

1

Course Objective

Provide comprehensive understanding of the core principles of Machine Learning with hands-on training on applying machine learning to solve real-world problems.

A learner who completes this course should be able to define a machine learning problem, understand the solution path, and display the ability to carry out the end-to-end process of building a machine learning application.

2

Machine Learning Career Prospectus

- Data Scientist
- AI Scientist
- ML/AI Engineer
- Data Engineer
- Data Analyst
- AI/ML Developer
- IoT Developer
- Solutions Architect
- Freelancer
- ...



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Schedule and Format

Duration: 60 hours

Schedule: 3-month program/12 weeks, two sessions per week.

Format: Live/Recorded Lectures, Demonstrations, Hands-on Exercises/Labs.

Evaluation: Quizzes (2), Project (1)

Additional Practice: Students must spend extra time on exercises and the capstone project.

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Prerequisites

- Basics of computer programming, mathematics, and statistics.
- Basic knowledge in computer applications:
 - Spreadsheet
 - word processor
 - presentation authoring

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Platform and Data for Hands-on Exercises and Project

Programming Language: Python 3 will be used as the primary programming language in teaching, practice examples and assignments.

Python Libraries: Scikit-learn, TensorFlow, Pandas, NumPy, Matplotlib, Seaborn, Flask.

Applications/Tools: Jupyter Notebook/Lab, IDE (Spyder/VS Code/Atom/PyCharm), Spreadsheet (MS Excel/LibreOffice Calc).

Data: Data for exercises, case studies, and projects will be obtained from open data repositories.

Computing Environment: Cloud platform (will be decided on class consensus and service availability) or locally installed Python distribution in student's PC.

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Session Topics

#	Topic Name	Training Week #
1	Introduction to Machine Learning (ML), History, and Applications	1
2	Setting up a Computing Environment, Python and Required Libraries.	2
3	Knowledge Foundations to ML (Computing, Statistics, and Mathematics) *	2-3
4	Exploratory Data Analysis (EDA) and Feature Engineering *	4-5
5	Supervised and Unsupervised Learning (concepts)	6
6	Machine Learning Algorithms *	6-7
7	Explaining ML Models and Predictions (introduction) *	7
8	Deep Learning and Neural Networks (introduction) *	8
9	Design and Develop and Deploy ML Solutions *	9-10
10	Capstone Project *	11-12

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Evaluations and Grading

Completion Requirement:

- 80 % Attendance (at least 19 out of 24 sessions)
- Final Grade > 70 %

Completion with Distinction:

- Final Grade > 90 %

	Topic #	%
Quiz1 (Basic Concepts)	1-6	20
Quiz 2 (Advanced Concepts, Deep Learning and Application Building)	7-9	20
Deliverable and Project Report	10	50
Presentation (video narration)	10	10
		100

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Introduction to Machine Learning

History and core concepts of ML to navigate the future lessons.

Applications of ML.



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Example: Identify Objects

What facts you consider to identify these objects?



Pineapple



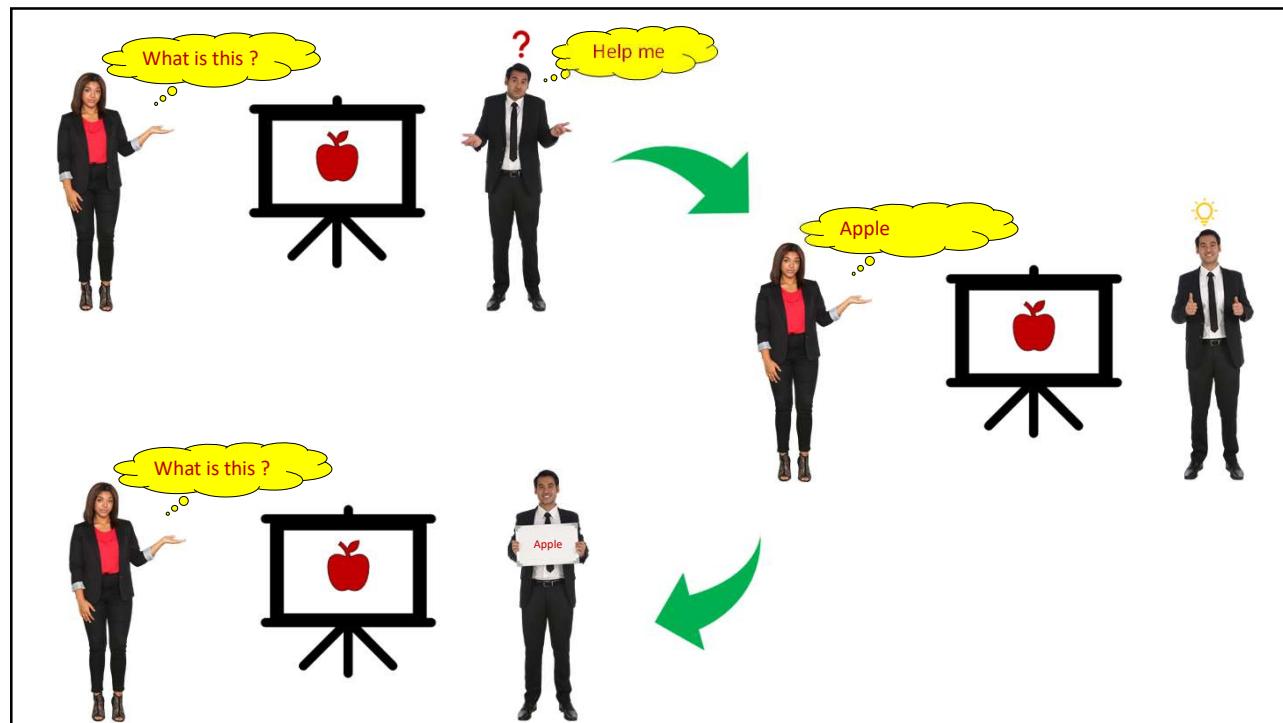
Apple

10

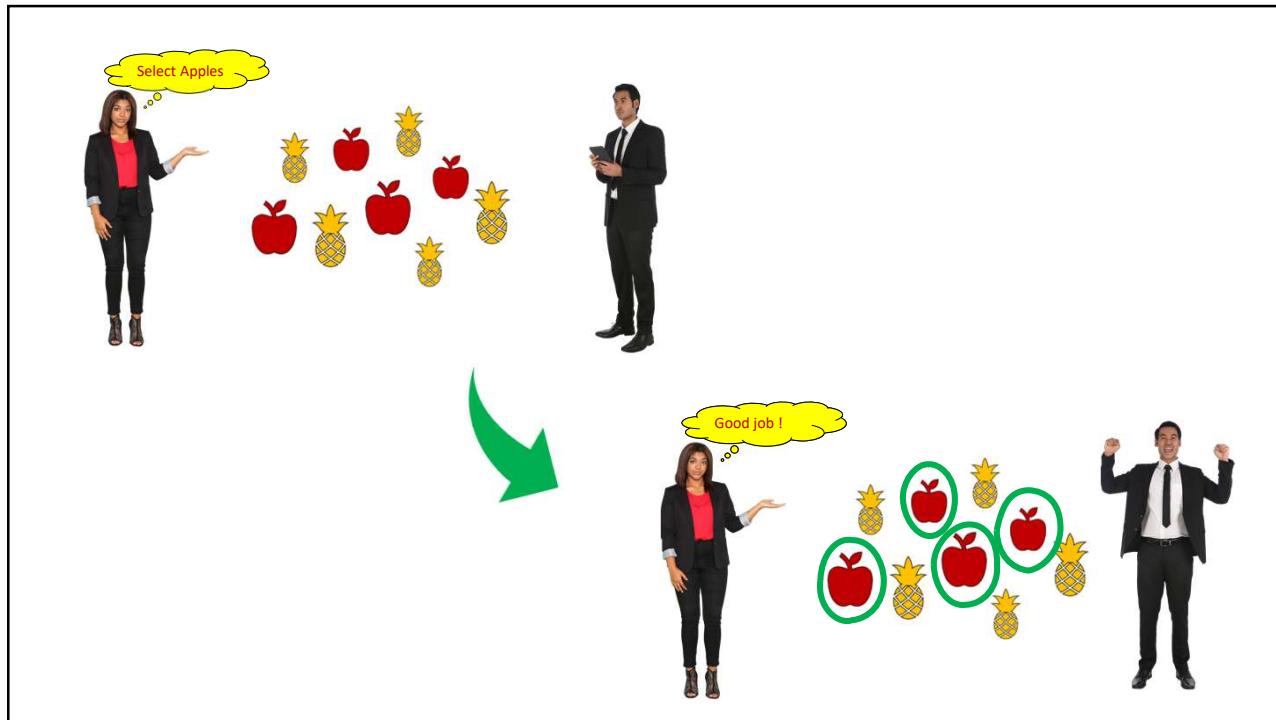
How we Learn ?

- Memorize Facts
 - Declarative Knowledge
 - Limited by memory and time to observe
- Infer (deduce new information from previously known facts)
 - Imperative Knowledge
 - Limited by accuracy of predictions and drifts (present is not behaving the same way as past)

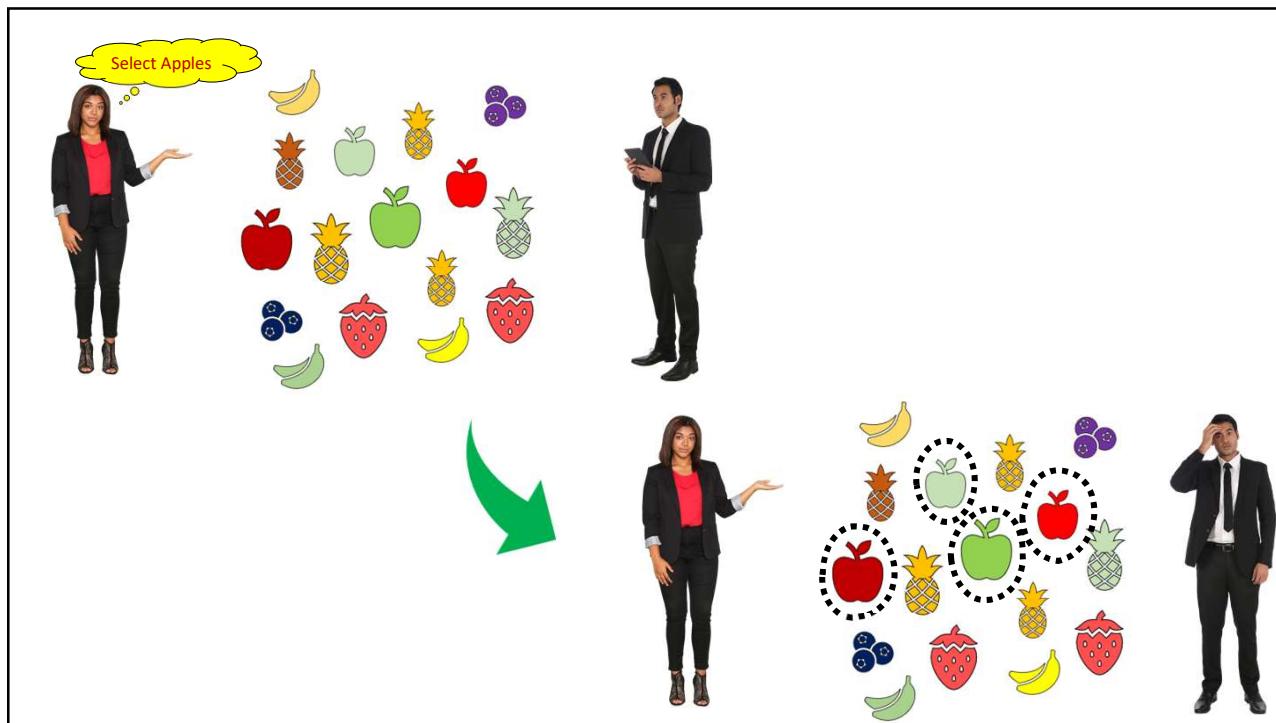
11



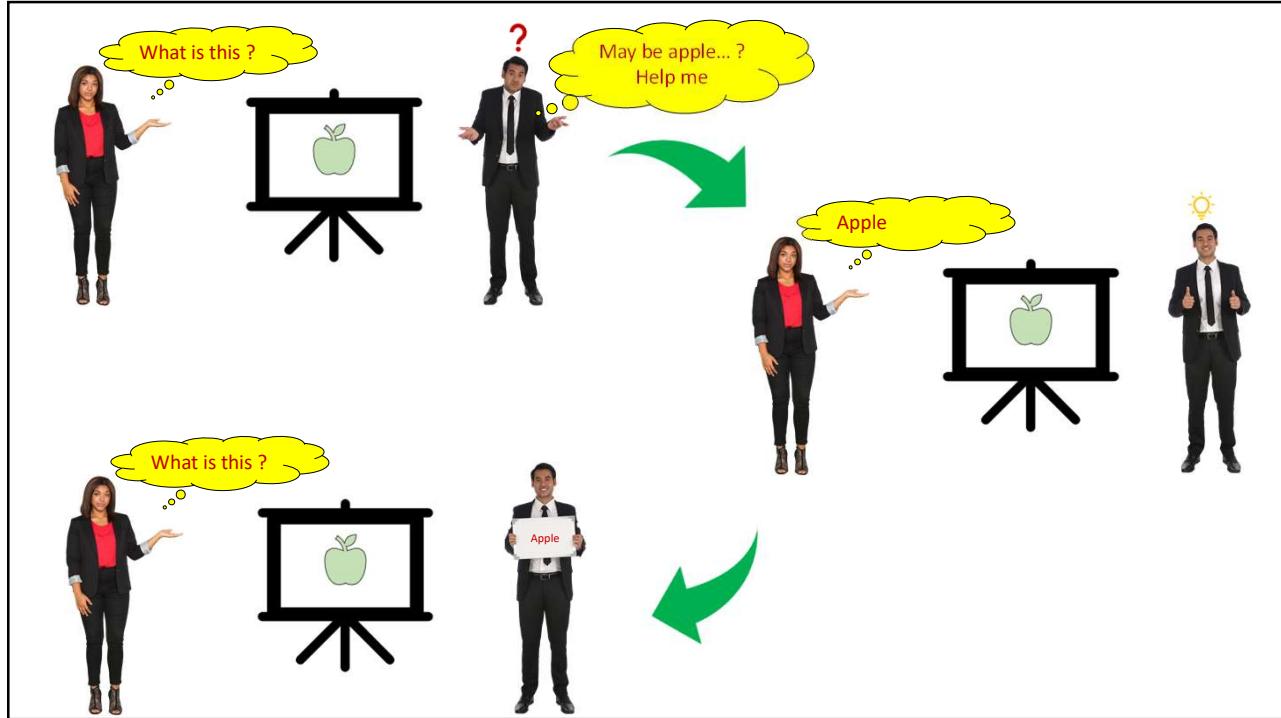
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Exercise

What are the Observations/Measurements can be used to make a determination.

Design a simple classifier logic.

Is it easy or difficult to convert this logic to a computer program (code)?

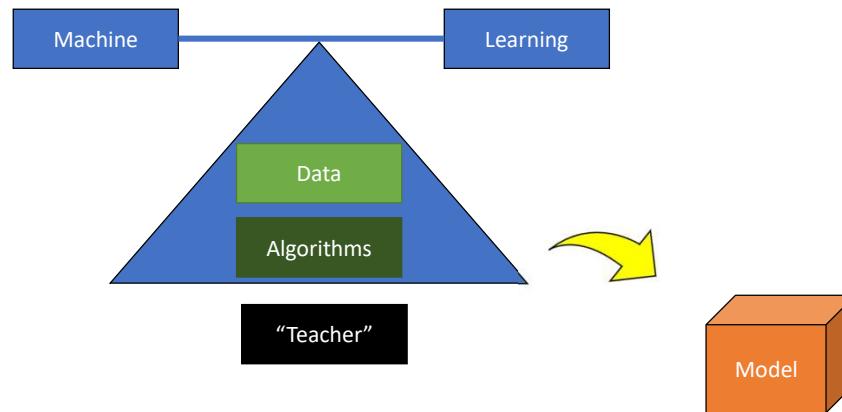
What are the considerations when converting this logic to a computer program (code)?

What are the points of failures in this approach?

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What is Machine Learning ?

- Learn from Data



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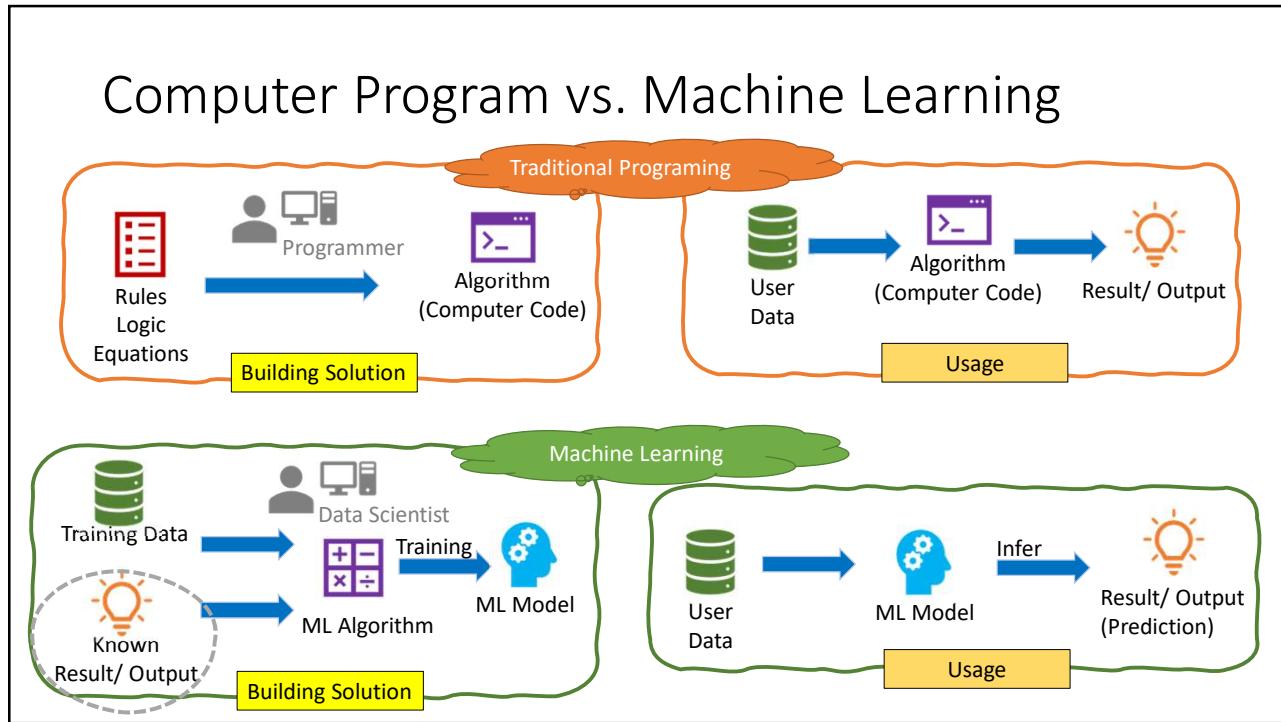
Machine learning model



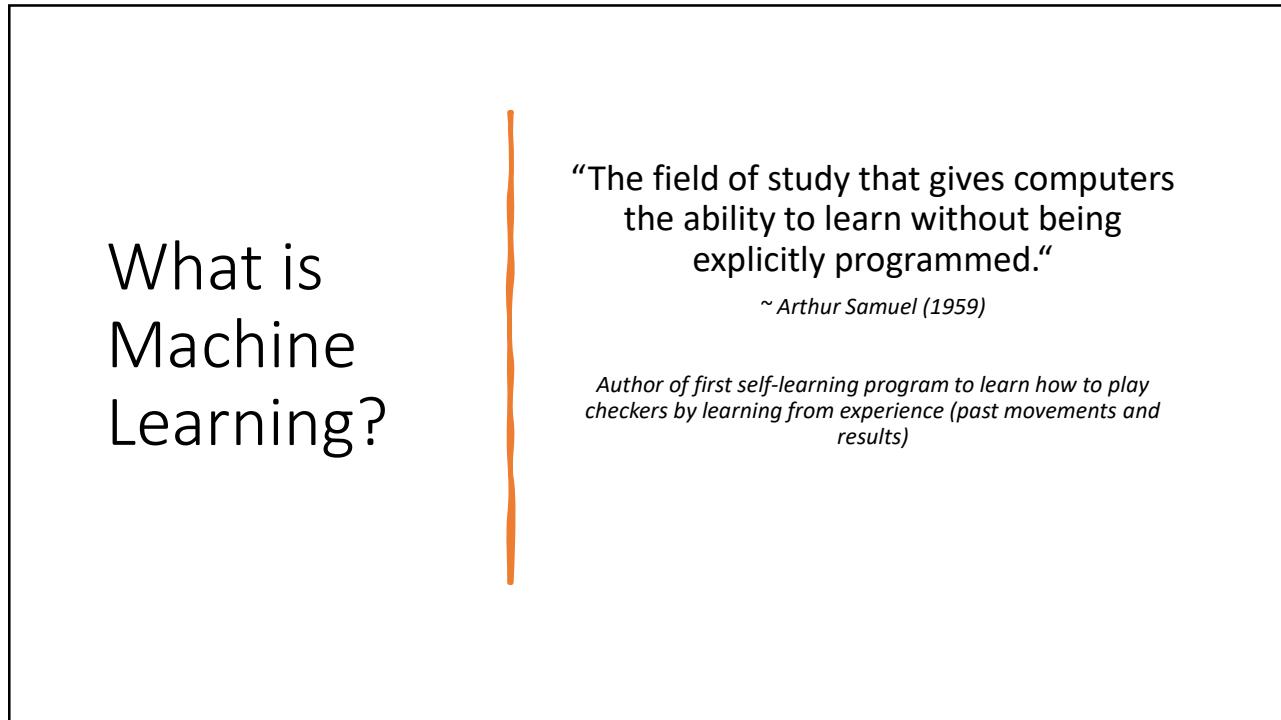
“Machine learning models are built on mathematical algorithms and are trained using data and human expertise to help us accurately predict outcomes based on input data such as images, text, or language.”

<https://developer.nvidia.com/ai-models>

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What is Machine Learning?

“ A computer program is said to learn from experience E with respect to some class of **tasks T** and **performance measure P**, if its performance at tasks in T, as measured by P, improves with **experience E**.“

~ Tom Mitchell (1997)

Example: playing checkers.

- E = the experience of playing many games of checkers
- T = the task of playing checkers.
- P = the probability that the program will win the next game.

Mitchell, T. (1997). *Machine Learning*. McGraw Hill.
p. 2. [ISBN 978-0-07-042807-2](#).

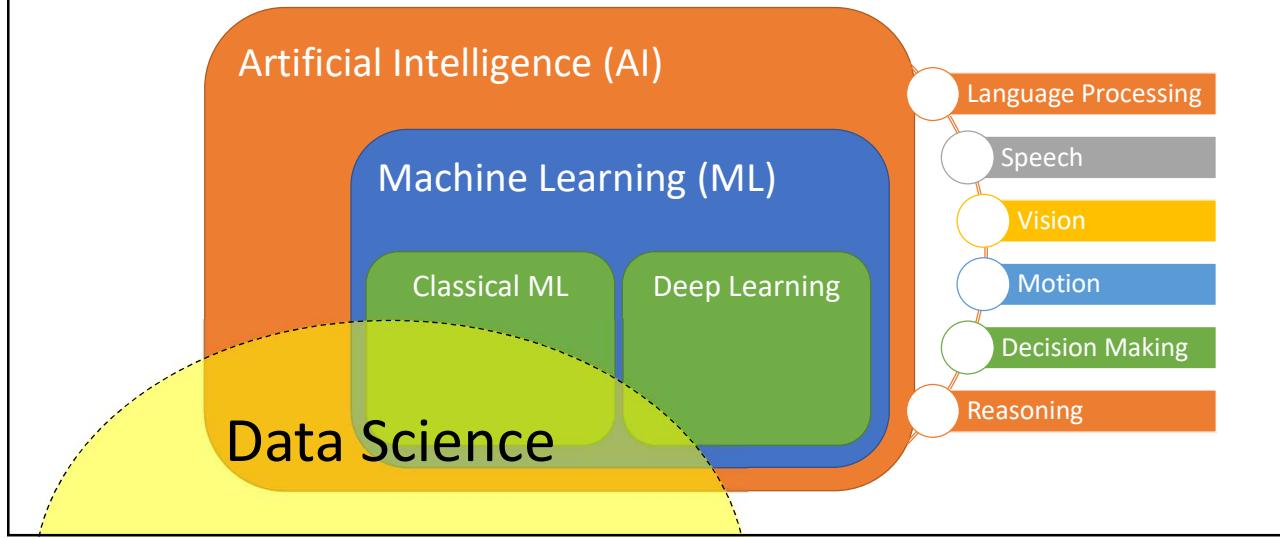
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AI and Machine Learning

- AI (Mimic Cognitive Functions of Human)
 - Computer Vision
 - Speech Recognition and Synthesis
 - Language Processing and Understanding
 - Motion
 - Decision Making
 - Prescribe or Predict
 - Reasoning
- Machine Learning (ML)
 - Machines learn on Data/Prior Knowledge
 - Statistical Modeling/Algorithms
 - Backbone of AI is Machine Learning
 - Algorithms to Find meanings of data
 - Find Relationships
 - Making Predictions
 - Problem-Solution Types
 - Classification
 - Regression
 - Clustering

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Machine Learning/Deep Learning/AI



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Levels of AI

Artificial General Intelligence (AGI) known as “Strong AI”

- AGI is the ability to solve *any* problem rather than finding a solution to a particular problem.
- Machine can understand or learn any intellectual task that a human being can.
- The machine can think and perform tasks on its own, just like a human being.
- In the Movies! We are not there yet.

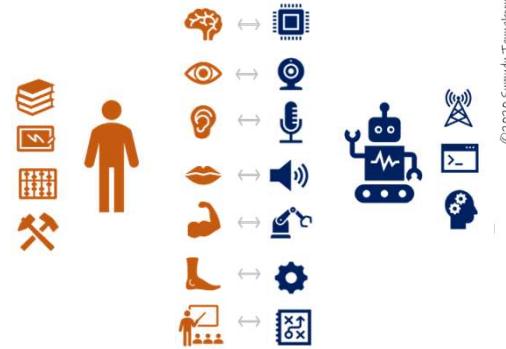
Weak Artificial Intelligence (Weak AI),

- Implements a limited part of human cognitive abilities.
- **Narrow AI** is a special case of Weak AI focused on a specific problem or task.
- Currently, existing AI systems are likely operating as a narrow AI.
- devices cannot follow these tasks independently but are made to look intelligent (simulate human behavior).

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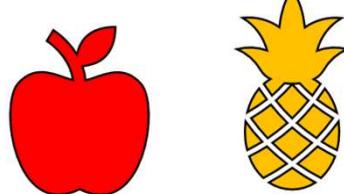
Building Blocks of an AI System

- Image recognition (computer vision)
- Signal processing (sound, sensor data feed, etc.)
- Speech Recognition (Speech to text/STT)
- Natural language processing (NLP)
- Visual Synthesis (Computer Graphics)
- Sound Synthesis (Text to Speech/TTS)
- Software/Algorithms
- Applications (Anomaly Detection, Classification, Prediction, Pattern Recognition)
- Memory (Storage, RAM, Cache)
- Processor (GPU, CPU, TPU)
- Connectivity (Wi-Fi, Satellite, 5G, ethernet, etc.)
- Hardware (Computer, Mechanical Components, etc.)



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Exercise



What are Observations/Measurements
can be used to make a determination.

Design a simple classifier logic.

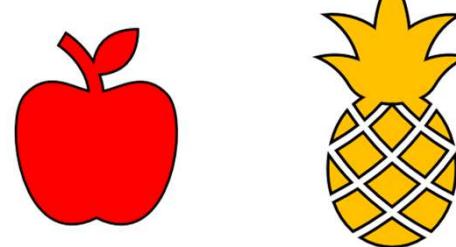
Is it easy or difficult to converting this
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What are the considerations when
converting this logic to a code?

What are the points of failure in this
approach?

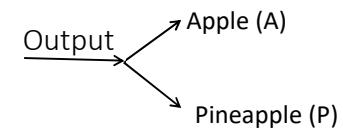
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Input and Output ?



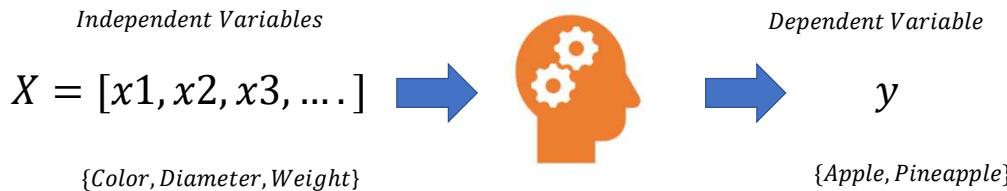
Input →
(Features/Attributes)

Color	red	yellow
Weight	50 g	200 g
Diameter	10 cm	20 cm

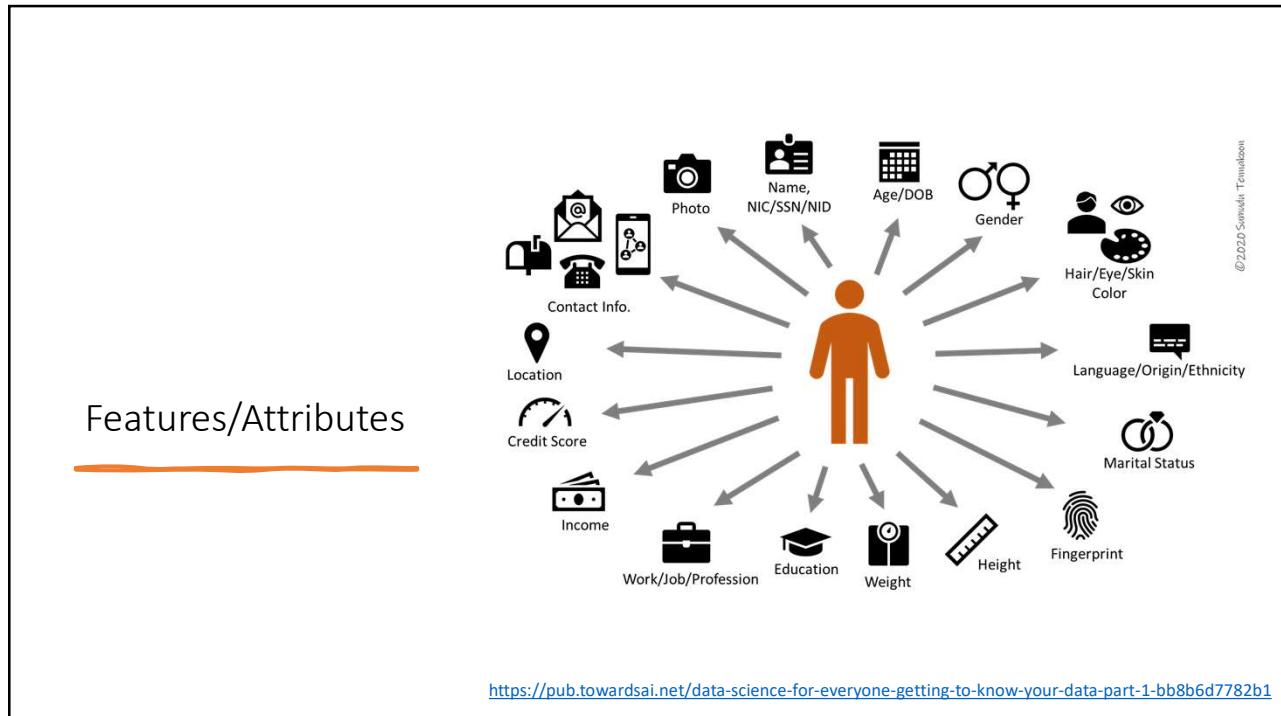


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$$y = f(X)$$



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Tabular Data

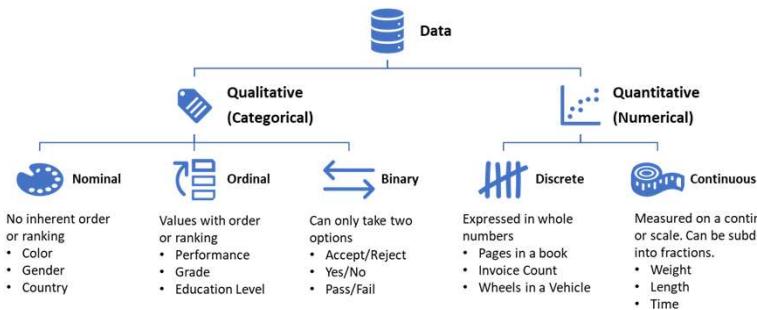
	Column 1	Column 2	Column 3	
	ID	Name	DOB	← Column Names
Row 1	10001	John Doe	1988-01-01	
Row 2	10002	Jane Doe	1990-12-31	Row (Record)
Row 3	

Column
(Data Field)

<https://pub.towardsai.net/data-science-for-everyone-getting-to-know-your-data-part-1-bb8b6d7782b1>

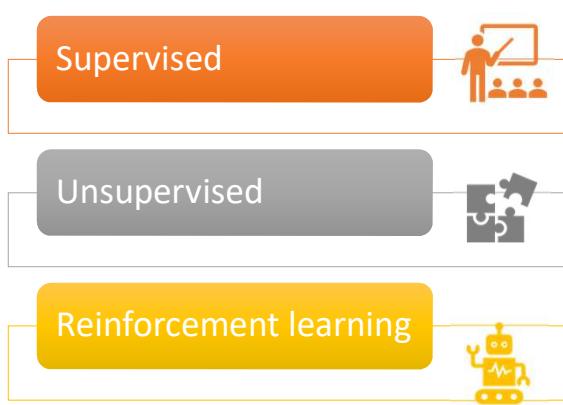
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Understanding Data

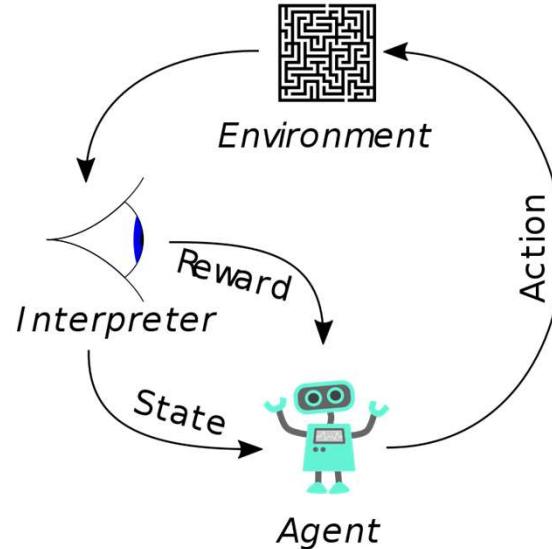


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Machine Learning Approaches



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https://en.wikipedia.org/wiki/Reinforcement_learning

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Types of ML Algorithms



Regression

Linear

Polynomial



Classification

Tree Classifiers

Logistic Regression

Support Vector Machines (SVM)

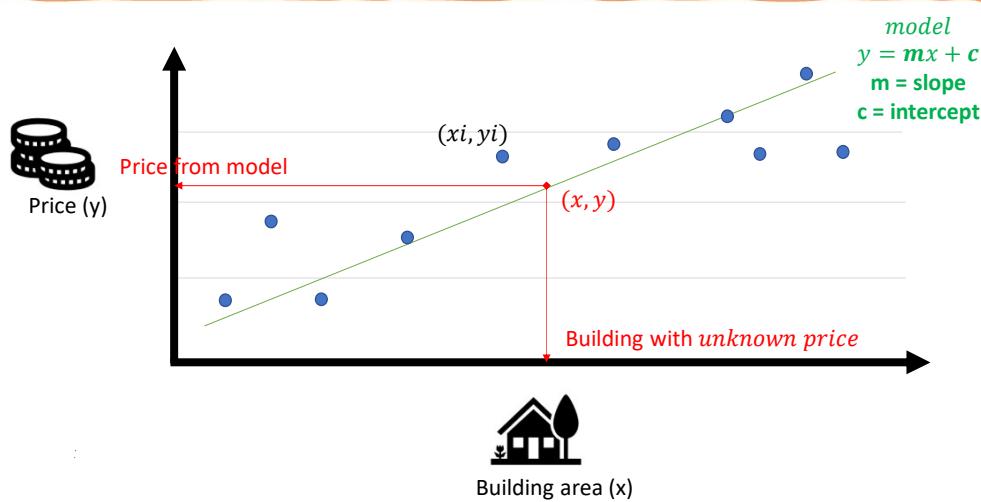


Clustering

K-Means

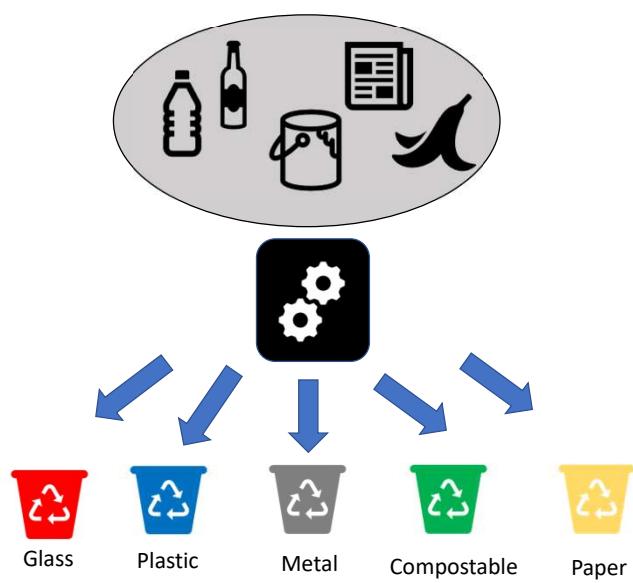
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Regression



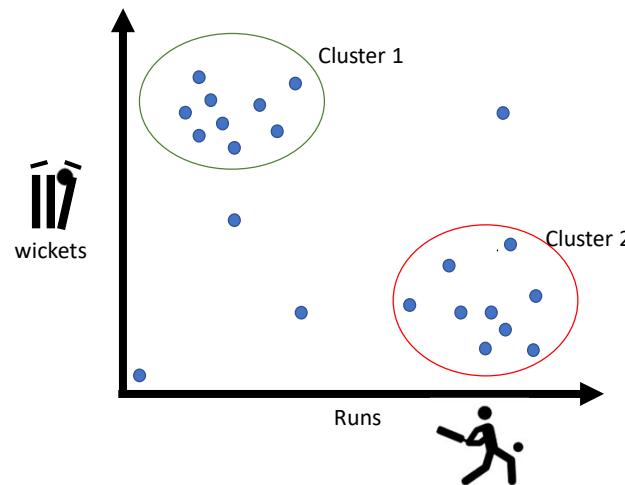
35

Classification



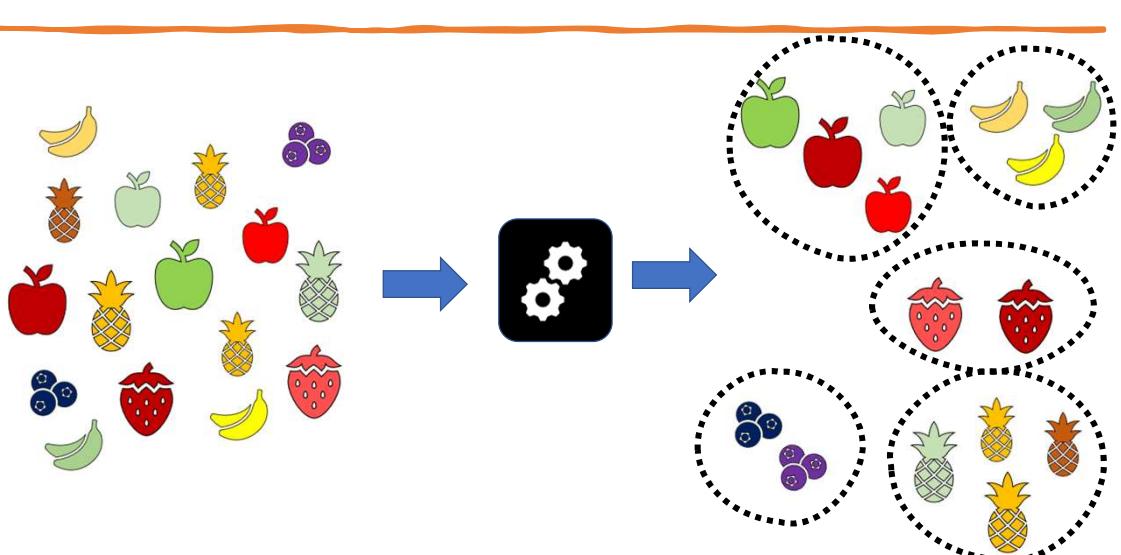
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Clustering



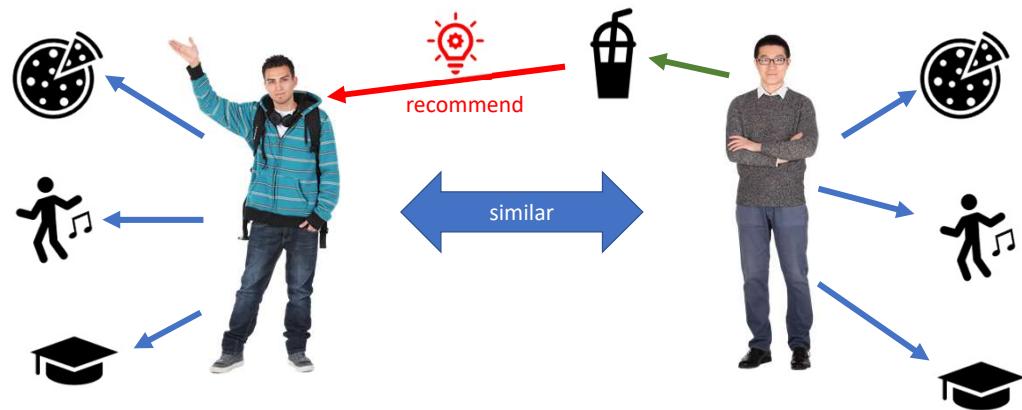
37

Clustering



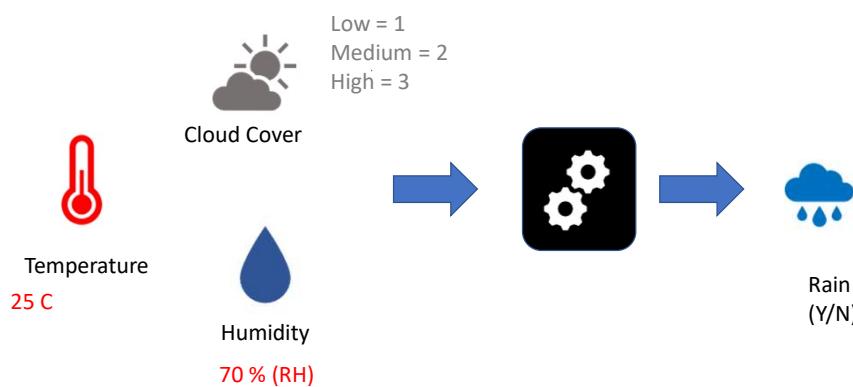
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Recommender Systems



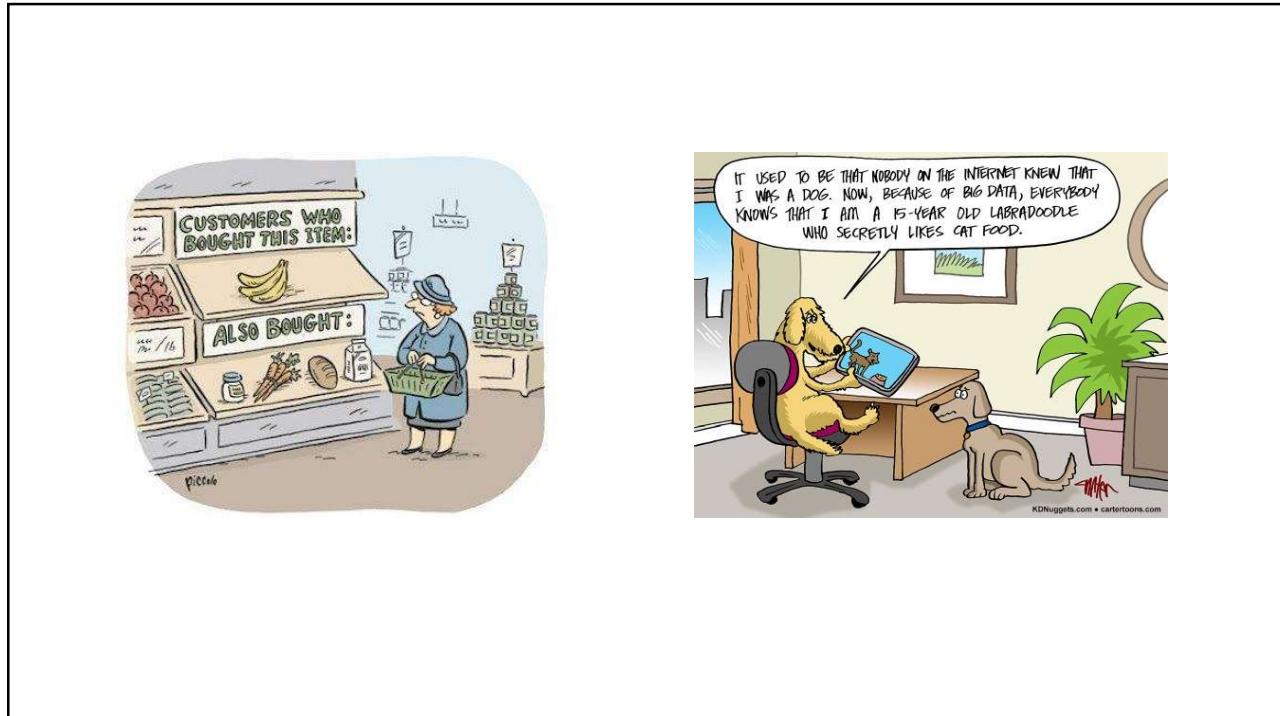
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Prediction



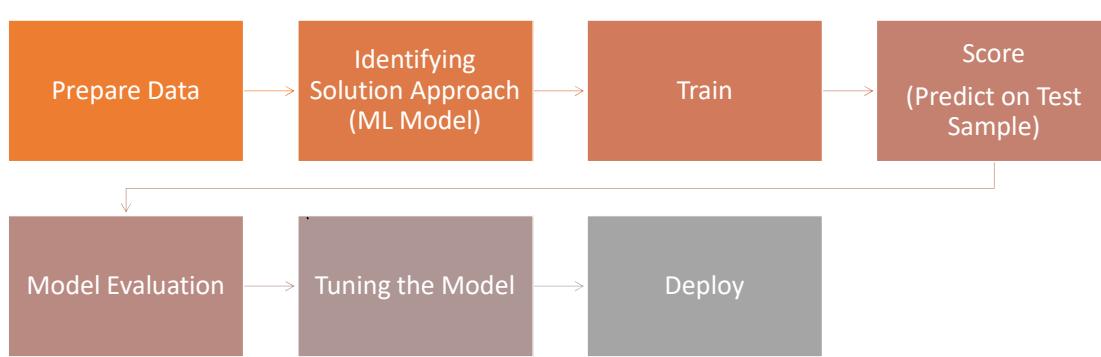
$$y = f(X) ?$$

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Steps of Building ML Model



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Why we need Machine Learning?

Simulate human intelligence

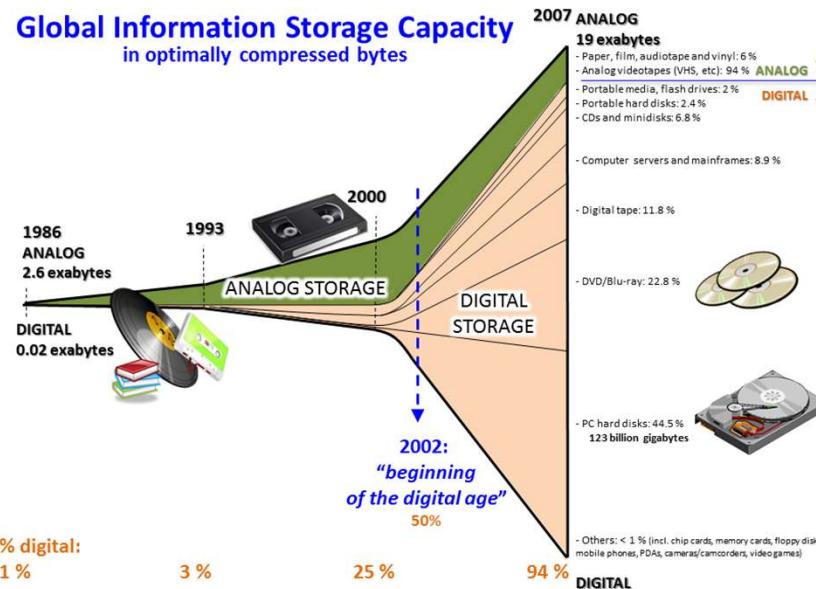
Automation

Help humans with informed decision making

Solve multidimensional problems

Predict future outcome based on historical observations

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Scholz, R.. "Sustainable Digital Environments: What Major Challenges Is Humankind Facing?" *Sustainability* 8 (2016): 1-31.

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Machine Learning Applications

- Spam Email Filtering
- Approve or Reject Loan Application
- Predicting Stock Price
- Credit Card Fraud Detection
- Recommending Items to Purchase (Advertising)

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Application Areas

- Finance
- Marketing
- Information Technology
- Cyber Security
- Agriculture
- Government
- Automobile
- Manufacturing
- Retail
- Entertainment
- ...
- Everywhere!

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Why should everyone get familiar with ML?



Applications of machine learning are all around us.



ML is used in many industries and domains.



Job opportunities.



It is fun to learn and helps train your brain.



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Machine Learning Career Prospectus

- Data Scientist
- AI Scientist
- ML/AI Engineer
- Data Engineer
- Data Analyst
- AI/ML Developer
- IoT Developer
- Solutions Architect
- ...

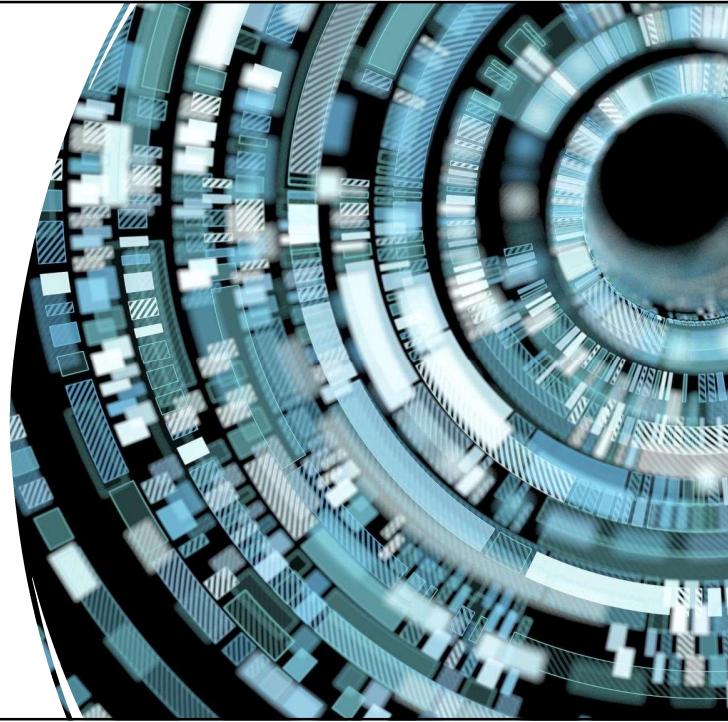


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Setting up Computing Environment

Cloud Computing Platform
(Google Colab)

Python (Install, Libraries)



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Install Python in Local Computer

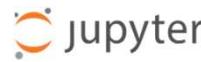
- Python:
<https://www.python.org/downloads/>
- Anaconda Python:
<https://www.anaconda.com/products/individual>
- Python: Libraries:
<https://www.anaconda.com/open-source>



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Python Libraries

- Data Handling
 - Pandas
 - Dask (distributed)
- Machine Learning
 - Scikit-learn
 - TensorFlow
 - PyTorch
- Visualizing
 - Matplotlib
 - Seaborn
- Numerical and Scientific Computing
 - SciPy
 - NumPy
- Machine Learning Model Interpretation
 - LIME
 - SHAP
- Web Services/API
 - Flask
 - Django



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Install Python Libraries

- Using pip package manager for Python
 - <https://packaging.python.org/tutorials/installing-packages>
 - pip install some-package-name
 - NumPy: pip install numpy
 - Pandas: pip install pandas
 - Scikit-Learn: pip install scikit-learn
 - Matplotlib: pip install matplotlib
- Using Conda package manager
 - <https://docs.conda.io/projects/conda/en/latest/commands/install.html>
 - conda install -c conda-forge some-package-name
 - NumPy: conda install -c conda-forge numpy
 - Pandas: conda install -c conda-forge pandas
 - Scikit-Learn: conda install -c conda-forge scikit-learn
 - Matplotlib: conda install -c conda-forge matplotlib

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Jupyter Notebook/Lab



A screenshot of the Jupyter Notebook interface. On the left, there's a file tree showing notebooks, images, and other files. In the center, a notebook cell is open with Python code for linear regression. To the right, there are several tabs for different kernels: Python 3, C++, C/C++, C/C++, Julia, and R. The R tab shows a scatter plot of satellite weather data from 2013-2015. Other tabs include a launcher, a console, and a camera module.

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Google Colab

<https://colab.research.google.com>

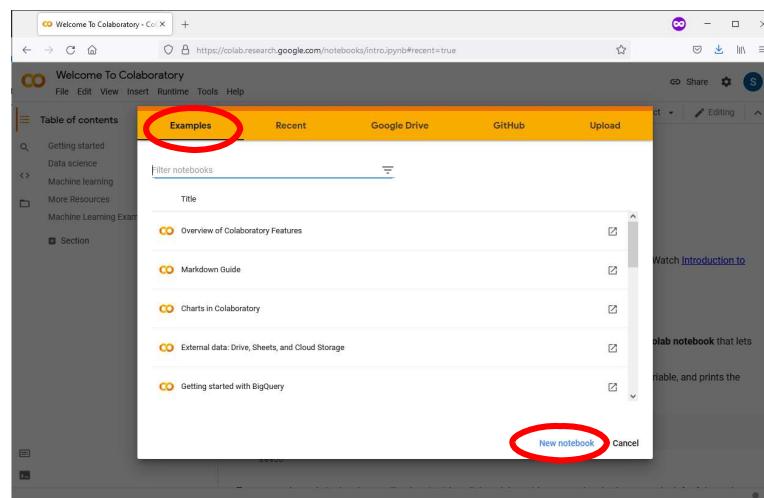
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Getting Started

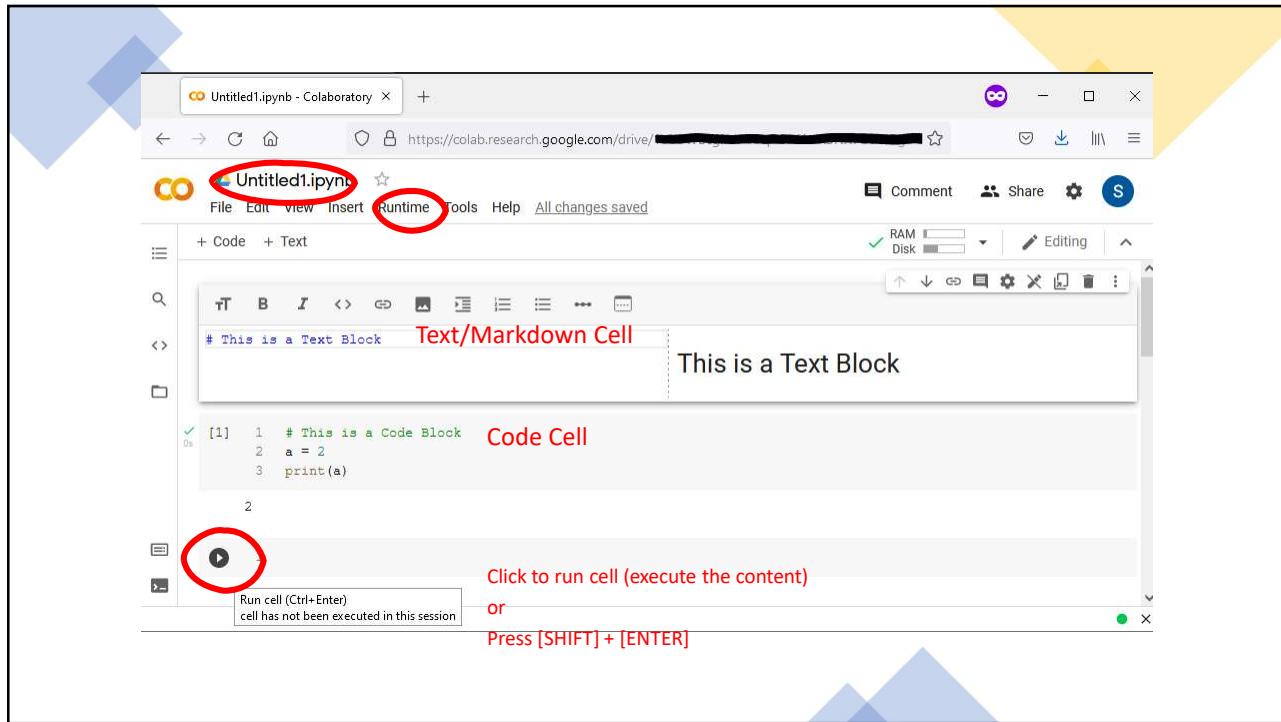
- Creating New Notebook
- Opening Notebook from GitHub
- Opening Notebook from file

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Creating New Notebook

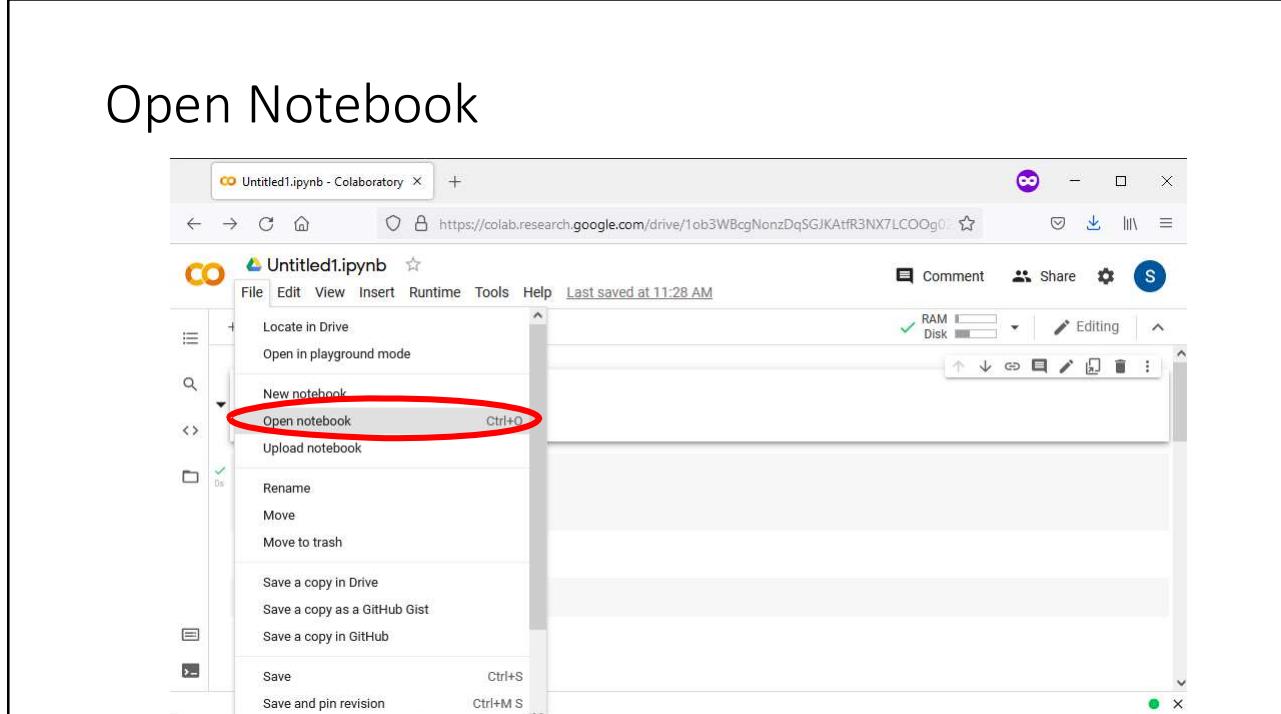


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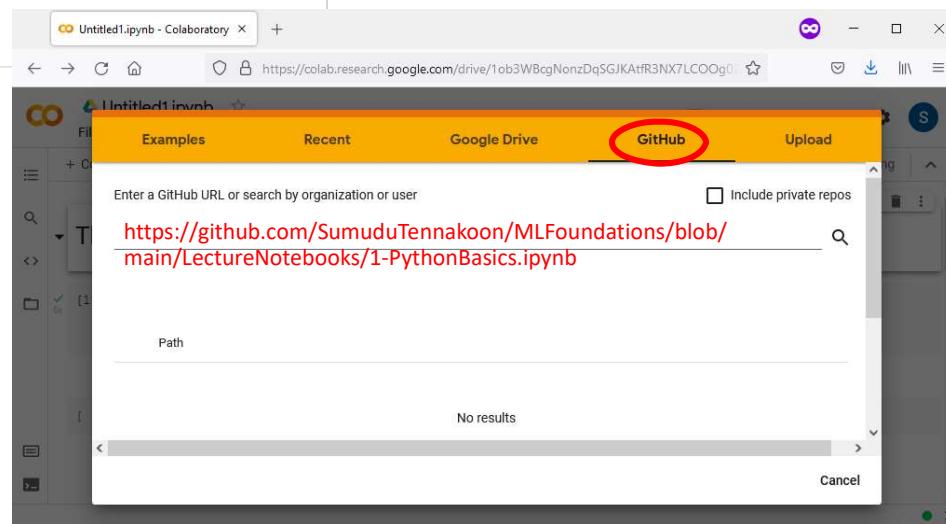
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Open Notebook



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Open Notebook from GitHub



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https://github.com/SumuduTennakoon/MLFoundations/blob/main/LectureNotebooks/1-PythonBasics.ipynb

Repository: SumuduTennakoon/MLFoundations Branch: main

Path

LectureNotebooks/1-PythonBasics.ipynb

Open notebook in new tab

Cancel

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The screenshot shows the Google Colab interface. The top navigation bar includes File, Edit, View, Insert, Runtime, Tools, Help, Share, and Settings. A sidebar on the left titled 'Table of contents' lists sections such as Python Basics, Variables, Data Types, Lists, Tuples, Sets, Dictionary, Conditions, Functions, and Exercise 1. The main content area displays the title 'Machine Learning Foundations' and author 'Sumudu Tennakoon, PhD'. Below this, a section titled 'Python Basics' is expanded, containing text about exploring basic features of Python for those with prior programming experience, a note to refer to Python.org, and a link to 'www.python.org'.

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The screenshot shows the Google Colab interface with a modal dialog box titled 'Upload Notebook (.ipynb file)'. The dialog has tabs for Examples, Recent, Google Drive, GitHub, and Upload, with 'Upload' highlighted and circled in red. It contains a 'Browse...' button and a message 'No file selected.' The background shows the Colab interface with a notebook titled 'Untitled1.ipynb'.

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The screenshot shows a Google Colab notebook titled "1-PythonBasics.ipynb". The left sidebar displays a "Table of contents" with sections like Machine Learning Foundations, Python Basics, Variables, Data Types, Lists, Tuples, Sets, Dictionary, Conditions, Functions, and Exercise 1. The main content area is titled "Machine Learning Foundations" by Sumudu Tennakoon, PhD. It contains a section titled "Python Basics" with a description of basic features for programmers and links to Python.org.

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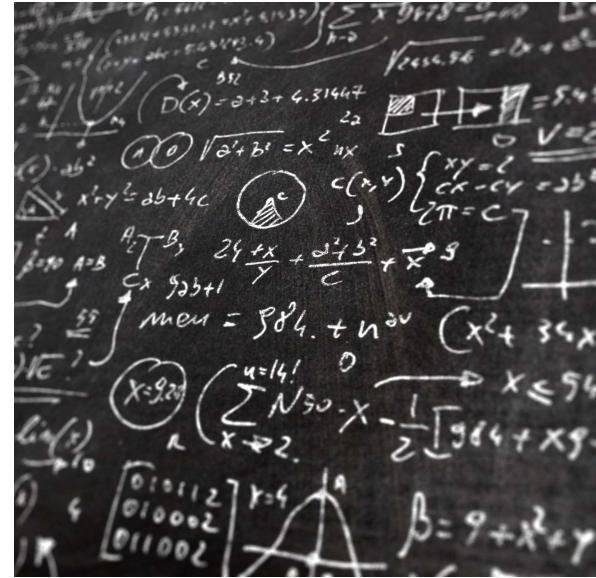
GitHub Link to Lecture Notebooks

Folder:
<https://github.com/sumudutennakoon/mlfoundations/tree/main/lecturenotebooks>

Python basics notebook:
<https://github.com/sumudutennakoon/mlfoundations/blob/main/lecturenotebooks/1-pythonbasics.ipynb>

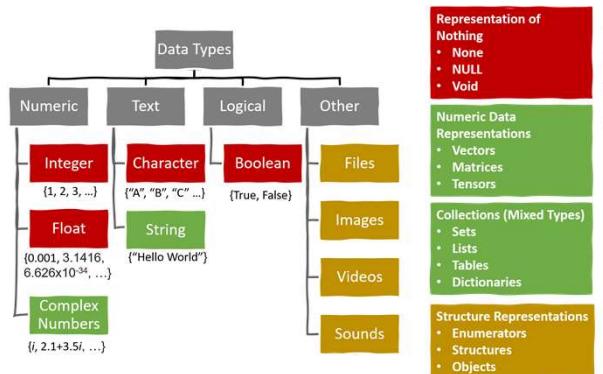
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Mathematics for Machine Learning



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Data Types and Representations



<https://pub.towardsai.net/data-science-for-everyone-getting-to-know-your-data-part-1-bb8b6d7782b1>

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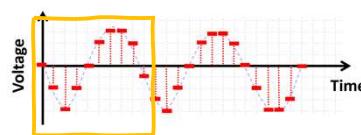
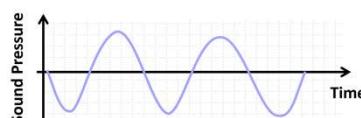
Data Representations

- In computing everything must convert into numbers !
- Numeric Data Structures:
 - Scalars: [3.14](#)
 - Vectors: [\[1,2,3\]](#)
 - Matrices: [\[\[1,2\], \[3,4\] \]](#)
 - Tensors: [\[\[\[1,2\], \[3,4\] \], \[\[5,6\], \[7,8\] \] \]](#)

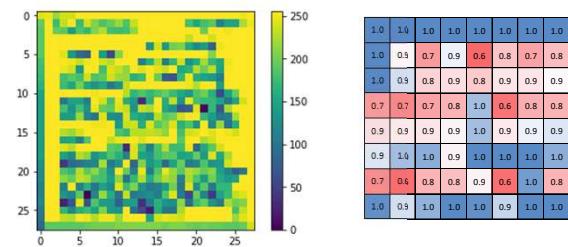


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Why we need Vectors, Matrices and Tensors in Machine Learning?



```
signal = [0 -2 -4 -2 0 2 3 3 2 -1 -3]
time   = [0 1 2 3 4 5 6 7 8 9 10]
```



<https://pub.towardsai.net/data-science-for-everyone-getting-to-know-your-data-part-1-bb8b6d7782b1>

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Vectors and Matrices

The diagram illustrates the hierarchy of tensors:

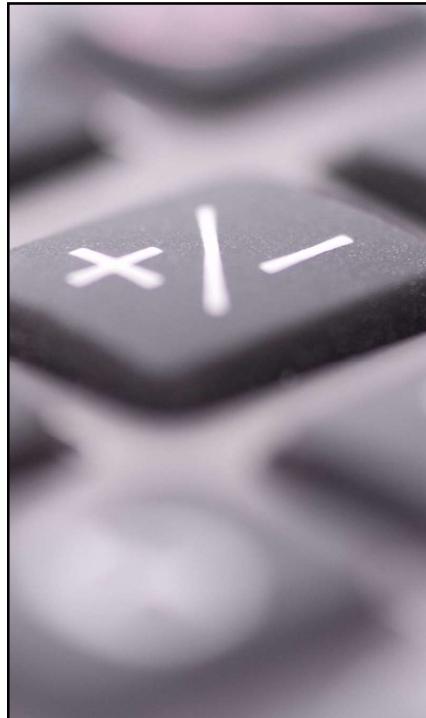
- Scalar:** A single value, represented by a red-bordered box containing the number 1.
- Vector:** A one-dimensional array. It can be a **Column Vector** (3 rows) or a **Row Vector** (1 row).
 - Column Vector: $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}_{3 \times 1}$
 - Row Vector: $[1 \ 2 \ 3]_{1 \times 3}$
- Matrix:** A two-dimensional array. It is labeled as 2×3 (rows x columns).
 - Matrix: $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}_{2 \times 3}$
 - Matrix: $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2 \times 2}$
- Tensor:** A multi-dimensional array. It is labeled as $2 \times 2 \times 2$.
 - Tensor: $\begin{bmatrix} [1 \ 2] & [3 \ 4] \\ [5 \ 6] & [7 \ 8] \end{bmatrix}_{2 \times 2 \times 2}$

• *Temperature*
• *Mass*
• *Speed*

• *Distance [x, y, z]*
• *Velocity [Vx, Vy, Vz]*

[What's a Tensor? - YouTube](#)

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Matrix Algebra

- Transpose
- Sum
- Diagonal
- Determinant
- Adjugate
- Inverse

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Matrix Notation

- Matrix Notation

$$\bullet A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix}_{m \times n}$$

- Representation of elements in python index numbers

$$\bullet A = \begin{bmatrix} A[0][0] & A[0][1] & \dots & A[0][n-1] \\ A[1][0] & A[1][1] & \dots & A[1][n-1] \\ \vdots & \vdots & \ddots & \vdots \\ A[m-1][0] & A[m-1][1] & \dots & A[m-1][n-1] \end{bmatrix}$$

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Transpose, Sum, Diagonal

• $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} = [a_{ij}]$

• $A^T = \begin{bmatrix} a_{11} & a_{21} & a_{31} & a_{41} \\ a_{12} & a_{22} & a_{32} & a_{42} \\ a_{13} & a_{23} & a_{33} & a_{43} \\ a_{14} & a_{24} & a_{34} & a_{44} \end{bmatrix} = [a_{ji}]$

• $\text{Sum}(A) = \sum_{ij} a_{ij}$

• $\text{Diagonal}(A) = [a_{11} \ a_{22} \ a_{33} \ a_{44}]$

• $\text{Trace}(A) = a_{11} + a_{22} + a_{33} + a_{44}$

Columns (j)
Rows (i)
Diagonal
Rows \leftrightarrow Columns

E.g.,

• $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$

• $A^T = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$

• $\text{Sum}(A) = 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 = 45$

• $\text{Diagonal}(A) = [1 \ 5 \ 9]$

• $\text{Trace}(A) = 1 + 5 + 9 = 15$

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Determinant

$$\det \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

$$\det \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = 1 \times 4 - 2 \times 3 = -2$$

$$\det \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} = a \begin{vmatrix} e & f \\ h & i \end{vmatrix} - b \begin{vmatrix} d & f \\ g & i \end{vmatrix} + c \begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

$$\det \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} = 1 \begin{vmatrix} 5 & 6 \\ 8 & 9 \end{vmatrix} - 2 \begin{vmatrix} 4 & 6 \\ 7 & 9 \end{vmatrix} + 3 \begin{vmatrix} 4 & 5 \\ 7 & 8 \end{vmatrix} = 0$$

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Inverse Matrix

- $A^{-1} = \frac{1}{|A|} adj(A)$

- $A^{-1}A = AA^{-1} = I$

- $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

- $|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

- $adj(A) = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

- $A^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

https://en.wikipedia.org/wiki/Invertible_matrix

https://en.wikipedia.org/wiki/Adjugate_matrix

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Properties of Matrix

$$\begin{aligned} A + B &= B + A \\ A + 0 &= A \\ AB &\neq BA \end{aligned}$$

$$\begin{aligned} A(BC) &= (AB)C \\ A(B+C) &= AB+AC \\ (A+B)C &= AC+BC \end{aligned}$$

$$\begin{aligned} \alpha(A+B) &= \alpha A + \alpha B \\ (\alpha + \beta)A &= \alpha A + \beta A \end{aligned}$$

$$\begin{aligned} 0.A &= 0 \\ A.0 &= 0 \end{aligned}$$

$$\begin{aligned} (A^T)^T &= A \\ (A+B)^T &= A^T + B^T \\ (AB)^T &= B^T A^T \end{aligned}$$

$$\begin{aligned} (A^{-1})^{-1} &= A \\ A^{-1}A &= AA^{-1} = I \end{aligned}$$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$$

$$\begin{aligned} (A^T)^{-1} &= (A^{-1})^T \\ (AB)^{-1} &= B^{-1}A^{-1} \end{aligned}$$

$$\begin{aligned} |A^{-1}| &= \frac{1}{|A|} \\ |AB| &= |A||B| \end{aligned}$$

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Special Vectors/Matrices

- Square Matrix: $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2 \times 2}$

- Symmetric Matrix: $S = \begin{bmatrix} 1 & 4 & 5 \\ 4 & 2 & 6 \\ 5 & 6 & 3 \end{bmatrix}$

- Zero Matrix: $0 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

- Unit Vector: $\hat{x} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

- Diagonal Matrix: $D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

- Identity Matrix: $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

- Scalar Diagonal Matrix: $D = \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} = \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \lambda I$

- Upper Triangular matrix: $U = \begin{bmatrix} 1 & 4 & 5 \\ 0 & 2 & 6 \\ 0 & 0 & 3 \end{bmatrix}$

- Lower Triangular Matrix: $L = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 2 & 0 \\ 5 & 6 & 3 \end{bmatrix}$

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Matrix Operations and Applications

- Addition/Subtraction
- Scalar Multiplication
- Matrix Multiplication (Dot Product)
- Matrix-Vector Multiplication
- Row operations
- Solving Linear Equations
- Linear transformations
- Decomposition

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Matrix Multiplication

$$\bullet A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\bullet B = \begin{bmatrix} p & q \\ r & s \end{bmatrix}$$

$$\bullet A \cdot B = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} p & q \\ r & s \end{bmatrix}$$

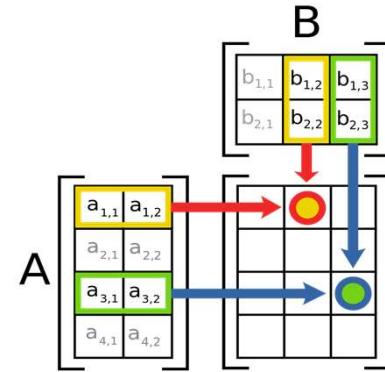
$$\bullet A \cdot B = \begin{bmatrix} ap + br & aq + bs \\ cp + dr & cq + ds \end{bmatrix}$$

https://en.wikipedia.org/wiki/Matrix_multiplication

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Matrix Multiplication (Dot Product)

$$\begin{array}{c} \text{m=3} \\ | \quad \quad \quad | \\ \text{A}_{5 \times 3} \end{array} \cdot \begin{array}{c} n \\ | \quad \quad \quad | \\ \text{B}_{3 \times 4} \end{array} = \begin{array}{c} n \\ | \quad \quad \quad | \\ \text{C}_{5 \times 4} \end{array}$$



https://en.wikipedia.org/wiki/Matrix_multiplication

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Solving Linear Equations

Problem:

$$\begin{aligned} 3x + 2y &= 12 \rightarrow (1) \\ 4x + 5y &= 23 \rightarrow (2) \end{aligned}$$

$$\begin{aligned} x &=? \\ y &=? \end{aligned}$$

Solution Approach:

$$\begin{aligned} x &= \frac{12 - 2y}{3} = 4 - \frac{2}{3}y \\ 4\left(4 - \frac{2}{3}y\right) + 5y &= 23 \\ 16 - \frac{8}{3}y + 5y &= 16 + \frac{15 - 8}{3}y = 16 + \frac{7}{3}y = 23 \\ y &= \frac{3(23 - 16)}{7} = \frac{3(7)}{7} = 3 \\ x &= 4 - \frac{2}{3}(3) = 4 - 2 = 2 \end{aligned}$$

$$\begin{aligned} x &= 2 \\ y &= 3 \end{aligned}$$

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Formulating the Problem in Matrix Form

$$\begin{aligned}x_1 &= x \\x_2 &= y\end{aligned}$$

$$3x_1 + 2x_2 = 12 \rightarrow (1)$$

$$4x_1 + 5x_2 = 23 \rightarrow (2)$$

Solve using Matrices:

$$\begin{bmatrix} 3 \\ 4 \end{bmatrix} x_1 + \begin{bmatrix} 2 \\ 5 \end{bmatrix} x_2 = \begin{bmatrix} 12 \\ 23 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 2 \\ 4 & 5 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 12 \\ 23 \end{bmatrix}$$

$$A = \begin{bmatrix} 3 & 2 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{11} \\ a_{11} & a_{11} \end{bmatrix}$$

$$\begin{bmatrix} a_{11} & a_{11} \\ a_{11} & a_{11} \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$Ax = b$$

$$b = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} 12 \\ 23 \end{bmatrix}$$

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Obtaining the Solution in Matrix Form

$$\begin{aligned}Ax &= b \\A^{-1}Ax &= A^{-1}b \\Ix &= A^{-1}b \\x &= A^{-1}b\end{aligned}$$

$$\text{Example } A = \begin{bmatrix} 3 & 2 \\ 4 & 5 \end{bmatrix}$$

$$b = \begin{bmatrix} 12 \\ 23 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} adj(A)$$

$$|A| = 3 \times 5 - 4 \times 2 = 7$$

$$\begin{aligned}x &= A^{-1}b \\x &= \left(\frac{1}{|A|} \times adj(A) \right) b\end{aligned}$$

$$A^{-1} = \frac{1}{7} \begin{bmatrix} 5 & -2 \\ -4 & 3 \end{bmatrix}$$

$$x = \frac{1}{7} \begin{bmatrix} 5 & -2 \\ -4 & 3 \end{bmatrix} \cdot \begin{bmatrix} 12 \\ 23 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

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Solving Linear Equations (General Form)

$$Ax = b$$

$$\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix}_{m \times n} \cdot \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}_{n \times 1} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}_{m \times 1}$$

$m = \text{number of linearly independent equations}$

$n = \text{number of unknowns } (x_i)$

$m = n \rightarrow \text{unique solution}$

$m < n \rightarrow \text{infinitely many solutions}$

$$x = A^{-1}b$$

$$\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}_{n \times 1} = \underbrace{\begin{bmatrix} a'_{11} & a'_{12} & \dots & a'_{1m} \\ a'_{21} & a'_{22} & \dots & a'_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ a'_{n1} & a'_{n2} & \dots & a'_{nm} \end{bmatrix}_{n \times m}}_{A^{-1}} \cdot \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}_{m \times 1}$$

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Linear Transformations

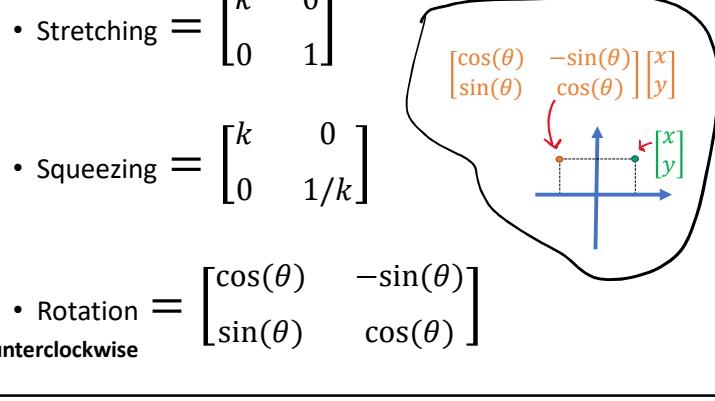
$$p = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$p' = T \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x' \\ y' \end{bmatrix}$$

- Stretching = $\begin{bmatrix} k & 0 \\ 0 & 1 \end{bmatrix}$

- Squeezing = $\begin{bmatrix} k & 0 \\ 0 & 1/k \end{bmatrix}$

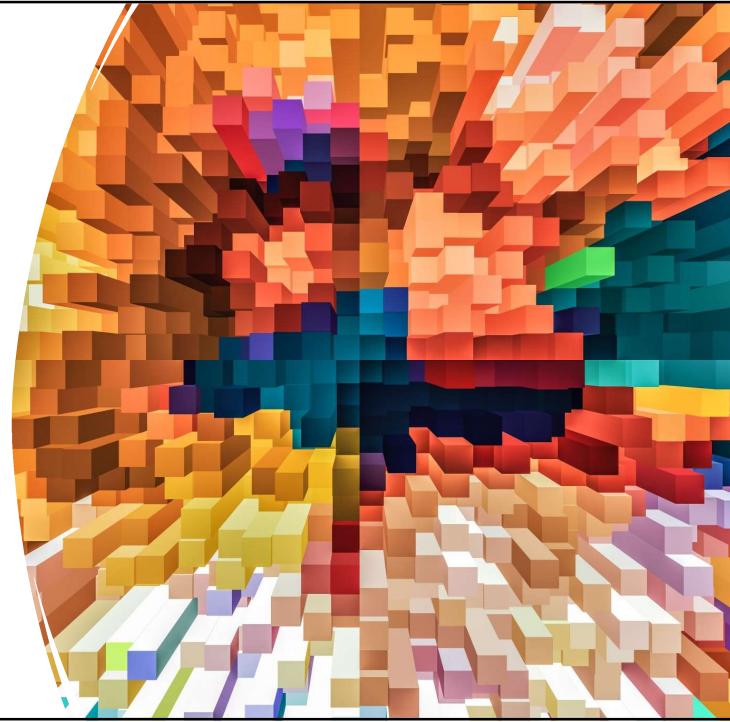
- Rotation = $\begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$
counterclockwise



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Sets

- Basics of Set Theory
- Set Operators
 - Union
 - Intersection
 - Complement
 - Difference



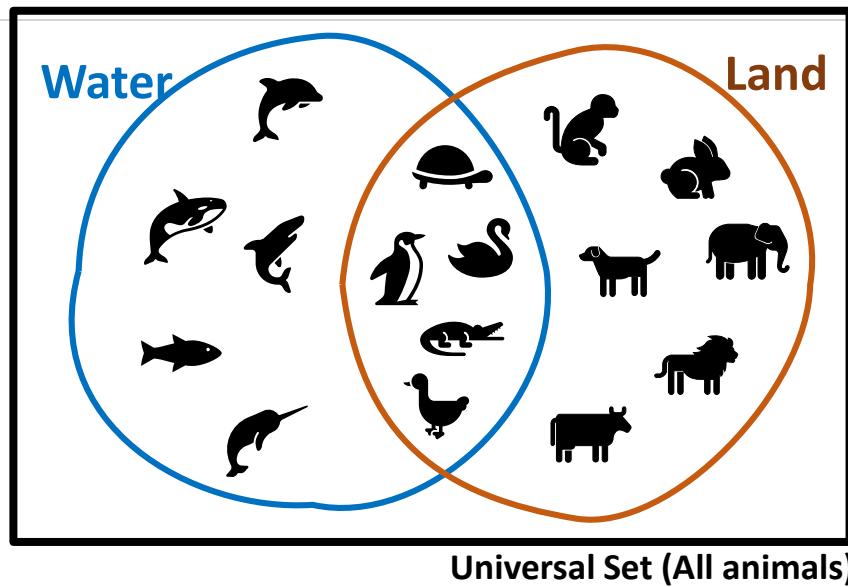
85

Basics of Set Theory

- A set is a collection of things (elements).
 - $A = \{\clubsuit, \diamond, \heartsuit, \spadesuit\}$
 - $B = \{\times, \div, +, -\}$
 - $C = \{apple, orange, mango, banana\}$
 - $D = \{x | x \text{ satisfies some property}\}$
 - $\mathbb{N} = \{1, 2, 3, 4, \dots\}$ is set of natural numbers
 - $\mathbb{Z} = \{\dots, -3, -2, -1, 0, 1, 2, 3, 4, \dots\}$ is set of integers
 - $E = \{x | x \in \mathbb{Z}, -2 \leq x < 10\}$
 - $\emptyset = \{\}$ is Null set
- Items belongs to a set (element of)
 - $\heartsuit \in A$
 - $\div \in B$
 - $apple \in C$
- Items not belongs to a set (not an element of)
 - $\mathbb{X} \notin A$
 - $*$ $\notin B$
 - $strawberry \notin C$
- Subset
 - $\{\diamond, \heartsuit\} \subset A$
 - $\mathbb{N} \subset \mathbb{Z}$

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Venn Diagrams



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Cardinality (size of the set)

$$A = \{\clubsuit, \diamondsuit, \heartsuit, \spadesuit\}$$

$$|A| = 4$$

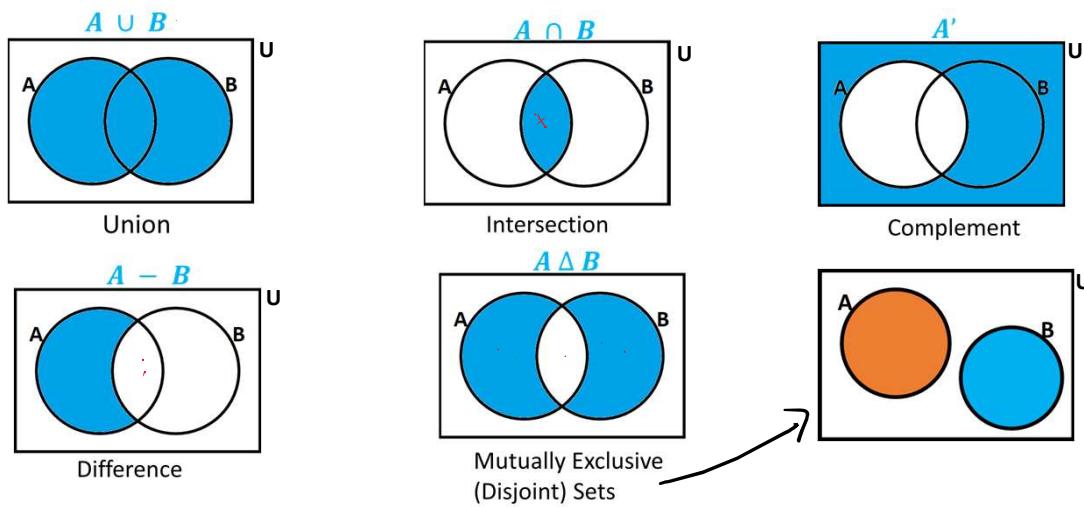
$$B = \{1, 2, 3, 4, 5, 6\}$$

$$|B| = 6$$

<https://en.wikipedia.org/wiki/Cardinality>

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Set Operations



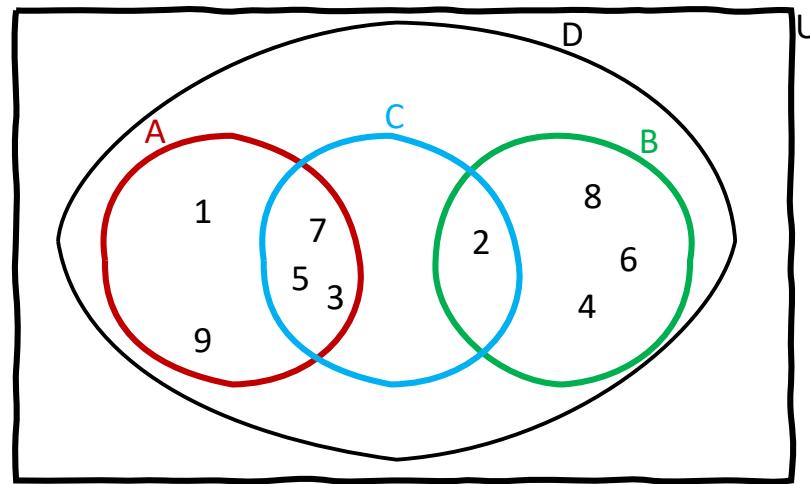
89

$$A = \{x | x \in \mathbb{Z}, 1 \leq x < 10, x \text{ is an odd number}\}$$

$$B = \{x | x \in \mathbb{Z}, 1 \leq x < 10, x \text{ is an even number}\}$$

$$C = \{x | x \in \mathbb{Z}, 1 \leq x < 10, x \text{ is a prime number}\}$$

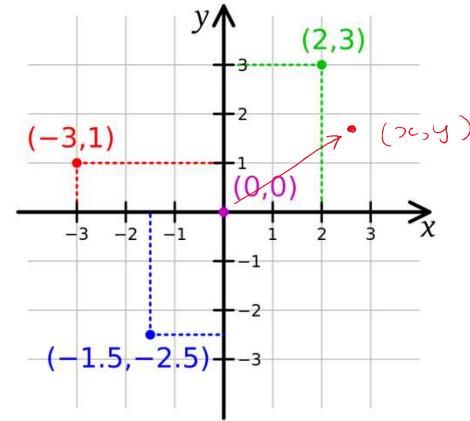
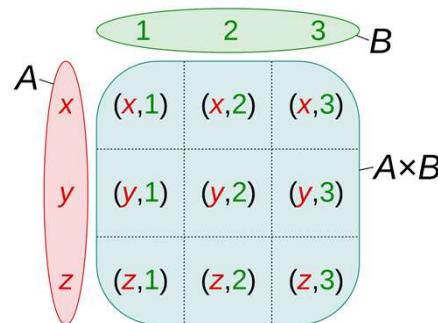
$$D = \{x | x \in \mathbb{Z}, 1 \leq x < 10\}$$



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Cartesian product

- $A \times B = \{(a, b) | a \in A \text{ and } b \in B\}$
- $A \times B \neq B \times A$

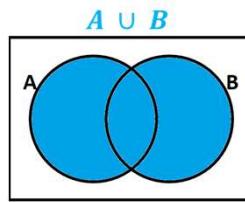


https://en.wikipedia.org/wiki/Cartesian_product

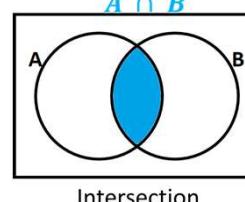
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Laws of Set Theory

- Identity
 - $A \cap U = A$
 - $A \cup \emptyset = A$



- Dominance
 - $A \cap \emptyset = \emptyset$
 - $A \cup U = U$



- Idempotence
 - $A \cap A = A$
 - $A \cup A = A$

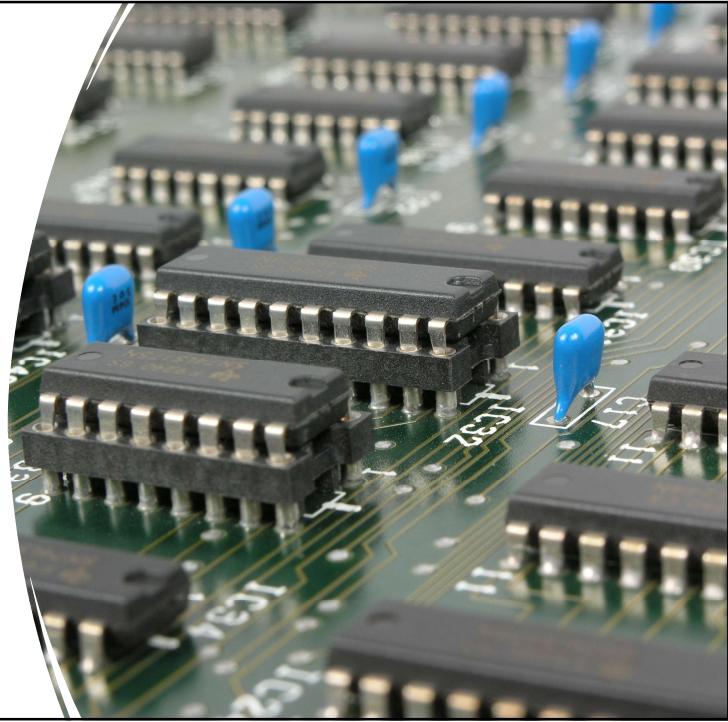
- Complement
 - $A \cap A' = \emptyset$
 - $A \cup A' = U$

- Double Compliment
 - $(A')' = A$

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Logic Gates

- Logical Statements
- Logical Operators
 - AND
 - OR
 - NOT



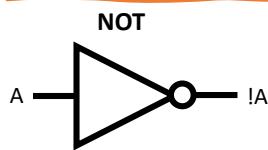
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Logical Statements and Binary Logic

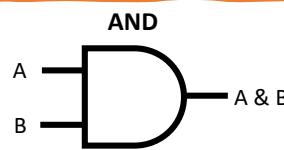
- TRUE = 1
- FALSE = 0
- “5 is an odd number” is **TRUE**
- “4 is a prime number” is **FALSE**
- “Kandy is the Capital City of Sri Lanka” is **FALSE**
- “A triangle has three sides” is **TRUE**

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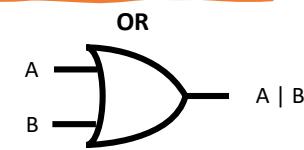
Logic Gates



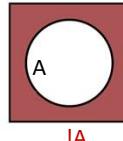
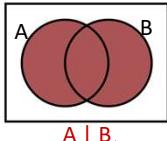
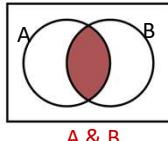
A	!A
1	0
0	1



A	B	A & B
1	1	1
1	0	0
0	1	0
0	0	0



A	B	A B
1	1	1
1	0	1
0	1	1
0	0	0



https://en.wikipedia.org/wiki/Boolean_algebra

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Laws of Boolean Algebra

- Identity
 - $A \& 1 = A$
 - $A | 0 = A$

• A is a Boolean variable
• A can take either 1 or 0

- Annihilator
 - $A \& 0 = 0$
 - $A | 1 = 1$

- Idempotence
 - $A \& A = A$
 - $A | A = A$

- Associativity
 - $A \& (B \& C) = (A \& B) \& C$
 - $A | (B | C) = (A | B) | C$

- Commutativity
 - $A \& B = B \& A$
 - $A | B = B | A$

- Distributivity
 - $A \& (B | C) = (A \& B) | (A \& C)$
 - $A | (B \& C) = (A | B) \& (A | C)$

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Laws of Boolean Algebra

- Complement
 - $A \& !A = 0$
 - $A | !A = 1$
- Double Negation
 - $!(!A) = A$
- De Morgan's laws:
 - $!A \& !B = !(A | B)$
 - $!A | !B = !(A \& B)$

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Exponents and Logarithms

	Exponent Representation	Logarithmic representation
Base n	$n^x = y$	$\log_n y = x$
Base 2	$2^3 = 8$	$\log_2 8 = 3$
Base 10	$10^3 = 1000$	$\log_{10} 1000 = 3$
Base e	$e^{6.907755} \approx 1000$	$\log_e 1000 \approx 6.907755$

$$\log_e(1000) = \ln(1000)$$

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Functions

- $y = f(x)$
- $f(x) = \frac{1}{x}$
- $f(x) = a_0 + a_1x + a_2x^2$
- $f(x) = a_0 + a_1x_1 + a_2x_2$
- $f(x) = a_0 + a_1x_1 + a_2x_2$
- *Step Functions*
 - $f(x) = \begin{cases} 0, & x < 0 \\ x, & x \geq 0 \end{cases}$
- *Logistic Function*
 - $f(x) = \frac{L}{1+e^{-k(x-x_0)}}$

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Series

- $S_n = \sum_{i=1}^n a_i = a_1 + a_2 + \dots + a_n$
- Example
 - Let $a_i = 2(i - 1) + 1$
 - $a_1 = 1$
 - $a_2 = 2 + 1 = 3$
 - $a_3 = 2 \times 2 + 1 = 5$
 - $a_4 = 2 \times 3 + 1 = 7$
 - $S_4 = 1 + 3 + 5 + 7 = 16$

[https://en.wikipedia.org/wiki/Series_\(mathematics\)](https://en.wikipedia.org/wiki/Series_(mathematics))

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Sequence

- $(a_i)_{i=1}^n = [a_0, a_1, a_2, \dots, a_n]$

- Example

- Let $a_i = i^2$
- $a_1 = 1$
- $a_2 = 2^2 = 4$
- $a_3 = 3^2 = 9$
- $a_4 = 4^2 = 16$

- $(i^2)_{i=1}^4 = 1, 4, 9, 16$

[https://en.wikipedia.org/wiki/Series_\(mathematics\)](https://en.wikipedia.org/wiki/Series_(mathematics))

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Trigonometric Functions

- $\sin(x)$
 - $\cos(x)$
 - $\tan(x)$
-
- Sound Signal:
 - amplitude = $\sin(\omega t) = \sin(2\pi ft)$

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Take-home Exercise

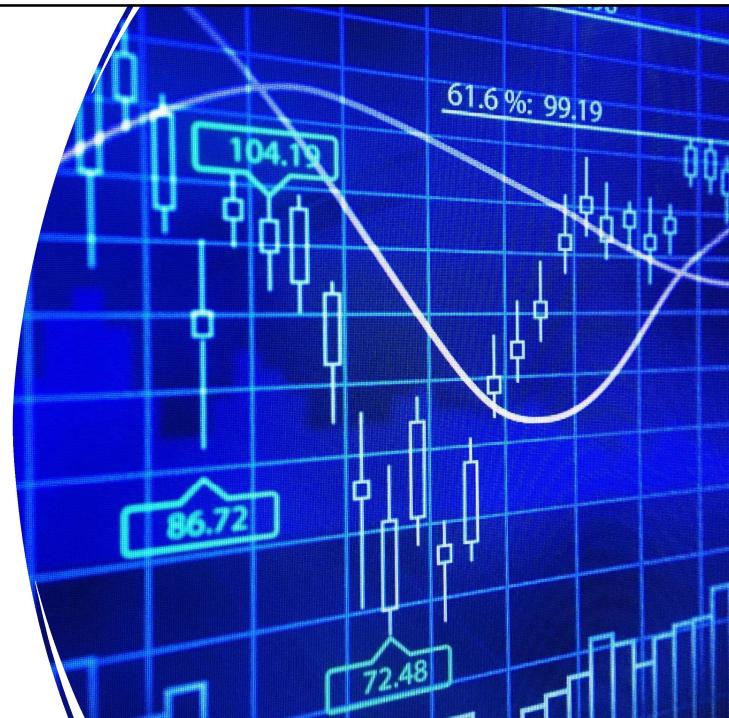
- Mention one machine learning problem (specify whether it is a classification, regression or clustering problem) and the training approach you would take (supervised or unsupervised).
- Your answer must be 240 characters or fewer.



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Probability and Statistics Concepts for Machine Learning

- Probability
- Statistical Distributions
- Descriptive Statistics



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Probability



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Probability Example

- Coin (single toss)
 - Possible outcomes: {Head, Tail}



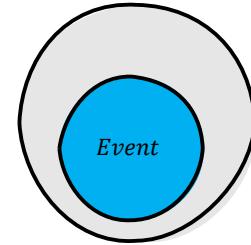
- Dice (single roll)
 - Possible outcomes: {1, 2, 3, 4, 5, 6!}



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Term Definitions

- **Experiment:** A procedure that can be repeated and has a well-defined set of possible outcomes.
 - **Random experiment:** outcome is unknown before the experiment. Results different outcomes when repeating the experiment in the same manner.
 - **Deterministic experiment:** outcome may be predicted with certainty beforehand. Has a definite outcome. Outcome is predictable.
- **Sample Space (Ω):** Set of all possible outcomes or results of an experiment.
- **Outcome:** Possible result of an experiment. An element of Ω .
- **Event:** A set of selected outcomes of an experiment. Subset of the sample space.
 - Independent Event: probabilities of occurrence do not depend on one another
- **Random Variable:** A variable that can take different values of the sample space (describes the outcomes of a random experiment).



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Probability

- Let $\Omega = \{\omega_1, \omega_2, \dots, \omega_n\}$ be a sample space of an experiment.
- Let the event $A \subset \Omega$
- Probability (How likely the event A can occur) : $P(A) = \frac{|A|}{|\Omega|}$
- $0 \leq P(A) \leq 1$
- Random Variable (X): $\{\omega \in \Omega | u < X(\omega) \leq v\}$
- Cumulative Probability: $\sum_x P(X = x) = 1$

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Probability Example

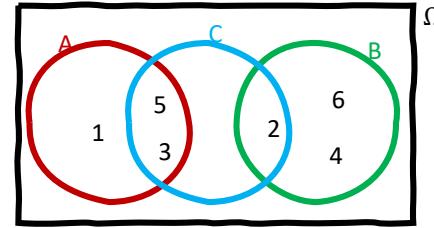
- Coin (single toss)

- Two possible outcomes
- $\Omega = \{H, T\}$
- $|\Omega| = 2$
- $P(A = \{H\}) = \frac{1}{2}$
- $P(A = \{T\}) = \frac{1}{2}$



- Dice (single roll)

- Six possible outcomes
- $\Omega = \{1, 2, 3, 4, 5, 6\}$
- $|\Omega| = 6$
- $P(A = \{1\}) = \frac{1}{6}$
- $P(A = \{6\}) = \frac{1}{6}$
- $P(A = \{1,3,5\}) = \frac{3}{6} = \frac{1}{2}$ (get odd number as outcome)



$$\text{Odd number: } P(A) = \frac{|A|}{|\Omega|} = \frac{1}{6} = \frac{1}{2}$$

$$\text{Even number: } P(B) = \frac{|B|}{|\Omega|} = \frac{1}{2}$$

$$\text{Prime number: } P(C) = \frac{|C|}{|\Omega|} = \frac{1}{2}$$

$$P(A \cup C) = \frac{|A \cup C|}{|\Omega|} = \frac{4}{6} = \frac{2}{3}$$

$$P(A \cap C) = \frac{|A \cap C|}{|\Omega|} = \frac{2}{6} = \frac{1}{3}$$

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Conditional Probability

- Probability of A given B : $P(A|B) = \frac{P(A \cap B)}{P(B)}$, if $P(B) \neq 0$

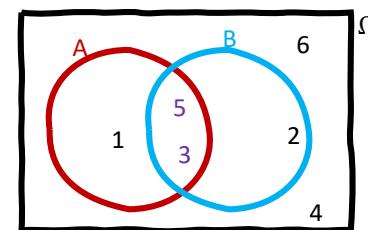
- Example

- $A = \{2, 3, 5\}$, prime numbers
- $B = \{1, 3, 5\}$, odd numbers



- If the outcome is an odd number, probability of that Number being a prime.

- $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{3}}{\frac{1}{2}} = \frac{2}{3}$
- $P(A \cap B) = \frac{|A \cap B|}{|\Omega|} = \frac{|\{3, 5\}|}{|\{1, 2, 3, 4, 5, 6\}|} = \frac{2}{6} = \frac{1}{3}$
- $P(B) = \frac{|B|}{|\Omega|} = \frac{|\{1, 3, 5\}|}{|\{1, 2, 3, 4, 5, 6\}|} = \frac{3}{6} = \frac{1}{2}$



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Conditional Probability: Example

- A: Selected person is vaccinated
- B: Age of the selected person ≥ 30
- If a person with age ≥ 30 is selected, probability that the person is vaccinated?
 - $P(A) = \frac{|A|}{|\Omega|} = \frac{55}{100}$
 - $P(A \cap B) = \frac{|A \cap B|}{|\Omega|} = \frac{45}{100}$
 - $P(B) = \frac{|B|}{|\Omega|} = \frac{50}{100}$
 - $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{45}{100}}{\frac{50}{100}} = \frac{45}{50} = 0.9$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

B

	Vaccinated (YES)	Vaccinated (NO)	Total
Age < 30	10	40	50
Age ≥ 30	45	5	50
Total	55	45	100

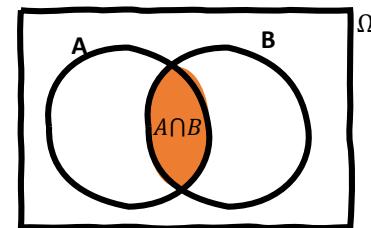
A

90 %

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Bayes Rule

- Conditional Probability of A given B
 - $P(A|B) = \frac{P(A \cap B)}{P(B)}$, if $P(B) \neq 0 \rightarrow (1)$
- Commutativity in Set theory
 - $P(A \cap B) = P(B \cap A) \rightarrow (2)$
- (2) in (1)
 - $P(B \cap A) = P(A|B) \cdot P(B) \rightarrow (3)$
- Conditional Probability of B given A
 - $P(B|A) = \frac{P(B \cap A)}{P(A)} \rightarrow (4)$
- (3) in (4)
 - $P(B|A) = \frac{P(A|B) \cdot P(B)}{P(A)}$, if $P(A) \neq 0$



https://en.wikipedia.org/wiki/Bayes%27_theorem

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Conditional Probability: Example

Events

- I : Infected
- N : Not infected
- T_P : Positive Test Result
- T_N : Negative Test Result

Measurements/Probabilities

- known*
- Infected population ratio: $P(I) = 0.04 = 4\%$
 - Positive Test Result if infected: $P(T_P|I) = 0.98 = 98\%$
 - Positive Test Result if not infected: $P(T_P|N) = 0.01 = 1\%$
 - A person get positive result regardless of the infection status: $P(T_P)$
 - $P(N) = 1 - P(I)$
 - $P(T_P) = \sum_{x=\{I,N\}} P(x)P(T_P|x)$
 - $P(T_P) = P(I)P(T_P|I) + P(N)P(T_P|N)$
 - $P(T_P) = 0.04 \times 0.98 + 0.96 \times 0.01 = 0.0488$

John got a positive test result. What is probability that John is infected.

$$P(I|T_P) = \frac{P(T_P|I) \cdot P(I)}{P(T_P)} = \frac{0.98 \times 0.04}{0.0488} = 0.803$$

80 %

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Probability Distributions

Discrete

- Probability distribution of a random variable that can take only a finite set of values.
- Probability mass function (PMF)
 - $\sum_{x \in X} P(x) = 1$

Continuous

- Probability distribution of a random variable that can take an infinite number of values.
- Probability density function (PDF)
 - $\int P(x) dx = 1$

https://en.wikipedia.org/wiki/Probability_distribution

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Combinatorics

- Permutations

- Order matters
- Take without repetition
- $P_x^n = \frac{n!}{(n-x)!}$

Example

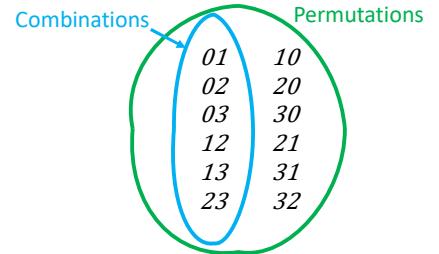
Set of Digits

- $S = \{0, 1, 2, 3\}$
- $|S| = 4$

Pick 2 elements (digits at a time) without repetition.

- Combinations

- Order does not matter
- Can take with or without repetitions
- $C_x^n = \frac{n!}{x! (n-x)!}$



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Permutations and Combinations

Set of Digits

- $S = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$
- $|S| = 10$

$$P_x^n = \frac{n!}{(n-x)!}$$

$$C_x^n = \frac{n!}{x! (n-x)!}$$

Pick 4 elements (digits at a time)

E.g.,

- $A = 1234$
- $B = 2341$

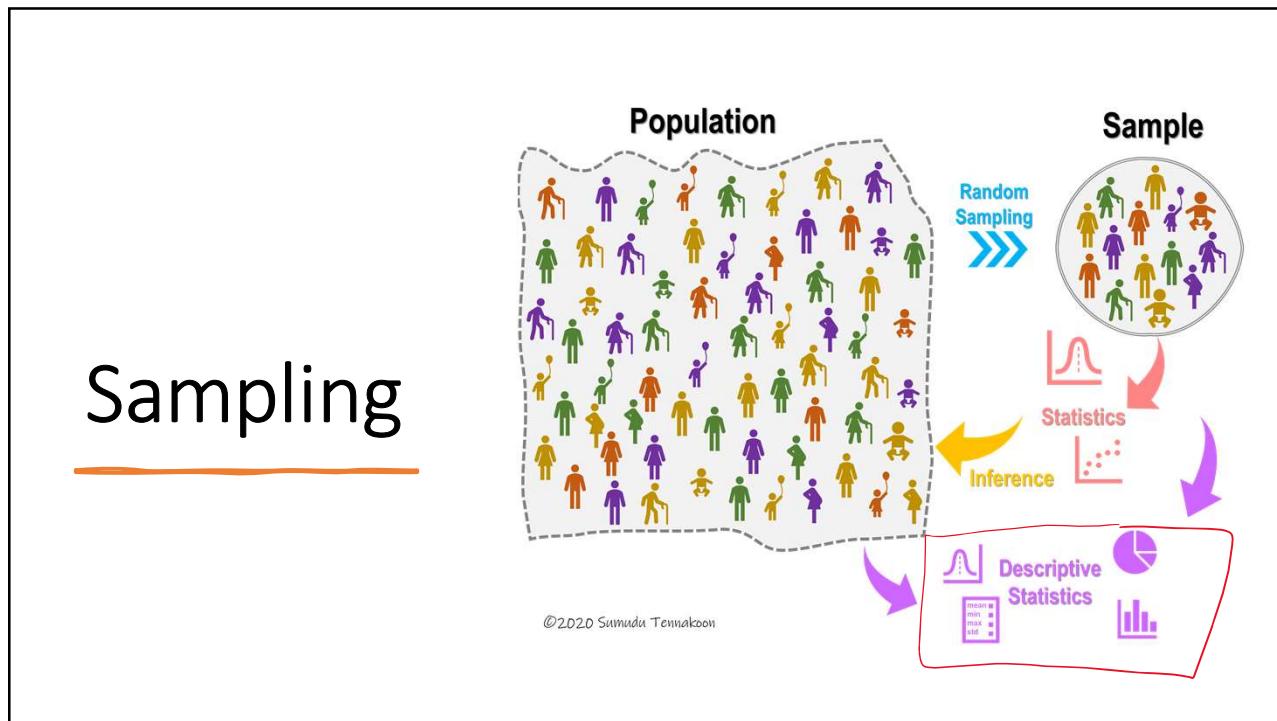
How many permutations can be made?

How many combinations can be made?

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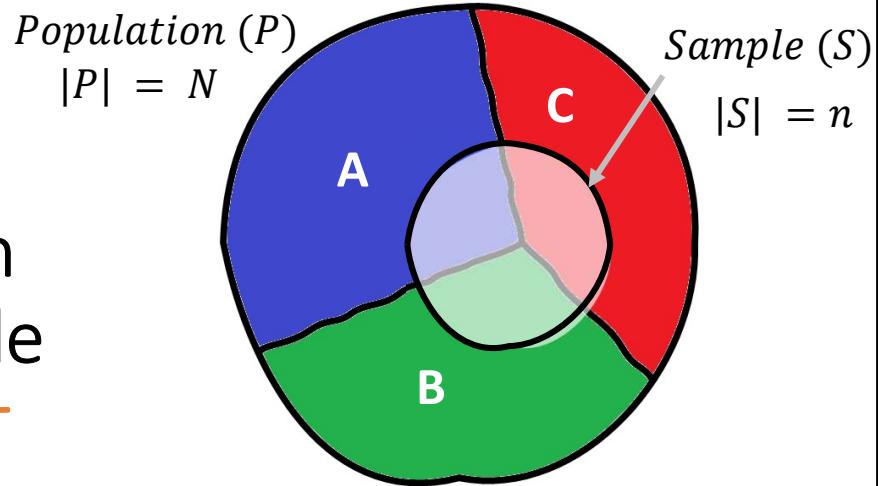


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Population and Sample



- $P = A \cup B \cup C$
- $S \subset P$
- $S = (A \cap S) \cup (B \cap S) \cup (C \cap S)$
- $|S| = n = |A \cap S| + |B \cap S| + |C \cap S| = a + b + c$

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Sampling Method depends on

- Nature of population
- Regulatory Restrictions
- Recourses available
 - Time
 - Money
 - Data Collectors/Probes

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Sampling Considerations

Unbiased: each element is equally likely to be chosen

Representative of the Population

Minimize the **Sampling Error**

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Probabilistic Sampling Techniques

Simple Random Sampling

Stratified Sampling

Systematic sampling

Cluster Sampling

...

[https://en.wikipedia.org/wiki/Sampling_\(statistics\)](https://en.wikipedia.org/wiki/Sampling_(statistics))
https://en.wikipedia.org/wiki/Stratified_sampling

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Non- Probabilistic Sampling Techniques

Convenience Sampling

Judgmental Sampling

Quota sampling

Snowball Sampling

...

[https://en.wikipedia.org/wiki/Sampling_\(statistics\)](https://en.wikipedia.org/wiki/Sampling_(statistics))

https://en.wikipedia.org/wiki/Stratified_sampling

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Types of Statistics

Descriptive Statistics: Organizing and summarizing data. Use data from the population or sample.

Mean, Median, Mode, Range, Variance

Inferential Statistics: Use the sample data to make an inference or draw the population's conclusion. It uses probability to find out the confidence of the predictions we make.

Distributions, Regression, ...

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Types of Analytics

Descriptive: What do we have now/ had

Predictive: What can happen if the conditions are set to

Prescriptive: Define rules/ constraints

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Descriptive Statistics

- Range
- Mean
- Median
- Mode
- Variance
- Standard Deviation
- Coefficient of Variation
- Co-variance
- Co-relation Coefficient

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Range, Mean, Median and Mod

- Range
 - Max Value - Min value
- Mean
 - Population Mean: $\mu = \frac{\sum_{i=1}^n x_i}{N}$
 - Sample Mean: $\bar{x} = \frac{\sum_{i=1}^n x_i}{n}$
 - Has influence of outliers
- Weighted Mean (w_i s are weights)
 - $\bar{x}_w = \frac{\sum_{i=1}^n w_i x_i}{\sum_{i=1}^n w_i}$
- Geometric Mean:
 - $\bar{x}_g = \sqrt[n]{\prod_{i=1}^n x_i}$
- Median:
 - Mid value when ordered
 - Removes influence of outliers
- Mode
 - Value that occur most often

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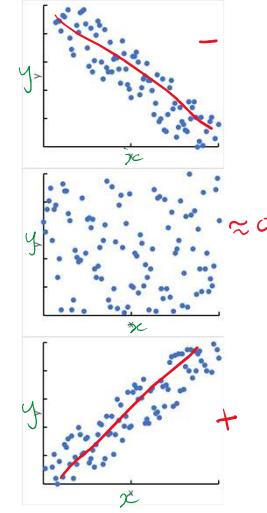
Variance , Standard Deviation and Coefficient of variance

- Variance
 - How data spread around mean
- Population : $\sigma^2 = \frac{\sum_{i=1}^n (x_i - \mu)^2}{N}$
- Sample : $S^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1}$
- Standard Deviation (STD)
 - Population : $\sigma = \sqrt{\frac{\sum_{i=1}^n (x_i - \mu)^2}{N}}$
 - Sample : $S = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1}}$
- Coefficient of variance (CV)
 - Also known as relative standard deviation (RSD)
 - Population : $CV = \frac{\sigma}{\mu}$
 - Sample : $CV = \frac{S}{\bar{x}}$

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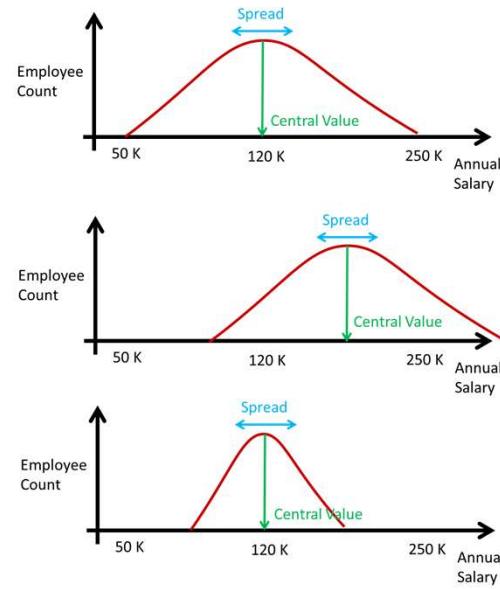
Co-variance and Co-relation Coefficient

- Co-variance:
 - Check whether 2 variables moving together
 - $\text{COV}(x,y) = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{n-1}$
 - $\text{COV} \begin{cases} < 0, & \text{if variables moving opposite direction} \\ = 0, & \text{if independent variables} \\ > 0, & \text{if variables moving together} \end{cases}$
- Co-relation Coefficient
 - $r = \frac{\text{COV}(x,y)}{S(x) \cdot S(y)}$
 - $-1 \leq r \leq 1$
 - $r = 1$ and $r = -1$: perfect corelation
 - $r \begin{cases} < 0, & \text{if negative corelation} \\ = 0, & \text{if independent variables} \\ > 0, & \text{if positive corelation} \end{cases}$



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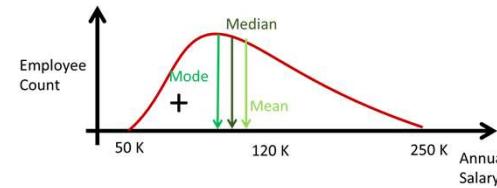
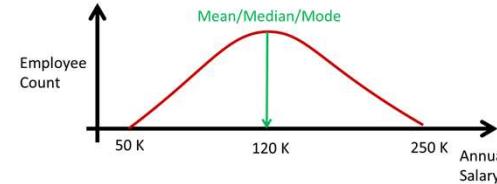
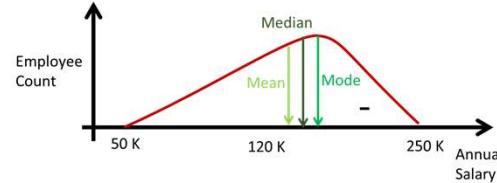
Distributions Central value and and Spread



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Distributions

Skewness



<https://en.wikipedia.org/wiki/Skewness>

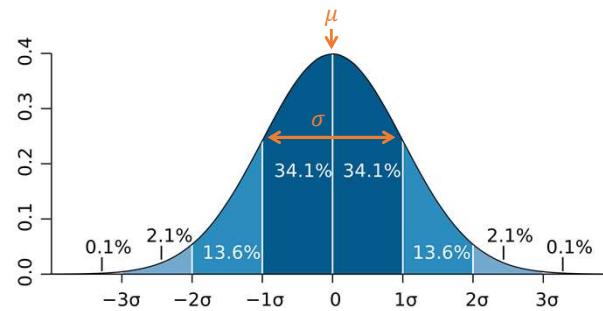
<https://docs.scipy.org/doc/scipy/reference/generated/scipy.stats.skew.html>

<https://mathworld.wolfram.com/Pearson'sSkewnessCoefficients.html>

131

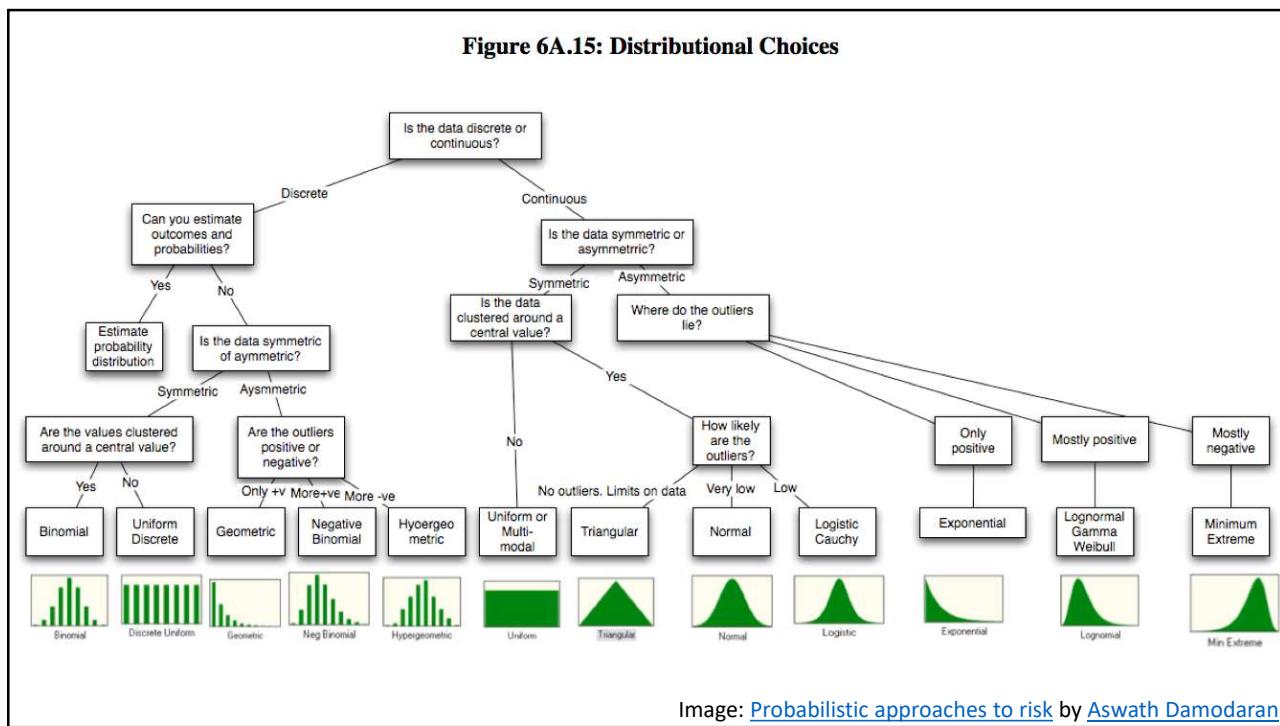
Standard Normal Distribution

- Standard Deviation (σ) = 1
- Mean (μ) = 0 = Median = Mode



https://en.wikipedia.org/wiki/Probability_distribution

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Figure 6A.15: Distributional ChoicesImage: [Probabilistic approaches to risk](#) by [Aswath Damodaran](#)

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