Theory of Computation

CS-312 **Fall 2023**

Exam 3 **DUE FRIDAY, 15 December 2023, NOON**

Instructions

- Complete all problems separately; each problem indicates the number of points possible. Show your work; partial credit may be awarded.

- This exam is *open-book*, *open notes*. You are permitted to consult the textbook, your own class notes, class handouts, and class homework and solution sets. All information available from the course Blackboard site is permitted.

  You may use JFLAP to develop your solutions, *except where using JFLAP would defeat the purpose of the question.* You may *always* use the JFLAP automata editor to draw your solutions.

  No other outside sources, including Internet sources or generative AI tools, are permitted. No collaboration with others is permitted. Violations are subject to severe penalties, up to and including course failure.

  EXCEPTION: You may use outside sources to solve the extra credit problem.

- Your answers must be submitted electronically via Blackboard by NOON on Friday, 15 December 2023 (11th Friday). ***Retain your electronic copy of your completed answers, in case of difficulties.***

By submitting your examination, you certify that you have neither given nor received any unauthorized assistance on this examination.

1. *15 points.* Let $w \in \{0,1\}^*$ be a non-empty binary string. Give a detailed description (*i.e.* a state-machine diagram) of a deterministic one-tape Turing machine which, when started with w on the tape as input, halts with the binary string ww on its tape. For example, when started with 01101 on its tape, the machine should halt with 0110101101 on its tape.

2. *15 points.* Give a detailed description (*i.e.* a state-machine diagram) of a deterministic one-tape Turing machine which decides the language $\{0^n 1^n 0^n 1^n : n \geq 0\}$. For example, 000111000111 is in this language.

3. *15 points.* A *Chalkboard Turing Machine (CTM)* is a Turing machine with the following restriction: the machine may never overwrite a non-blank symbol with a blank symbol. A CTM may leave an existing blank symbol unchanged; however, once a blank symbol has been overwritten by a non-blank symbol, it cannot be restored to a blank symbol again. (Symbolically: if $\delta(q, \alpha) = (r, \Box, d)$ is a transition of the CTM, then $\alpha = \Box$.)

   Show that Chalkboard Turing Machines are equivalent to classic Turing machines.

   (Note that this is an "iff" claim and thus requires two proofs.)

4. Let L be the set of Turing machines that reject at leastn two different input strings.

   (a) *15 points.* Show that L is recursively-enumerable (*i.e.*, Turing-acceptable).

   (b) *15 points.* Show that L is not recursive (*i.e.*, undecidable).

5. Recall the definition of the $\mathcal{NP}$-complete problem SATISFIABILITY: given a Boolean propositional logic formula ϕ, does ϕ have a satisfying truth assignment?

   We define a related problem called DOUBLE-SAT: given a Boolean propositional logic formula ϕ, does ϕ have two different satisfying truth assignments?

   (a) *10 points.* Show that DOUBLE-SAT is in $\mathcal{NP}$.

   (b) *15 points.* Show that DOUBLE-SAT is $\mathcal{NP}$-complete, by reducing SATISFIABILITY to DOUBLE-SAT. That is, show how to use an algorithm that solves DOUBLE-SAT to solve SATISFIABILITY.

6. *Extra Credit (worth 1 point added to final course grade):*

   The five homework groups for this course were named after colors: black, blue, green, red, and yellow. These five colors are used together in a commonly-recognized symbol. What is that symbol?

   (**Hint**: sometimes, white is considered to be a sixth color belonging to this group.)

   (**Reminder**: you are permitted to use outside sources to answer this question.)