihood ratio (LR+) $= \frac{TPR}{FPR}$	Negative likelihood ratio (LR−) $= \frac{FNR}{TNR}$
	Accuracy (ACC) $= \frac{TP + TN}{P + N}$	False discovery rate (FDR) $= \frac{FP}{PP} = 1 - PPV$	Negative predictive value (NPV) $= \frac{TN}{PN} = 1 - FOR$	Markedness (MK), deltaP (Δp) $= PPV + NPV - 1$	Diagnostic odds ratio (DOR) $= \frac{LR+}{LR-}$
	Balanced accuracy (BA) $= \frac{TPR + TNR}{2}$	F_1 score $= \frac{2PPV \times TPR}{PPV + TPR} = \frac{2TP}{2TP + FP + FN}$	Fowlkes–Mallows index (FM) $= \sqrt{PPV \times TPR}$	Matthews correlation coefficient (MCC) $= \sqrt{TPR \times TNR \times PPV \times NPV} - \sqrt{FNR \times FPR \times FOR \times FDR}$	Threat score (TS), critical success index (CSI), Jaccard index $= \frac{TP}{TP + FN + FP}$

https://en.wikipedia.org/wiki/Confusion_matrix

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Evaluating Classification Models

Accuracy (ACC):

$$\frac{TP + TN}{P + N}$$

Precision: Positive Predictive Value (PPV)

$$\frac{TP}{TP + FP}$$

Recall: Sensitivity/Hit rate/True Positive Rate (TPR)

$$\frac{TP}{P}$$

F1 score:

$$2 \times \frac{PPV \times TPR}{PPV + TPR} = \frac{2 \times TP}{2 \times TP + TN + FP + FN}$$

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Visual Methods

Receiver operating characteristic(ROC) curve

Precision Recall Curve

Sample Mean vs. Predicted Mean

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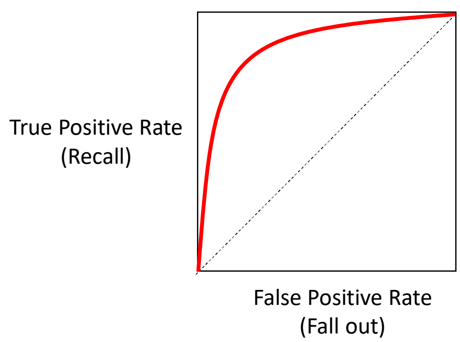
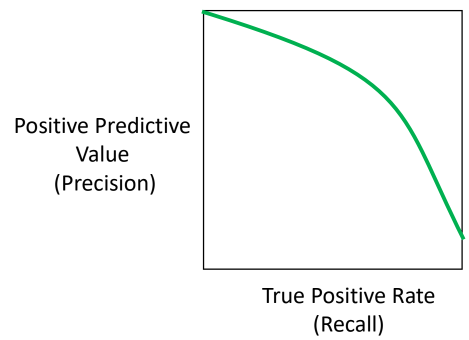
Receiver operating characteristic (ROC) curve



Precision Recall Curve



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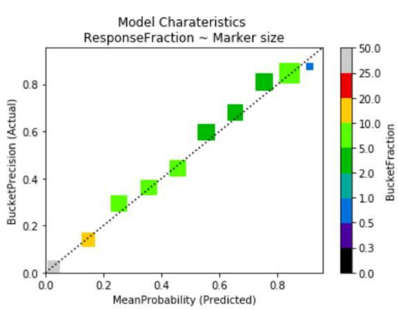
Sample Mean vs. Predicted Mean



Quantile	Mean Probability	Bucket Count	Response Count	Bucket Fraction	Response Fraction	Bucket Precision
1	0.0027	659	4	0.101	0.0026	0.0061
2	0.008	646	5	0.0992	0.0032	0.0077
3	0.0181	649	13	0.0996	0.0083	0.02
4	0.0376	651	23	0.1	0.0147	0.0353
5	0.0801	652	57	0.1	0.0365	0.0874
6	0.152	651	94	0.1	0.0602	0.144
7	0.252	651	191	0.1	0.122	0.293
8	0.385	651	253	0.1	0.162	0.389
9	0.561	651	381	0.1	0.244	0.585
10	0.8	652	540	0.1	0.346	0.828
DataSet	**0.23**	**6513**	**1561**	**1**	**1**	**0.24**

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Evaluating Regression Models

Mean Absolute Error (MAE):

$$MAE = \frac{1}{N}\sum_{i=1}^{N}|y_i - \hat{y}_i|$$

Mean Squared Error (MSE):

$$MSE = \frac{1}{N}\sum_{i=1}^{N}(y_i - \hat{y}_i)^2$$

Root Mean Squared Error (RMSE):

$$RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(y_i - \hat{y}_i)^2}$$

Coefficient of Determination (R-squared):

$$RMSE = 1 - \frac{\sum_{i=1}^{N}(y_i - \hat{y}_i)^2}{\sum_{i=1}^{N}(y_i - \bar{y})^2}$$

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Machine Learning Algorithms

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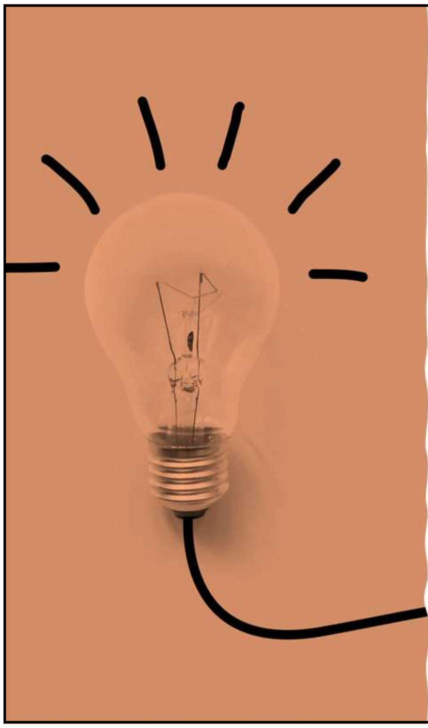

Supervised and Unsupervised Machine Learning

Supervised learning

- $y = f(x_1, x_{2,\dots,} x_n)$
- $x_1, x_{2,\dots,} x_n$ are independent variables used to analyze the dependent variable (y) and the relation between them.

Unsupervised Learning

- No dependent variable.
- Starts with a collection of variables $(x_1, x_{2,\dots,} x_n)$ to find out similarity between them and classify them into clusters.

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Supervised Machine Learning

Regression

- Linear Regression
- Polynomial Regression
- Random Forest Regression

Classification

- Logistic Regression
- Decision Trees
- Random Forest Classifier
- Support Vector Machine (SVM)
- K-Nearest Neighbors (KNN)

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Unsupervised Machine Learning

Clustering
- K-Means
- DBSCAN

Dimensionality Reduction
- PCA
- Autoencoder

Generative adversarial networks (GANs)

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Explaining ML Models and Predictions

Explainable AI (XAI)

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Why do we need to Explain Models and their Predictions?

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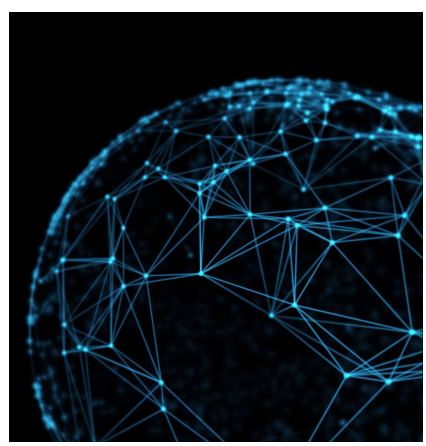
Explaining ML Models and Predictions

- Global
  - Overall understanding of how the model makes a prediction.
  - Feature Importance
- Local
  - Understand how model makes a prediction for a given observation.
  - Positive and Negative contribution of x variables (features) to the prediction.
  - Scale of contribution could be very different from model feature Importance
  - What are the variables contributed the most.



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Explainable AI local methods

- Local interpretable model-agnostic explanation (LIME)
- Kernel Shapley additive explanations (KernalSHAP)
- Integrated gradients (IG)
- Explainable explanations through AI (XRAI)

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Explainability

Models can be easily explained

- Regression
- Decision Trees

Models cannot be easily explained (Blackbox Models)

- Ensemble Models (e.g., Random Forest)
- Neural Network (e.g., CNN, RNN)

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Deep Learning - Introduction

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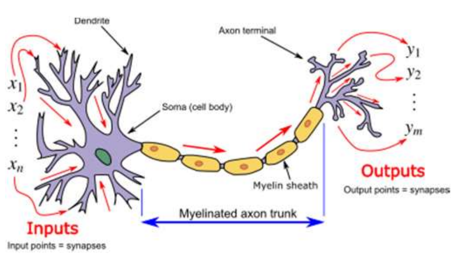
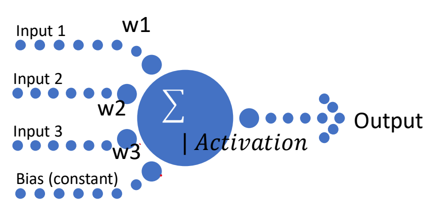
Neuron and Perceptron

- We have approximately 86 billion neurons in our brain.





https://en.wikipedia.org/wiki/Neuron

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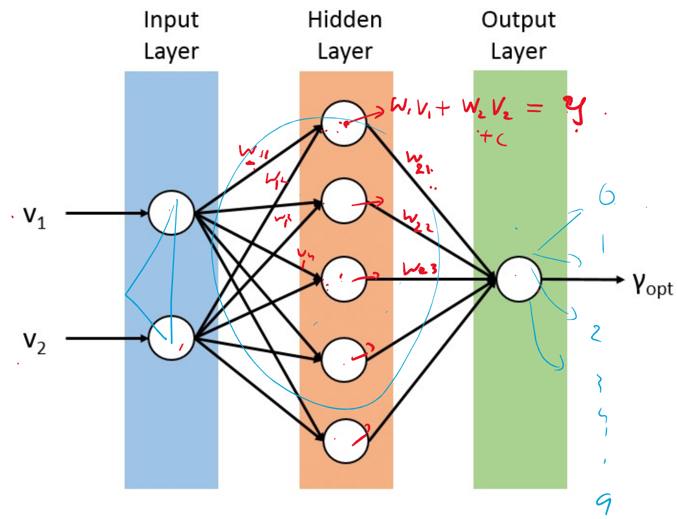
Neural Network



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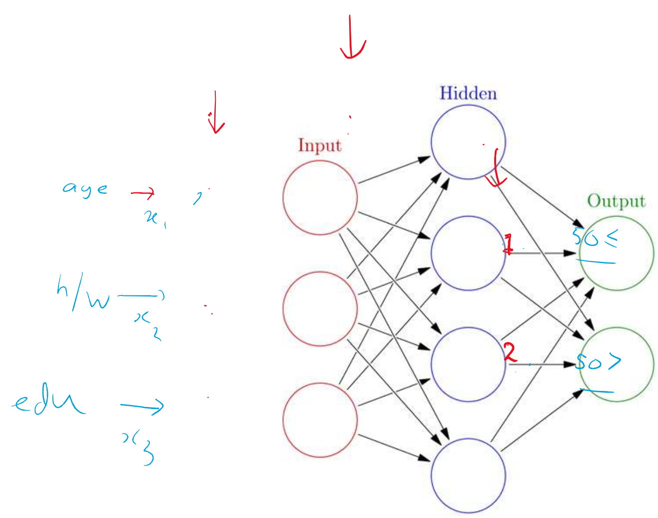

https://en.wikipedia.org/wiki/Artificial_neural_network

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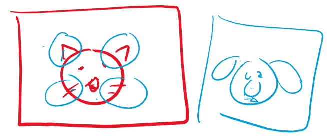
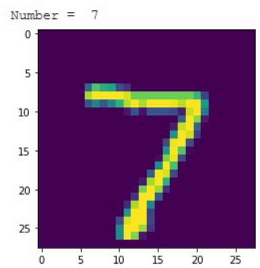
Traditional ML and Deep Learning

- Feature engineering
- Modeling
- Solving Complex Problems





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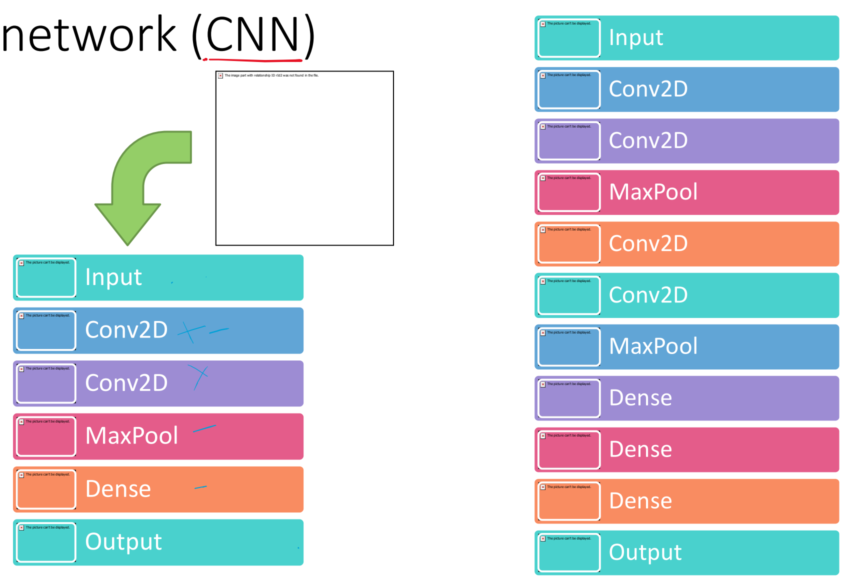
Convolutional neural network (CNN)



Basic Model

Model with added layers

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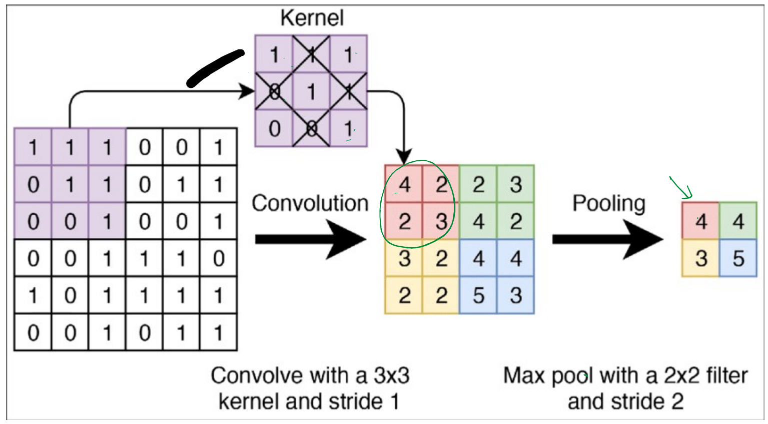
CNN



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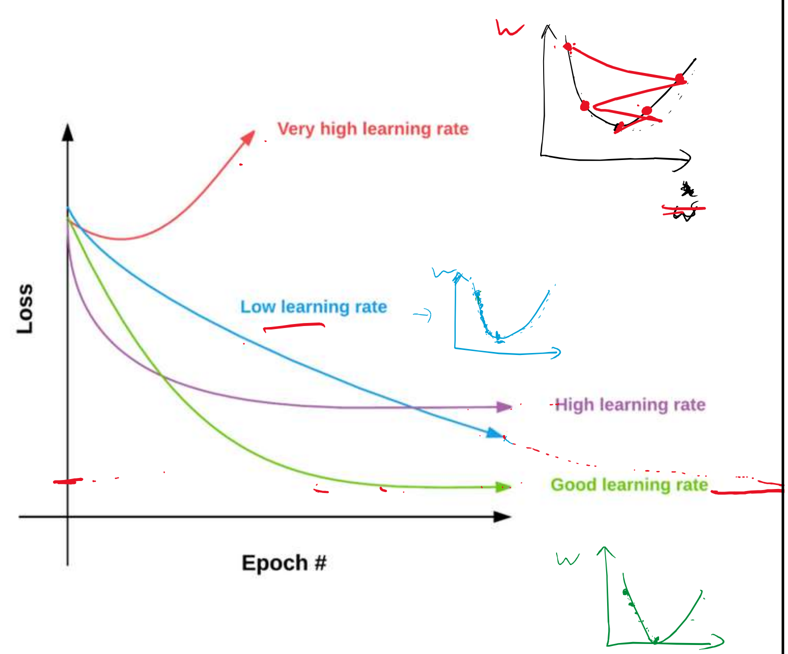
Learning Rate



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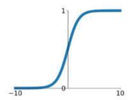
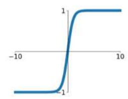
Activation functions

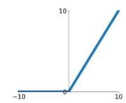
Sigmoid
$\sigma(x) = \frac{1}{1+e^{-x}}$



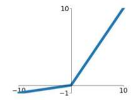
tanh
$\tanh(x)$



ReLU
$\max(0, x)$

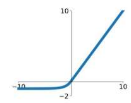


Leaky ReLU
$\max(0.1x, x)$



Maxout
$\max(w_1^T x + b_1, w_2^T x + b_2)$

ELU
$\begin{cases} x & x \geq 0 \\ \alpha(e^x - 1) & x < 0 \end{cases}$



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Regularization

$$\hat{y} = w_1 x_1 + w_2 x_2 + ... + w_N x_N + b$$

$$Loss = Error(y, \hat{y})$$

Loss function with no regularisation

$$Loss = Error(y, \hat{y}) + \lambda \sum_{i=1}^{N} |w_i|$$

Loss function with L1 regularisation

$$Loss = Error(y, \hat{y}) + \lambda \sum_{i=1}^{N} w_i^2$$

Loss function with L2 regularisation

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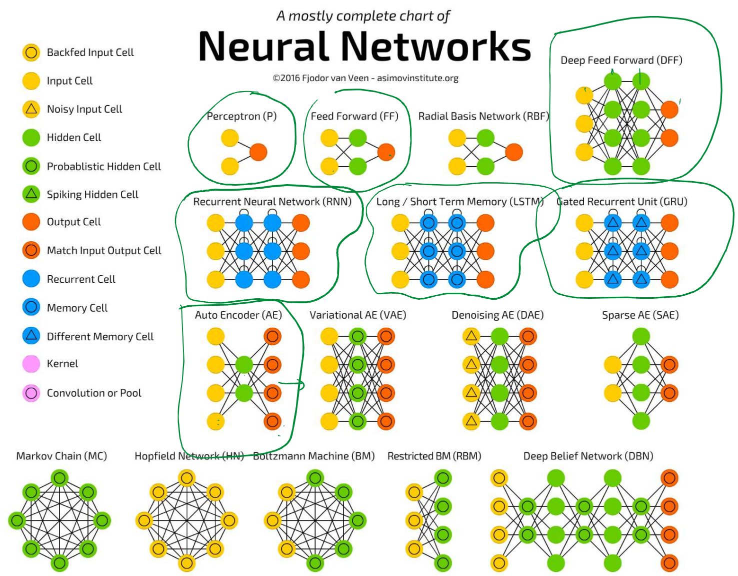
A mostly complete chart of
Neural Networks
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Backfed Input Cell
Input Cell
Noisy Input Cell
Hidden Cell
Probablistic Hidden Cell
Spiking Hidden Cell
Output Cell
Match Input Output Cell
Recurrent Cell
Memory Cell
Different Memory Cell
Kernel
Convolution or Pool
Perceptron (P)
Feed Forward (FF)
Radial Basis Network (RBF)
Deep Feed Forward (DFF)
Recurrent Neural Network (RNN)
Long / Short Term Memory (LSTM)
Gated Recurrent Unit (GRU)
Auto Encoder (AE)
Variational AE (VAE)
Denoising AE (DAE)
Sparse AE (SAE)
Markov Chain (MC)
Hopfield Network (HN)
Boltzmann Machine (BM)
Restricted BM (RBM)
Deep Belief Network (DBN)

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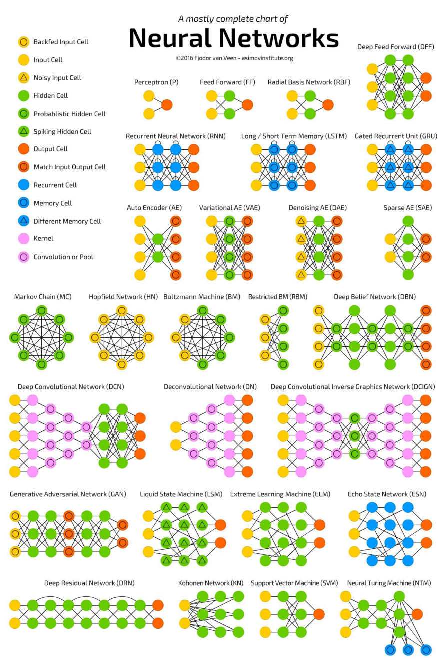
A mostly complete chart of
Neural Networks
Deep Convolutional Network (DCN)
Deconvolutional Network (DN)
Deep Convolutional Inverse Graphics Network (DCIGN)
Generative Adversarial Network (GAN)
Liquid State Machine (LSM)
Extreme Learning Machine (ELM)
Echo State Network (ESN)
Deep Residual Network (DRN)
Kohonen Network (KN)
Support Vector Machine (SVM)
Neural Turing Machine (NTM)

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