CE-210 Digital Systems I
Solution of Assignment 1

1- Expand the following decimal number as a sum of weighted decimal digits:

$215692 = 2 \times 10^0 + 9 \times 10^1 + 6 \times 10^2 + 5 \times 10^3 + 1 \times 10^4 + 2 \times 10^5$

2- Convert the following binary numbers to decimal. Show your work.

10 000 000 000 (this is called 1K)

$10\ 000\ 000\ 000 = 0 \times 2^0 + 0 \times 2^1 + 0 \times 2^2 + 0 \times 2^3 + 0 \times 2^4 + 0 \times 2^5 + 0 \times 2^6 + 0 \times 2^7 + 0 \times 2^8 + 0 \times 2^9 + 1 \times 2^{10} = 1024$

$\boxed{10\ 000\ 000\ 000 = 1024}$

$100101 = 1 \times 2^0 + 0 \times 2^1 + 1 \times 2^2 + 0 \times 2^3 + 0 \times 2^4 + 1 \times 2^5$

$\boxed{100101 = 37}$

$= 1 + 4 + 32 = 37$

$10100101 = 1 \times 2^0 + 0 \times 2^1 + 1 \times 2^2 + 0 \times 2^3 + 0 \times 2^4 + 1 \times 2^5 + 0 \times 2^6 + 1 \times 2^7$

$= 1 + 4 + 32 + 128 = (165)_{ten}$

$\boxed{10100101 = (165)_{ten}}$

3- Using the "successive division" algorithm, convert the following decimal numbers to binary. Show your work.

$199 = (11\ 000\ 111)_2$, $69 = (1000101)_2$, and $19 = (10011)_2$

4- How many numbers are there in the closed interval [147:319]? Notice that the interval includes 147 and 319.

$319 – 147 + 1 = 173$

173

5- How many different 8-bit patterns are there? Show it in decimal.

$2^8 = 256$

256

6- What is the largest 7-bit number? Show it in binary and decimal.

$2^7 – 1$ = decimal 127 = binary 111 1111

$(127)_{ten} = (111\ 1111)_{two}$

7- How many bits (n) do we need to assign one unique bit pattern (address) to each room in a 74-room building? Show your work.

Substitute 74 for P in (2) on page 4, Chapter 1:

$n = \log_2 74$.

One way to manually obtain (estimate) $\log_2 74$ (or $\log_2$ of any small number) is *trial and error* after we take a reasonable initial guess. (See Example 10 on page 4, Chapter 1)

$\log_2 74 = 6. \ldots \rightarrow 7$ bits

Here is a different approach:

Remember that to represent 74 choices we need as many bits as the number of bits that we need to represent 74 – 1.

74 – 1 = 73 = 1 0 0 1 0 0 1, which needs 7 bits.

No of bits = 7

Proof:

We assume that the 74 rooms are numbered 0 to 73 (number of choices = 73 + 1). Therefore, the largest number that we need to represent is 73. But 73 is equivalent to 1 0 0 1 0 0 1; so we need 7 bits to assign one unique bit pattern to each room.

8- A digital adder adds two numbers, A and B, each up to 63 (decimal). Determine the number of choices that we, as the user of this adder, have for each input, A or B, and also the number of choices that we have for the output (result). Notice that the output includes the final carry bit. How wide (in term of bits) is each input (A or B) of this adder? How wide is the result (output)? Show your work.

Range of each input = [0:63]

No of choices for each input = 64

No of choices for each input = 63 + 1 = 64

Result min = 0

Result max = 63 + 63 = 126

No of choices for result = 127

Range of result = [0:126]

No of choices for result = 126 + 1 = 127

No of bits for each input = $\log_2 64 = 6$

No of bits for result = $\log_2 127 = 6. \ldots \rightarrow 7$

Width of each input = 6 bits

Output width = 7 bits

9- Refereeing to Figure 11*a* in Chapter 1, circle the right choice for each of the following statements:

- A high temperature and a high pressure (together) are not sufficient to sound the alarm. True False
- The unattended plant would necessarily make the alarm sound. True False
- To shut off the alarm the operator's supervision is sufficient. True False

10- A digital comparator takes two 2-bit operands (A: a_1a_0 & B: b_1b_0), compares them and generates three single-bit outputs, A>B, A=B, and A<B, with obvious meanings. Fill in the following truth table (with three outputs) to describe this comparator. As an example output = 0 1 0 means A = B.

Row	a_1 a_0	b_1 b_0	A>B	A= B	A<B	Row	a_1 a_0	b_1 b_0	A>B	A=B	A<B
0	0 0	0 0	0	1	0	8	1 0	0 0	1	0	0
1	0 0	0 1	0	0	1	9	1 0	0 1	1	0	0
2	0 0	1 0	0	0	1	10	1 0	1 0	0	1	0
3	0 0	1 1	0	0	1	11	1 0	1 1	0	0	1
4	0 1	0 0	1	0	0	12	1 1	0 0	1	0	0
5	0 1	0 1	0	1	0	13	1 1	0 1	1	0	0
6	0 1	1 0	0	0	1	14	1 1	1 0	1	0	0
7	0 1	1 1	0	0	1	15	1 1	1 1	0	1	0