CS 482-Machine Learning
Formulas and Math Tutorial

1. Regression metric RMSE

$$\text{MSE} = \frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2, \qquad \text{RMSE} = \sqrt{MSE}$$

2. Regression metric R^2

$$R^2 = 1 - \frac{\sum (y_i - \hat{y}_i)^2}{\sum (y_i - \vec{y})^2}$$

3. Correlation r

$$r = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum (x_i - \bar{x})^2 \sum (y_i - \bar{y})^2}}$$

$\bar{x}$ is the mean of the values in vector x and $\overline{y}$ Is the mean of the values in vector y

4. Linear Model

$$\beta = (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T y$$

5. Distance Formula between two points

Distance formula

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

d = distance

(x_1, y_1) = coordinates of the first point

(x_2, y_2) = coordinates of the second point

6. Softmax probability p_l

Whenever one wants to change a set of predicted values $\hat{y}_l$ values into probabilities so they all add to 1, softmax formula given below can be used.

$$\hat{p}_\ell^* = \frac{e^{\hat{y}_\ell}}{\sum_{l=1}^{C} e^{\hat{y}_l}}$$

7. Classification Metrics

$$\text{Accuracy} = \frac{\text{TP+TN}}{\text{TP+TN + FP + FN}} \qquad \text{Precision} = \frac{\text{TP}}{\text{TP+FP}}$$

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$$\text{Recall} = \frac{\text{TP}}{\text{TP+FN}} \qquad \text{Specificity} = \frac{\text{TN}}{\text{TN + FP}}$$

$$\text{F} = 2 \cdot \frac{\text{precision} \cdot \text{recall}}{\text{precision+recall}}$$

8. Standard Deviation

$$\sigma = \sqrt{\frac{\sum (x_i - \mu)^2}{N}}$$

μ is the mean of the vector *x* of length N

9. Covariance

$$\text{cov(x,y)} = \frac{\sum_{i=1}^{i=n} (x_i - \bar{x}) \,.(y_i - \bar{y})}{(\text{n} - 1)}$$

Covariance between two vectors of length n and means $\bar{x}$ and $\bar{y}$

10. Dot product of two vectors

Let u = $\begin{bmatrix} u_1 \\ u_2 \\ \cdots \\ \\ u_n \end{bmatrix}$ v = $\begin{bmatrix} v_1 \\ v_2 \\ \cdots \\ \\ v_n \end{bmatrix}$

Then $u \circ v = u^T * v = u_1.v_1 + u_2.v_2 + u_3.v_3 + u_4 .v_4 + \ldots + u_n.v_n$
Note that the $u^T = (u_1, u_2, u_3 \ldots u_n)$ The result would be 1 by 1

$$u \circ v = u^T * v = [u_1\ u_2\ \ u_3\ \ \ldots\ u_n\ ] * \begin{bmatrix} v_1 \\ v_2 \\ \cdots \\ \\ v_n \end{bmatrix} = u_1 * v_1 + u_2 * v_2 + u_3 * v_3 + \ldots + u_n * v_n$$

11. Multiplying matrices

To multiply two matrices, you need the number of columns of the first matrix equal the number of rows of second matrix.
$A_{m\times,n} * B_{n\times,p} = C_{m\times,p}$

$C_{ij} = A_{i1} * A_{1j} + A_{i2} * B_{2j} + A_{i3} * B_{3j} + \ldots + A_{in} * B_{nj}$

Basically i^{th} **row** from A and j^{th} **column** from B are used to *multiply* and then *add* the results to get ij^{th} element.

<u>2 X 2 Matrix Multiplication</u>

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

$$C = A * B = \begin{bmatrix} a_{11} * b_{11} + a_{12} * b_{21} & a_{11} * b_{12} + a_{12} * b_{22} \\ a_{21} * b_{11} + a_{22} * b_{21} & a_{21} * b_{12} + a_{22} * b_{22} \end{bmatrix}$$

2 X 2 Matrix Multiplication-Example

$$\mathbf{C} = \begin{bmatrix} 7 & 5 \\ 6 & 3 \end{bmatrix} \qquad \mathbf{D} = \begin{bmatrix} 2 & 1 \\ 5 & 1 \end{bmatrix}$$

Type equation here.

$$\mathbf{C\,x\,D} = \begin{bmatrix} 7*2+5*5 & 7*1+5*1 \\ 6*2+3*5 & 6*1+3*1 \end{bmatrix} = \begin{bmatrix} 39 & 12 \\ 27 & 9 \end{bmatrix}$$

3 X 3 Matrix Multiplication

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \quad B = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$$

$$C = A * B = \begin{bmatrix} a_{11} * b_{11} + a_{12} * b_{21} + a_{13} * b_{31} & a_{11} * b_{12} + a_{12} * b_{22} + a_{13} * b_{32} & a_{11} * b_{13} + a_{12} * b_{23} + a_{13} * b_{33} \\ a_{21} * b_{11} + a_{22} * b_{21} + a_{23} * b_{31} & a_{21} * b_{12} + a_{22} * b_{22} + a_{23} * b_{32} & a_{21} * b_{13} + a_{22} * b_{23} + a_{23} * b_{33} \\ a_{31} * b_{11} + a_{32} * b_{21} + a_{33} * b_{31} & a_{31} * b_{12} + a_{32} * b_{22} + a_{33} * b_{32} & a_{31} * b_{13} + a_{32} * b_{23} + a_{33} * b_{33} \end{bmatrix}$$

3 X 3 Matrix Multiplication-Example

$$\begin{pmatrix} 1 & 2 & -1 \\ 3 & 2 & 0 \\ -4 & 0 & 2 \end{pmatrix}\begin{pmatrix} 3 & 4 & 2 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1(3)+2(0)+(-1)(-2) & 1(4)+2(1)+(-1)0 & 1(2)+2(0)+(-1)(1) \\ 3(3)+2(0)+(0)(-2) & 3(4)+2(1)+(0)0 & 3(2)+2(0)+(0)(1) \\ -4(3)+0(0)+(2)(-2) & -4(4)+0(1)+(2)0 & -4(2)+0(0)+(2)(1) \end{pmatrix}$$

$$= \begin{pmatrix} 5 & 6 & 1 \\ 9 & 14 & 6 \\ -16 & -16 & -6 \end{pmatrix}$$

<u>Multiplication of non-square matrix</u>

Here is an example of multiplying matrices of sizes 3 X 2 and 2 X 1.

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 1 \end{pmatrix} . \begin{pmatrix} 2 \\ \\ 4 \end{pmatrix} = \begin{pmatrix} (1\times 2)+(2\times 4) \\ (3\times 2)+(4\times 4) \\ (5\times 2)+(1\times 4) \end{pmatrix} = \begin{pmatrix} 2+8 \\ 6+16 \\ 10+4 \end{pmatrix} = \begin{pmatrix} 10 \\ 22 \\ 14 \end{pmatrix}$$

12. Finding minor of an element

The minor of an element is the value or matrix you get by deleting the row and column containing that element.

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

Minor of element a_{11} is $\begin{bmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{bmatrix}$

Minor of element a_{12} is $\begin{bmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{bmatrix}$

Minor of element a_{13} is $\begin{bmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$

Minor of element a_{21} is $\begin{bmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{bmatrix}$

Minor of element a_{22} is $\begin{bmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{bmatrix}$

Minor of element a_{23} is $\begin{bmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{bmatrix}$

Minor of element a_{31} is $\begin{bmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{bmatrix}$

Minor of element a_{32} is $\begin{bmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{bmatrix}$

Minor of element $a_{33}$is $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

$$\text{Minor(A)} = \begin{bmatrix} Minor\ of\ a_{11} & Minor\ of\, a_{12} & Minor\ of\, a_{13} \\ Minor\ of\ a_{21} & Minor\ of\ a_{22} & Minor\ of\ a_{23} \\ Minor\ of\ a_{31} & Minor\ of\ a_{32} & Minor\ of\ a_{33} \end{bmatrix}$$

$$\text{Minor(A)} = \begin{bmatrix} \begin{bmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{bmatrix} & \begin{bmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{bmatrix} & \begin{bmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix} \\ \begin{bmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{bmatrix} & \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} & \begin{bmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{bmatrix} \\ \begin{bmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{bmatrix} & \begin{bmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{bmatrix} & \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \end{bmatrix}$$

13. Finding determinant of a matrix

If A is a matrix, determinant of A exists only if it is a square matrix, written as |A| (with vertical bars around matrix) or det(A).

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$$\det(A) = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11} * a_{22} - a_{12} * a_{21}$$

<u>Example</u>

$$\det\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = (1)(4) - (2)(3)$$
$$= 4 - 6$$
$$= -2 \checkmark$$

Given 3 by 3 matrix determinant is calculated as follows:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$\det(A) = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11} * \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12} * \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13} * \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

<u>EXAMPLE</u>

$$\det\begin{bmatrix} 2 & -3 & 1 \\ 2 & 0 & -1 \\ 1 & 4 & 5 \end{bmatrix} = 2\cdot\det\begin{bmatrix} 0 & -1 \\ 4 & 5 \end{bmatrix} - (-3)\cdot\det\begin{bmatrix} 2 & -1 \\ 1 & 5 \end{bmatrix} + 1\cdot\det\begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}$$

$$\begin{aligned}
&= 2\left[0-(-4)\right] + 3\left[10-(-1)\right] + 1\left[8-0\right] \\
&= 2(0+4) + 3(10+1) + 1(8) \\
&= 2(4) + 3(11) + 8 \\
&= 8 + 33 + 8 \\
&= 49 \checkmark
\end{aligned}$$

14. Finding inverse of a 2 by 2 matrix

Let $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

$$A^{-1} = \frac{1}{\det(A)}\begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix} = \frac{1}{a_{11} * a_{22} - a_{12} * a_{21}}\begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix}$$

<u>Example</u>

$$\begin{bmatrix} 4 & 7 \\ 2 & 6 \end{bmatrix}^{-1} = \frac{1}{4\times6-7\times2}\begin{bmatrix} 6 & -7 \\ -2 & 4 \end{bmatrix}$$

$$= \frac{1}{10}\begin{bmatrix} 6 & -7 \\ -2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 0.6 & -0.7 \\ -0.2 & 0.4 \end{bmatrix}$$

15. Finding inverse of a 3 by 3 matrix

$$\text{Let A} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$Minor(\mathrm{A}) = \begin{bmatrix} \begin{bmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{bmatrix} & \begin{bmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{bmatrix} & \begin{bmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix} \\ \begin{bmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{bmatrix} & \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} & \begin{bmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{bmatrix} \\ \begin{bmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{bmatrix} & \begin{bmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{bmatrix} & \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \end{bmatrix}$$

$$Determinant\ of\ Minors(A) = \begin{bmatrix} det\begin{bmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{bmatrix} & det\begin{bmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{bmatrix} & det\begin{bmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix} \\ det\begin{bmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{bmatrix} & det\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} & det\begin{bmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{bmatrix} \\ det\begin{bmatrix} a_{11} & a_{12} \\ a_{22} & a_{23} \end{bmatrix} & det\begin{bmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{bmatrix} & det\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \end{bmatrix}$$

Co-factor(A) = Is the matrix obtained from *Determinant of Minors(A)* with the sign of an element is flipped if there is a minus in the corresponding position in the matrix below.

$$\begin{pmatrix} + & - & + \\ - & + & - \\ + & - & + \end{pmatrix}$$

Inverse of A = $A^{-1} = \frac{1}{\det(A)}$ * *Transpose*(*Co-factor(A)*)

<u>EXAMPLE</u>

Let us find the inverse of the following matrix.

$$A = \begin{bmatrix} 0 & -3 & -2 \\ 1 & -4 & -2 \\ -3 & 4 & 1 \end{bmatrix}$$

Step1 : Find the *Determinant of minors(A)* matrix.

$$\begin{pmatrix} (-4)\times 1-(-2)\times 4 & 1\times 1-(-2)\times(-3) & 1\times 4-(-4)\times(-3) \\ (-3)\times 1-(-2)\times 4 & 0\times 1-(-2)\times(-3) & 0\times 4-(-3)\times(-3) \\ (-3)\times(-2)-(-2)\times(-4) & 0\times(-2)-(-2)\times 1 & 0\times(-4)-(-3)\times 1 \end{pmatrix} = \begin{pmatrix} 4 & -5 & -8 \\ 5 & -6 & -9 \\ -2 & 2 & 3 \end{pmatrix}$$

Step 2: Change the matrix above by changing appropriate signs to obtain the **co-factor** matrix of A shown

$$\text{Co-factor(A)} = \begin{pmatrix} 4 & 5 & -8 \\ -5 & -6 & 9 \\ -2 & -2 & 3 \end{pmatrix}$$

Step 3 : Now transpose the *Co-factor(A)* as follows :

$$\textit{Transpose(Co-factor(A))} = \begin{pmatrix} 4 & -5 & -2 \\ 5 & -6 & -2 \\ -8 & 9 & 3 \end{pmatrix}.$$

Step 4: Find the determinant of the original matrix

$$\det\begin{pmatrix} 0 & -3 & -2 \\ 1 & -4 & -2 \\ -3 & 4 & 1 \end{pmatrix} = 0 \times \det\begin{pmatrix} -4 & -2 \\ 4 & 1 \end{pmatrix} + 3 \times \det\begin{pmatrix} 1 & -2 \\ -3 & 1 \end{pmatrix} - 2 \times \det\begin{pmatrix} 1 & -4 \\ -3 & 4 \end{pmatrix} = 1,$$

Step 5: Divide the *Transpose(Co-factor(A))* by the determinant to obtain inverse!

$$A^{-1} = \begin{pmatrix} 4 & -5 & -2 \\ 5 & -6 & -2 \\ -8 & 9 & 3 \end{pmatrix}.$$

15. **NEURAL NETWORKS FOR REGRESSION**

Assume that the NN has input with P predictors, M hidden units with one hidden layer and one output unit. Z_j represents the output from the hidden unit j. Y_1 is the output from output unit.

$$z_j(x) = \sigma\left(\alpha_{0j} + \sum_{i=1}^{P} x_i \alpha_{ij}\right), \ j \in \{1..M\}$$

$$\sigma(u) = \frac{1}{1 + e^u}$$

$$Y_1 = f(x) = \beta_0 + \sum_{j=1}^{M} \beta_j z_j$$

16. NEURAL NETWORKS FOR CLASSIFICATION

Assume that the NN has input with P predictors, M hidden units with one hidden layer. Z_j represents the output from the hidden unit j. T_k is the output from the output class Y_k.

$$z_j(x) = \sigma\left(\alpha_{0j} + \sum_{i=1}^{P} x_i \alpha_{ij}\right), \quad j \in \{1..M\}$$

$$T_k = \beta_{0k} + \sum_{j=1}^{M} \beta_{jk} z_j \quad k \in \{1..K\}$$

$$f_k(x) = g_k(T), \text{ with } T = [T_1, T_2, .. T_K]$$

$$g_k(T) = \frac{e^{T_k}}{\sum_{\ell=1}^{K} e^{T_\ell}}.$$

17.The D(u) for SVM

y_i represents the class the data belongs to, α_i represents the weight given to the sample, x_i represents the sample point, and u represents the unknown data. β_0 is the constant given by the SVM. The value D(u) determines which class the unknown data point belongs to and is given by:

$$= \beta_0 + \sum_{i=1}^{n} y_i \alpha_i \mathbf{x}_i' \mathbf{u}$$

18.Find eigen values of a matrix:

Let A be the covariance matrix.

Identity matrix *I,* of size n is a n X n matrix in which diagonals are 1 and non diagonal elements are 0.

To find eigen values of a matrix A of size n X n,

Step 1: Take the identity matrix I whose size is same as A
Step 2: Multiply every element with scalar λ to obtain λI
Step3: Subtract λI from A to get A- λI.
Step 4: Find the det(A- λI)
Step 5: Set det(A -λI) to 0 and solve for λ
Step 6: For each solution λ_i, solve $(A\, v_i - \lambda_i\, v_i) = 0$ to find the corresponding eigenvector v_i
Step 7: For each simultaneous equation you will get the ratio of components of the vector and you can use any vector that has the given ratio.

Example:

$$\mathbf{A} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$$

Step 1: $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$,
Step 2: $\lambda I = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$

Step 3 ,4 & 5:

$$|\mathbf{A} - \lambda \cdot \mathbf{I}| = \left|\begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}\right| = 0$$

$$\left|\begin{bmatrix} -\lambda & 1 \\ -2 & -3-\lambda \end{bmatrix}\right| = \lambda^2 + 3\lambda + 2 = 0$$

$\lambda_1=-1$, $\lambda_2=-2$

Steps 6 & 7 for λ_1 = -1

$$\mathbf{A} \cdot \mathbf{v}_1 = \lambda_1 \cdot \mathbf{v}_1$$

$$(\mathbf{A} - \lambda_1) \cdot \mathbf{v}_1 = 0$$

$$\begin{bmatrix} -\lambda_1 & 1 \\ -2 & -3-\lambda_1 \end{bmatrix} \cdot \mathbf{v}_1 = 0$$

$$\begin{bmatrix} 1 & 1 \\ -2 & -2 \end{bmatrix} \cdot \mathbf{v}_1 = \begin{bmatrix} 1 & 1 \\ -2 & -2 \end{bmatrix} \cdot \begin{bmatrix} v_{1,1} \\ v_{1,2} \end{bmatrix} = 0$$

$$v_{1,1} + v_{1,2} = 0, \quad \text{so}$$

$$v_{1,1} = -v_{1,2}$$

$$-2 \cdot v_{1,1} + -2 \cdot v_{1,2} = 0, \quad \text{so again}$$

$$v_{1,1} = -v_{1,2}$$

$$\mathbf{v}_1 = k_1 \begin{bmatrix} +1 \\ -1 \end{bmatrix}$$

Steps 6 & 7 for λ_2 = -2

$$\mathbf{A} \cdot \mathbf{v}_2 = \lambda_2 \cdot \mathbf{v}_2$$

$$(\mathbf{A} - \lambda_2) \cdot \mathbf{v}_2 = \begin{bmatrix} -\lambda_2 & 1 \\ -2 & -3-\lambda_2 \end{bmatrix} \cdot \mathbf{v}_2 = \begin{bmatrix} 2 & 1 \\ -2 & -1 \end{bmatrix} \cdot \begin{bmatrix} v_{2,1} \\ v_{2,2} \end{bmatrix} = 0 \quad \text{so}$$

$$2 \cdot v_{2,1} + 1 \cdot v_{2,2} = 0 \quad (\text{or from bottom line: } -2 \cdot v_{2,1} - 1 \cdot v_{2,2} = 0)$$

$$2 \cdot v_{2,1} = -v_{2,2}$$

$$\mathbf{v}_2 = k_2 \begin{bmatrix} +1 \\ -2 \end{bmatrix}$$