CS 482 Machine Learning

TEST 1

Chapters 1- Introduction, Chapter 2: KNN, Chapter 3: Linear Models
Time Allocated: 45 minutes for CS-482
60 minutes for CS-682

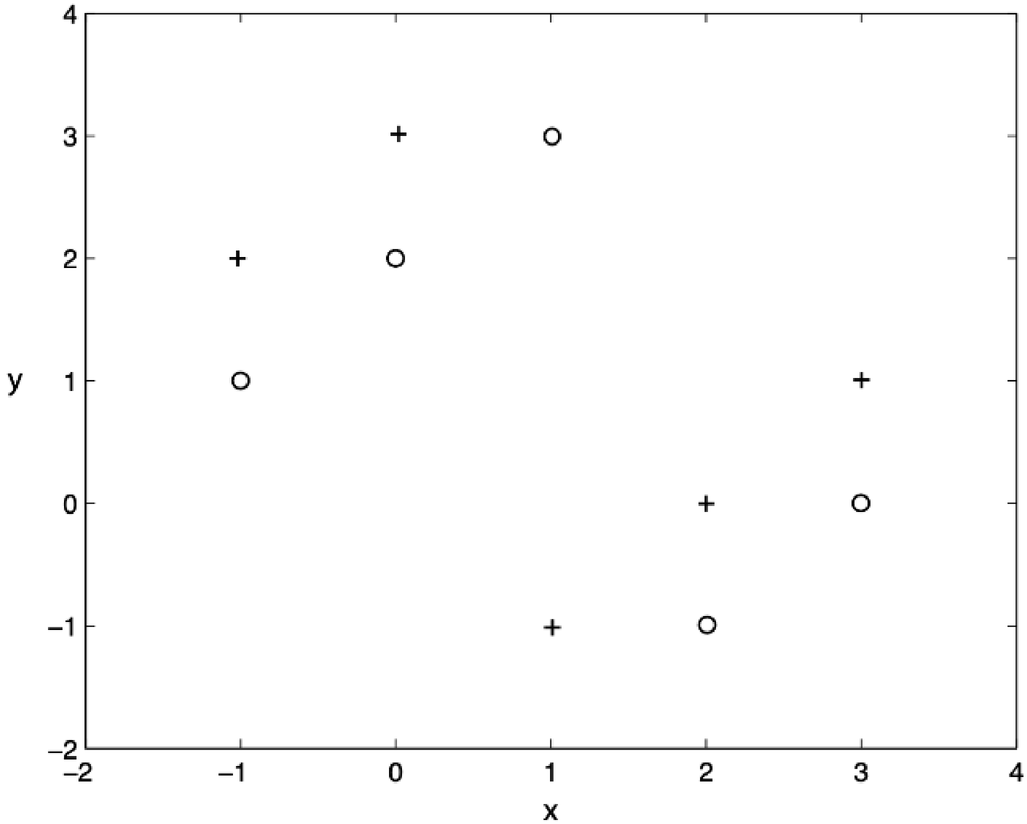
Your Name: __KEY_________________________

You can use the formula sheet (printed version) and a non-programmable calculator.

1. (4 points) Define the terms overfitting and underfitting. How would you check if your model is overfitting and how would you check if your model is underfitting?

   Overfitting occurs when the model is too "close" to the data and does not have the big picture of the data. Underfitting is occurs when the model does not capture the trends in the data. When the model performs extremely well on training set but does poorly on test set, then there is overfitting. When the model does poorly on training set then the model is underfitting

2. (2 points) Consider K-NN using Euclidean distance on the following data set with points shown in + and o. (each point belongs to one of two classes: + and o).



a) Classify the unknown point (1.7, -1) using 1-NN
b) Classify the unknown point (1.7, -1) using 3-NN

a) The point (1.7, -1) will be o (circle) with 1-NN
b) The point (1.7, -1) sill be + (plus) with 3-NN

1. (5 points) Given the following table

Predicted	Observed	
	Cats	Dogs
Cats	*3(TP)*	*4(FP)*
Dogs	*5(TN)*	*4 (FN)*

Compute Accuracy, Recall, Precision and Specificity and f-statistic (provide answers up to two decimal places)

Accuracy = (FP+FN)/(TP+TN+FP + FN) = (5+4) / (5+4+3+4) = 9/16 =.56
Recall = TP/(TP+FN) = 3/(3+4) = 3/7 = .42
Precision = TP/(TP+FP) = 3/(3+4) =3/7 =.42
Specificity = TN/(TN+FP) = 5/(5+4) = 5/9 =.55
f-statistic = 2 * [precision*recall] /(precision+recall) = 2 * (.42 *.42)/(.42 + .42) = 2(.18/.84) =0.42

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3. (6+ 1+ 2 = 9 points) Given the following predictor matrix P with 1 predictor and sample size of 2 for linear regression,

Predictor matrix, P = $\begin{bmatrix} 2 \\ 4 \end{bmatrix}$ and the target y, y = $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$

a) Find the parameters of the linear model β

b) Make prediction for the newdata [3] and [1].

c) Using the 2 and 4 are target values for the test data in b), find the metrics R^2 and RMSE

SOLUTION

a) $\beta = (X^TX)^{(-1)}\, X^Ty$

$X = \begin{bmatrix} 1 & 2 \\ 1 & 4 \end{bmatrix}$

$X^T = \begin{bmatrix} 1 & 1 \\ 2 & 4 \end{bmatrix}$

$X^T * X = \begin{bmatrix} 1 & 1 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 1*1+1*1 & 1*2+1*4 \\ 2*1+4*1 & 2*2+4*4 \end{bmatrix} = \begin{bmatrix} 2 & 6 \\ 6 & 20 \end{bmatrix}$

$(X^T * X)^{(-1)} = \frac{1}{2*20-36} * \begin{bmatrix} 20 & -6 \\ -6 & 2 \end{bmatrix} = \frac{1}{4} * \begin{bmatrix} 20 & -6 \\ -6 & 2 \end{bmatrix}$

$X^T * y = \begin{bmatrix} 1 & 1 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 8 \end{bmatrix}$

$\beta = (X^TX)^{(-1)}\, X^Ty =) = \frac{1}{4} * \begin{bmatrix} 20 & -6 \\ -6 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 8 \end{bmatrix} = \frac{1}{4} * \begin{bmatrix} 12 \\ -2 \end{bmatrix} = = \begin{bmatrix} 3 \\ -1/2 \end{bmatrix}$

$\hat{y} = 3 - (1/2)x_1$

b) Prediction for 3 $\quad \hat{y} = 3 - (1/2)3 = 3 - 3/2 = 3/2$
Prediction for 1 is $\quad \hat{y} = 3 - (1/2) = 5/2$

c) The correct values are 2 and 4 $\quad R^2 = 1 - \sum_{i=1}^{i=2} \frac{(y_i - \hat{y}_i)^2}{(y_i - \bar{y})^2}$

Mean of y = 3/2 = 1.5

R^2 =1 – [(3-3/2)2 + (4-5/2)2]/[(2.-1.5)2 + (1-1.5)2 = 1 – [(1.5)2 + (1.5)2] /[.25 + .25]
= 1 – [(2.25 + 2.25)/.5] = 1 – 4.5/.5 = -8

d) RMSE = 4.5

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4. (6 + 1 + 2= 9 points) Given the following predictor matrix X with 2 predictors and sample size of 3 for linear regression,

Predictor matrix, $P = \begin{bmatrix} 2 & 2 \\ 1 & 2 \\ 2 & 1 \end{bmatrix}$ and the target y, $y = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$

a) Find the parameters of the linear model β

b) Make prediction for the new data [3 2] and [1 4].

c) Using the 3 and 1 are target values for the test data in b), find the metrics R^2 and RMSE

a) $\beta = (X^TX)^{(-1)} X^Ty$

$X = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 1 & 9 & 1 \end{bmatrix}$

$X^T = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 4 & 9 \\ 3 & 2 & 1 \end{bmatrix}$

$X^T * X = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 4 & 9 \\ 3 & 2 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 1 & 9 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 15 & 6 \\ 15 & 101 & 23 \\ 6 & 23 & 14 \end{bmatrix}$

$\det(X^T * X) = 3*(101*14 - 23*23) – 15 *(15*14 - 6*23) + 6*(15*23 - 101*6) = 2655 - 1080 - 1566 = 9$

$$\text{Co-factor matrix} = \begin{bmatrix} 885 & -72 & -261 \\ -72 & 6 & 21 \\ -261 & 21 & 78 \end{bmatrix}$$

$$(X^T * X)^{(-1)} = \frac{1}{9} * \begin{bmatrix} 885 & -72 & -261 \\ -72 & 6 & 21 \\ -261 & 21 & 78 \end{bmatrix} == \begin{bmatrix} 98.33 & -8 & -29 \\ -8 & 0.66 & 2.33 \\ -29 & 2.33 & 8.66 \end{bmatrix}$$

$$X^T * y = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 4 & 9 \\ 3 & 2 & 1 \end{bmatrix} * \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix} = \begin{bmatrix} 7 \\ 44 \\ 12 \end{bmatrix}$$

$$\beta = (X^T X)^{(-1)} X^T y =) = \begin{bmatrix} 98.33 & -8 & -29 \\ -8 & 0.66 & 2.33 \\ -29 & 2.33 & 8.66 \end{bmatrix} * \begin{bmatrix} 7 \\ 44 \\ 12 \end{bmatrix} = \begin{bmatrix} -35/13 \\ 4/3 \\ 11/3 \end{bmatrix} = \begin{bmatrix} -11.66 \\ 1.33 \\ 3.66 \end{bmatrix}$$

b)

$$R^2 = 1 - \sum_{i=1}^{i=3} \frac{(y_i - \hat{y}_i)^2}{(y_i - \bar{y})^2}$$