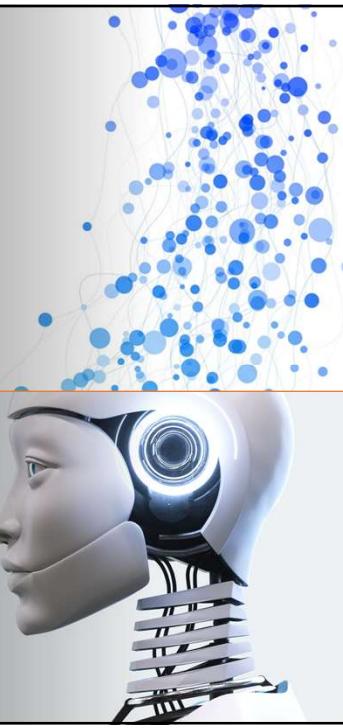


Machine Learning Foundations

An Introduction to your AI Journey

Sumudu Tennakoon, PhD



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1

Course Objective

Provide comprehensive understanding of the core principles of Machine Learning with hands-on training on applying machine learning to solve real-world problems.

A learner who completes this course should be able to define a machine learning problem, understand the solution path, and display the ability to carry out the end-to-end process of building a machine learning application.

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2

Machine Learning Career Prospectus

- Data Scientist
- AI Scientist
- ML/AI Engineer
- Data Engineer
- Data Analyst
- AI/ML Developer
- IoT Developer
- Solutions Architect
- Freelancer
- ...

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3

Schedule and Format

Duration: 60 hours

Schedule: 3-month program/12 weeks, two sessions per week.

Format: Live/Recorded Lectures, Demonstrations, Hands-on Exercises/Labs.

Evaluation: Quizzes (2), Project (1)

Additional Practice: Students must spend extra time on exercises and the capstone project.

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Prerequisites

- Basics of computer programming, mathematics, and statistics.
- Basic knowledge in computer applications:
 - Spreadsheet
 - word processor
 - presentation authoring

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5

Platform and Data for Hands-on Exercises and Project

Programming Language: Python 3 will be used as the primary programming language in teaching, practice examples and assignments.

Python Libraries: Scikit-learn, TensorFlow, Pandas, NumPy, Matplotlib, Seaborn, Flask.

Applications/Tools: Jupyter Notebook/Lab, IDE (Spyder/VS Code/Atom/PyCharm), Spreadsheet (MS Excel/LibreOffice Calc).

Data: Data for exercises, case studies, and projects will be obtained from open data repositories.

Computing Environment: Cloud platform (will be decided on class consensus and service availability) or locally installed Python distribution in student's PC.

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6

Session Topics

#	Topic Name	Training Week #
1	Introduction to Machine Learning (ML), History, and Applications	1
2	Setting up a Computing Environment, Python and Required Libraries.	2
3	Knowledge Foundations to ML (Computing, Statistics, and Mathematics) *	2-3
4	Exploratory Data Analysis (EDA) and Feature Engineering *	4-5
5	Supervised and Unsupervised Learning (concepts)	6
6	Machine Learning Algorithms *	6-7
7	Explaining ML Models and Predictions (introduction) *	7
8	Deep Learning and Neural Networks (introduction) *	8
9	Design and Develop and Deploy ML Solutions *	9-10
10	Capstone Project *	11-12

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Evaluations and Grading

Completion Requirement:

- 80 % Attendance (at least 19 out of 24 sessions)
- Final Grade > 70 %

Completion with Distinction:

- Final Grade > 90 %

	Topic #	%
Quiz1 (Basic Concepts)	1-6	20
Quiz 2 (Advanced Concepts, Deep Learning and Application Building)	7-9	20
Deliverable and Project Report	10	50
Presentation (video narration)	10	10
		100

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8

Introduction to Machine Learning

History and core concepts of ML to navigate the future lessons.

Applications of ML.

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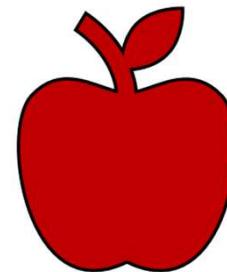
9

Example: Identify Objects

What facts you consider to identify these objects?



Pineapple



Apple

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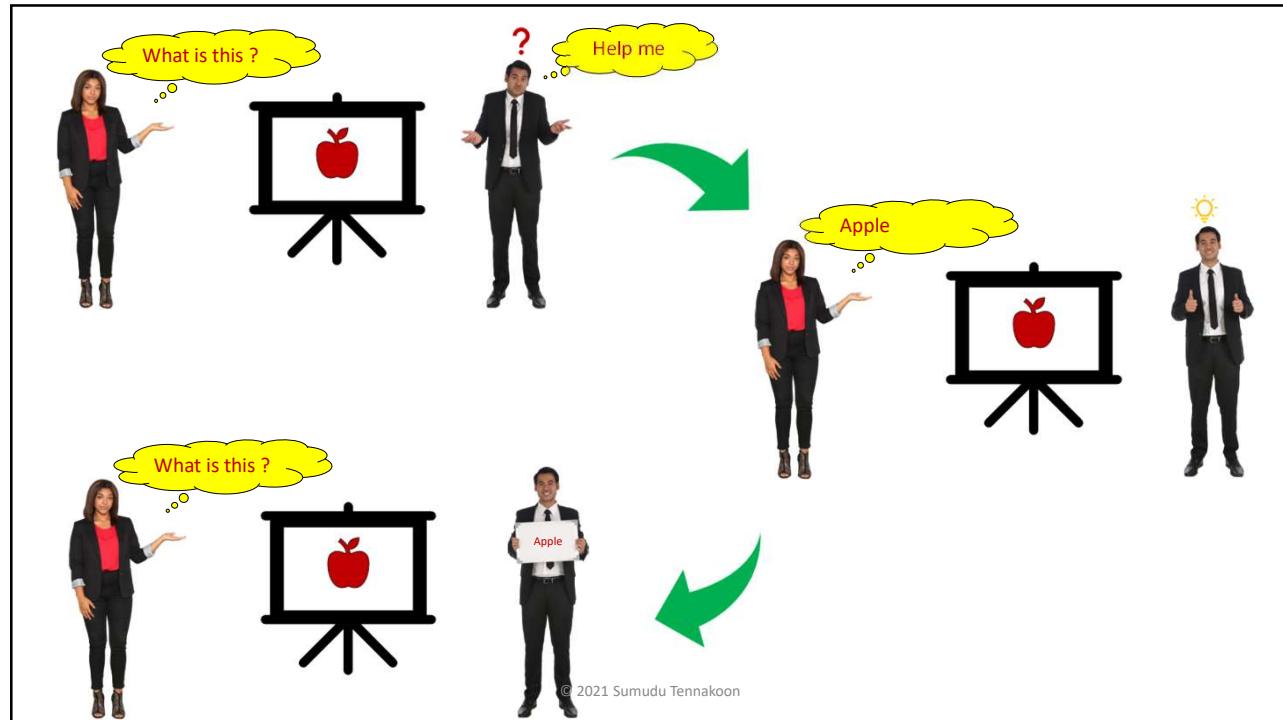
10

How we Learn ?

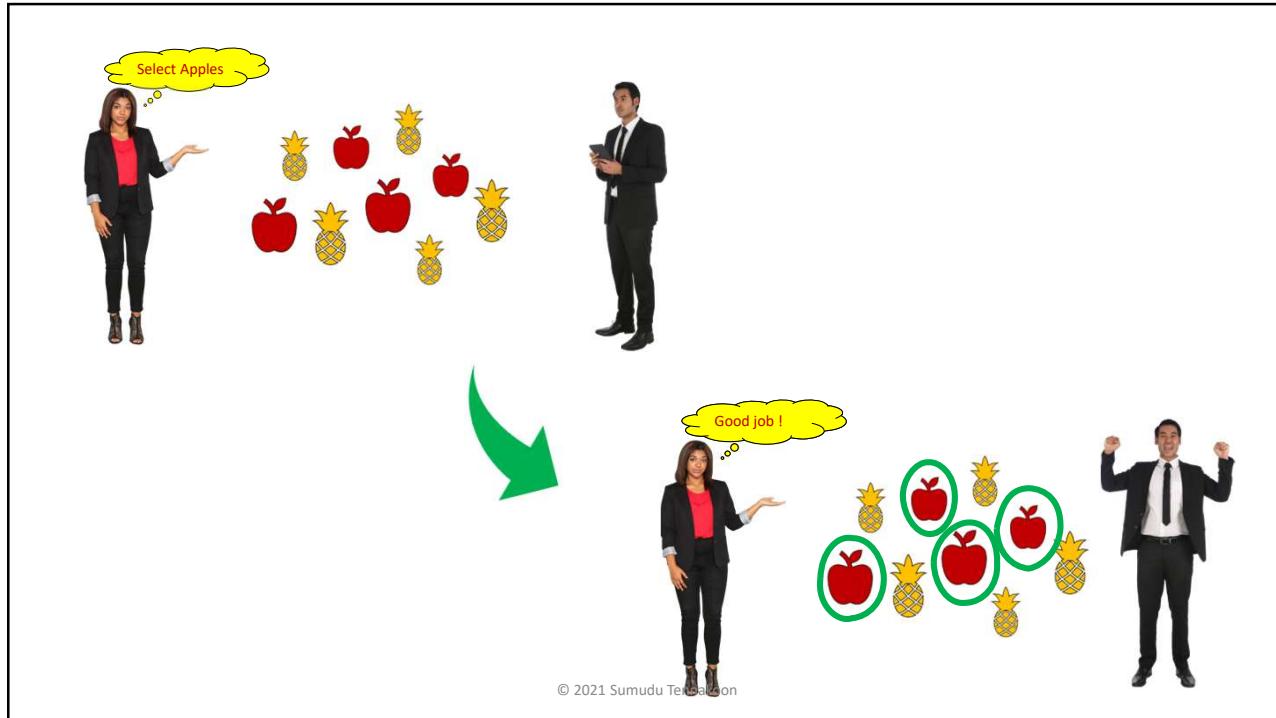
- Memorize Facts
 - Declarative Knowledge
 - Limited by memory and time to observe
- Infer (deduce new information from previously known facts)
 - Imperative Knowledge
 - Limited by accuracy of predictions and drifts (present is not behaving the same way as past)

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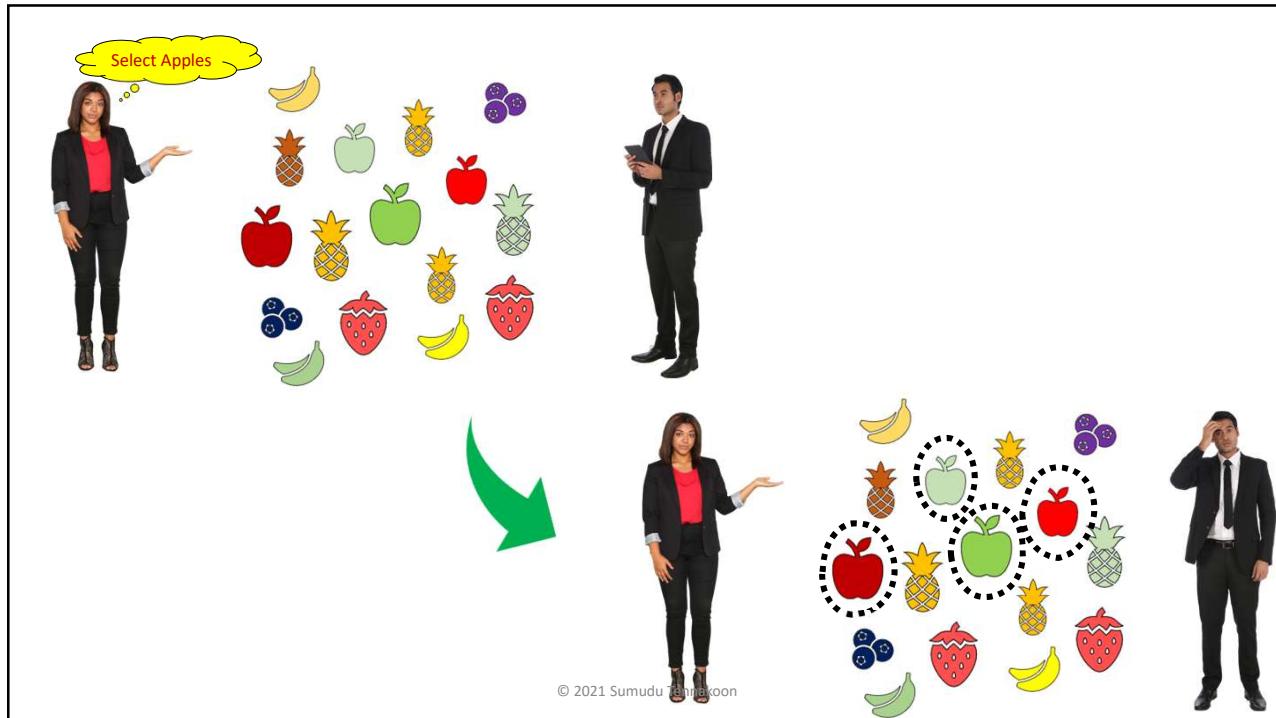
11



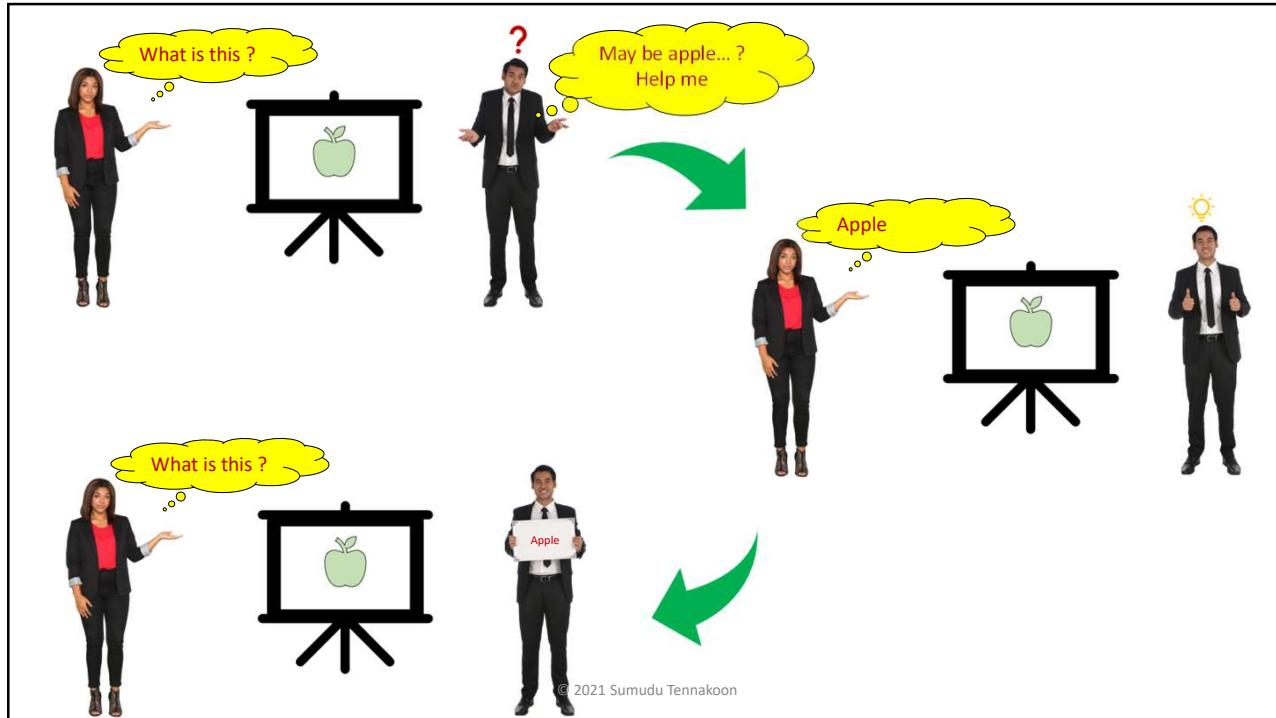
12



13



14



15

Exercise

What are the Observations/Measurements can be used to make a determination.

Design a simple classifier logic.

Is it easy or difficult to convert this logic to a computer program (code)?

What are the considerations when converting this logic to a computer program (code)?

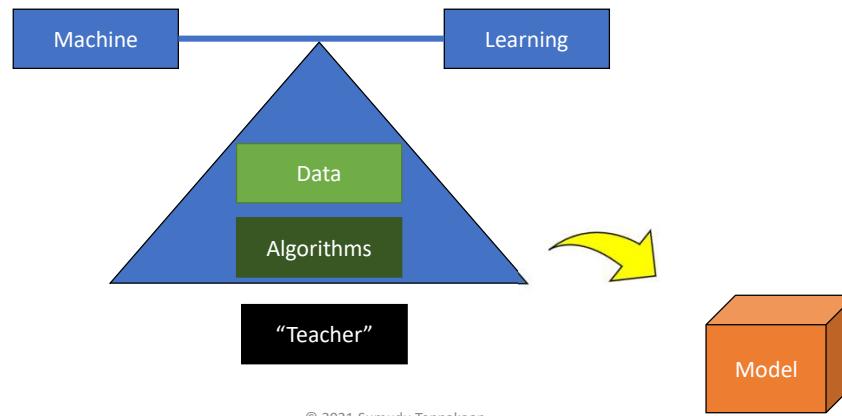
What are the points of failures in this approach?

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What is Machine Learning ?

- Learn from Data



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Machine learning model

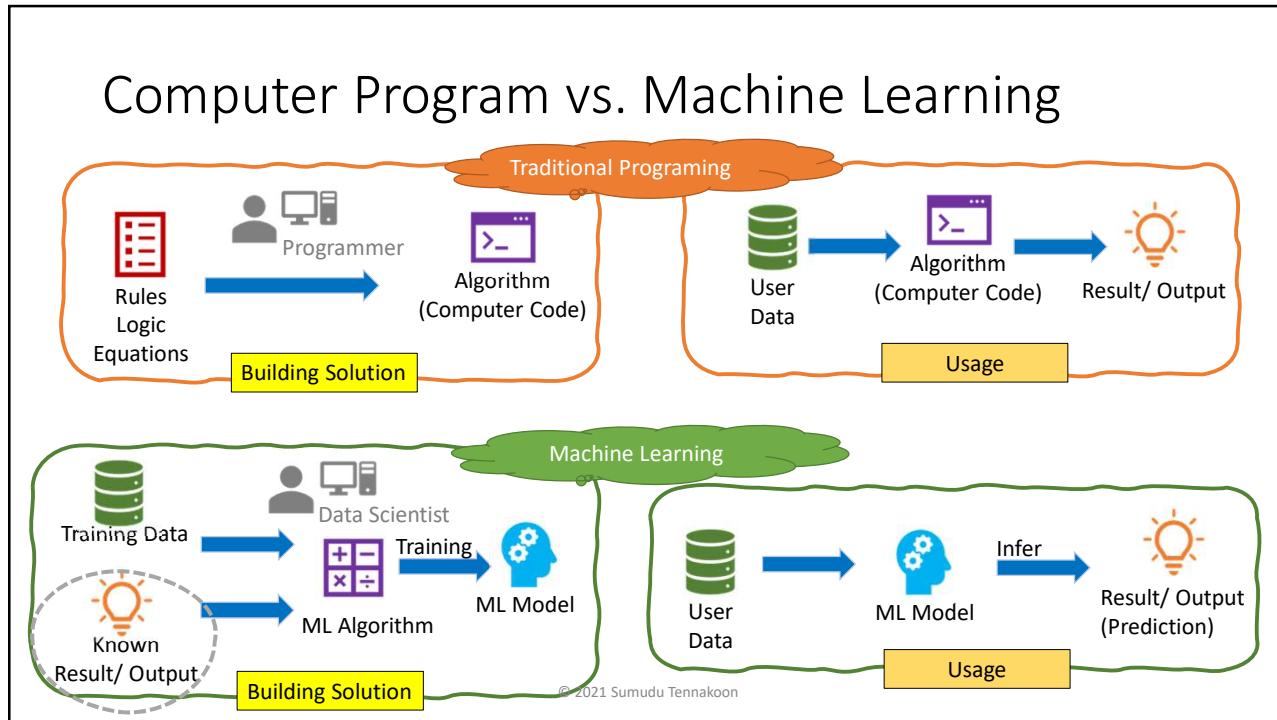


“Machine learning models are built on mathematical algorithms and are trained using data and human expertise to help us accurately predict outcomes based on input data such as images, text, or language.”

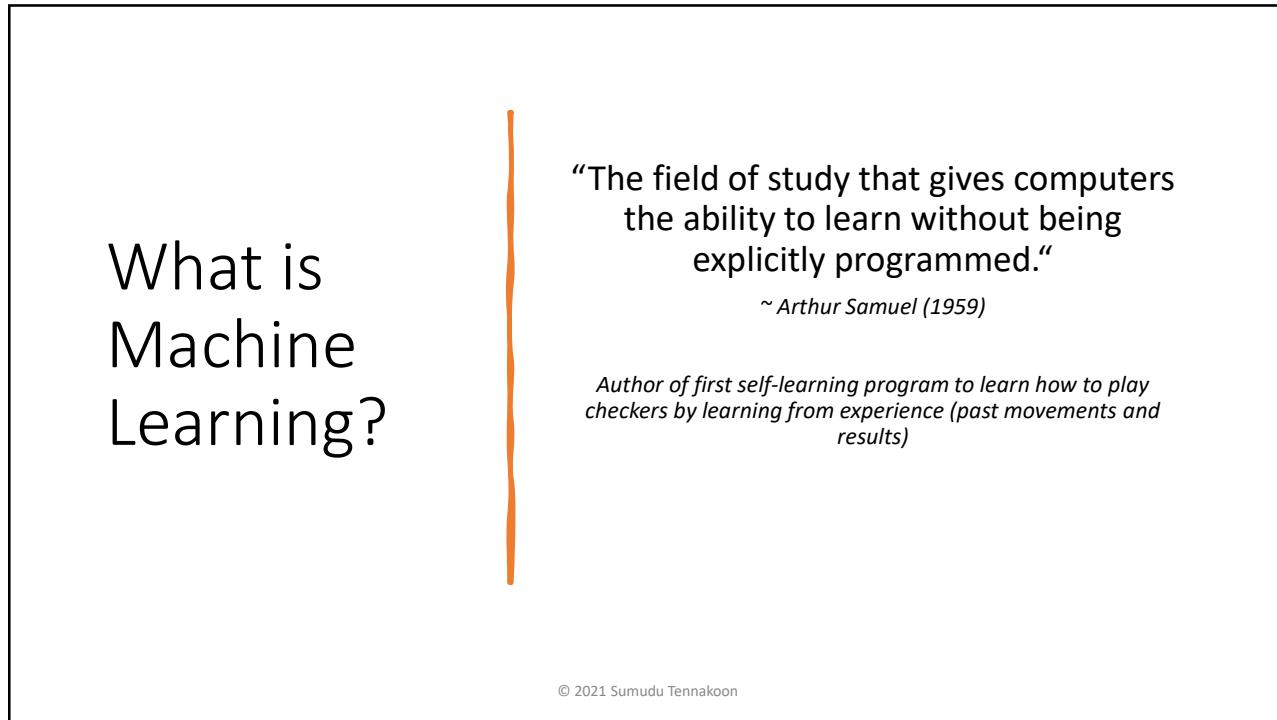
<https://developer.nvidia.com/ai-models>

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20

What is Machine Learning?

“A computer program is said to learn from experience E with respect to some class of **tasks T** and **performance measure P**, if its performance at tasks in T, as measured by P, improves with **experience E**.“

~ Tom Mitchell (1997)

Example: playing checkers.

- E = the experience of playing many games of checkers
- T = the task of playing checkers.
- P = the probability that the program will win the next game.

Mitchell, T. (1997). *Machine Learning*. McGraw Hill.
© 2021 Sumudu Tennakoon
p. 2. [ISBN 978-0-07-042807-2](#).

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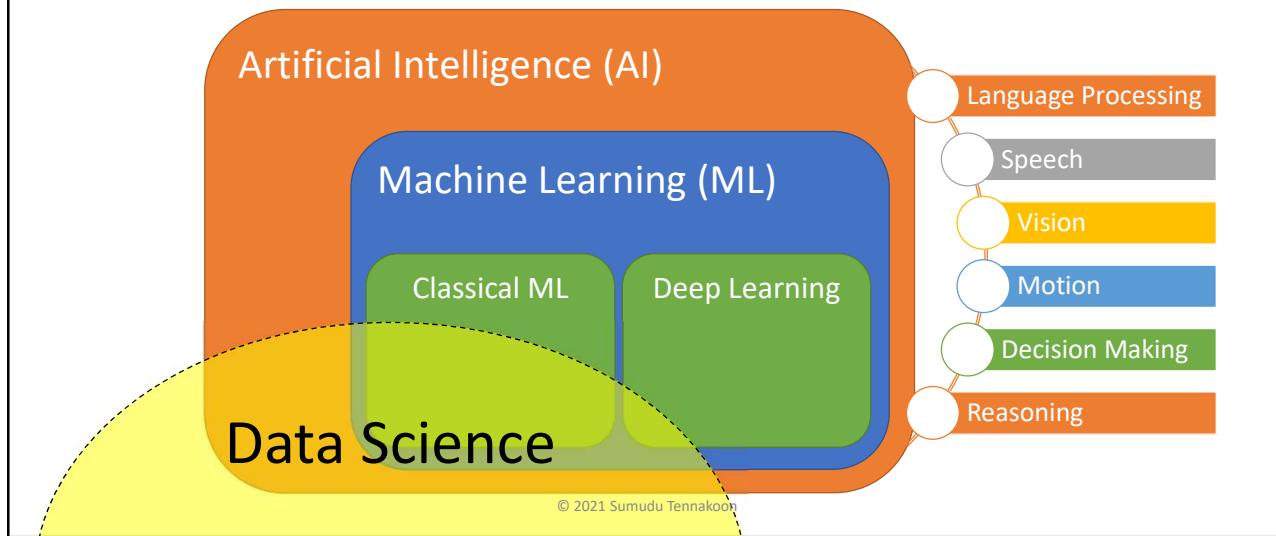
AI and Machine Learning

- AI (Mimic Cognitive Functions of Human)
 - Computer Vision
 - Speech Recognition and Synthesis
 - Language Processing and Understanding
 - Motion
 - Decision Making
 - Prescribe or Predict
 - Reasoning
- Machine Learning (ML)
 - Machines learn on Data/Prior Knowledge
 - Statistical Modeling/Algorithms
 - Backbone of AI is Machine Learning
 - Algorithms to Find meanings of data
 - Find Relationships
 - Making Predictions
 - Problem-Solution Types
 - Classification
 - Regression
 - Clustering

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Machine Learning/Deep Learning/AI



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Levels of AI

Artificial General Intelligence (AGI) known as "Strong AI"

- AGI is the ability to solve *any* problem rather than finding a solution to a particular problem.
- Machine can understand or learn any intellectual task that a human being can.
- The machine can think and perform tasks on its own, just like a human being.
- In the Movies! We are not there yet.

Weak Artificial Intelligence (Weak AI),

- Implements a limited part of human cognitive abilities.
- **Narrow AI** is a special case of Weak AI focused on a specific problem or task.
- Currently, existing AI systems are likely operating as a narrow AI.
- devices cannot follow these tasks independently but are made to look intelligent (simulate human behavior).

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Building Blocks of an AI System

- Image recognition (computer vision)
- Signal processing (sound, sensor data feed, etc.)
- Speech Recognition (Speech to text/STT)
- Natural language processing (NLP)
- Visual Synthesis (Computer Graphics)
- Sound Synthesis (Text to Speech/TTS)
- Software/Algorithms
- Applications (Anomaly Detection, Classification, Prediction, Pattern Recognition)
- Memory (Storage, RAM, Cache)
- Processor (GPU, CPU, TPU)
- Connectivity (Wi-Fi, Satellite, 5G, ethernet, etc.)
- Hardware (Computer, Mechanical Components, etc.)



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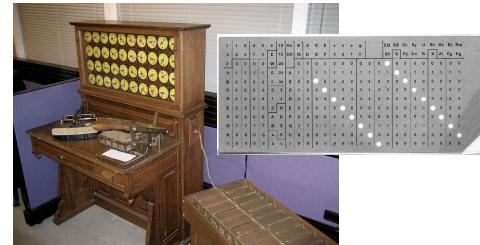
Precursor to Machine Learning



Pascal's calculator (1642)



Arithmomètre (1887)



Tabulating machine & punched card (1890)

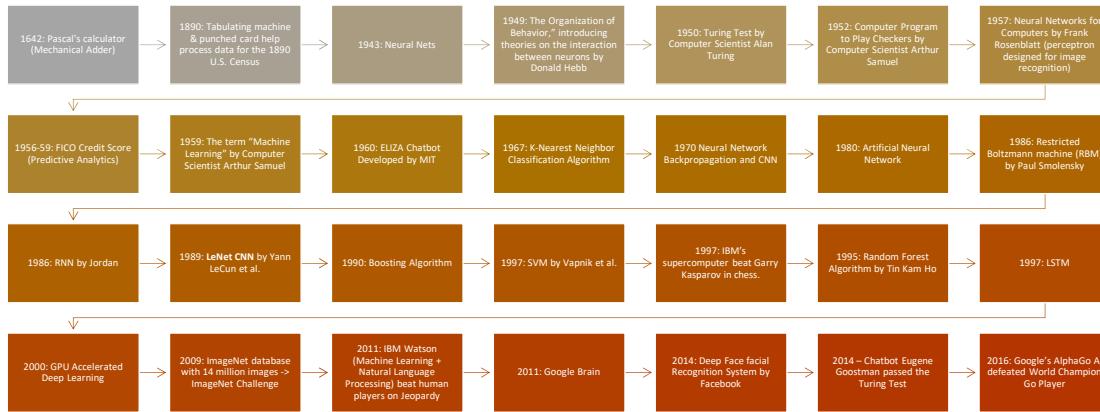


IBM 405 Alphabetical Accounting Machine (1934).

https://en.wikipedia.org/wiki/Pascal%27s_calculator
<https://en.wikipedia.org/wiki/Arithmometer>
https://en.wikipedia.org/wiki/Tabulating_machine
<http://www.columbia.edu/cu/computinghistory/405.html>

26

(Incomplete) Timeline of Machine Learning

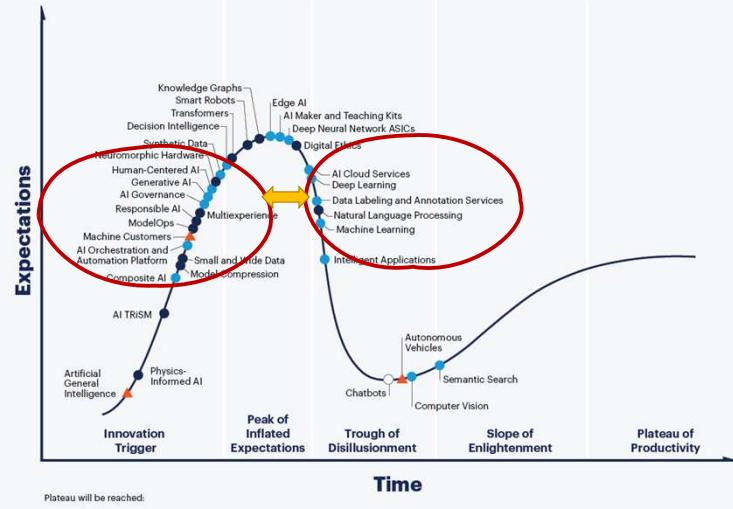


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Hype Cycle for Artificial Intelligence, 2021

Hype Cycle for Emerging Tech

<https://www.gartner.com/en/articles/the-4-trends-that-prevail-on-the-gartner-hype-cycle-for-ai-2021>



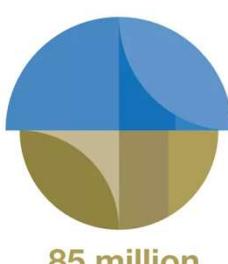
28

WORLD ECONOMIC FORUM

Job Landscape

Job landscape

By 2025, new jobs will emerge and others will be displaced by a shift in the division of labour between humans and machines, affecting:



Decreasing job demand:

1. Data Entry Clerks
2. Administrative and Executive Secretaries
3. Accounting, Bookkeeping and Payroll Clerks
4. Accountants and Auditors
5. Assembly and Factory Workers
6. Business Services and Administration Managers
7. Client Information and Customer Service Workers
8. General and Operations Managers
9. Mechanics and Machinery Repairers
10. Material-Recording and Stock-Keeping Clerks

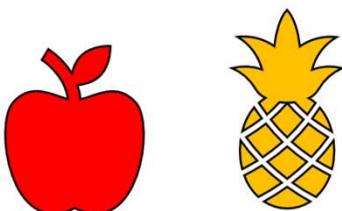
Growing job demand:

1. Data Analysts and Scientists
2. AI and Machine Learning Specialists
3. Big Data Specialists
4. Digital Marketing and Strategy Specialists
5. Process Automation Specialists
6. Business Development Professionals
7. Digital Transformation Specialists
8. Information Security Analysts
9. Software and Applications Developers
10. Internet of Things Specialists

Source: Future of Jobs Report 2020, World Economic Forum.

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Exercise



What are Observations/Measurements can be used to make a determination.

Design a simple classifier logic.

Is it easy or difficult to converting this logic to a computer program (code)?

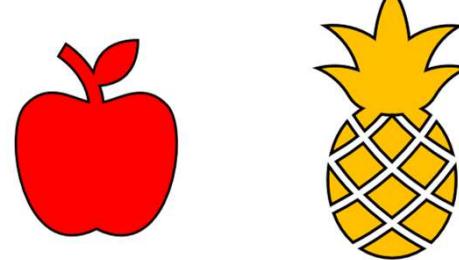
What are the considerations when converting this logic to a code?

What are the points of failure in this approach?

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Input and Output ?



Input →
(Features/Attributes)

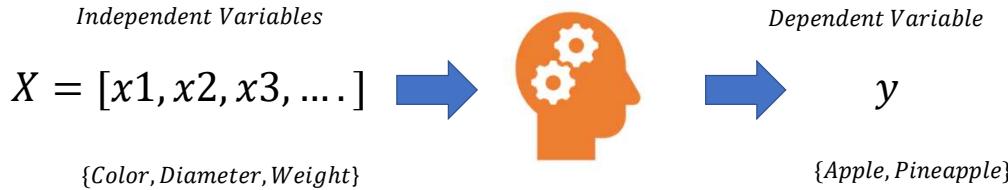
Color	red	yellow
Weight	50 g	200 g
Diameter	10 cm	20 cm

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Output →
Apple (A)
Pineapple (P)

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$$y = f(X)$$



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Tabular Data

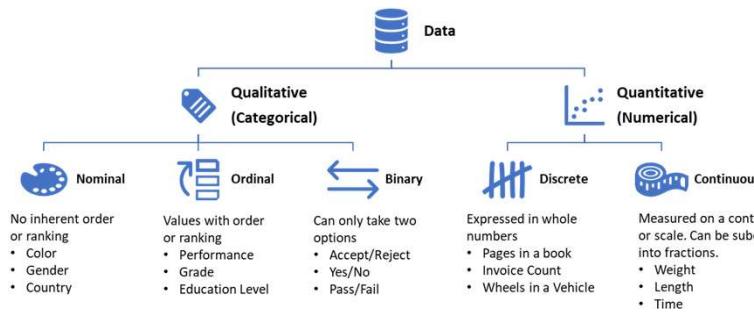
	Column 1	Column 2	Column 3	
	ID	Name	DOB	← Column Names
Row 1	10001	John Doe	1988-01-01	
Row 2	10002	Jane Doe	1990-12-31	Row (Record)
Row 3	

Column
(Data Field)

<https://pub.towardsai.net/data-science-for-everyone-getting-to-know-your-data-part-1-bb8b6d7782b1>

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Understanding Data



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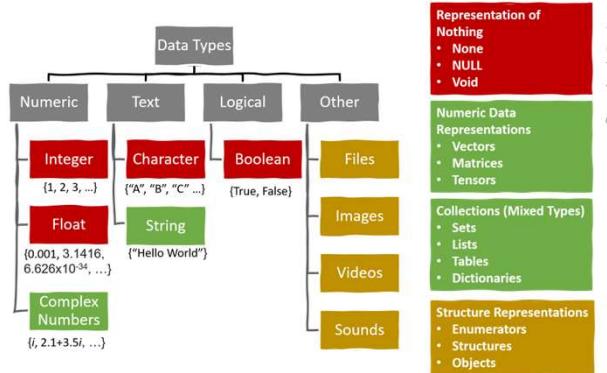
35



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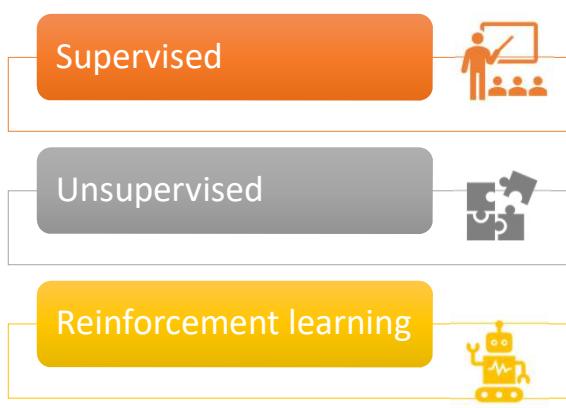
Data Types and Representations



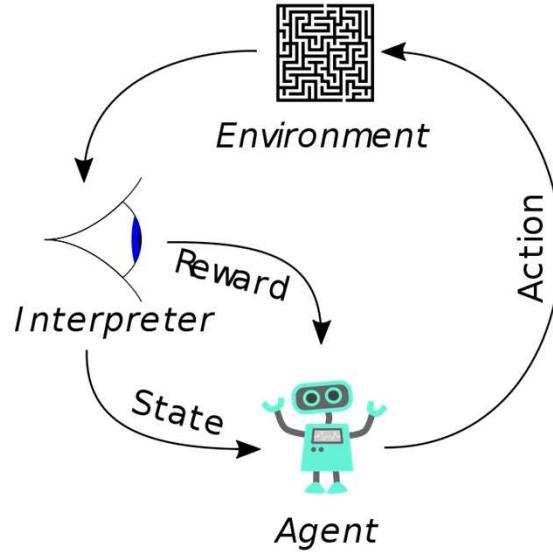
<https://pub.towardsai.net/data-science-for-everyone-getting-to-know-your-data-part-1-bb8b6d7782b1>

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Machine Learning Approaches



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© 2021 Sumudu Tennakoon https://en.wikipedia.org/wiki/Reinforcement_learning

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Types of ML Algorithms



Regression

Linear

Polynomial



Classification

Tree Classifiers

Logistic Regression

Support Vector Machines (SVM)



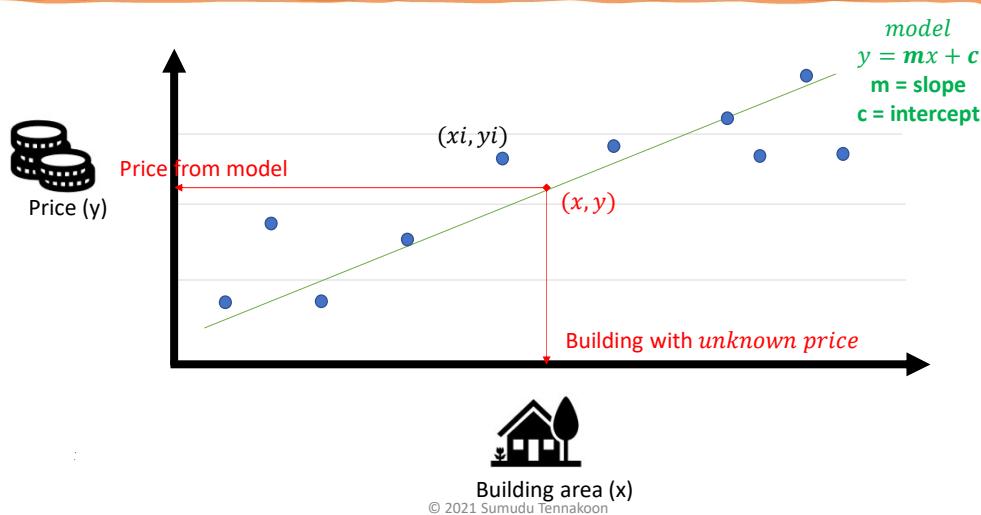
Clustering

K-Means

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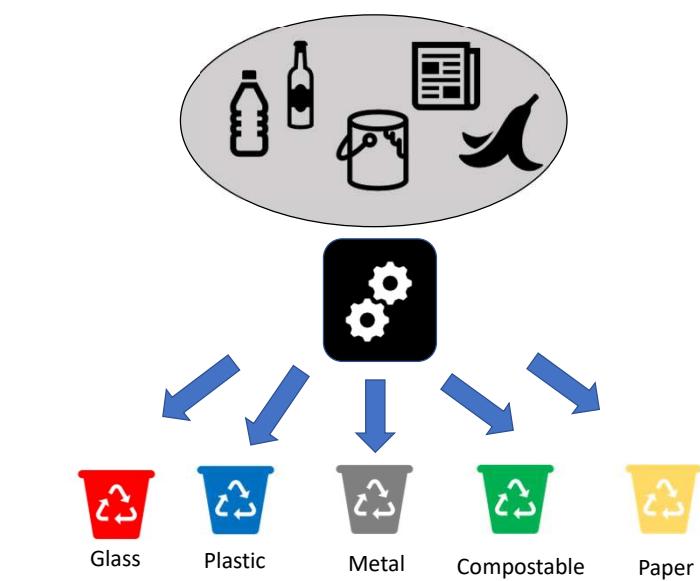
40

Regression



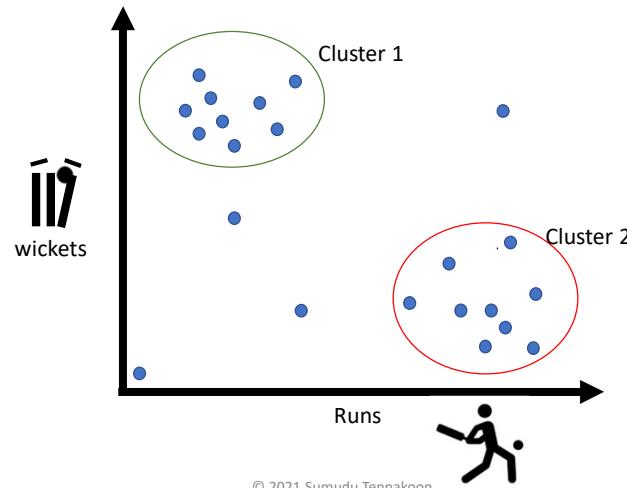
41

Classification



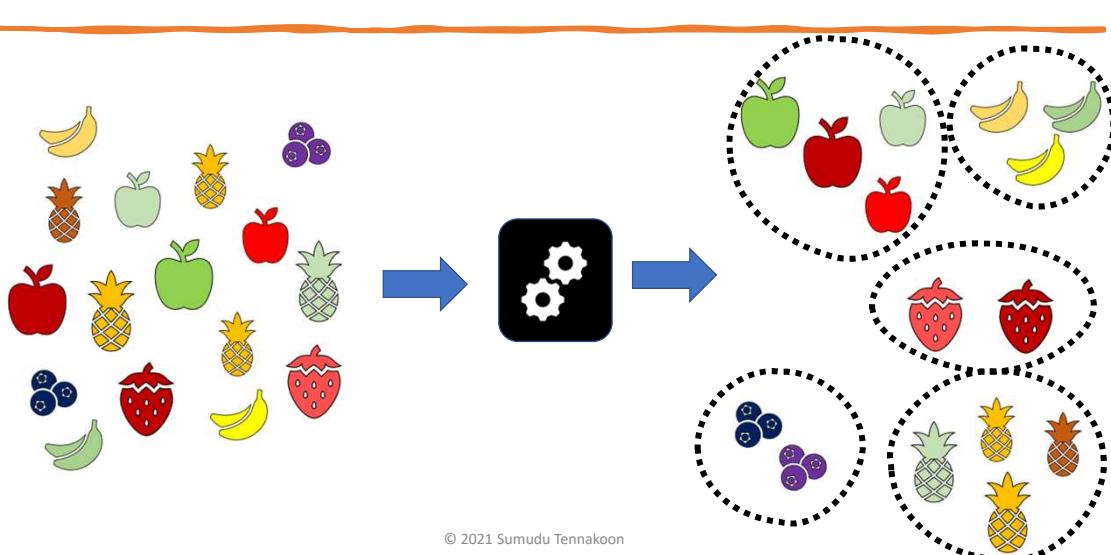
42

Clustering



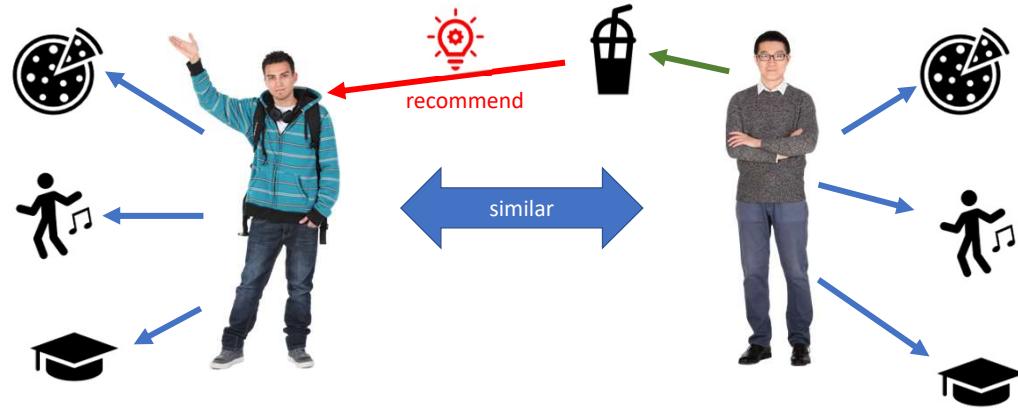
43

Clustering



44

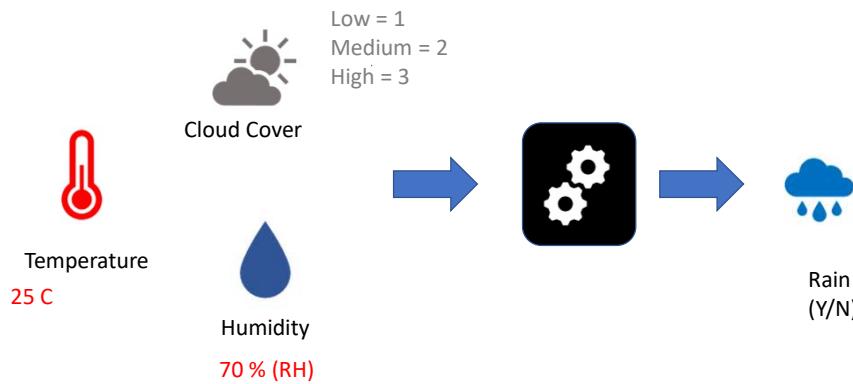
Recommender Systems



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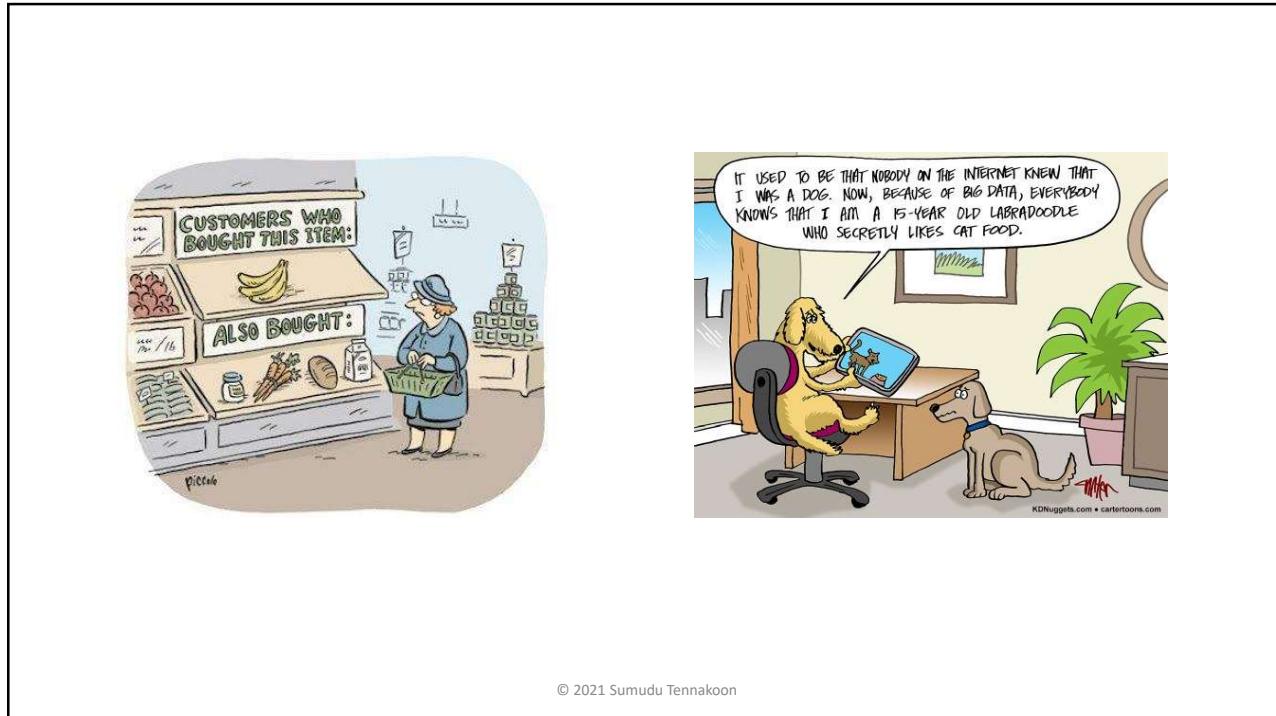
Prediction



$$y = f(X) ?$$

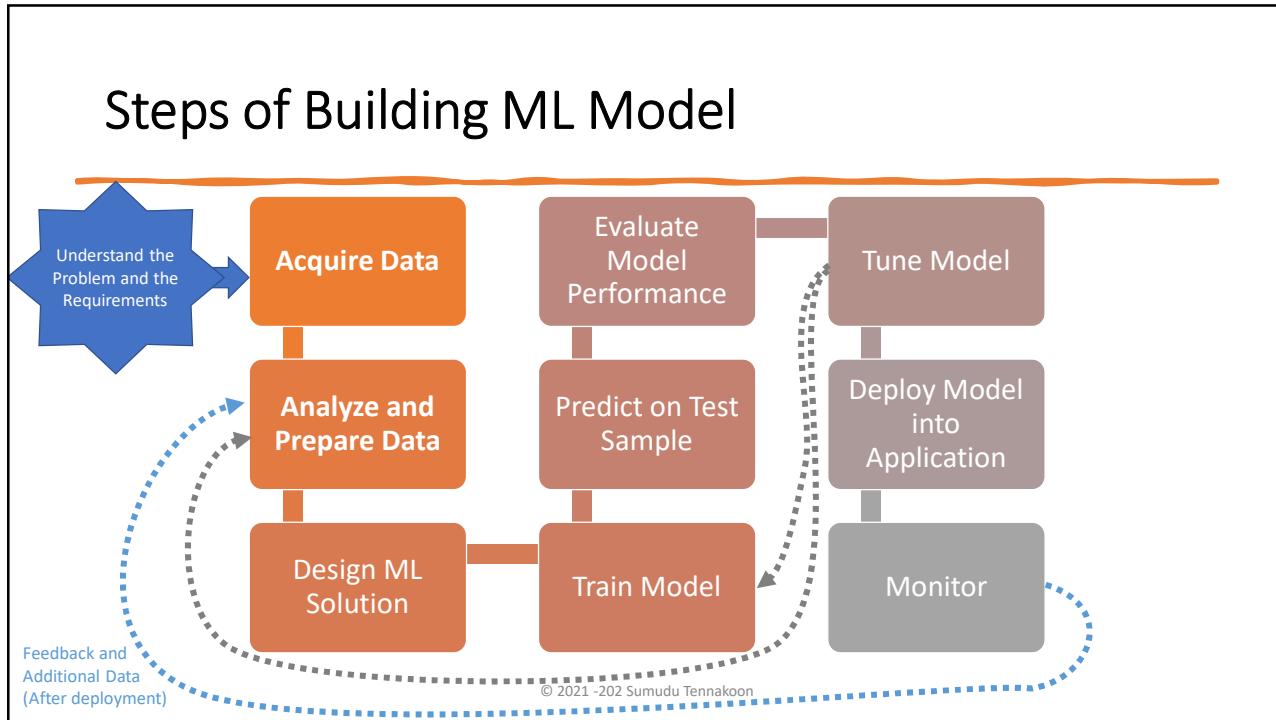
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Why we need Machine Learning?

Simulate human intelligence

Automation

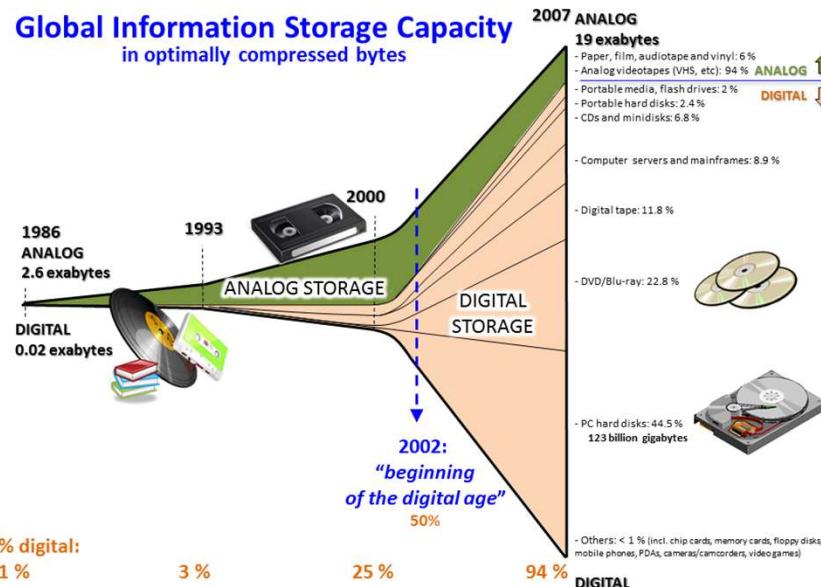
Help humans with informed decision making

Solve multidimensional problems

Predict future outcome based on historical observations

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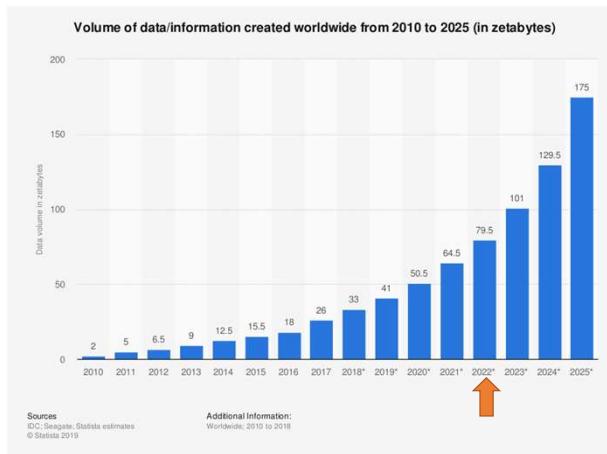


Source: Hilbert, M., & López, P. (2011). The World's Technological Capacity to Store, Communicate, and Compute Information. *Science*, 332(6025), 60–65. <http://www.martinhilbert.net/WorldInfoCapacity.html>

Scholz, R.. "Sustainable Digital Environments: What Major Challenges Is Humankind Facing?" *Sustainability* 8 (2016): 1-31.

50

Growth of World Data



zetta $1 \text{ ZB} = 10^{21} \text{ B}$

Annual volume of data and information created worldwide
Image: IDC; Seagate; Statista estimates

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Machine Learning Applications

Spam Email Filtering

Approve or Reject Loan Application

Predicting Stock Price

Credit Card Fraud Detection

Recommending Items to Purchase
(Advertising)

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Application Areas

Finance
Marketing
Information Technology
Cyber Security
Agriculture
Government
Automobile
Manufacturing
Retail
Entertainment
...
Everywhere!

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Why should everyone get familiar with ML?



Applications of machine learning are all around us.



ML is used in many industries and domains.



Job opportunities.



It is fun to learn and helps train your brain.



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Machine Learning Career Prospectus

- Data Scientist
- AI Scientist
- ML/AI Engineer
- Data Engineer
- Data Analyst
- AI/ML Developer
- IoT Developer
- Solutions Architect
- ...

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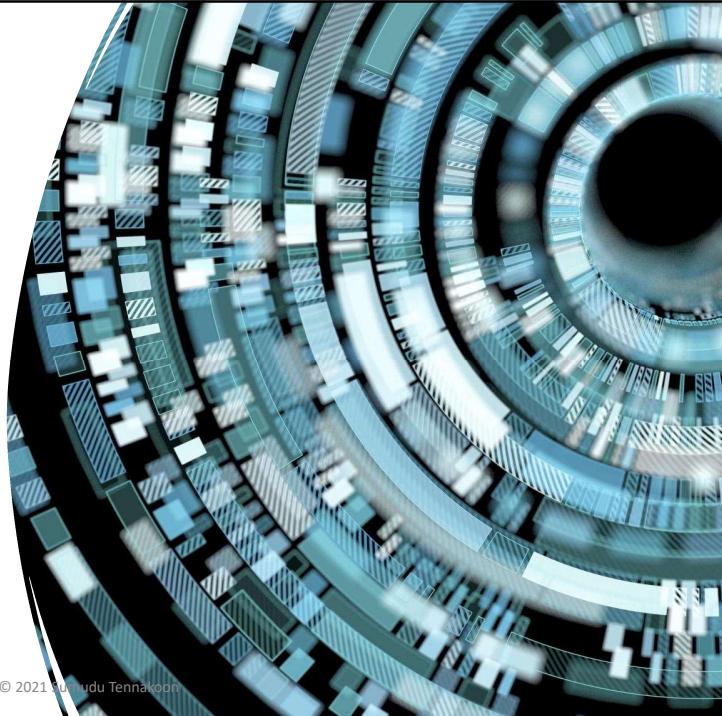


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Setting up Computing Environment

Cloud Computing Platform
(Google Colab)
Python (Install, Libraries)

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Install Python in Local Computer

- Python: <https://www.python.org/downloads/>
- Anaconda Python: <https://www.anaconda.com/products/individual>
- Python: Libraries: <https://www.anaconda.com/open-source>



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Python Libraries

- Data Handling
 - Pandas
 - Dask (distributed)
- Machine Learning
 - Scikit-learn
 - TensorFlow
 - PyTorch
- Visualizing
 - Matplotlib
 - Seaborn
- Numerical and Scientific Computing
 - SciPy
 - NumPy
- Machine Learning Model Interpretation
 - LIME
 - SHAP
- Web Services/API
 - Flask
 - Django



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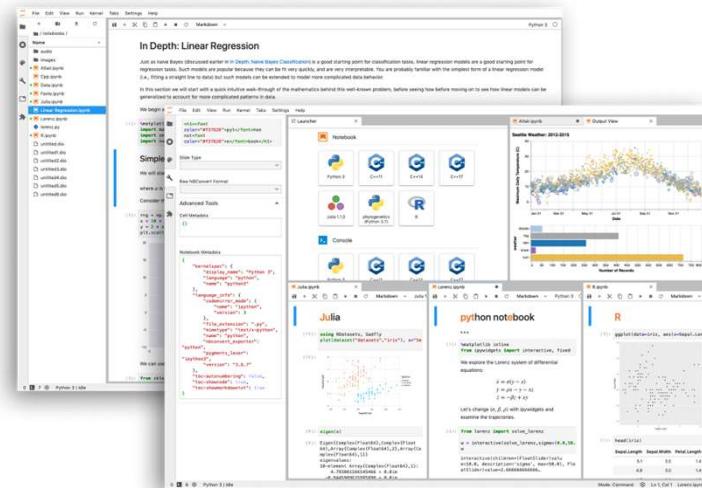
Install Python Libraries

- **Using pip package manager for Python**
 - <https://packaging.python.org/tutorials/installing-packages>
 - `pip install some-package-name`
 - NumPy: `pip install numpy`
 - Pandas: `pip install pandas`
 - Scikit-Learn: `pip install scikit-learn`
 - Matplotlib: `pip install matplotlib`
 - **Using Conda package manager**
 - <https://docs.conda.io/projects/conda/en/latest/commands/install.html>
 - `conda install -c conda-forge some-package-name`
 - NumPy: `conda install -c conda-forge numpy`
 - Pandas: `conda install -c conda-forge pandas`
 - Scikit-Learn: `conda install -c conda-forge scikit-learn`
 - Matplotlib: `conda install -c conda-forge matplotlib`

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Jupiter Notebook/Lab



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Google Colab

<https://colab.research.google.com>

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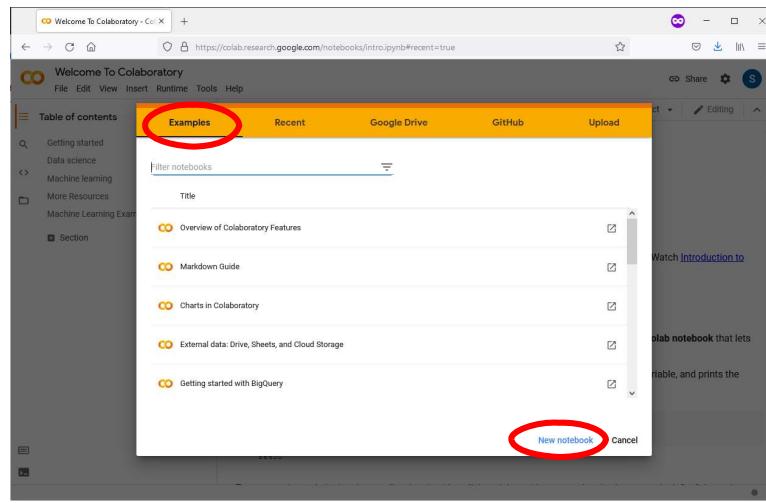
Getting Started

- Creating New Notebook
- Opening Notebook from GitHub
- Opening Notebook from file

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Creating New Notebook



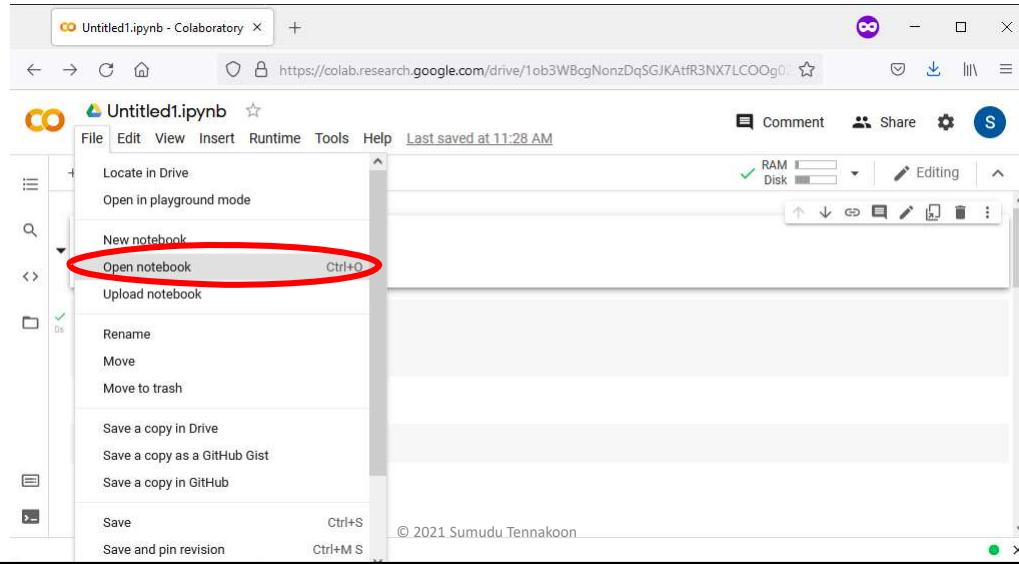
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A screenshot of the Google Colab interface showing a new notebook titled 'Untitled1.ipynb'. The top navigation bar includes 'File', 'Edit', 'view', 'Insert', 'Runtime' (which is circled in red), 'Tools', 'Help', and 'All changes saved'. The main workspace shows a 'Text/Markdown Cell' containing '# This is a Text Block' and a 'Code Cell' containing the Python code: # This is a Code Block, a = 2, print(a). Below the code cell is a play button icon (circled in red) with the text 'Click to run cell (execute the content)' and 'or Press [SHIFT] + [ENTER]'. The bottom status bar indicates '© 2021 Sumudu Tennakoon'.

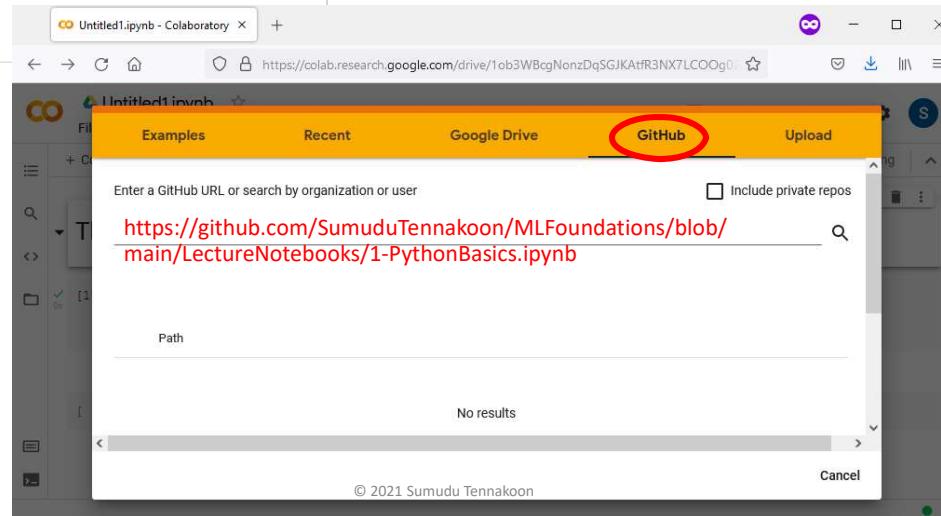
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Open Notebook

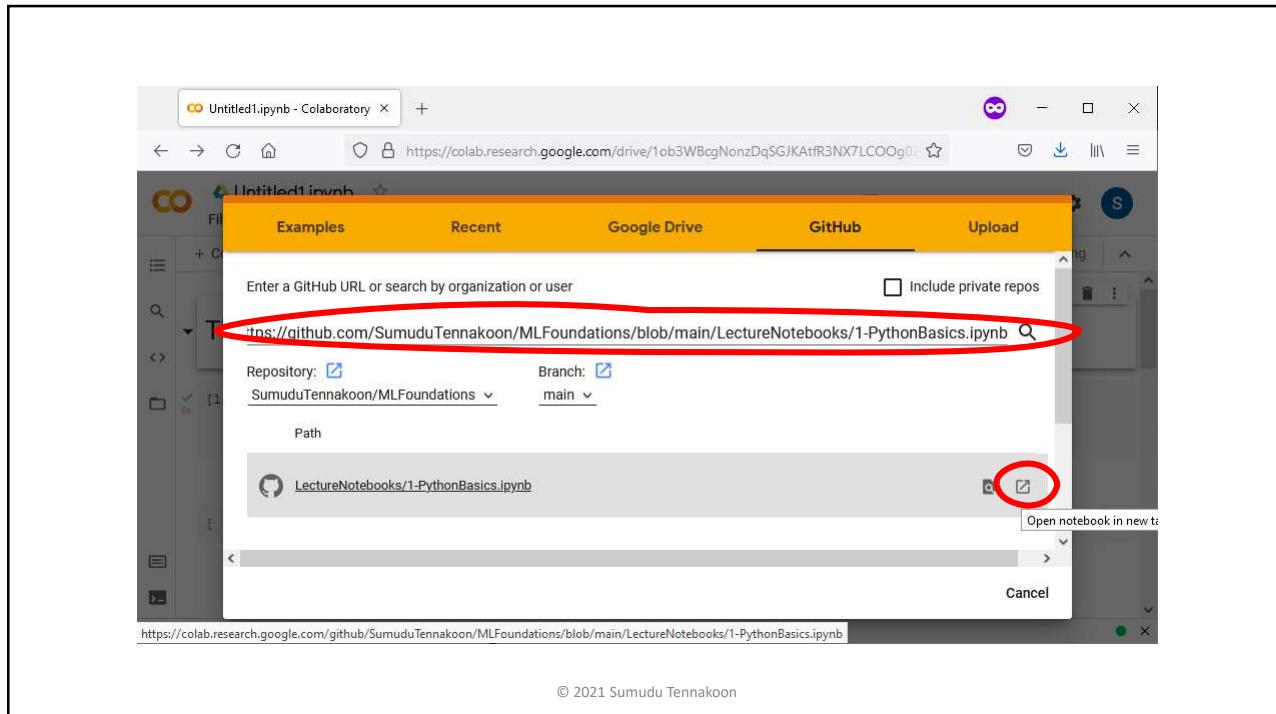


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Open Notebook from GitHub



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A screenshot of a Python notebook titled "Machine Learning Foundations" by Sumudu Tennakoon, PhD. The notebook content includes:

- Table of contents:**
 - Machine Learning Foundations
 - Python Basics
 - Variables
 - Data Types
 - Lists
 - Tuples
 - Sets
 - Dictionary
 - Conditions
 - Functions
 - Exercise 1:

Machine Learning Foundations

Sumudu Tennakoon, PhD

Python Basics

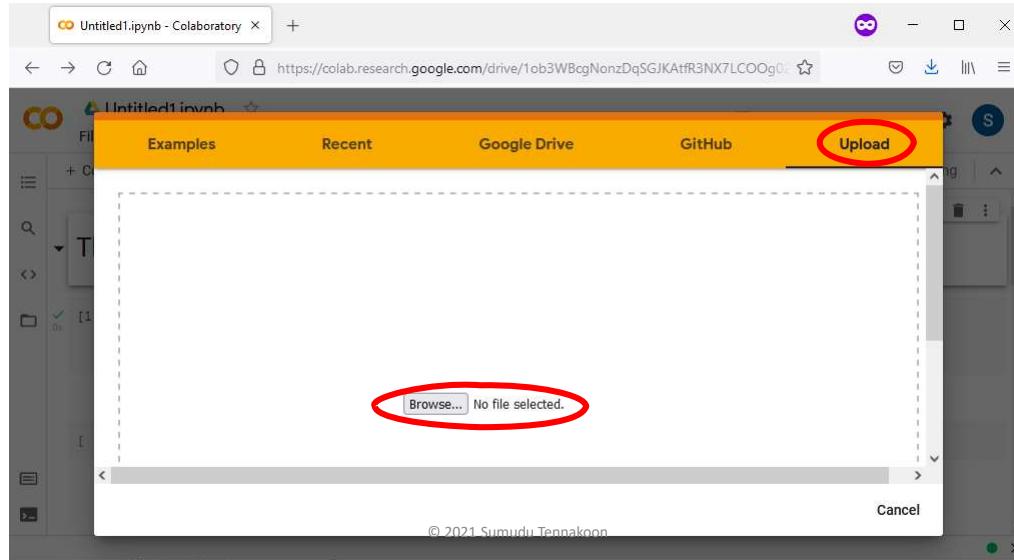
In this notebook we will explore some basic features on Python programming language for those who have a prior programming experience.

To learn more about Python, refer to the following websites

 - Python : www.python.org

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Upload Notebook (.ipynb file)



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A screenshot of a completed Python Basics notebook in Google Colab. The title bar shows '1-PythonBasics.ipynb - Colab'. The left sidebar contains a 'Table of contents' with sections like 'Machine Learning Foundations' (which is expanded) and 'Python Basics' (which is collapsed). The main content area displays the 'Machine Learning Foundations' section, which includes the author's name 'Sumudu Tennakoon, PhD'. Below it is the 'Python Basics' section, which contains a brief introduction, a note about learning more from websites, and a bullet point about Python with a link to www.python.org.

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GitHub Link to Lecture Notebooks

**Folder:**

<https://github.com/sumudutennakoon/mlfoundations/tree/main/lecturenotebooks>

Python basics notebook:

<https://github.com/sumudutennakoon/mlfoundations/blob/main/lecturenotebooks/1-pythonbasics.ipynb>

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Mathematics for Machine Learning

The blackboard contains several mathematical derivations and formulas:

- $D(x) = \dots + 4.31447$
- $V(x) = \dots$
- $x^2 + y^2 = ab + 4c$
- $c(x,y) = \begin{cases} xy = c \\ cx - cy = 3b \\ cz = c \end{cases}$
- $\text{men} = 384. + n \cdot v$
- $x = 920 \quad \left(\sum N_{50} \cdot x - \frac{1}{2} [984 + x] \right)$
- $\beta = 9 + x^2 + y^2$

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Data Representations

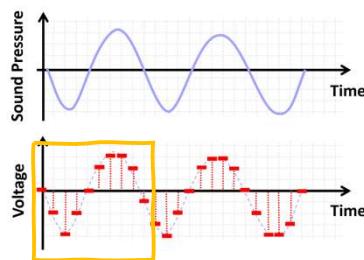
- In computing everything must convert into numbers !
- Numeric Data Structures:
 - Scalars: [3.14](#)
 - Vectors (1D): [\[1,2,3\]](#)
 - Matrices (2D): [\[\[1,2\], \[3,4\] \]](#)
 - Tensors (3D+): [\[\[\[1,2\], \[3,4\] \], \[\[5,6\], \[7,8\] \] \]](#)

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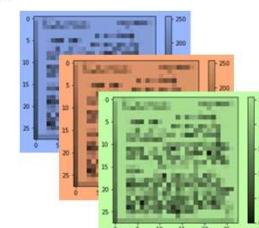
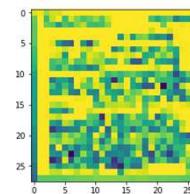


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Why we need Vectors, Matrices and Tensors in Machine Learning?



```
signal = [0 -2 -4 -2 0 2 3 3 2 -1 -3]
time   = [0 1 2 3 4 5 6 7 8 9 10]
```

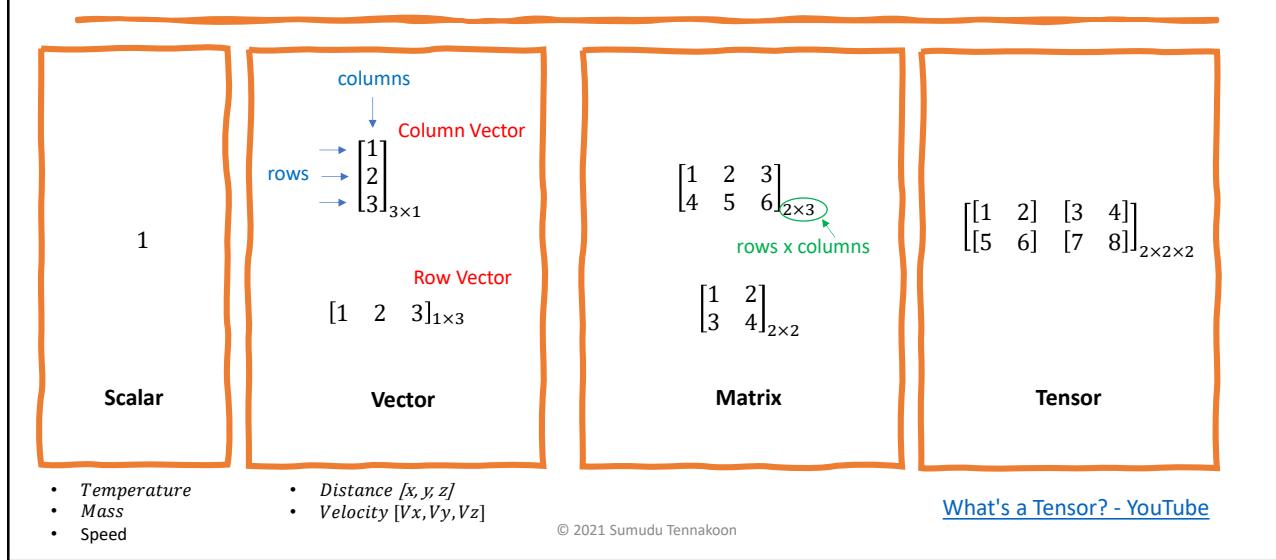


1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
1.0	0.9	0.7	0.9	0.6	0.8	0.7	0.8	0.9	0.9
0.7	0.7	0.7	0.8	1.0	0.6	0.8	0.8	0.9	0.9
0.9	0.9	0.9	0.9	1.0	0.9	0.9	0.9	0.9	0.9
0.9	1.0	1.0	0.9	1.0	1.0	1.0	1.0	1.0	1.0
0.7	0.6	0.8	0.8	0.9	0.6	1.0	0.8	0.7	0.8
1.0	0.9	1.0	1.0	1.0	0.9	1.0	1.0	1.0	1.0
1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
1.0	0.9	0.7	0.9	0.6	0.8	0.7	0.8	0.9	0.9
1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0

<https://pub.towardsai.net/data-science-for-everyone-getting-to-know-your-data-part-1-bb8b6d7782b1>

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Vectors and Matrices



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Matrix Algebra

- Transpose
- Sum
- Diagonal
- Determinant
- Adjugate
- Inverse

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Matrix Notation

- Matrix Notation

$$\bullet A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix}_{m \times n}$$

- Representation of elements in python index numbers

$$\bullet A = \begin{bmatrix} A[0][0] & A[0][1] & \dots & A[0][n-1] \\ A[1][0] & A[1][1] & \dots & A[1][n-1] \\ \vdots & \vdots & \ddots & \vdots \\ A[m-1][0] & A[m-1][1] & \dots & A[m-1][n-1] \end{bmatrix}$$

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Transpose, Sum, Diagonal

• $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} = [a_{ij}]$

• $A^T = \begin{bmatrix} a_{11} & a_{21} & a_{31} & a_{41} \\ a_{12} & a_{22} & a_{32} & a_{42} \\ a_{13} & a_{23} & a_{33} & a_{43} \\ a_{14} & a_{24} & a_{34} & a_{44} \end{bmatrix} = [a_{ji}]$

• $\text{Sum}(A) = \sum_{i,j} a_{ij}$

• $\text{Diagonal}(A) = [a_{11} \ a_{22} \ a_{33} \ a_{44}]$

• $\text{Trace}(A) = a_{11} + a_{22} + a_{33} + a_{44}$

E.g.,

• $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$

• $A^T = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$

• $\text{Sum}(A) = 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 = 45$

• $\text{Diagonal}(A) = [1 \ 5 \ 9]$

• $\text{Trace}(A) = 1 + 5 + 9 = 15$

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Determinant

$$\det \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

$$\det \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = 1 \times 4 - 2 \times 3 = -2$$

$$\det \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} = a \begin{vmatrix} e & f \\ h & i \end{vmatrix} - b \begin{vmatrix} d & f \\ g & i \end{vmatrix} + c \begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

$$\det \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} = 1 \begin{vmatrix} 5 & 6 \\ 8 & 9 \end{vmatrix} - 2 \begin{vmatrix} 4 & 6 \\ 7 & 9 \end{vmatrix} + 3 \begin{vmatrix} 4 & 5 \\ 7 & 8 \end{vmatrix} = 0$$

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Inverse Matrix

- $A^{-1} = \frac{1}{|A|} adj(A)$

- $A^{-1}A = AA^{-1} = I$

- $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

- $|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

- $adj(A) = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

- $A^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

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https://en.wikipedia.org/wiki/Invertible_matrix

https://en.wikipedia.org/wiki/Adjugate_matrix

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Properties of Matrix

$$\begin{aligned} A + B &= B + A \\ A + 0 &= A \\ AB &\neq BA \end{aligned}$$

$$\begin{aligned} A(BC) &= (AB)C \\ A(B+C) &= AB+AC \\ (A+B)C &= AC+BC \end{aligned}$$

$$\begin{aligned} \alpha(A+B) &= \alpha A + \alpha B \\ (\alpha + \beta)A &= \alpha A + \beta A \end{aligned}$$

$$\begin{aligned} 0.A &= 0 \\ A.0 &= 0 \end{aligned}$$

$$\begin{aligned} (A^T)^T &= A \\ (A+B)^T &= A^T + B^T \\ (AB)^T &= B^T A^T \end{aligned}$$

$$\begin{aligned} (A^{-1})^{-1} &= A \\ A^{-1}A &= AA^{-1} = I \end{aligned}$$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$$

$$\begin{aligned} (A^T)^{-1} &= (A^{-1})^T \\ (AB)^{-1} &= B^{-1}A^{-1} \end{aligned}$$

$$\begin{aligned} |A^{-1}| &= \frac{1}{|A|} \\ |AB| &= |A||B| \end{aligned}$$

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Special Vectors/Matrices

- Square Matrix: $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2 \times 2}$

- Symmetric Matrix: $S = \begin{bmatrix} 1 & 4 & 5 \\ 4 & 2 & 6 \\ 5 & 6 & 3 \end{bmatrix}$

- Zero Matrix: $0 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

- Unit Vector: $\hat{x} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

- Diagonal Matrix: $D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

- Identity Matrix: $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

- Scalar Diagonal Matrix: $D = \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} = \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \lambda I$

- Upper Triangular matrix: $U = \begin{bmatrix} 1 & 4 & 5 \\ 0 & 2 & 6 \\ 0 & 0 & 3 \end{bmatrix}$

- Lower Triangular Matrix: $L = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 2 & 0 \\ 5 & 6 & 3 \end{bmatrix}$

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Matrix Operations and Applications

- Addition/Subtraction
- Scalar Multiplication
- Matrix Multiplication (Dot Product)
- Matrix-Vector Multiplication
- Row operations
- Solving Linear Equations
- Linear transformations
- Decomposition

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Matrix Multiplication

$$\bullet A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\bullet B = \begin{bmatrix} p & q \\ r & s \end{bmatrix}$$

$$\bullet A \cdot B = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} p & q \\ r & s \end{bmatrix}$$

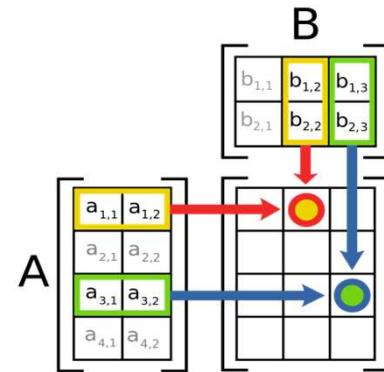
$$\bullet A \cdot B = \begin{bmatrix} ap + br & aq + bs \\ cp + dr & cq + ds \end{bmatrix}$$

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Matrix Multiplication (Dot Product)

$$\begin{array}{c}
 \text{m} = 3 \\
 \text{A} \underset{5 \times 3}{\cdot} \underset{3 \times 4}{B} = \underset{5 \times 4}{C}
 \end{array}$$



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Solving Linear Equations

Problem:

$$\begin{aligned}
 3x + 2y &= 12 \rightarrow (1) \\
 4x + 5y &= 23 \rightarrow (2)
 \end{aligned}$$

$$\begin{aligned}
 x &=? \\
 y &=?
 \end{aligned}$$

Solution Approach:

$$\begin{aligned}
 x &= \frac{12 - 2y}{3} = 4 - \frac{2}{3}y \\
 4\left(4 - \frac{2}{3}y\right) + 5y &= 23 \\
 16 - \frac{8}{3}y + 5y &= 16 + \frac{15 - 8}{3}y = 16 + \frac{7}{3}y = 23 \\
 y &= \frac{3(23 - 16)}{7} = \frac{3(7)}{7} = 3
 \end{aligned}$$

$$x = 4 - \frac{2}{3}(3) = 4 - 2 = 2$$

$$\begin{aligned}
 x &= 2 \\
 y &= 3
 \end{aligned}$$

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Formulating the Problem in Matrix Form

$$\begin{aligned}x_1 &= x \\x_2 &= y\end{aligned}$$

$$3x_1 + 2x_2 = 12 \rightarrow (1)$$

$$4x_1 + 5x_2 = 23 \rightarrow (2)$$

Solve using Matrices:

$$\begin{bmatrix} 3 \\ 4 \end{bmatrix} x_1 + \begin{bmatrix} 2 \\ 5 \end{bmatrix} x_2 = \begin{bmatrix} 12 \\ 23 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 2 \\ 4 & 5 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 12 \\ 23 \end{bmatrix}$$

$$A = \begin{bmatrix} 3 & 2 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

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$$Ax = b$$

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$b = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} 12 \\ 23 \end{bmatrix}$$

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Obtaining the Solution in Matrix Form

$$\begin{aligned}Ax &= b \\A^{-1}Ax &= A^{-1}b \\Ix &= A^{-1}b \\x &= A^{-1}b\end{aligned}$$

$$\text{Example} \quad A = \begin{bmatrix} 3 & 2 \\ 4 & 5 \end{bmatrix}$$

$$b = \begin{bmatrix} 12 \\ 23 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{adj}(A)$$

$$|A| = 3 \times 5 - 4 \times 2 = 7$$

$$\begin{aligned}x &= A^{-1}b \\x &= \left(\frac{1}{|A|} \times \text{adj}(A) \right) b\end{aligned}$$

$$A^{-1} = \frac{1}{7} \begin{bmatrix} 5 & -2 \\ -4 & 3 \end{bmatrix}$$

$$x = \frac{1}{7} \begin{bmatrix} 5 & -2 \\ -4 & 3 \end{bmatrix} \cdot \begin{bmatrix} 12 \\ 23 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

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Solving Linear Equations (General Form)

$$Ax = b$$

$$\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix}_{m \times n} \cdot \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}_{n \times 1} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}_{m \times 1}$$

$m = \text{number of linearly independent equations}$

$n = \text{number of unknowns } (x_i)$

$m = n \rightarrow \text{unique solution}$

$m < n \rightarrow \text{infinitely many solutions}$

$$x = A^{-1}b$$

$$\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}_{n \times 1} = \underbrace{\begin{bmatrix} a'_{11} & a'_{12} & \dots & a'_{1m} \\ a'_{21} & a'_{22} & \dots & a'_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ a'_{n1} & a'_{n2} & \dots & a'_{nm} \end{bmatrix}_{n \times m}}_{A^{-1}} \cdot \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}_{m \times 1}$$

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Linear Transformations

$$p = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$p' = T \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x' \\ y' \end{bmatrix}$$

- Stretching = $\begin{bmatrix} k & 0 \\ 0 & 1 \end{bmatrix}$

- Squeezing = $\begin{bmatrix} k & 0 \\ 0 & 1/k \end{bmatrix}$

- Rotation = $\begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$

counterclockwise

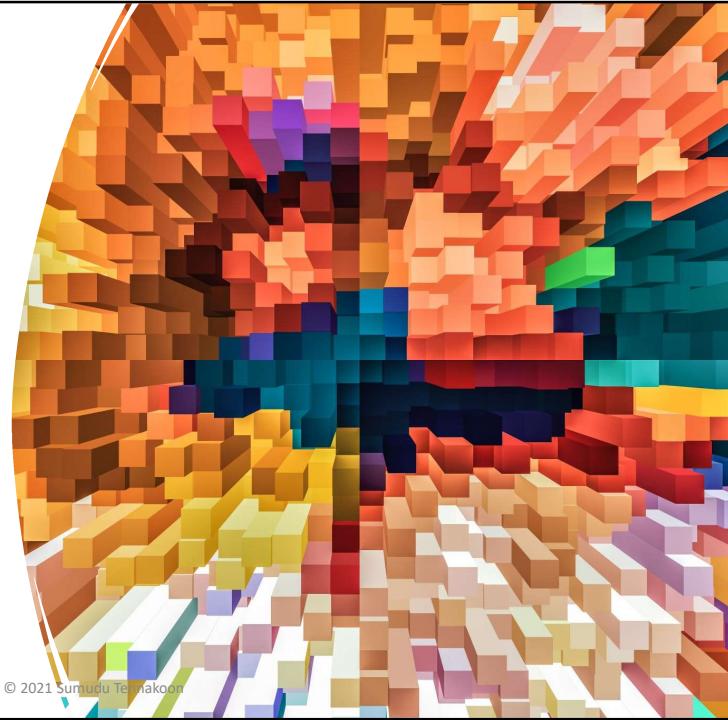
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Sets

- Basics of Set Theory
- Set Operators
 - Union
 - Intersection
 - Complement
 - Difference



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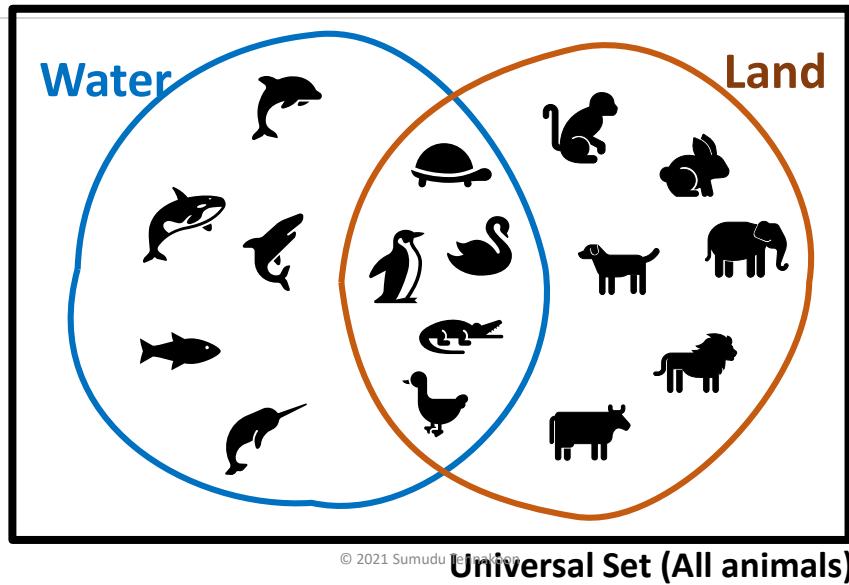
Basics of Set Theory

- A set is a collection of things (elements).
 - $A = \{\clubsuit, \diamond, \heartsuit, \spadesuit\}$
 - $B = \{\times, \div, +, -\}$
 - $C = \{apple, orange, mango, banana\}$
 - $D = \{x | x \text{ satisfies some property}\}$
 - $\mathbb{N} = \{1, 2, 3, 4, \dots\}$ is set of natural numbers
 - $\mathbb{Z} = \{\dots, -3, -2, -1, 0, 1, 2, 3, 4, \dots\}$ is set of integers
 - $E = \{x | x \in \mathbb{Z}, -2 \leq x < 10\}$
 - $\emptyset = \{\}$ is Null set
- Items belongs to a set (element of)
 - $\heartsuit \in A$
 - $\div \in B$
 - $apple \in C$
- Items not belongs to a set (not an element of)
 - $\blacksquare \notin A$
 - $*$ $\notin B$
 - $strawberry \notin C$
- Subset
 - $\{\diamond, \heartsuit\} \subset A$
 - $\mathbb{N} \subset \mathbb{Z}$

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Venn Diagrams



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Cardinality (size of the set)

$$A = \{\clubsuit, \diamondsuit, \heartsuit, \spadesuit\}$$

$$|A| = 4$$

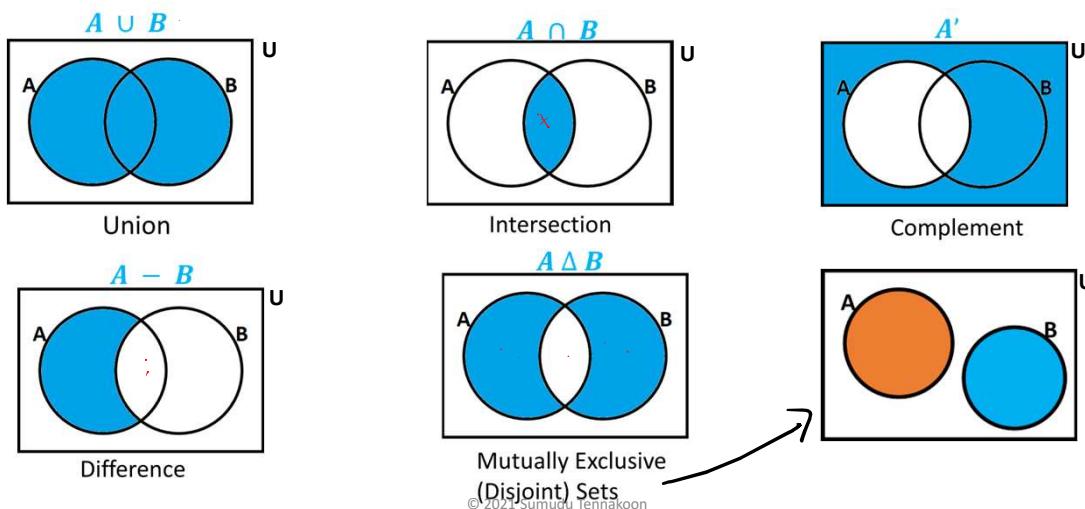
$$B = \{1, 2, 3, 4, 5, 6\}$$

$$|B| = 6$$

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Set Operations



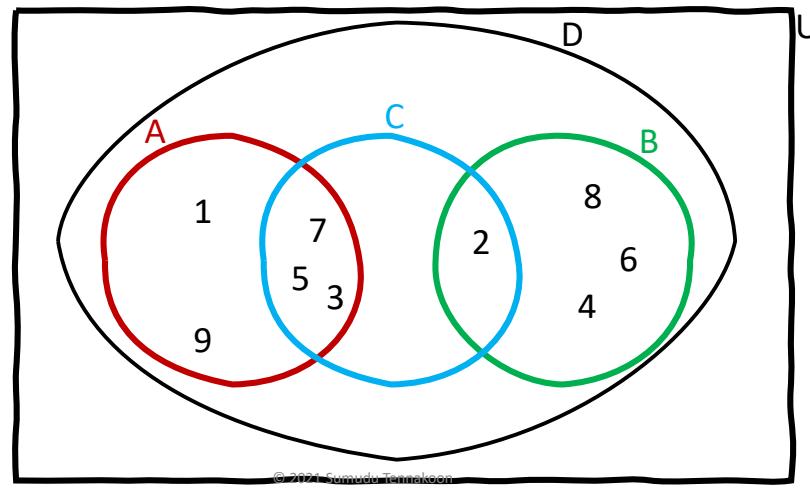
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$$A = \{x | x \in \mathbb{Z}, 1 \leq x < 10, x \text{ is an odd number}\}$$

$$B = \{x | x \in \mathbb{Z}, 1 \leq x < 10, x \text{ is an even number}\}$$

$$C = \{x | x \in \mathbb{Z}, 1 \leq x < 10, x \text{ is a prime number}\}$$

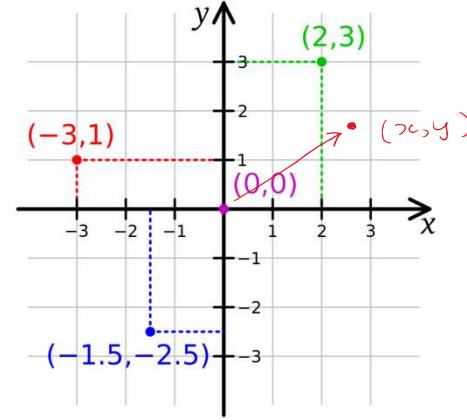
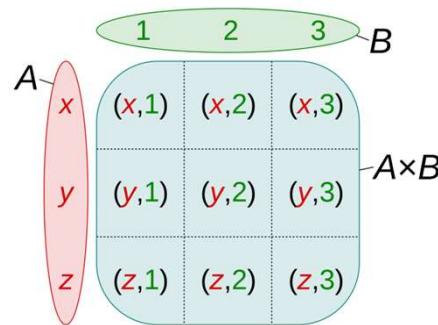
$$D = \{x | x \in \mathbb{Z}, 1 \leq x < 10\}$$



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Cartesian product

- $A \times B = \{(a, b) | a \in A \text{ and } b \in B\}$
- $A \times B \neq B \times A$

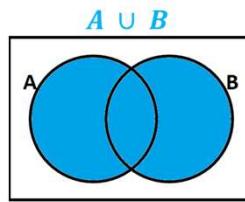


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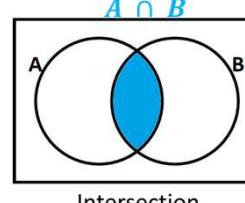
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Laws of Set Theory

- Identity
 - $A \cap U = A$
 - $A \cup \emptyset = A$



- Dominance
 - $A \cap \emptyset = \emptyset$
 - $A \cup U = U$
- Idempotence
 - $A \cap A = A$
 - $A \cup A = A$



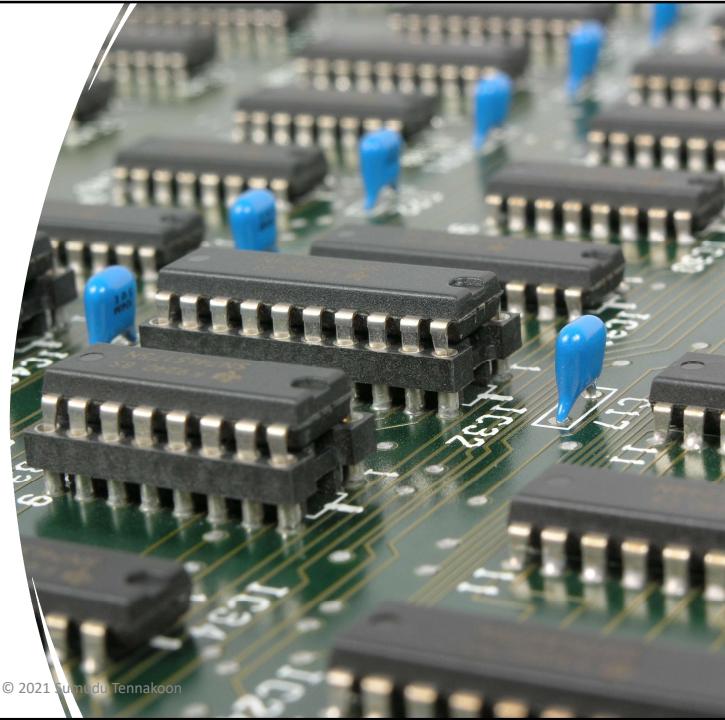
- Complement
 - $A \cap A' = \emptyset$
 - $A \cup A' = U$
- Double Compliment
 - $(A')' = A$

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Logic Gates

- Logical Statements
- Logical Operators
 - AND
 - OR
 - NOT



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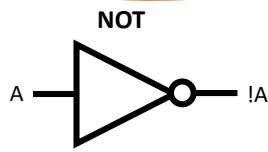
Logical Statements and Binary Logic

- TRUE = 1
- FALSE = 0
- “5 is an odd number” is **TRUE**
- “4 is a prime number” is **FALSE**
- “Kandy is the Capital City of Sri Lanka” is **FALSE**
- “A triangle has three sides” is **TRUE**

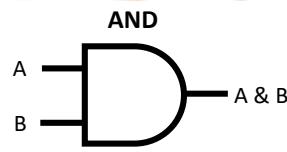
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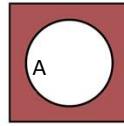
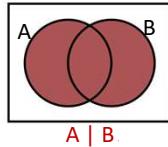
Logic Gates



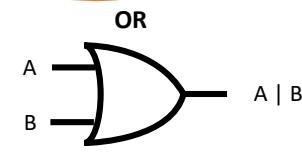
A	!A
1	0
0	1



A	B	A & B
1	1	1
1	0	0
0	1	0
0	0	0



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A	B	A B
1	1	1
1	0	1
0	1	1
0	0	0

https://en.wikipedia.org/wiki/Boolean_algebra

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Laws of Boolean Algebra

- Identity
 - $A \& 1 = A$
 - $A | 0 = A$

A is a Boolean variable
A can take either 1 or 0

- Annihilator
 - $A \& 0 = 0$
 - $A | 1 = 1$

- Idempotence
 - $A \& A = A$
 - $A | A = A$

- Associativity
 - $A \& (B \& C) = (A \& B) \& C$
 - $A | (B | C) = (A | B) | C$

- Commutativity
 - $A \& B = B \& A$
 - $A | B = B | A$

- Distributivity
 - $A \& (B | C) = (A \& B) | (A \& C)$
 - $A | (B \& C) = (A | B) \& (A | C)$

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Laws of Boolean Algebra

- Complement
 - $A \& !A = 0$
 - $A | !A = 1$
- Double Negation
 - $!(!A) = A$
- De Morgan's laws:
 - $!A \& !B = !(A | B)$
 - $!A | !B = !(A \& B)$

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Exponents and Logarithms

	Exponent Representation	Logarithmic representation
Base n	$n^x = y$	$\log_n y = x$
Base 2	$2^3 = 8$	$\log_2 8 = 3$
Base 10	$10^3 = 1000$	$\log_{10} 1000 = 3$
Base e	$e^{6.907755} \approx 1000$	$\log_e 1000 \approx 6.907755$

$$\log_e(1000) = \ln(1000)$$

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Functions

- $y = f(x)$
- $f(x) = \frac{1}{x}$
- $f(x) = a_0 + a_1x + a_2x^2$
- $f(x) = a_0 + a_1x_1 + a_2x_2$
- $f(x) = a_0 + a_1x_1 + a_2x_2$
- *Step Functions*

$$f(x) = \begin{cases} 0, & x < 0 \\ x, & x \geq 0 \end{cases}$$
- *Logistic Function*

$$f(x) = \frac{L}{1+e^{-k(x-x_0)}}$$

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Series

- $S_n = \sum_{i=1}^n a_i = a_1 + a_2 + \dots + a_n$
- Example
 - Let $a_i = 2(i - 1) + 1$
 - $a_1 = 1$
 - $a_2 = 2 + 1 = 3$
 - $a_3 = 2 \times 2 + 1 = 5$
 - $a_4 = 2 \times 3 + 1 = 7$
 - $S_4 = 1 + 3 + 5 + 7 = 16$

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Sequence

- $(a_i)_{i=1}^n = [a_0, a_1, a_2, \dots, a_n]$

- Example

- Let $a_i = i^2$
- $a_1 = 1$
- $a_2 = 2^2 = 4$
- $a_3 = 3^2 = 9$
- $a_4 = 4^2 = 16$

- $(i^2)_{i=1}^4 = 1, 4, 9, 16$

© 2021 Sumudu Tennakoon [https://en.wikipedia.org/wiki/Series_\(mathematics\)](https://en.wikipedia.org/wiki/Series_(mathematics))

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Trigonometric Functions

- $\sin(x)$
 - $\cos(x)$
 - $\tan(x)$
-
- Sound Signal:
 - amplitude = $\sin(\omega t) = \sin(2\pi f t)$

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Take-home Exercise

- Mention one machine learning problem (specify whether it is a classification, regression or clustering problem) and the training approach you would take (supervised or unsupervised).
- Your answer must be 240 characters or fewer.



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Probability and Statistics Concepts for Machine Learning

- Probability
- Statistical Distributions
- Descriptive Statistics



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Probability



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Probability Example

- Coin (single toss)
 - Possible outcomes: {Head, Tail}



- Dice (single roll)
 - Possible outcomes: {1, 2, 3, 4, 5, 6l}

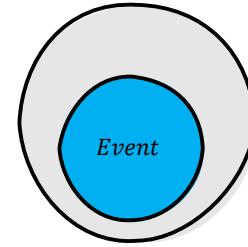


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Term Definitions

- **Experiment:** A procedure that can be repeated and has a well-defined set of possible outcomes.
 - **Random experiment:** outcome is unknown before the experiment. Results different outcomes when repeating the experiment in the same manner.
 - **Deterministic experiment:** outcome may be predicted with certainty beforehand. Has a definite outcome. Outcome is predictable.
- **Sample Space (Ω):** Set of all possible outcomes or results of an experiment.
- **Outcome:** Possible result of an experiment. An element of Ω .
- **Event:** A set of selected outcomes of an experiment. Subset of the sample space.
 - Independent Event: probabilities of occurrence do not depend on one another
- **Random Variable:** A variable that can take different values of the sample space (describes the outcomes of a random experiment).



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Probability

- Let $\Omega = \{\omega_1, \omega_2, \dots, \omega_n\}$ be a sample space of an experiment.
- Let the event $A \subset \Omega$
- Probability (How likely the event A can occur) : $P(A) = \frac{|A|}{|\Omega|}$
- $0 \leq P(A) \leq 1$
- Random Variable (X): $\{\omega \in \Omega | u < X(\omega) \leq v\}$
- Cumulative Probability: $\sum_x P(X = x) = 1$

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Probability Example

- Coin (single toss)

- Two possible outcomes

- $\Omega = \{H, T\}$

- $|\Omega| = 2$

- $P(A = \{H\}) = \frac{1}{2}$

- $P(A = \{T\}) = \frac{1}{2}$



- Dice (single roll)

- Six possible outcomes

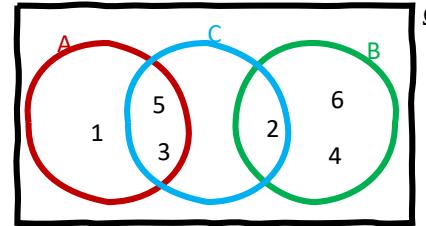
- $\Omega = \{1, 2, 3, 4, 5, 6\}$

- $|\Omega| = 6$

- $P(A = \{1\}) = \frac{1}{6}$

- $P(A = \{6\}) = \frac{1}{6}$

- $P(A = \{1, 3, 5\}) = \frac{3}{6} = \frac{1}{2}$ (get odd number as outcome)



Odd number: $P(A) = \frac{|A|}{|\Omega|} = \frac{1}{6} = \frac{1}{2}$

Even number: $P(B) = \frac{|B|}{|\Omega|} = \frac{1}{2}$

Prime number: $P(C) = \frac{|C|}{|\Omega|} = \frac{1}{2}$

$P(A \cup C) = \frac{|A \cup C|}{|\Omega|} = \frac{4}{6} = \frac{2}{3}$

$P(A \cap C) = \frac{|A \cap C|}{|\Omega|} = \frac{2}{6} = \frac{1}{3}$

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Conditional Probability

- Probability of A given B : $P(A|B) = \frac{P(A \cap B)}{P(B)}$, if $P(B) \neq 0$

- Example

- $A = \{2, 3, 5\}$, prime numbers
- $B = \{1, 3, 5\}$, odd numbers

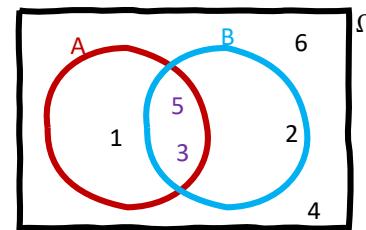


- If the outcome is an odd number, probability of that Number being a prime.

- $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{3}}{\frac{1}{2}} = \frac{2}{3}$

- $P(A \cap B) = \frac{|A \cap B|}{|\Omega|} = \frac{|\{3, 5\}|}{|\{1, 2, 3, 4, 5, 6\}|} = \frac{2}{6} = \frac{1}{3}$

- $P(B) = \frac{|B|}{|\Omega|} = \frac{|\{1, 3, 5\}|}{|\{1, 2, 3, 4, 5, 6\}|} = \frac{3}{6} = \frac{1}{2}$



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Conditional Probability: Example

- A: Selected person is vaccinated
- B: Age of the selected person ≥ 30
- If a person with age ≥ 30 is selected, probability that the person is vaccinated?
 - $P(A) = \frac{|A|}{|\Omega|} = \frac{55}{100}$
 - $P(A \cap B) = \frac{|A \cap B|}{|\Omega|} = \frac{45}{100}$
 - $P(B) = \frac{|B|}{|\Omega|} = \frac{50}{100}$
 - $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{45}{100}}{\frac{50}{100}} = \frac{45}{50} = 0.9$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

B

	Vaccinated (YES)	Vaccinated (NO)	Total
Age < 30	10	40	50
Age ≥ 30	45	5	50
Total	55	45	100

A

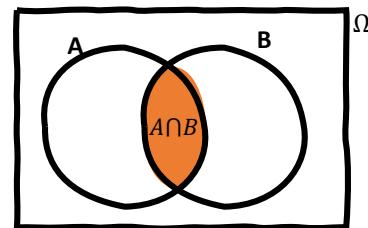
90 %

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Bayes Rule

- Conditional Probability of A given B
 - $P(A|B) = \frac{P(A \cap B)}{P(B)}$, if $P(B) \neq 0 \rightarrow (1)$
- Commutativity in Set theory
 - $P(A \cap B) = P(B \cap A) \rightarrow (2)$
- (2) in (1)
 - $P(B \cap A) = P(A|B) \cdot P(B) \rightarrow (3)$
- Conditional Probability of B given A
 - $P(B|A) = \frac{P(B \cap A)}{P(A)} \rightarrow (4)$
- (3) in (4)
 - $P(B|A) = \frac{P(A|B) \cdot P(B)}{P(A)}$, if $P(A) \neq 0$



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https://en.wikipedia.org/wiki/Bayes%27_theorem

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Conditional Probability: Example

Events

- I : Infected
- N : Not infected
- T_P : Positive Test Result
- T_N : Negative Test Result

Measurements/Probabilities

- known*
- Infected population ratio: $P(I) = 0.04 = 4\%$
 - Positive Test Result if infected: $P(T_P|I) = 0.98 = 98\%$
 - Positive Test Result if not infected: $P(T_P|N) = 0.01 = 1\%$
 - A person get positive result regardless of the infection status: $P(T_P)$
 - $P(N) = 1 - P(I)$
 - $P(T_P) = \sum_{x=\{I,N\}} P(x)P(T_P|x)$
 - $P(T_P) = P(I)P(T_P|I) + P(N)P(T_P|N)$
 - $P(T_P) = 0.04 \times 0.98 + 0.96 \times 0.01 = 0.0488$

John got a positive test result. What is probability that John is infected.

$$P(I|T_P) = \frac{P(T_P|I) \cdot P(I)}{P(T_P)} = \frac{0.98 \times 0.04}{0.0488} = 0.803$$

80 %

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Probability Distributions

Discrete

- Probability distribution of a random variable that can take only a finite set of values.
- Probability mass function (PMF)
 - $\sum_{x \in X} P(x) = 1$

Continuous

- Probability distribution of a random variable that can take an infinite number of values.
- Probability density function (PDF)
 - $\int P(x) dx = 1$

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Combinatorics

- Permutations

- Order matters
- Without repetition: $P_x^n = \frac{n!}{(n-x)!}$
- With repetitions: n^x

- Combinations

- Order does not matter
- Can take with or without repetitions
- Without repetitions: $C_x^n = \frac{n!}{x!(n-x)!}$
- With repetitions: $C_x^{n+x-1} = \frac{(n+x-1)!}{x!(n-x)!}$

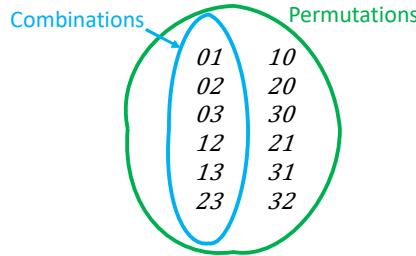
Example

Set of Digits

- $S = \{0, 1, 2, 3\}$
- $n = |S| = 4$

Pick 2 elements (digits at a time) without repetition.

- $x = 2$



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Permutations and Combinations

Set of Digits

- $S = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$
- $|S| = 10$

$$P_x^n = \frac{n!}{(n-x)!}$$

$$C_x^n = \frac{n!}{x!(n-x)!}$$

Pick 4 elements (digits at a time)

E.g.,

- $A = 1234$
- $B = 2341$

How many permutations can be made?

How many combinations can be made?

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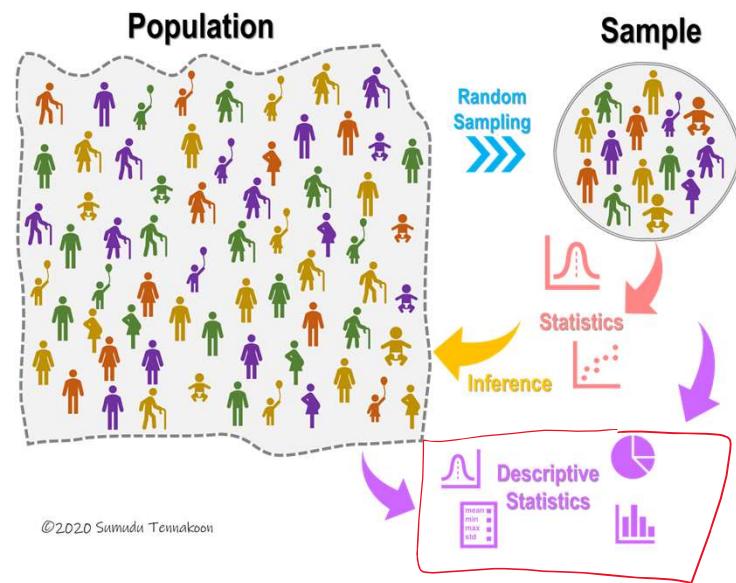
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Statistics



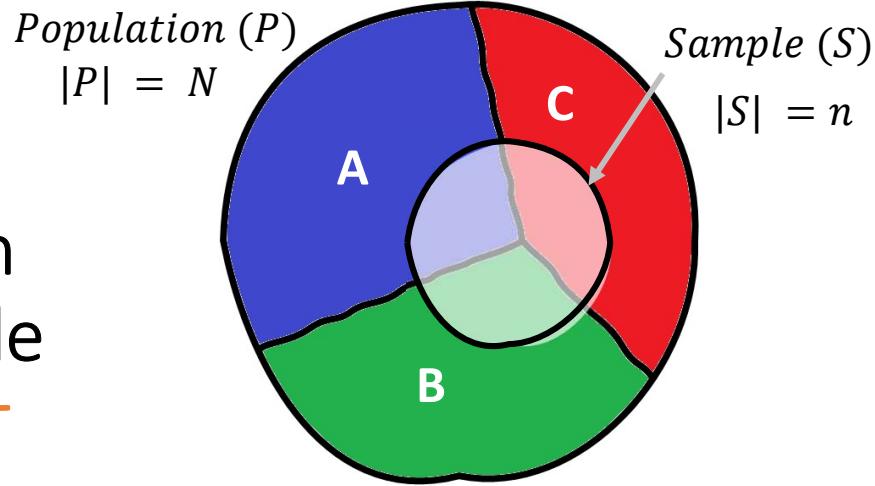
123

Sampling



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Population and Sample



- $P = A \cup B \cup C$
- $S \subset P$
- $S = (A \cap S) \cup (B \cap S) \cup (C \cap S)$
- $|S| = n = |A \cap S| + |B \cap S| + |C \cap S| = a + b + c$

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Sampling Method depends on

- Nature of population
- Regulatory Restrictions
- Resources available
 - Time
 - Money
 - Data Collectors/Probes

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Sampling Considerations

Unbiased: each element is equally likely to be chosen

Representative of the Population

Minimize the **Sampling Error**

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Probabilistic Sampling Techniques

Simple Random Sampling

Stratified Sampling

Systematic sampling

Cluster Sampling

...

[https://en.wikipedia.org/wiki/Sampling_\(statistics\)](https://en.wikipedia.org/wiki/Sampling_(statistics))

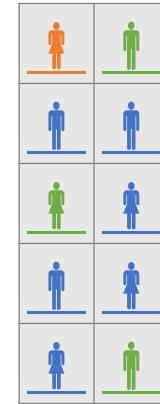
https://en.wikipedia.org/wiki/Stratified_sampling

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Simple Random Sample

	A	B	C	D	E	F	G	H	I	J
1	Blue	Blue	Blue	Blue	Green	Green ✓	Orange	Blue ✓	Blue	Blue
2	Orange ✓	Blue	Blue	Blue ✓	Blue	Blue	Blue	Orange	Orange	Green
3	Orange	Green ✓	Green	Green	Green	Blue	Blue	Green	Green	Green
4	Green	Blue	Blue	Green	Green	Blue	Blue ✓	Orange	Orange	Blue
5	Orange	Blue	Blue	Blue ✓	Orange	Blue ✓	Green	Green	Blue ✓	Blue



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Stratified Sampling

	A	B	C	D	E	F	G	H	I	J
1	Blue	Blue	Blue	Blue	Blue	Green	Green	Orange	Orange	Orange
2	Blue	Blue	Blue	Blue	Blue	Green	Green	Orange	Orange	Orange
3	Blue	Blue	Blue	Blue	Blue	Green	Green	Orange	Orange	Orange
4	Blue	Blue	Blue	Blue	Blue	Green	Green	Orange	Orange	Orange
5	Blue	Blue	Blue	Blue	Blue	Green	Green	Orange	Orange	Orange



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Cluster Sampling

	A	B	C	D	E	F	G	H	I	J
1	Blue	Blue	Blue	Blue	Green	Green	Orange	Blue	Blue	Blue
2	Orange	Blue	Blue	Blue	Blue	Blue	Blue	Orange	Orange	Green
3	Orange	Green	Green	Green	Green	Blue	Blue	Green	Green	Green
4	Green	Blue	Blue	Green	Blue	Blue	Blue	Orange	Orange	Blue
5	Orange	Blue	Blue	Blue	Blue	Blue	Blue	Green	Blue	Blue

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B	E
Blue	Green
Blue	Blue
Green	Green
Blue	Green
Blue	Orange

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Systematic sampling

	A	B	C	D	E	F	G	H	I	J
1	Blue	Blue	Blue	Blue	Green	Green	Orange	Blue	Blue	Blue
2	Orange	Blue	Blue	Blue	Blue	Blue	Blue	Orange	Orange	Green
3	Orange	Green	Green	Green	Green	Blue	Blue	Green	Green	Green
4	Green	Blue	Blue	Green	Green	Blue	Blue	Orange	Orange	Blue
5	Orange	Blue	Blue	Blue	Blue	Blue	Blue	Green	Blue	Blue

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Blue	Blue
Blue	Orange
Green	Green
Blue	Orange
Blue	Green

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Non- Probabilistic Sampling Techniques

Convenience Sampling

Judgmental Sampling

Quota sampling

Snowball Sampling

...

[https://en.wikipedia.org/wiki/Sampling_\(statistics\)](https://en.wikipedia.org/wiki/Sampling_(statistics))

https://en.wikipedia.org/wiki/Stratified_sampling

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Convenience Sampling

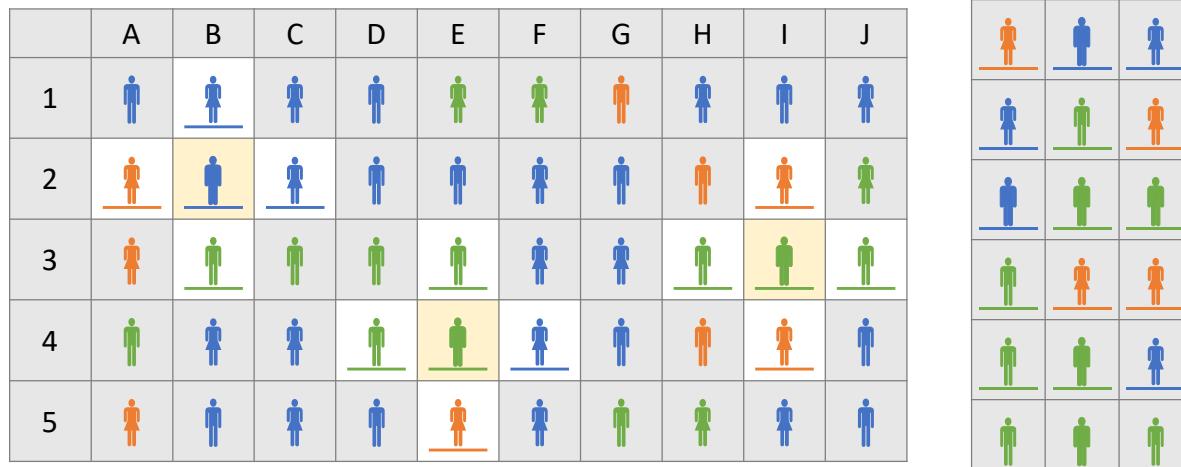
	A	B	C	D	E	F	G	H	I	J
1	Blue	Blue	Blue	Blue	Green	Green	Orange	Blue	Blue	Green
2	Orange	Blue	Blue	Blue	Blue	Blue	Blue	Blue	Blue	Green
3	Orange	Green	Green	Blue	Green	Blue	Blue	Green	Blue	Green
4	Green	Blue	Blue	Blue	Green	Blue	Blue	Orange	Orange	Blue
5	Orange	Blue	Blue	Blue	Orange	Blue	Green	Blue	Blue	Blue

Blue	Blue	Green
Blue	Blue	Green
Orange	Orange	Orange
Green	Green	Green
Orange	Orange	

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Snowball Sampling



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Types of Statistics

Descriptive Statistics: Organizing and summarizing data. Use data from the population or sample.

Mean, Median, Mode, Range, Variance

Inferential Statistics: Use the sample data to make an inference or draw the population's conclusion. It uses probability to find out the confidence of the predictions we make.

Distributions, Regression, ...

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Types of Analytics

Descriptive: What do we have now/ had

Diagnostic: Why it happened (cause?)

Predictive: What can happen if the conditions are set to

Prescriptive: Define rules/ constraints

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Descriptive Statistics

Range

Mean

Median

Mode

Variance

Standard Deviation

Coefficient of Variation

Co-variance

Co-relation Coefficient

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Range, Mean, Median and Mode

- Range
 - Max Value - Min value
- Mean
 - Population Mean: $\mu = \frac{\sum_{i=1}^n x_i}{N}$
 - Sample Mean: $\bar{x} = \frac{\sum_{i=1}^n x_i}{n}$
 - Has influence of outliers
- Weighted Mean (w_i s are weights)
 - $\bar{x}_w = \frac{\sum_{i=1}^n w_i x_i}{\sum_{i=1}^n w_i}$
- Geometric Mean:
 - $\bar{x}_g = \sqrt[n]{\prod_{i=1}^n x_i}$
- Median:
 - Mid value when ordered
 - Removes influence of outliers
- Mode
 - Value that occur most often

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Variance , Standard Deviation and Coefficient of variance

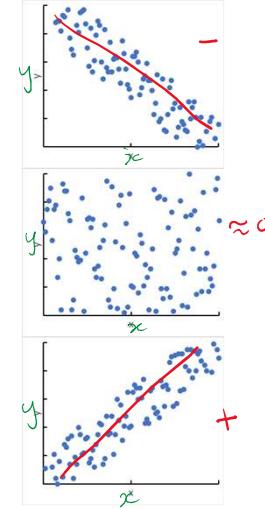
- Variance
 - How data spread around mean
- Population : $\sigma^2 = \frac{\sum_{i=1}^n (x_i - \mu)^2}{N}$
- Sample : $S^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1}$
- Standard Deviation (STD)
 - Population : $\sigma = \sqrt{\frac{\sum_{i=1}^n (x_i - \mu)^2}{N}}$
 - Sample : $S = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1}}$
- Coefficient of variance (CV)
 - Also known as relative standard deviation (RSD)
 - Population : $CV = \frac{\sigma}{\mu}$
 - Sample : $CV = \frac{S}{\bar{x}}$

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Co-variance and Co-relation Coefficient

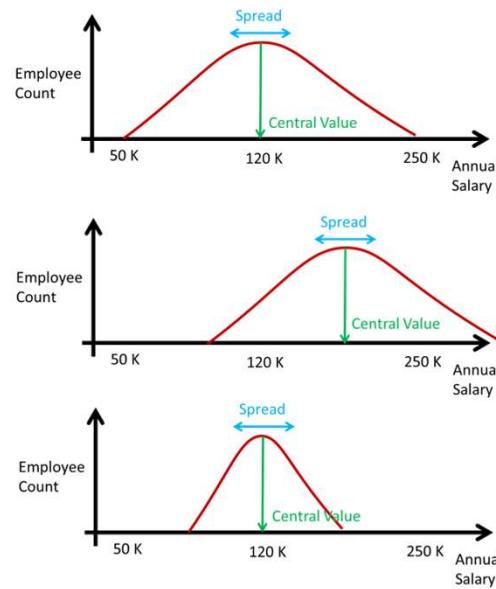
- Co-variance:
 - Check whether 2 variables moving together
 - $\text{COV}(x,y) = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{n-1}$
 - $\text{COV} \begin{cases} < 0, & \text{if variables moving opposite direction} \\ = 0, & \text{if independent variables} \\ > 0, & \text{if variables moving together} \end{cases}$
- Co-relation Coefficient
 - $r = \frac{\text{COV}(x,y)}{S(x) \cdot S(y)}$
 - $-1 \leq r \leq 1$
 - $r = 1$ and $r = -1$: perfect corelation
 - $r \begin{cases} < 0, & \text{if negative corelation} \\ = 0, & \text{if independent variables} \\ > 0, & \text{if positive corelation} \end{cases}$



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Distributions Central value and Spread

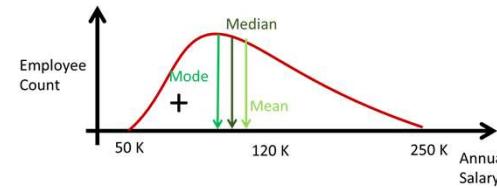
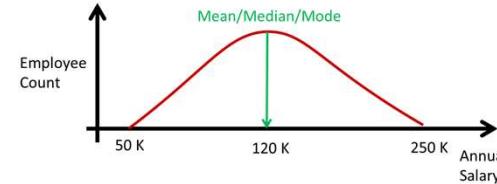
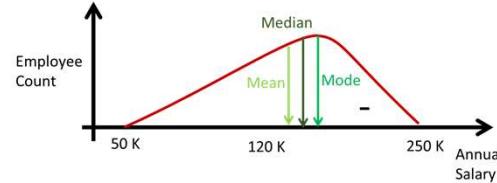


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Distributions

Skewness



<https://en.wikipedia.org/wiki/Skewness>

<https://docs.scipy.org/doc/scipy/reference/generated/scipy.stats.skew.html>

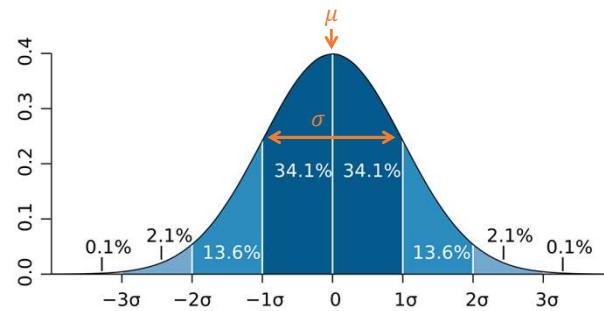
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<https://mathworld.wolfram.com/Pearson'sSkewnessCoefficients.html>

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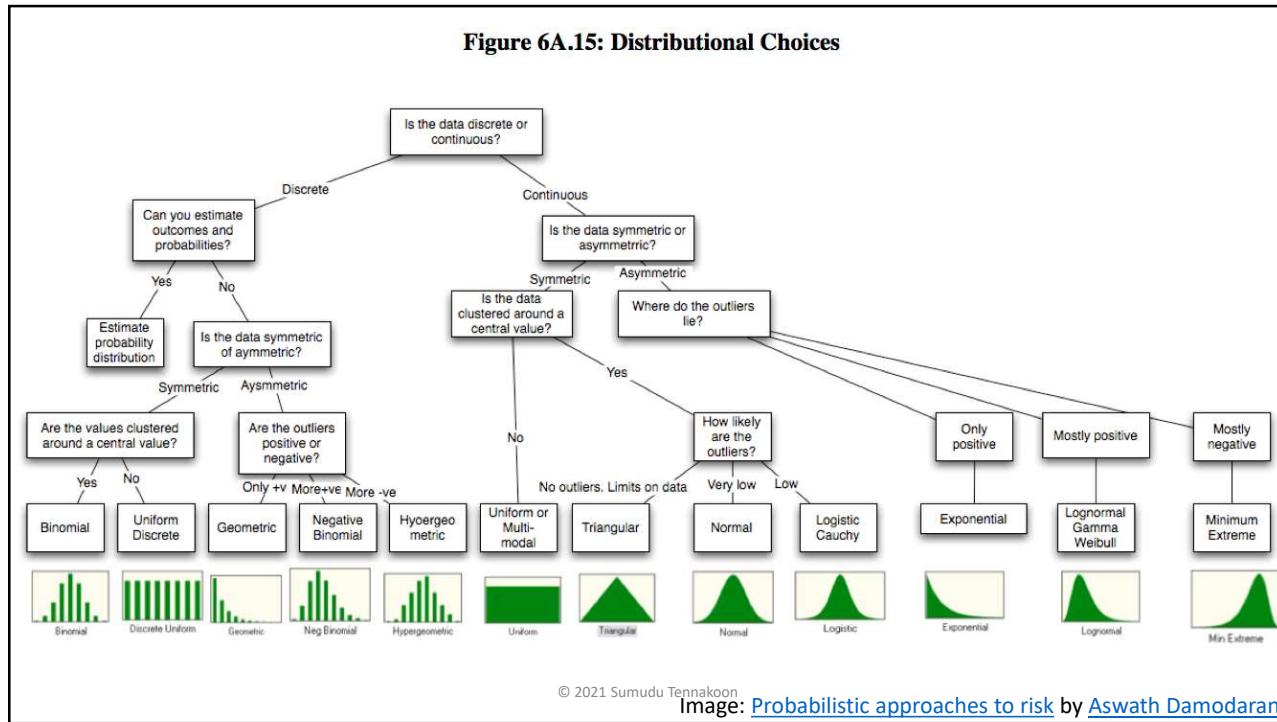
Standard Normal Distribution

- Standard Deviation (σ) = 1
- Mean (μ) = 0 = Median = Mode



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Exploratory Data Analysis and Feature Engineering

- Descriptive Statistics
- Univariate Analysis
- Multivariate Analysis



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Range, Mean, Median and Mode

- Range
 - Max Value - Min value
- Mean
 - Population Mean: $\mu = \frac{\sum_{i=1}^n x_i}{N}$
 - Sample Mean: $\bar{x} = \frac{\sum_{i=1}^n x_i}{n}$
 - Has influence of outliers
- Weighted Mean (w_i s are weights)
 - $\bar{x}_w = \frac{\sum_{i=1}^n w_i x_i}{\sum_{i=1}^n w_i}$
- Geometric Mean:
 - $\bar{x}_g = \sqrt[n]{\prod_{i=1}^n x_i}$
- Median:
 - Mid value when ordered
 - Removes influence of outliers
- Mod
 - Value that occur most often

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Variance , Standard Deviation and Coefficient of variance

- Variance
 - How data spread around mean
- Population : $\sigma^2 = \frac{\sum_{i=1}^n (x_i - \mu)^2}{N}$
- Sample : $S^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1}$
- Standard Deviation (STD)
 - Population : $\sigma = \sqrt{\frac{\sum_{i=1}^n (x_i - \mu)^2}{N}}$
 - Sample : $S = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1}}$
- Coefficient of variance (CV)
 - Also known as relative standard deviation (RSD)
 - Population : $CV = \frac{\sigma}{\mu}$
 - Sample : $CV = \frac{S}{\bar{x}}$

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Data Pre-Processing

Treat Missing Values

Treat Outliers

Scaling Variables

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Treat Missing Values

- Drop
 - Drop rows/columns with missing values ✗
- Impute
 - Impute (replacing with value)
 - Interpolate
 - Descriptive statistics: Median, Mean, Mode
 - Use machine learning model
 - Derive using other column (e.g., gender by title, name)
 - Replace with a dummy value (e.g., NA, N/A, null, -9999)
- Revisit
 - Revisit data collection
- Request
 - Request missing information from data provider

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Treating Outliers

- Detecting Outliers
 - Box Plot
 - Percentiles
- Remove Outliers
 - Drop rows/columns
 - Replace outlier values
 - Set cutoff value/clip (piecewise function)
 - Convert to ranges

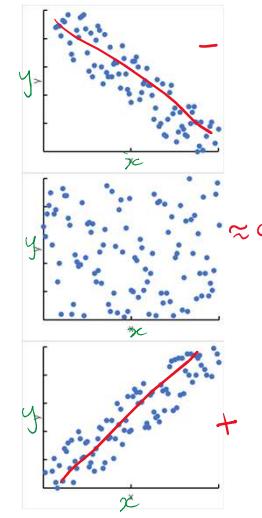
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Co-variance and Co-relation Coefficient

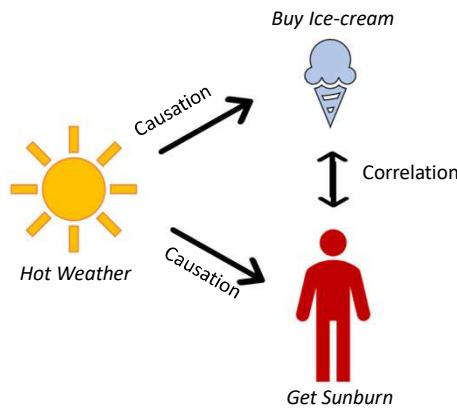
- Co-variance:
 - Check whether 2 variables moving together
 - $\text{COV}(x,y) = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{n-1}$
 - $\begin{cases} < 0, & \text{if variables moving opposite direction} \\ = 0, & \text{if independent variables} \\ > 0, & \text{if variables moving together} \end{cases}$
- Co-relation Coefficient
 - $r = \frac{\text{COV}(x,y)}{S(x) \cdot S(y)}$
 - $-1 \leq r \leq 1$
 - $r = 1$ and $r = -1$: perfect correlation
 - $\begin{cases} < 0, & \text{if negative correlation} \\ = 0, & \text{if independent variables} \\ > 0, & \text{if positive correlation} \end{cases}$

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Correlation & Causation



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Univariate Analysis

- Single Variable
- Techniques
 - Descriptive Statistics
 - Frequency Table
 - Count Plot/Bar plot (categorical)
 - Histogram (numerical)
 - Box Plot

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Bivariate Analysis

- Two variables
- Techniques
 - Cross tables (two-way tables)
 - Scatter Plot
 - Correlation coefficient
 - Stacked plot
 - Heatmaps
 - Pair Plot
 - Marginal Probability/Conditional Probability/Joint Probability
 - Regression

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Preparing Dataset for Machine Learning

- Dimensionality Reduction
- Feature Selection
 - Select best features based on highest predictive power
- Feature Engineering
 - Derive new features from existing features
 - Transform features
 - Combine multiple features (create interactions)

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Dimensionality Reduction

- Transform data from higher dimension space to lower dimension space (e.g., 4D to 3D, 3D to 2D)
- Retains characteristics of original data.
- Useful for
 - visualization purpose
 - Observe patterns more clearly
- Methods
 - Principle Component Analysis (PCA)
 - Linear discriminant analysis (LDA)
 - Kernel PCA
 - Autoencoder

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Feature Scaling

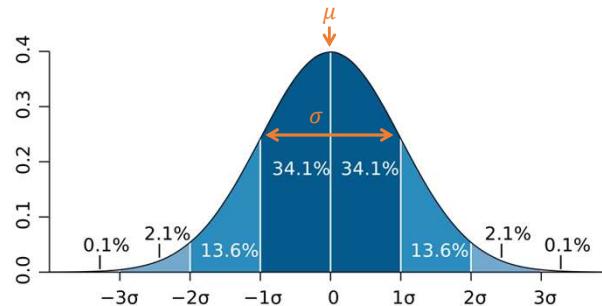
- Normalization (value between 0 and 1)
 - Min-max
 - Mean
- Standardization (scale based on standard normal distribution)
 - Z-score
- Other Scaling Techniques
 - Log transform
 - Power transform
- Not necessary in Tree based models

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Standard Normal Distribution

- Standard Deviation (σ) = 1
- Mean (μ) = 0 = Median = Mode



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Encoding

- Numerical Variables -> Categorical Variables
- Categorical Variables
 - One hot encoding (1/0, create dummy variables)
 - Mean Encoding (mean of each category)
 - Label Encoding (ordinal)
 - Target guided ordinal encoding (rank of mean)

ID#	Grade	A	B	C	D	N/A
1	A	1	0	0	0	0
2	B	0	1	0	0	0
3	C	0	0	1	0	0
4	D	0	0	0	0	0
5		0	0	0	0	1

ID#	Gender	Male	Female	Is_male?	Gender_ME	Gender_TGOE
1	M	1	0	1	0.48	2
2	F	0	1	0	0.52	1
3	F	0	1	0	0.52	1
4	M	1	0	1	0.48	2
5	M	1	0	1	0.48	1

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Feature Selection

- Based on Missing Values
 - Remove features with high % of missing values (define a threshold)
 - Create new feature to indicate missingness (binary column)
- Based on Variance
 - Remove features with zero variation
 - Remove features with low variation (define a threshold)
- Based on correlation
 - Analyze correlation between features (x variables) and those with the target (y) variable
 - Keep from a set of highly correlated variables
 - Keep one having highest correlation coefficient with the target (y) variable.
 - Drop features with low correlation with the target (y)
 - This could miss a useful feature, therefore use with caution

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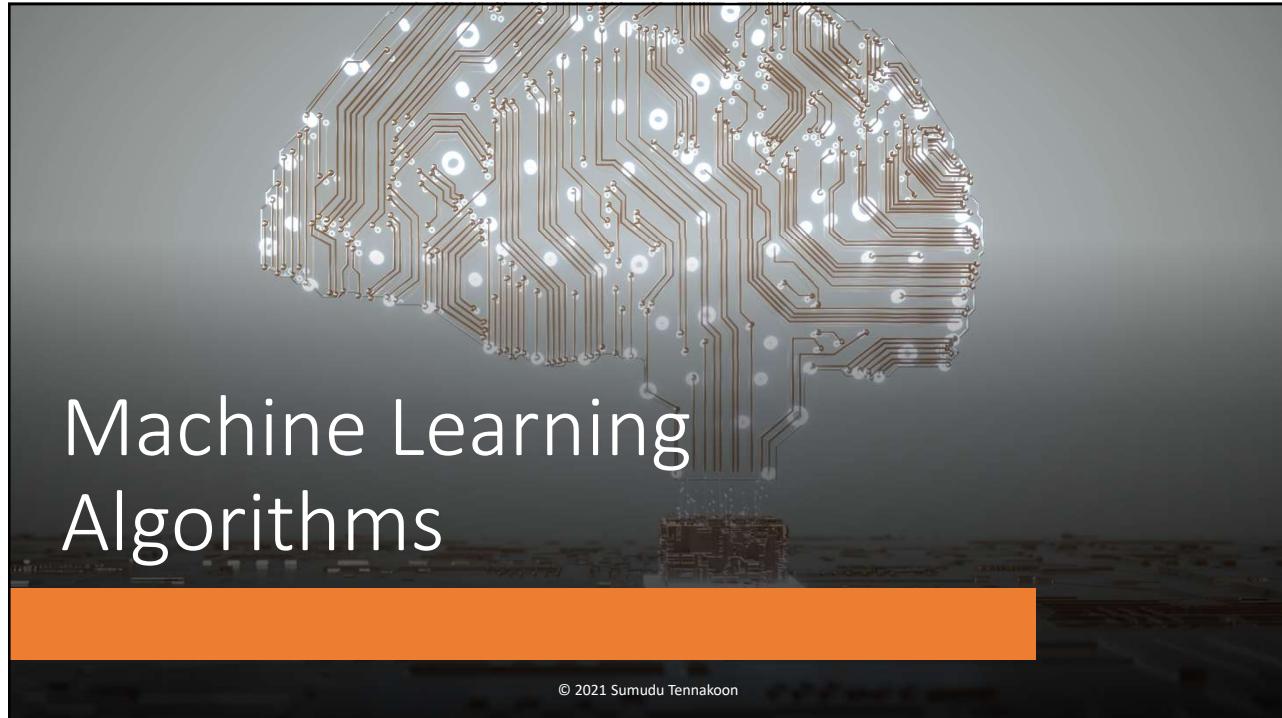
161

Feature Selection

- Forward Selection
 - Start with best feature (feature with most predictive power) to build model
 - Evaluate the model performance
 - Add next feature, build model and evaluate
 - If the model performance increases, keep feature if not, drop
- Backward Elimination
 - Start with all features, build model, evaluate
 - Drop least useful feature in each iteration

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Choosing Correct the Problem and Solution Approach



Features (X)

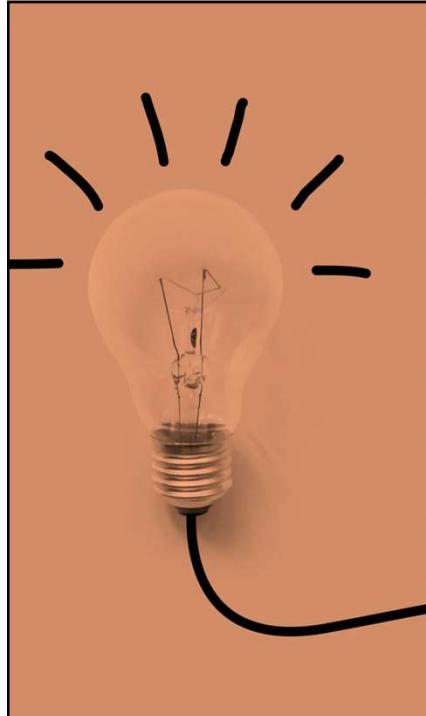
#	Day of The Week	Is Holiday?	Temperature (C)	Rainfall (mm)	Ice-creams Sold
1	Sun	0	23.5	20	52
2	Mon	0	25.6	0	120
...
50	Fri	1	29.8	10	155
...
728	Tue	1	16.0	0	45

Requirement?

- **A:** Predict How many ice-creams sold in a given day (predict numeric value)
- **B:** Predict whether or not > 100 ice-creams sold in a given day (predict binary value)
- **C:** Predict the range of ice-creams sold in a given day (predict class)

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Supervised and Unsupervised Machine Learning

Supervised learning

- $y = f(x_1, x_2, \dots, x_n)$
- x_1, x_2, \dots, x_n are independent variables used to analyze the dependent variable (y) and the relation between them.

Unsupervised Learning

- No dependent variable.
- Starts with a collection of variables (x_1, x_2, \dots, x_n) to find out similarity between them and classify them into clusters.

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Supervised Machine Learning

Regression

- Linear Regression
- Polynomial Regression
- Random Forest Regressor

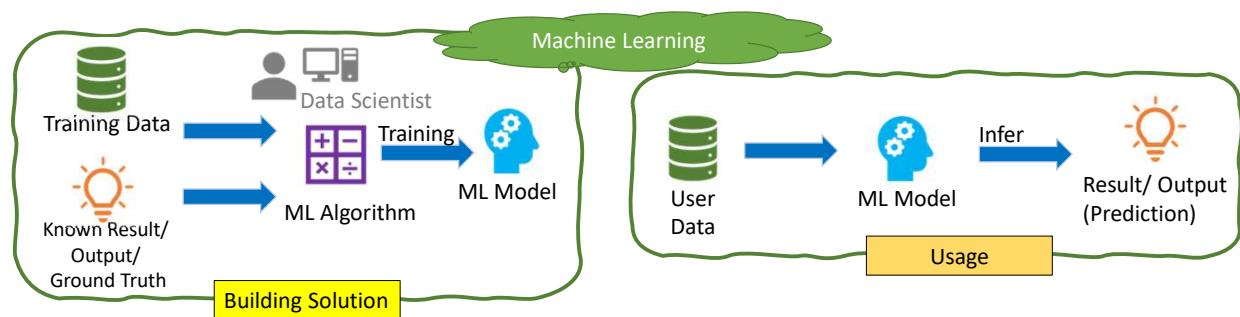
Classification

- Logistic Regression
- Decision Trees
- Random Forest Classifier
- Support Vector Machine (SVM)
- K-Nearest Neighbors (KNN)

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Supervised Machine Learning



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Types of ML Algorithms

Supervised



Regression

Linear
Polynomial



Classification

Tree Classifiers
Logistic Regression
Support Vector Machines (SVM)

Unsupervised



Clustering

K-Means
DBSCAN

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Regression

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Regression Equations

- Linear Regression

$$y = a_0 + a_1x_1 + \cdots + a_nx_n$$

- Logistic Regression (for binary classification)

$$y = \frac{1}{1 + e^{-z}} = \frac{1}{1 + e^{-(a_0 + a_1x_1 + \cdots + a_nx_n)}}$$

$$z = a_0 + a_1x_1 + \cdots + a_nx_n$$

- Polynomial Regression

$$y = a_0 + a_1x + a_2x^2 + \cdots + a_nx^n$$

$$x_1 = x$$

$$x_2 = x^2$$

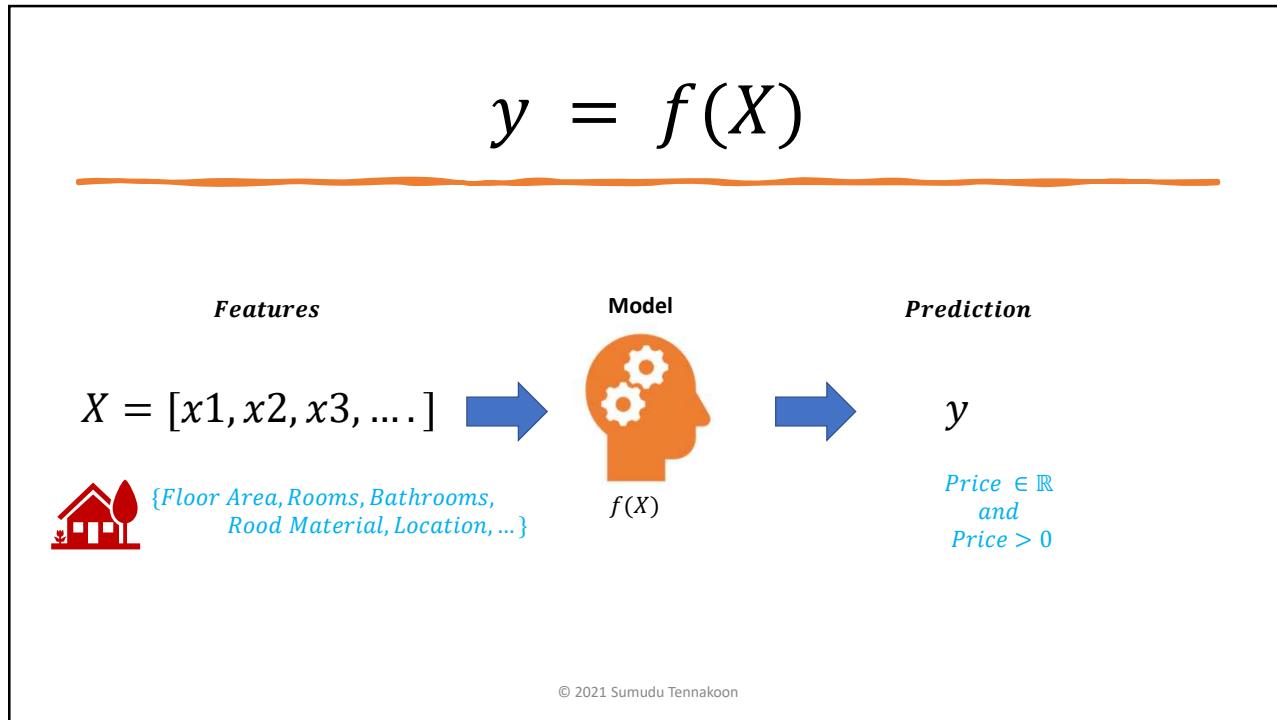
$$\vdots$$

$$x_n = x^n$$

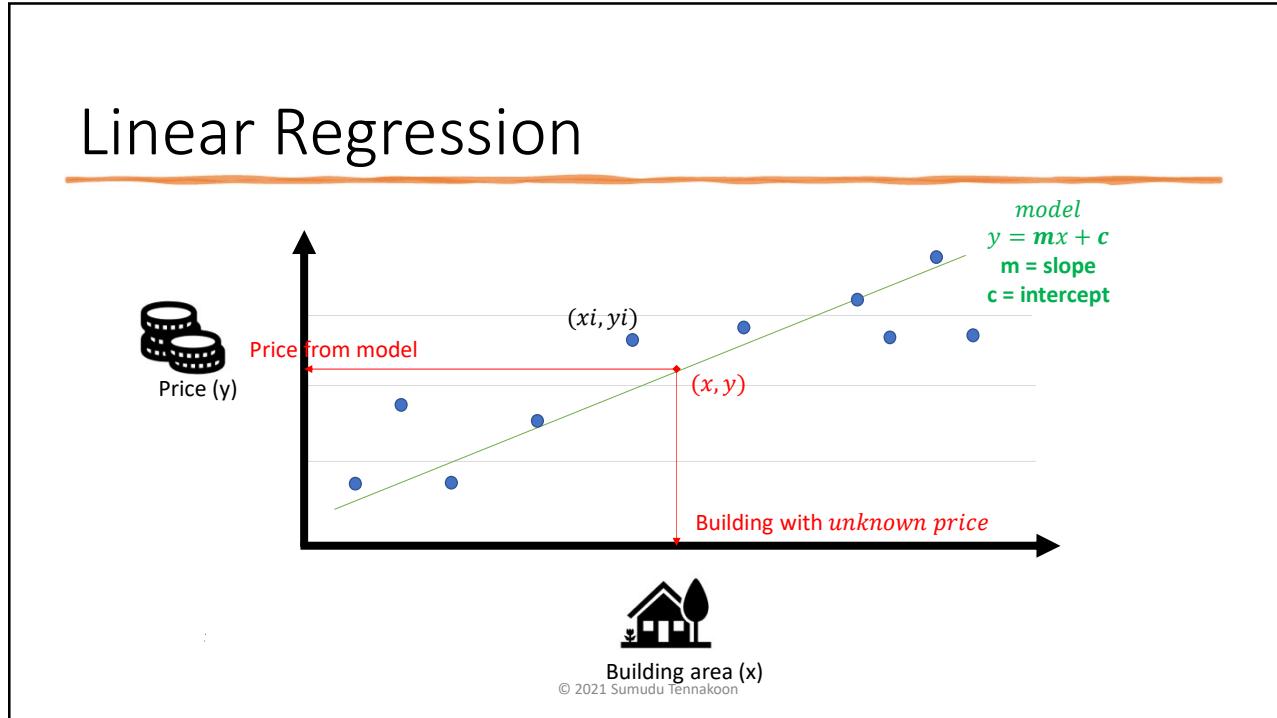
$$y = a_0 + a_1x_1 + \cdots + a_nx_n$$

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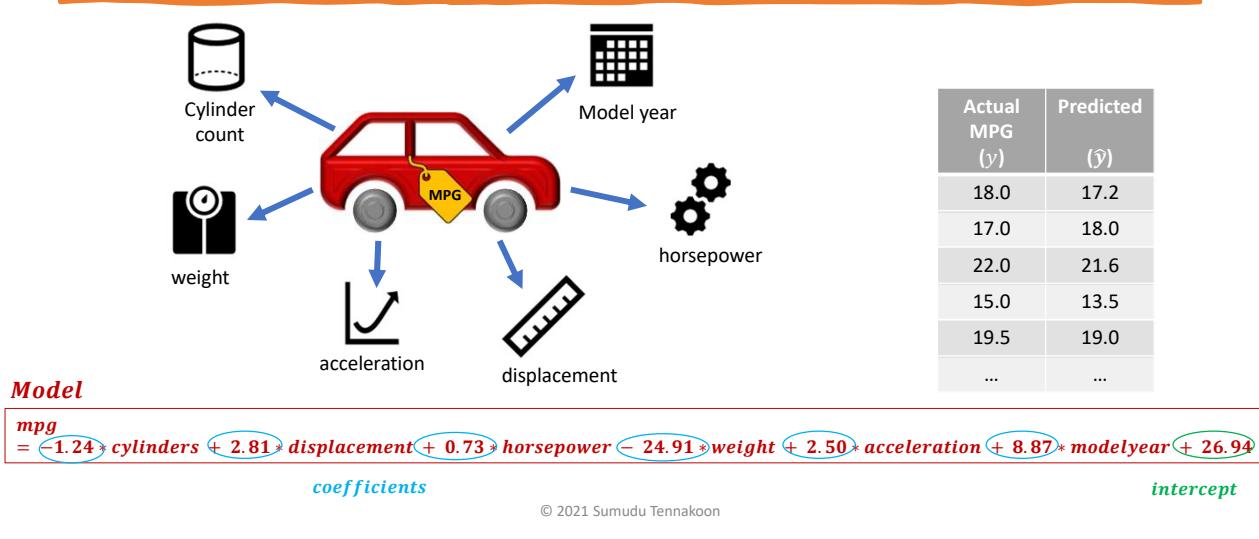


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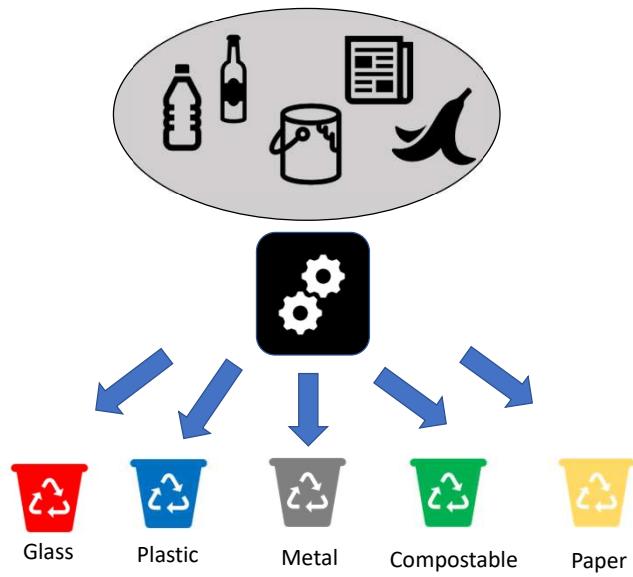
Linear Regression



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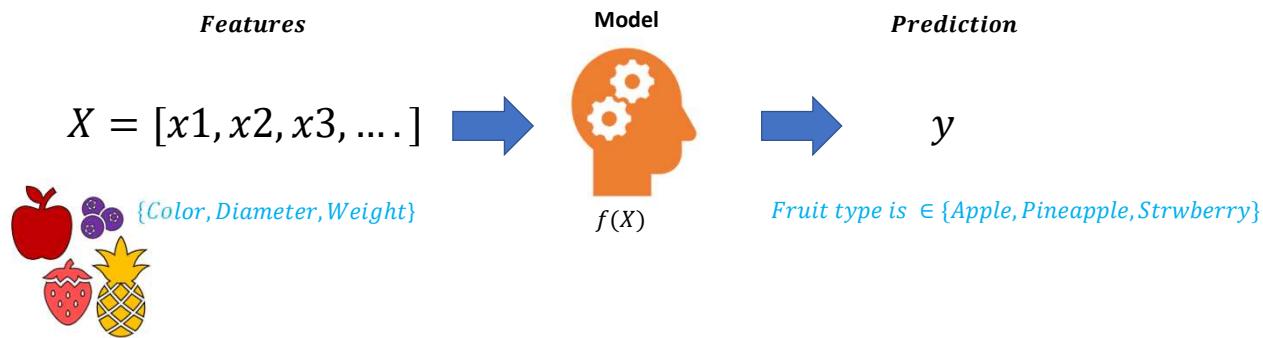
Classification

Classification



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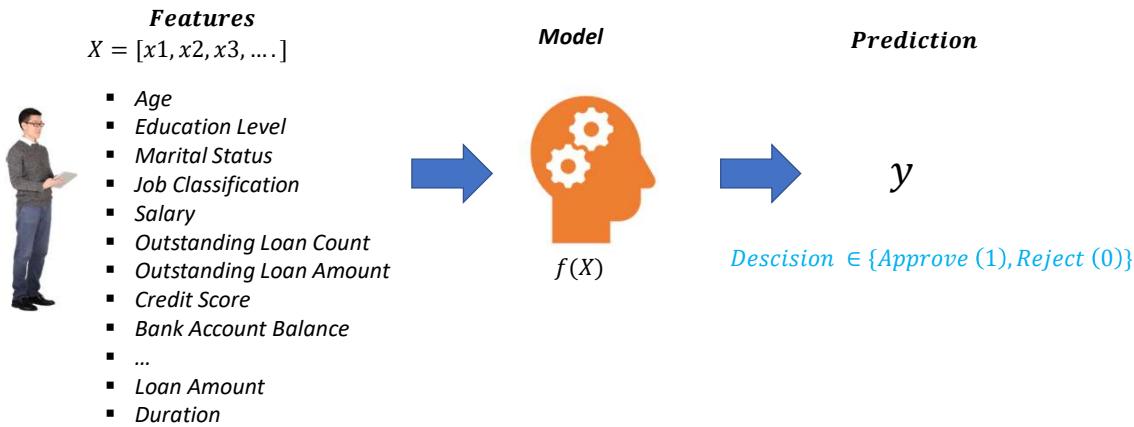
Multi-class Classification Example: Predicting Fruit Class



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Binary Classification Example: Loan Application



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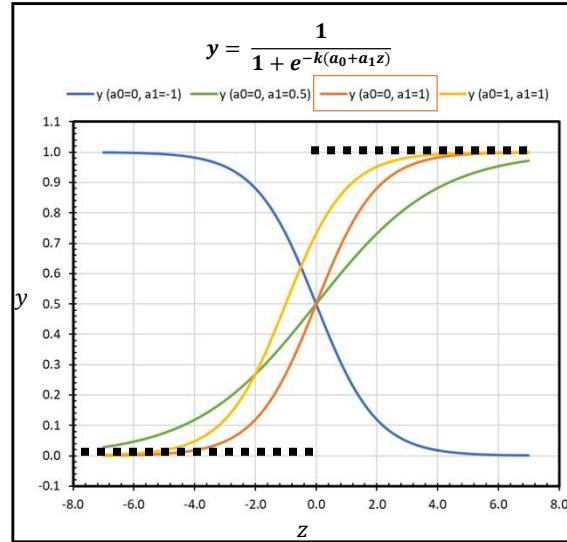
Logistic Regression

$$z = a_0 + a_1 x_1 + \dots + a_n x_n$$

$$y = \frac{1}{1 + e^{-(a_0 + a_1 x_1 + \dots + a_n x_n)}}$$

$$0.0 \leq y \leq 1.0$$

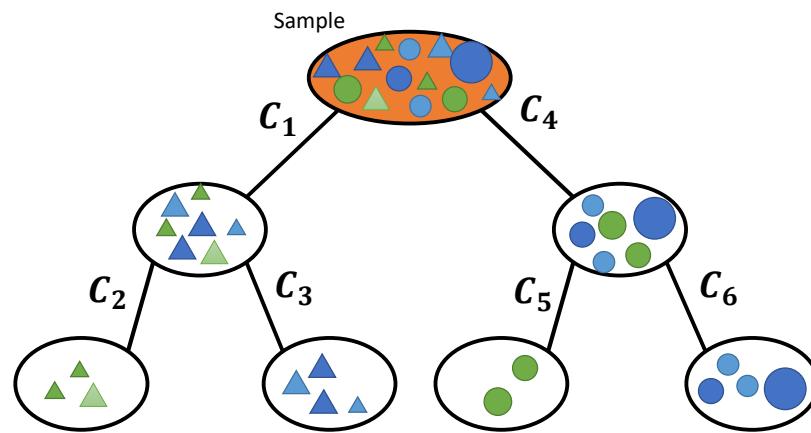
- $y = \text{range}/\text{output}$
- $e = \text{the natural logarithm base}$
- $a_0 = \text{intercept}$
- $a_1, a_2, \dots, a_n = \text{coefficients}$



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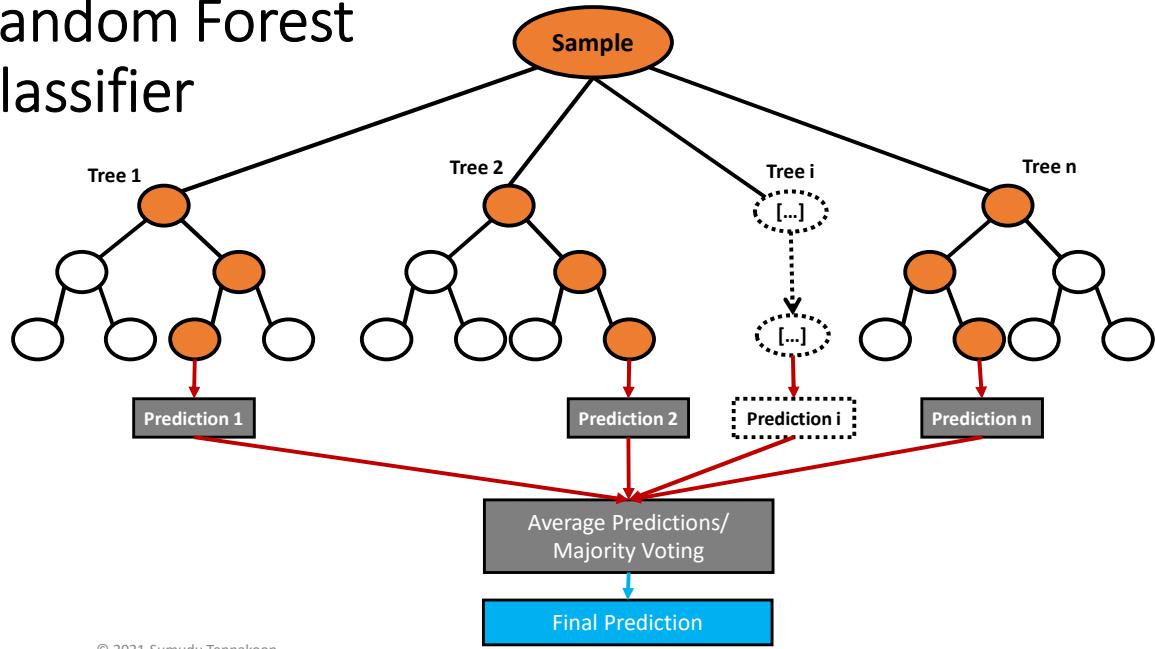
Decision Tree Classifier



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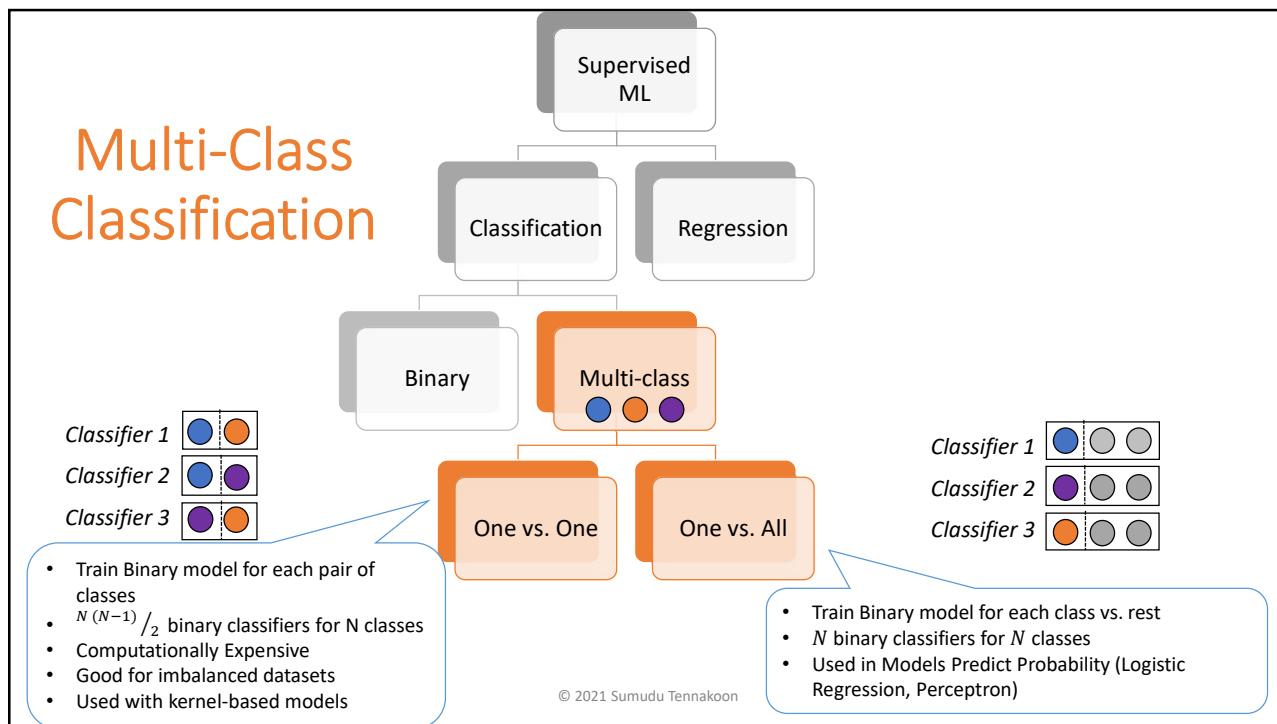
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Random Forest Classifier



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Choosing Correct the Problem and Solution Approach

The diagram illustrates the relationship between Features (X), Target Variables, and different types of classifiers.

#	Day of The Week	Is Holiday?	Temperature (C)	Rainfall (mm)	Ice-creams Sold	> 100 Ice-creams Sold	Ice-creams Sold Range
1	Sun	0	23.5	20	52	No	50-100
2	Mon	0	25.6	0	120	Yes	100-150
...
50	Fri	1	29.8	10	155	Yes	150-200
...
728	Tue	1	16.0	0	45	No	0-50

Labels above the table:

- Features (X)
- Regression Target (y)
- Binary Classifier Target (y)
- Multi-class Classifier Target (y)

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Unsupervised Machine Learning

Clustering

- K-Means
- DBSCAN

Dimensionality Reduction

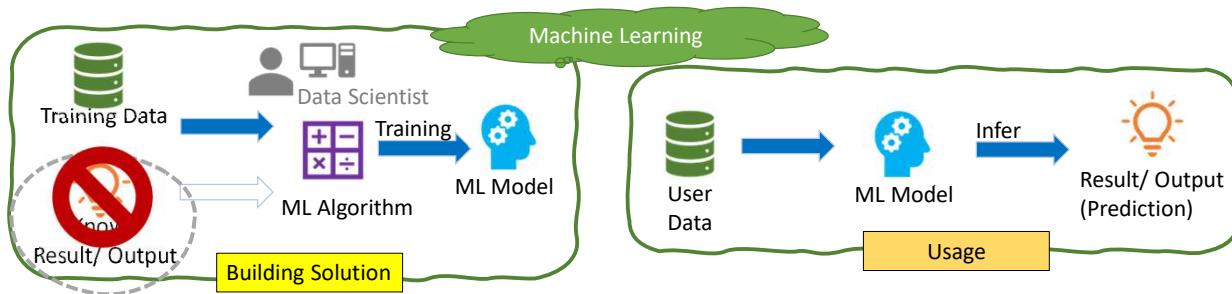
- PCA
- Autoencoder

Generative adversarial networks (GANs)

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Unsupervised Machine Learning



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Clustering

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Unsupervised learning

Group data points based on their similarities

Considerations

- Representation of a cluster with more than one data points.
- How we define the qualification for a datapoint to be a member of a cluster (nearness)
- When to stop the data point assignments to cluster

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Applications of Clustering

Market and Customer Segmentation

Recommendation engines

Anomaly Detection / Outlier Detection

Medical Data Analysis (Imaging, Disease Control, etc.)

Data Labeling (Cluster and Label)

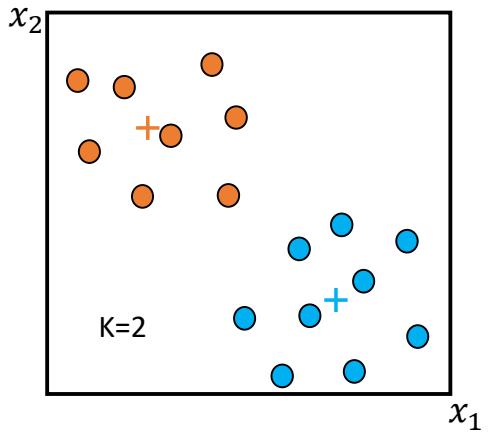
Understand the subgroups exists in data

- Number of groups
- Determine Group size
- Study group characteristics (aggregate values).
- Cohesion (togetherness) and separation
- Can those groups combine or further split?

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Centroid Based Clustering

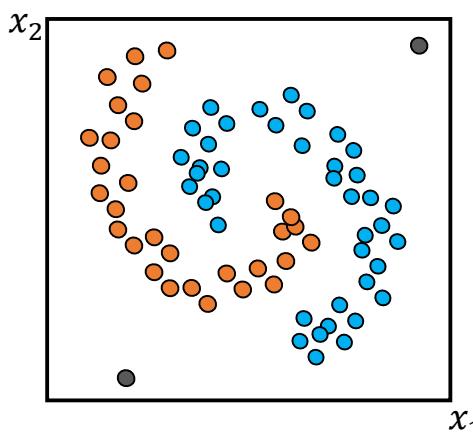


- Step 1: Place K centroids in random locations
- Step 2: Calculate the **distance** between each point to the centroid centers.
- Step 3: Find the nearest centroid to each data point using the get *min(distance)* and assign point to that centroid.
- Step 4: Calculate new center (mean of all assigned points) for the centroid based on the points.
- Loop: Repeat the Steps 2 -4 until converge (no points change cluster assignments)

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Density Based Clustering

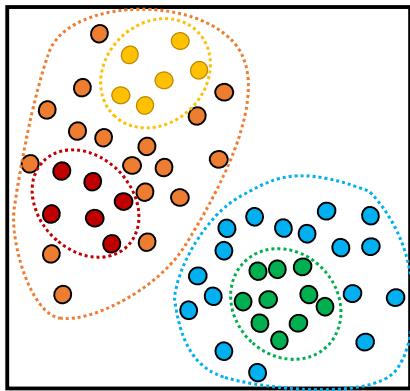


- Step 1: Select a random point previously unlabeled.
- Step 2: Count the number of other points (n) enclosed (density) in the defined region.
- Step 3:
 - If $n = 0$ where the current point is the only point in the region, mark the point as noise or outlier.
 - If $n < N_{\min}$
 - If no core point is included in the region, mark the point as noise or outlier.
 - If at least one other core point is included in the region, mark it as a border point.
 - If $n \geq N_{\min}$ mark the point as core point and initiate a cluster or assign the point to the current cluster.
- Step 4 (Loop):
 - If the point is a core point or border point, go to Step 2 covering all points in the cluster.
 - If the point is an outlier, go to Step 1
- Step 4 (Loop Break):
 - If all possible points covered in the current cluster (no more core points to be created), go to Step 1

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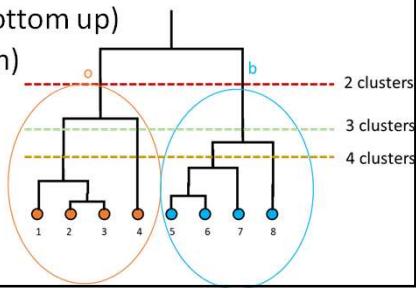
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Hierarchical Clustering



- Work well with data can be represented as heatmaps
- Dendrograms
- Methods
 - Agglomerative (bottom up)
 - Divisive (top-down)

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Explaining ML Models and Predictions

Explainable AI (XAI)

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Why do we need to Explain Models and their Predictions?

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Explaining ML Models and Predictions

- Global
 - Overall understanding of how the model makes a prediction.
 - Feature Importance
- Local
 - Understand how model makes a prediction for a given observation.
 - Positive and Negative contribution of x variables (features) to the prediction.
 - Scale of contribution could be very different from model feature Importance
 - What are the variables contributed the most.

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Explainable AI local methods

Local interpretable model-agnostic explanation (LIME)

Kernel Shapley additive explanations (KernelSHAP)

Integrated gradients (IG)

Explainable explanations through AI (XRAI)

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Explainability

Models can be easily explained

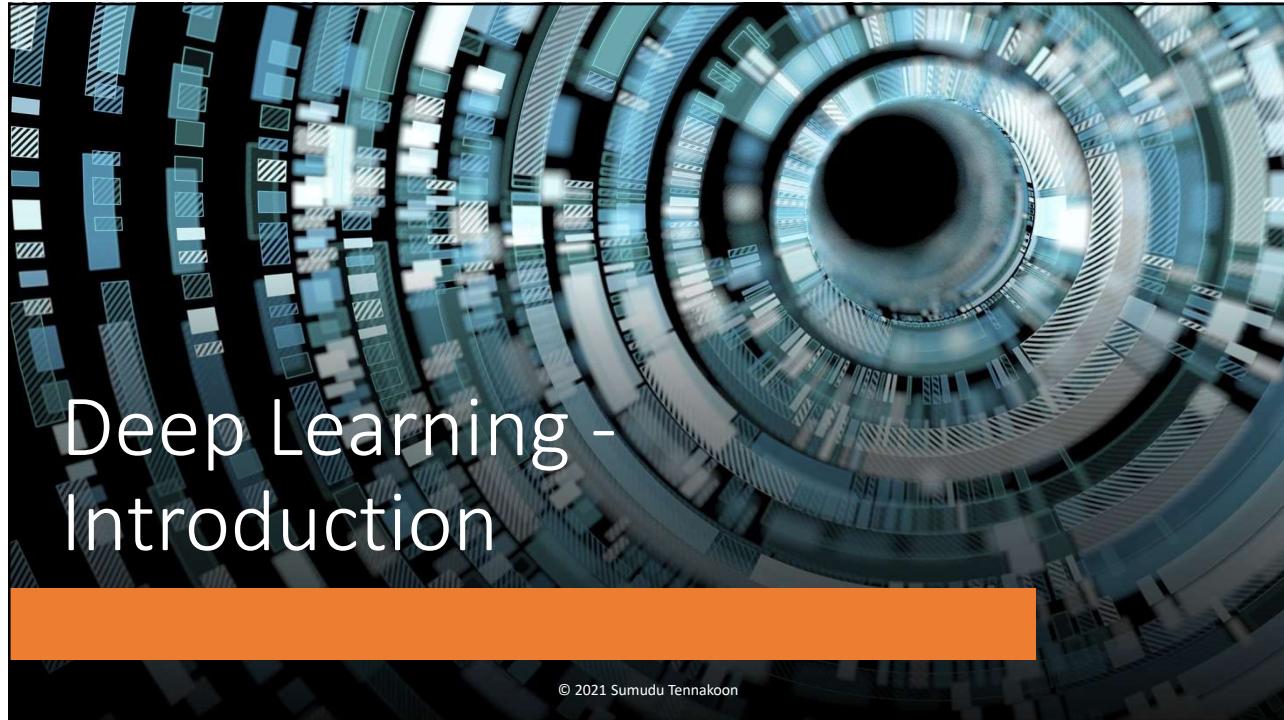
- Regression
- Decision Trees

Models cannot be easily explained (Blackbox Models)

- Ensemble Models (e.g., Random Forest)
- Neural Network (e.g., CNN, RNN)

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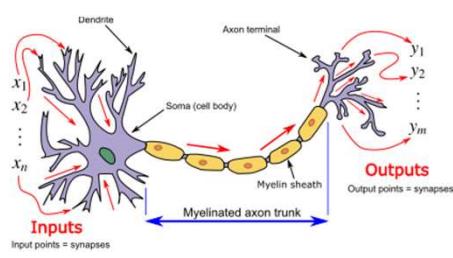
194



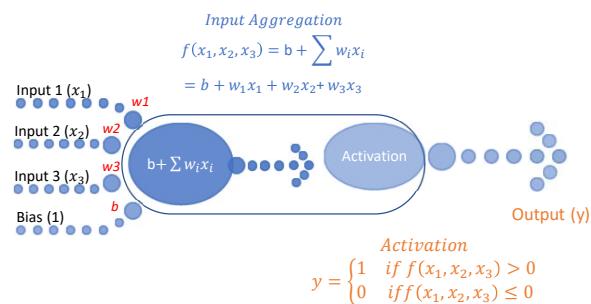
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Neuron and Perceptron

- We have approximately 86 billion neurons in our brain.



<https://en.wikipedia.org/wiki/Neuron>



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Activation functions

Input Aggregation

$$f(x_1, x_2, x_3) = b + \sum w_i x_i$$

$$= b + w_1 x_1 + w_2 x_2 + w_3 x_3$$

Activation

$$y = \begin{cases} 1 & \text{if } f(x_1, x_2, x_3) > 0 \\ 0 & \text{if } f(x_1, x_2, x_3) \leq 0 \end{cases}$$

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Sigmoid $\sigma(x) = \frac{1}{1+e^{-x}}$	Leaky ReLU $\max(0.1x, x)$
tanh $\tanh(x)$	Maxout $\max(w_1^T x + b_1, w_2^T x + b_2)$
ReLU $\max(0, x)$	ELU $\begin{cases} x & x \geq 0 \\ \alpha(e^x - 1) & x < 0 \end{cases}$

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Logistic Regression as Perceptron

Inputs

Summation

Activation

Output

$f(x_1, x_2, \dots, x_n) = \frac{1}{1 + e^{-(b + w_1 x_1 + \dots + w_n x_n)}}$

$f(x_1, x_2, \dots, x_n)$ $0 \leq y \leq 1$

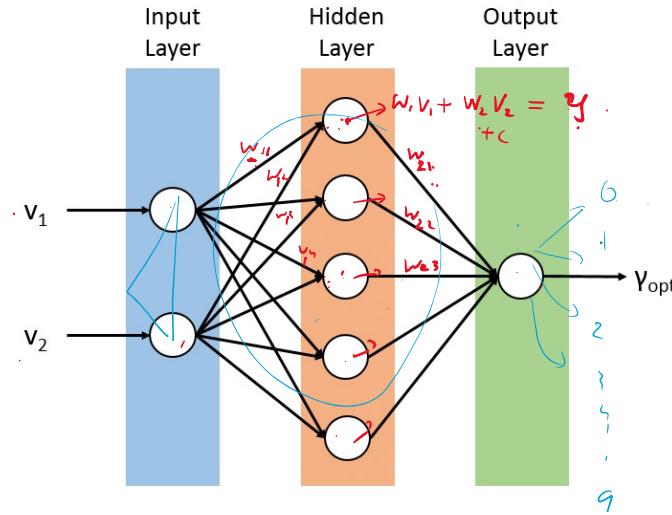
y

$z = a_0 + a_1 x_1 + \dots + a_n x_n$

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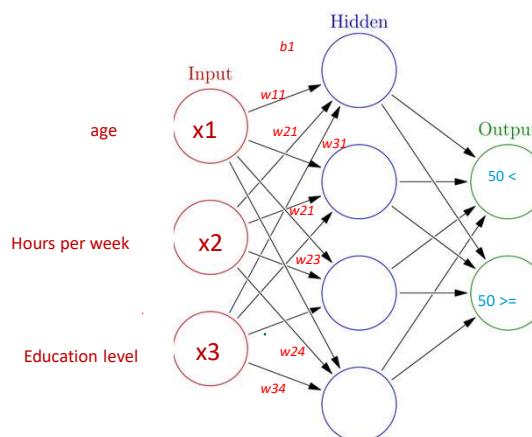
Neural Network



https://en.wikipedia.org/wiki/Artificial_neural_network

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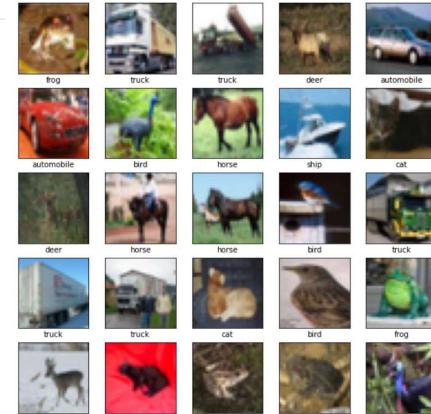
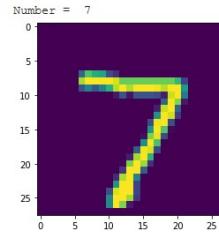
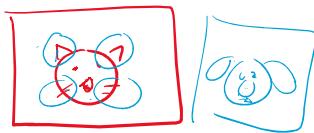


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Traditional ML vs. Deep Learning

- Feature Engineering Workload
- Model Training Recourses
- Amount of Training Data Required
- Solving Complex Problems

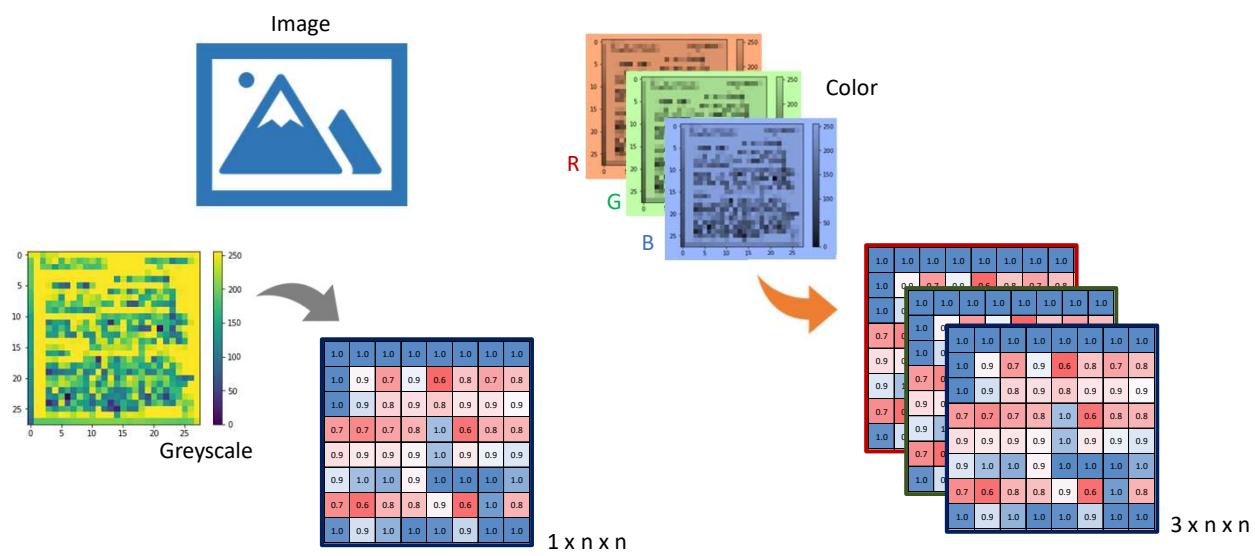


CIFAR10

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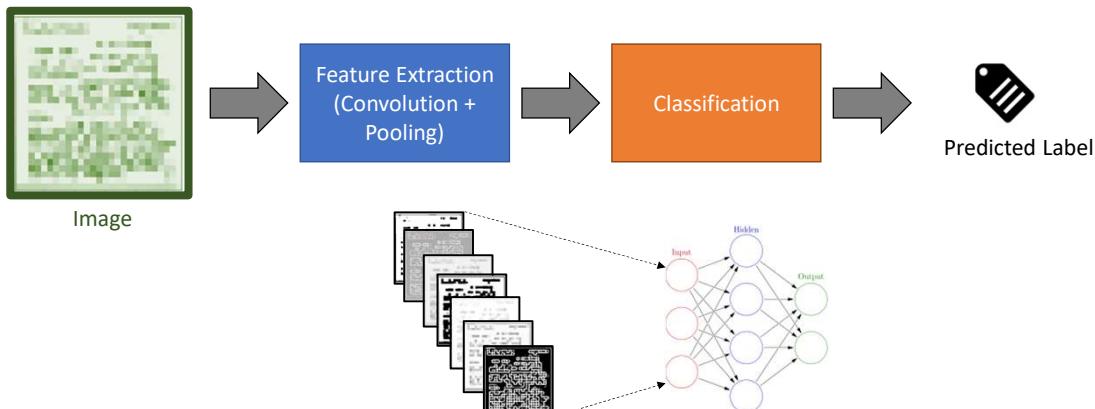
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Convolutional Neural Network



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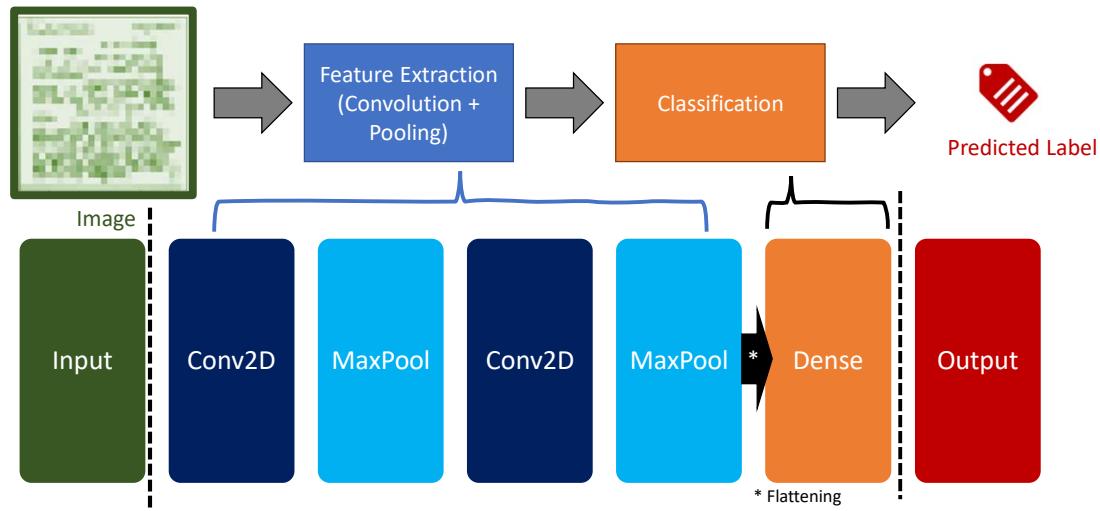
Convolutional Neural Network



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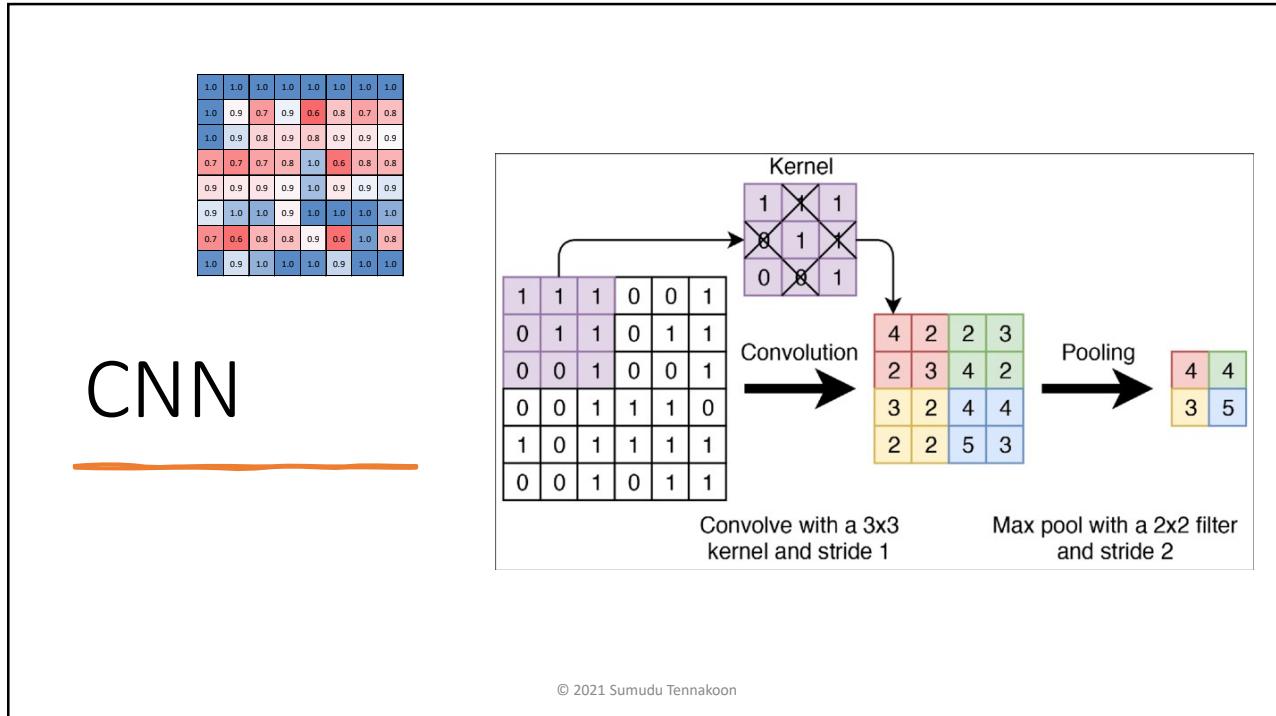
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Convolutional Neural Network

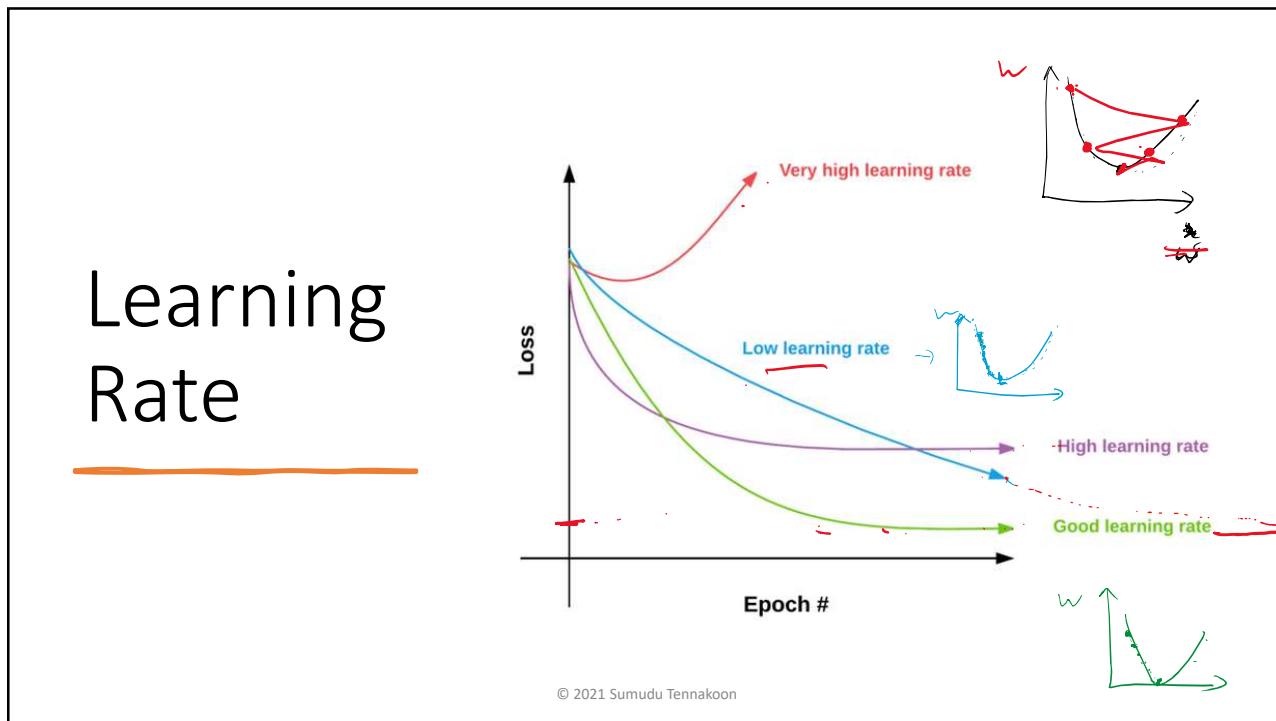


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Regularization

$$\hat{y} = w_1x_1 + w_2x_2 + \dots + w_Nx_N + b$$

$$Loss = Error(y, \hat{y})$$

Loss function with no regularisation

$$Loss = Error(y, \hat{y}) + \lambda \sum_{i=1}^N |w_i|$$

Loss function with L1 regularisation

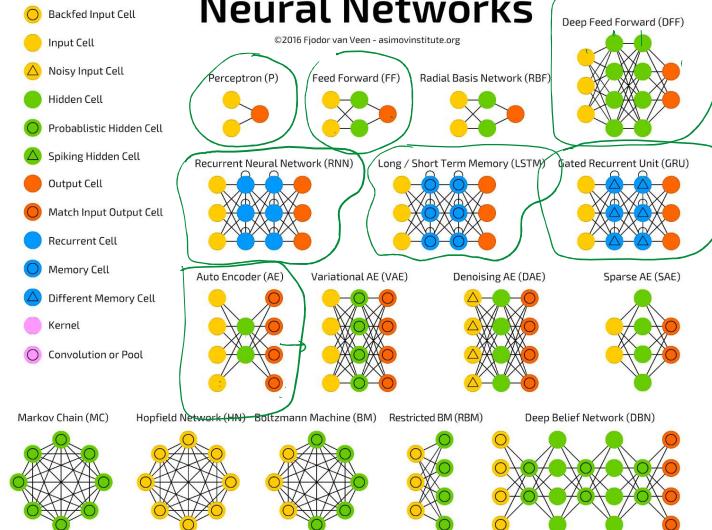
$$Loss = Error(y, \hat{y}) + \lambda \sum_{i=1}^N w_i^2$$

Loss function with L2 regularisation

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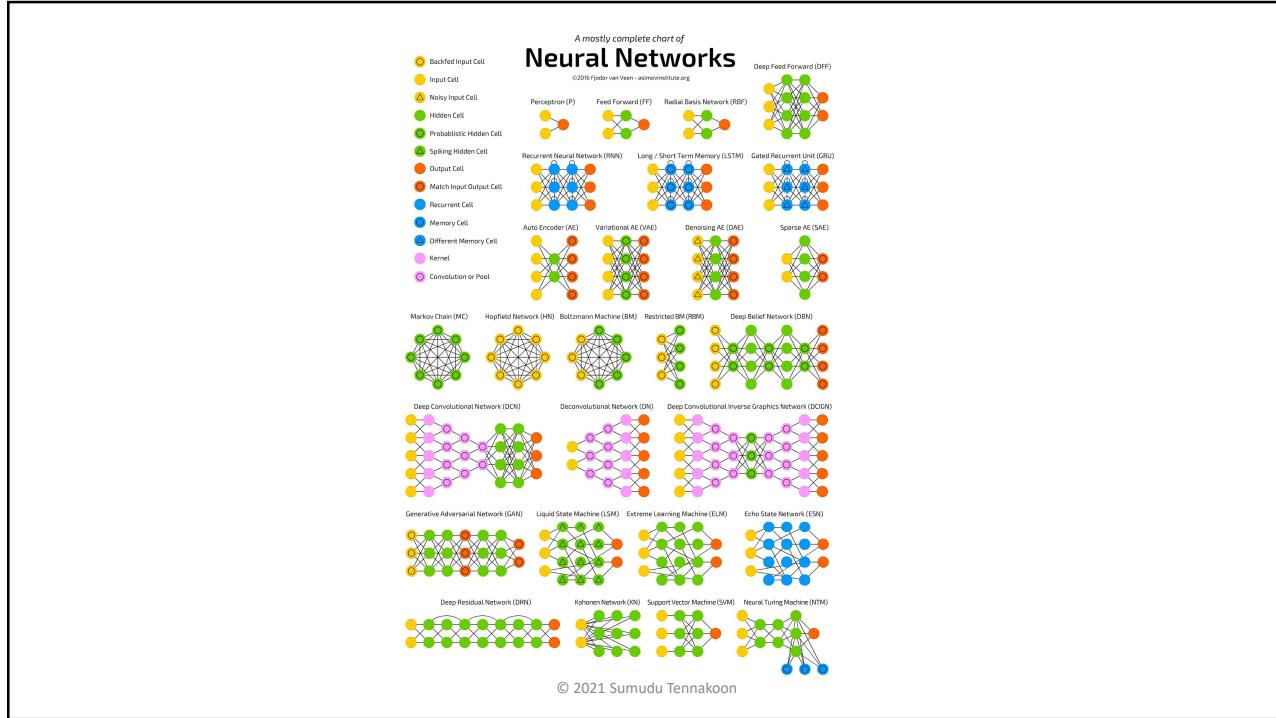
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A mostly complete chart of Neural Networks



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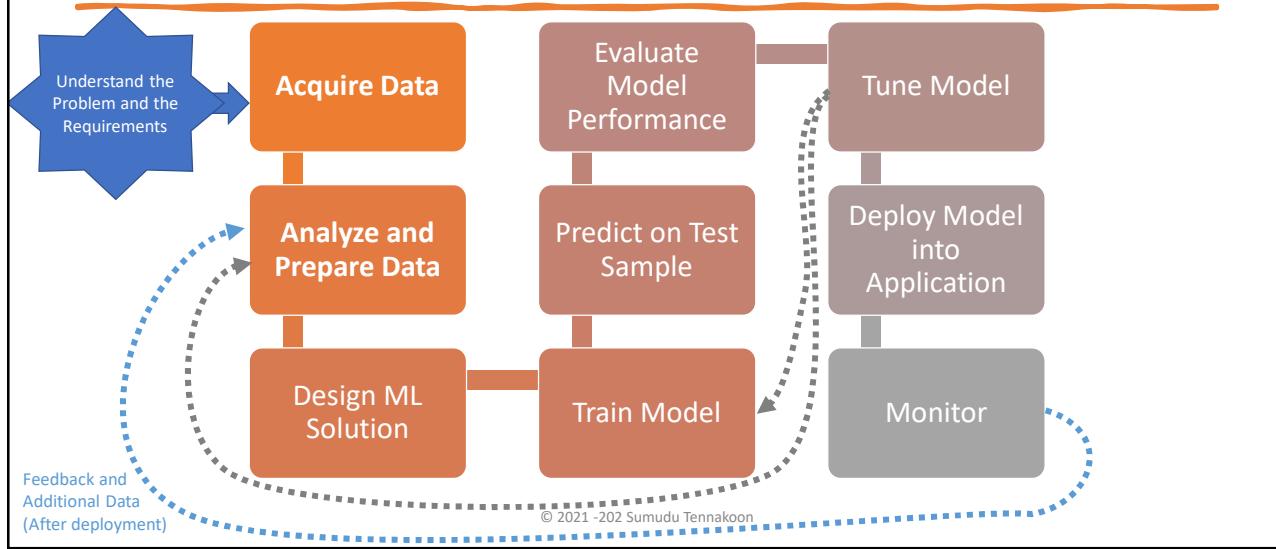


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Steps of Building ML Model



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Design, Develop and Deploy ML Solutions



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ML Solution Parts to Build

Data Pre-Processing Pipeline

- Input Module
 - Web form Interface
 - API
 - UI Application

- Pre-processing Pipeline
 - Feature Transformations
 - Column Selection

Machine Learning Model Building Pipeline

- Post-processing Pipeline
 - Transform Prediction

Machine Learning Model

- Scoring Module
 - Model Prediction

Prediction (Scoring) Module

- Post-processing Pipeline
 - Transform Prediction

Post Processing Pipeline

- Output Module
 - Return result to user interface

Interface to accept User Input

- Output Module
 - Return result to user interface

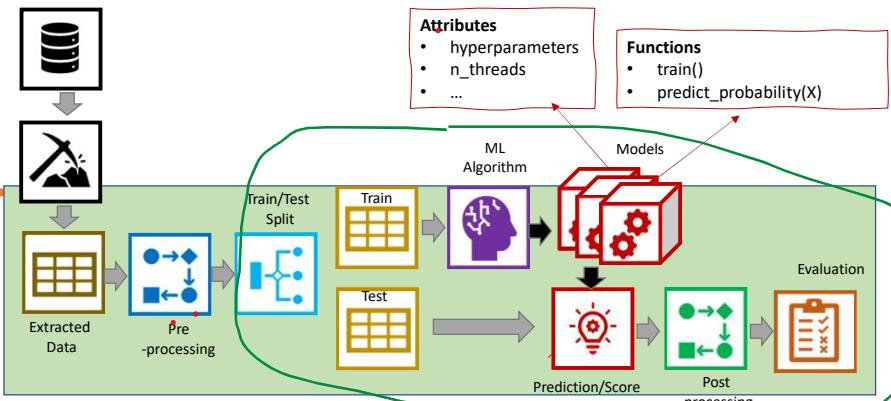
Interface to Return Prediction to User

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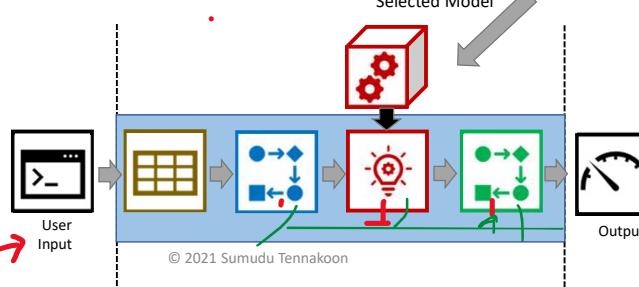
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Pipelines

Model Training Pipeline



Inference Pipeline



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Deployment

- Standalone Application
- Virtual Machines (VM)
- Containerized application
 - Docker
(<https://www.ibm.com/cloud/learn/docker>)
 - Kubernetes
(<https://www.ibm.com/cloud/learn/Kubernetes>)
- Locally hosted Web API
- Cloud API

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Introduction to Azure ML Platform for Machine Learning Workflow

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Evaluating Machine Learning Models



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	Positive	Negative	Total
Infected	48	2	50
Not Infected	1	49	50
Total	49	51	100

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Confusion Matrix: Terms

- condition positive (P)
 - the number of real positive cases in the data
- condition negative (N)
 - the number of real negative cases in the data
- true positive (TP)
- true negative (TN)
- false positive (FP)
- false negative (FN)

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Confusion Matrix

		Predicted condition		Sources: [13][14][15][16][17][18][19][20] view talk edit		
		Total population $= P + N$	Positive (PP)	Negative (PN)	Informedness, bookmaker informedness (BM) $= TPR + TNR - 1$	Prevalence threshold (PT) $= \frac{\sqrt{TPR \times FPR} - FPR}{TPR - FPR}$
Actual condition	Positive (P)	True positive (TP), hit	False negative (FN), type II error, miss, underestimation	True positive rate (TPR), recall, sensitivity (SEN), probability of detection, hit rate, power $= \frac{TP}{P} = 1 - FNR$	False negative rate (FNR), miss rate $= \frac{FN}{P} = 1 - TPR$	
	Negative (N)	False positive (FP), type I error, false alarm, overestimation	True negative (TN), correct rejection	False positive rate (FPR), probability of false alarm, fall-out $= \frac{FP}{N} = 1 - TNR$	True negative rate (TNR), specificity (SPC), selectivity $= \frac{TN}{N} = 1 - FPR$	
	Prevalence $= \frac{P}{P+N}$	Positive predictive value (PPV), precision $= \frac{TP}{P} = 1 - FDR$	False omission rate (FOR) $= \frac{FN}{P} = 1 - NPV$	Positive likelihood ratio (LR+) $= \frac{TPR}{FPR}$	Negative likelihood ratio (LR-) $= \frac{FNR}{TNR}$	
	Accuracy (ACC) $= \frac{TP + TN}{P + N}$	False discovery rate (FDR) $= \frac{FP}{P} = 1 - PPV$	Negative predictive value (NPV) $= \frac{TN}{P} = 1 - FOR$	Markedness (MK), deltaP (Δp) $= PPV + NPV - 1$	Diagnostic odds ratio (DOR) = $\frac{LR+}{LR-}$	
	Balanced accuracy (BA) $= \frac{TPR + TNR}{2}$	F ₁ score $= \frac{2PPV \times TPR}{PPV + TPR} = \frac{2TP}{2TP + FP + FN}$	Fowlkes–Mallows index (FM) $= \sqrt{PPV \times TPR}$	Matthews correlation coefficient (MCC) $= \sqrt{TPR \times TNR \times PPV \times NPV} - \sqrt{FNR \times FPR \times FOR \times FDR}$	Threat score (TS), critical success index (CSI), Jaccard index $= \frac{TP}{TP + FN + FP}$	

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https://en.wikipedia.org/wiki/Confusion_matrix

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Evaluating Classification Models

Accuracy (ACC):

$$\frac{TP + TN}{P + N}$$

Precision: Positive Predictive Value (PPV)

$$\frac{TP}{TP + FP}$$

Recall: Sensitivity/Hit rate/True Positive Rate (TPR)

$$\frac{TP}{P}$$

F1 score:

$$2 \times \frac{PPV \times TPR}{PPV + TPR} = \frac{2 \times TP}{2 \times TP + TN + FP + FN}$$

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Visual Methods

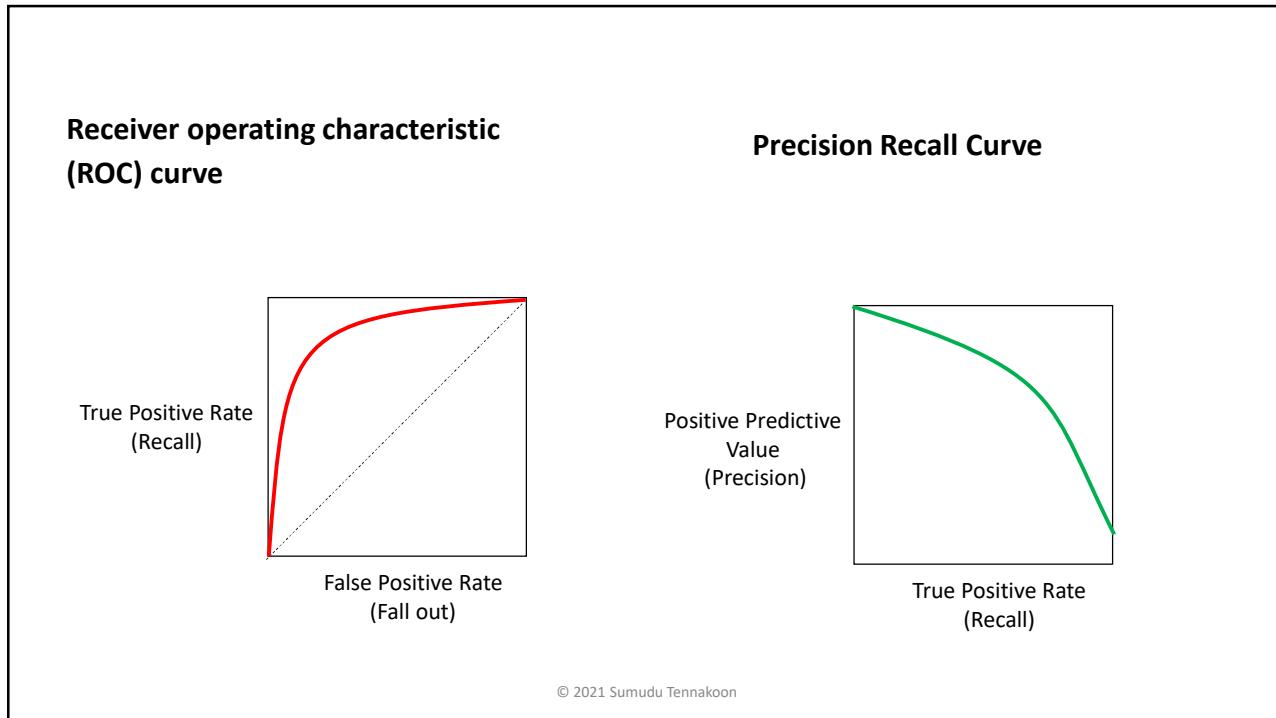
Receiver operating characteristic(ROC) curve

- Precision Recall Curve

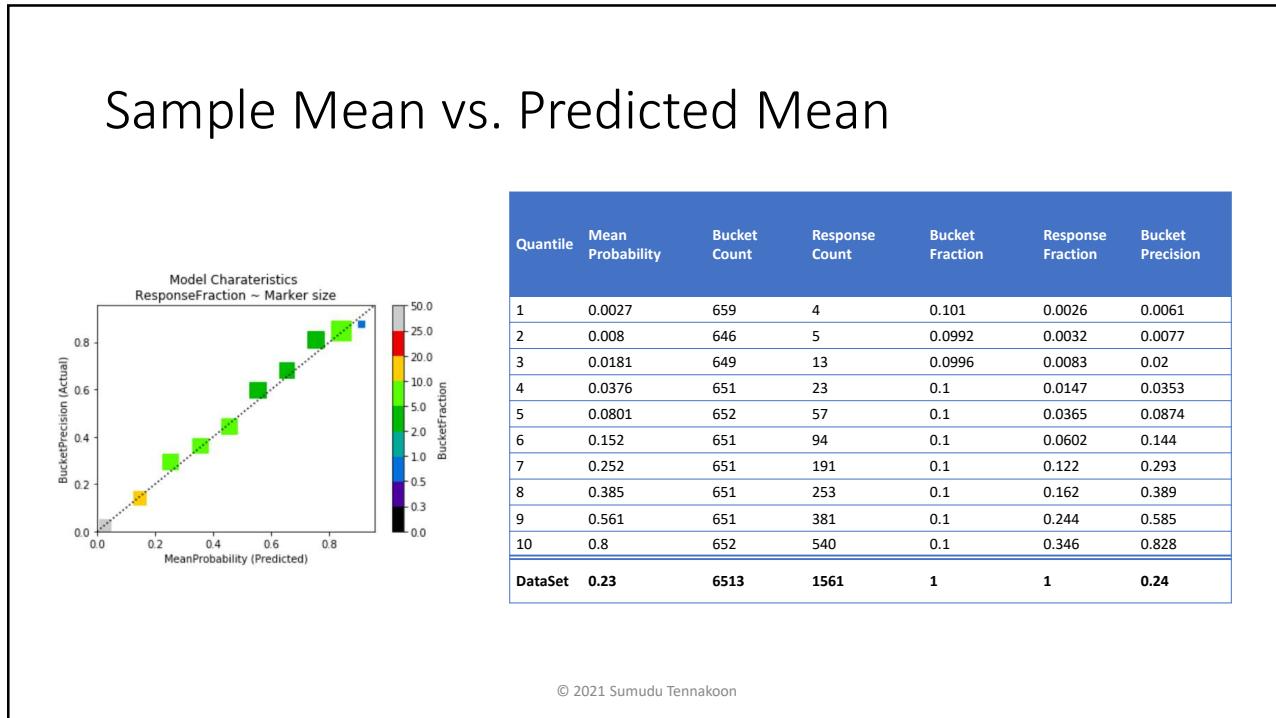
Sample Mean vs.
Predicted Mean

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Evaluating Regression Models

Mean Absolute Error (MAE):

$$MAE = \frac{1}{N} \sum_{i=1}^N |y_i - \hat{y}_i|$$

Mean Squared Error (MSE):

$$MSE = \frac{1}{N} \sum_{i=1}^N (y_i - \hat{y}_i)^2$$

Root Mean Squared Error (RMSE):

$$RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^N (y_i - \hat{y}_i)^2}$$

Coefficient of Determination (R-squared):

$$RMSE = 1 - \frac{\sum_{i=1}^N (y_i - \hat{y}_i)^2}{\sum_{i=1}^N (y_i - \bar{y})^2}$$

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Bias and Variance

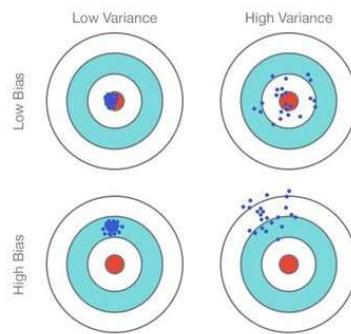


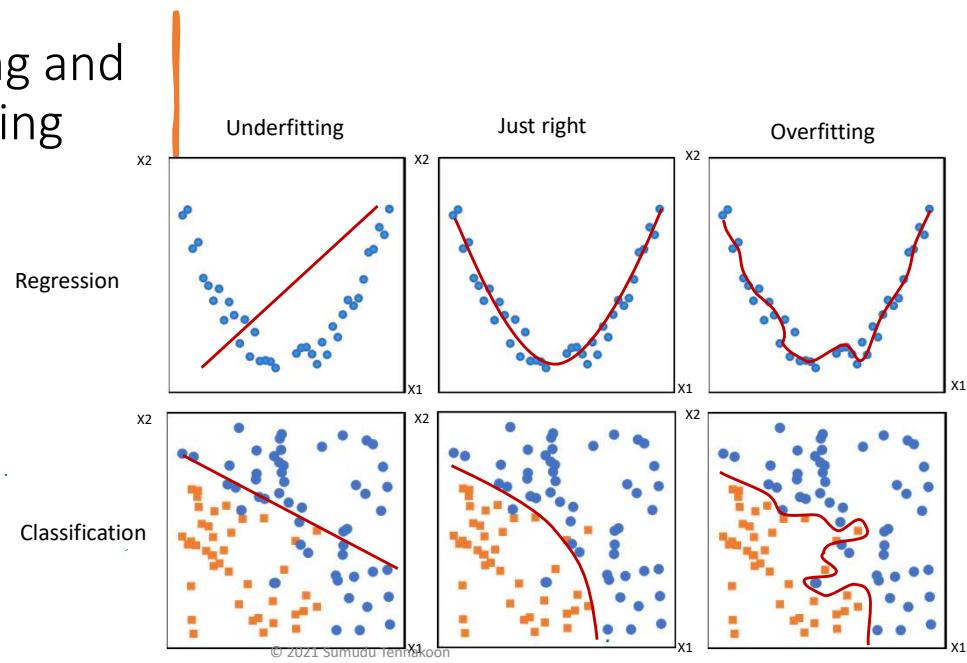
Fig. 1: Graphical Illustration of bias-variance trade-off, Source: Scott Fortmann-Roe., Understanding Bias-Variance Trade-off

- **Bias:** deviation from the actual value due to the model's simplistic assumptions in fitting the data.
 - High bias: model is unable to capture the patterns in the data and this results in **under-fitting**.
- **Variance:** deviation from the actual value due to the complex model trying to fit the data.
 - High variance: model passes through most of the data points, and it results in **over-fitting** the data.

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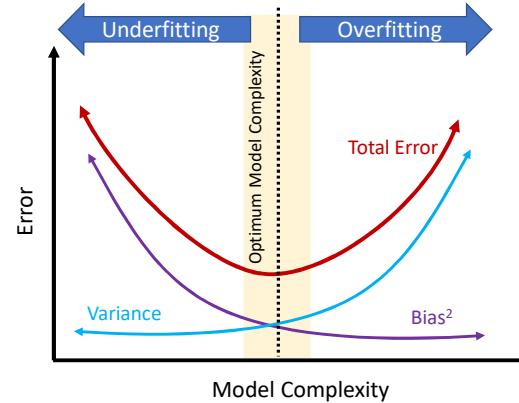
Overfitting and Underfitting



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Bias-Variance Tradeoff

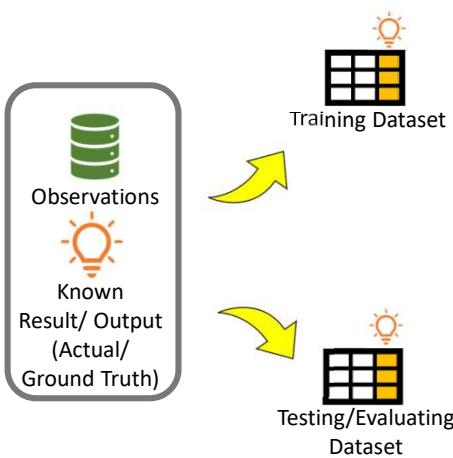
- Ideal Case for a machine learning model :
 - low variance and low bias.
- Reality
 - Bias-Variance trade-off
 - Find the model that performs well on both the train and unseen(test) data.



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Train-Test Split

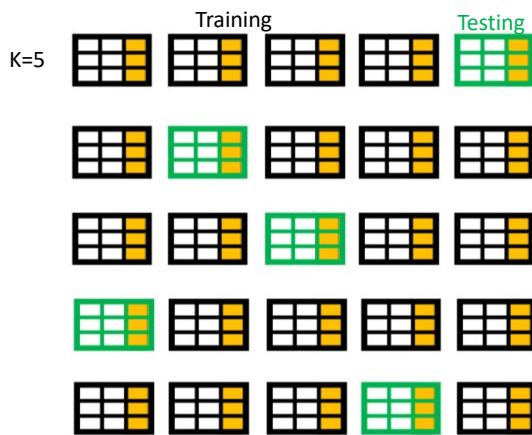


- No Training data should be used for Testing/Evaluating and vice versa.
- Keep Training and Testing Datasets Separate or properly labeled in the same dataset.
- Do train test split once and use the same two datasets over model tuning process.
- Use random Sampling
- Commonly used Train to Test ratio is 70:30

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Cross Validation

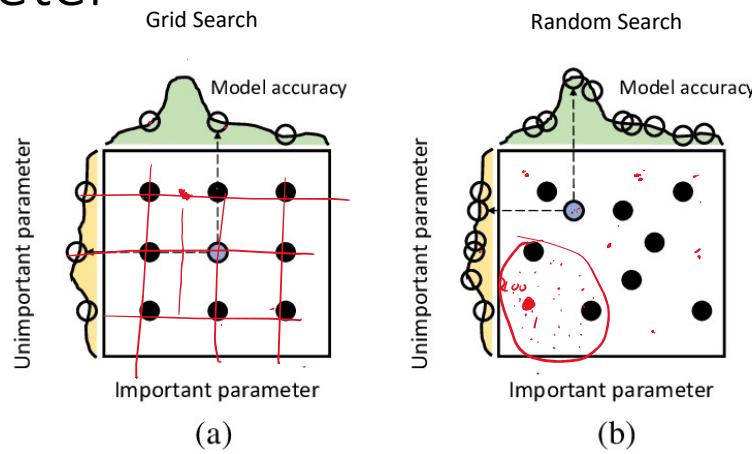


- If required to use all data in the Training Process
 - E.g., small number of observations available.
- K-Fold Cross validation
- Split Dataset into K number of Samples
- Create K number of configurations holding one out of K sample as the Testing Sample
- Train Models using K folds and average results

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Hyperparameter Tuning

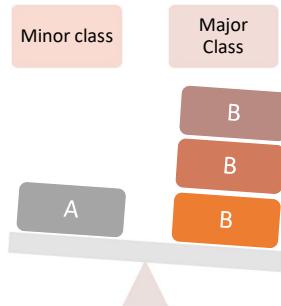


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Class Imbalance

- Majority Class Minority Class
- Examples
 - Fraud Detection.
 - Churn Prediction.
 - Spam Detection.
- Ways to treat Imbalance
 - Collect More Data
 - (Re)Sampling
 - Stratified sample
 - Over Sampling (multiple copies of some of the minority classes)
 - SMOTE: Synthetic Minority Over-sampling Technique.
 - Under Sampling (Randomly remove samples from the majority class)
 - Data Augmentation (Generate Synthetic Data)
 - Using proper evaluation metric
 - Precision, Recall, F1 Score, AUC (ROC)
 - Do not use accuracy !
 - Split Larger Class into sub classes.
 - Use Ensemble Models



<https://arxiv.org/abs/1106.1813>
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<https://link.springer.com/article/10.1007/s13748-016-0094-0>

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