Guide to Uncertainty in the Physics Lab

Created by Helen Mae Cothrel, Kettering University Physics

Last Updated August 2020

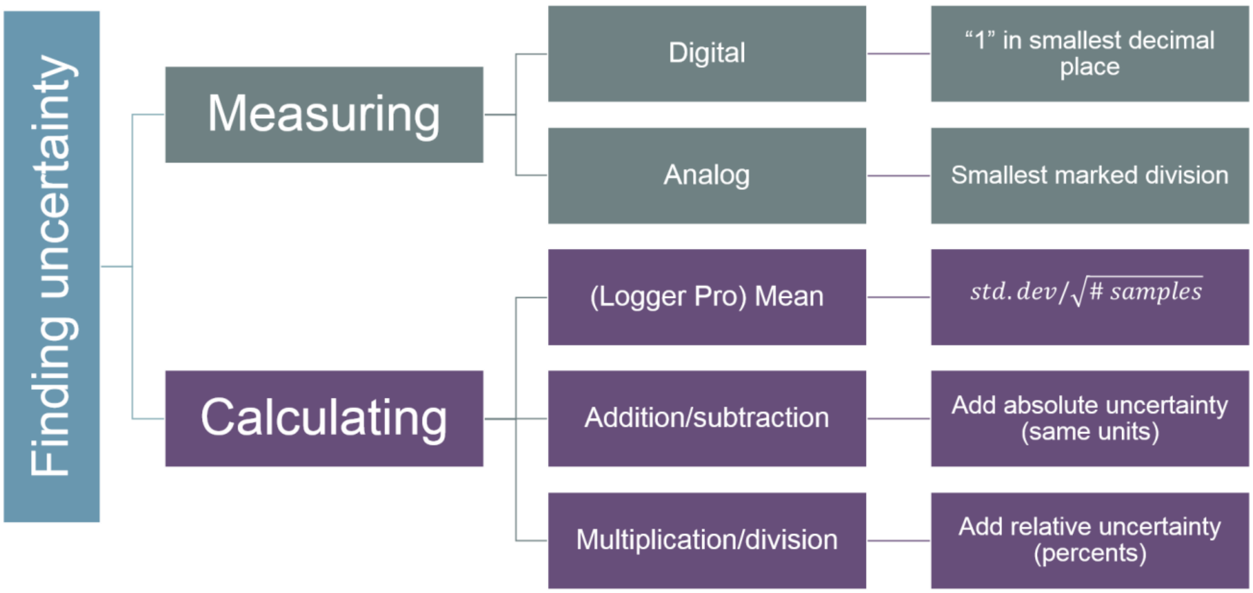

Figure 1: Uncertainty flow chart

WRITING UNCERTAINTY

- Measured and calculated values must be recorded and presented with uncertainty.
- Uncertainty in a variable is written as δx, where x is the variable and δ, lower-case Greek delta, means "uncertainty in".
- Round values to match their uncertainty. (The value shouldn't have more decimal places than the uncertainty.)
- Uncertainty should be a single digit, unless the first digit in an uncertainty is a 1. (See Equation 3 for an example. This is most common in calculated uncertainties.)

WRITING ABSOLUTE UNCERTAINTY

Absolute uncertainty has the same units as the measured value. It is typically determined directly from a measurement device or calculation. Values should be written as:

$$y = y_{best} \pm \delta y \quad (1)$$

where y_{best} is the measured or calculated value (which we recognize as being our best estimate of the value) and δy is the uncertainty in that value.

For example, if a measurement of $5.12\ m$ has an uncertainty of $0.03\ m$, this measurement would be recorded as:

$$y = (5.12 \pm 0.03)\ m \quad (2)$$

CALCULATING AND WRITING RELATIVE UNCERTAINTY

Relative (percent) uncertainty tells you what percent of the measured value is uncertainty. It is calculated as follows.

$$relative\ uncertainty = \frac{\delta y}{y_{best}} \times 100\% \quad (3)$$

For the example in Equation 2, this would result in

$$relative\ uncertainty = \frac{0.03\ m}{5.12\ m} \times 100\% = 0.6\% \quad (4)$$

And this would be written as

$$y = 5.12\ m \pm 0.6\% \quad (6)$$

UNCERTAINTY IN MEASUREMENTS

ANALOG MEASUREMENTS

When making a measurement with an analog measurement device such as a ruler, protractor, or analog meter, the uncertainty is the value of the smallest marked division on the device.

Examples:

A voltmeter marked every 2 Volts: $voltage = (6 \pm 2)\ V$

A voltmeter marked every 0.2 Volts: $voltage = (6.0 \pm 0.2)\ V$

A ruler marked every 0.5 cm: $length = (0.872 \pm 0.005)\ m$

DIGITAL MEASUREMENTS

When making a measurement from a digital readout, the uncertainty is the smallest reliable value measurable by that instrument. This is often the last visible decimal place. However, if the value fluctuates during measurements, you should estimate the fluctuation as the uncertainty.

Examples:

A digital ammeter which can read to 0.001 A: $current = (0.007 \pm 0.001)\ A$

A capacitance meter which can read to 0.01 F: $capacitance = (5.74 \pm 0.01)\ F$

A stopwatch which can read to 0.1 s: $time = (15.4 \pm 0.1)\ s$

A balance which can read to 1 mg: $mass = (1.234 \pm 0.001)\ g$

UNCERTAINTY IN CALCULATIONS

Multiplying by a number

If you are multiplying a measurement by a number with no uncertainty, multiply its uncertainty by the same number. For example, to find the circumference of a circle with a radius of 1.5 ± 0.1 cm, first you would calculate

$$Circumference = 2\pi r = 2 * \pi * (1.5\ cm)$$

And then the uncertainty would be

$$Uncertainty\ in\ Circ. = 2 * \pi * (0.1\ cm).$$

Then, your final result is $9.4 \pm 0.6\ cm$.

ADDING AND SUBTRACTING

When adding and subtracting, add the absolute uncertainties of each quantity. For example, adding two lengths measured from a ruler marked by centimeters may look like:

$$total\ length = (0.24 \pm 0.01)\ m + (0.07 \pm 0.01)\ m = (0.31 \pm 0.02) m$$

Always add uncertainties, even if you are subtracting measurements.

MULTIPLYING AND DIVIDING

When multiplying or dividing, add the percent uncertainties of each quantity. For example, dividing mass ($0.520\ kg \pm 0.1\%$) by volume ($0.272\ m^3 \pm 3\%$) to find density may look like:

$$\rho = \frac{0.520\ kg}{0.272\ m^3} \pm (0.1\% + 3\%) = 1.91 \frac{kg}{m^3} \pm 3.1\%$$

If all you want is a percent uncertainty, you can stop there. Otherwise, multiply the percent by your answer to get an uncertainty with the same units.

3.1% as a decimal is 0.031, so that calculation would look like:

$$\delta\rho = \left(1.91 \frac{kg}{m^3}\right) \times 0.031 = 0.06\ \frac{kg}{m^3}$$

And the final result would be:

$$\rho = (1.91 \pm 0.06)\ \frac{kg}{m^3}$$

ANYTHING ELSE

When using a function, $f(x, y, z)$, to determine a result, Q, the uncertainty is calculated by:

$$\delta Q = \left|\frac{\partial f}{\partial x}\right| \delta x + \left|\frac{\partial f}{\partial y}\right| \delta y + \left|\frac{\partial f}{\partial z}\right| \delta z$$

If using this formula, partial derivatives would be evaluated at x_{best}, y_{best}, z_{best} and so on.

UNCERTAINTIES IN GRAPHING

Uncertainties are relevant in graphing, too. Look at the following example:

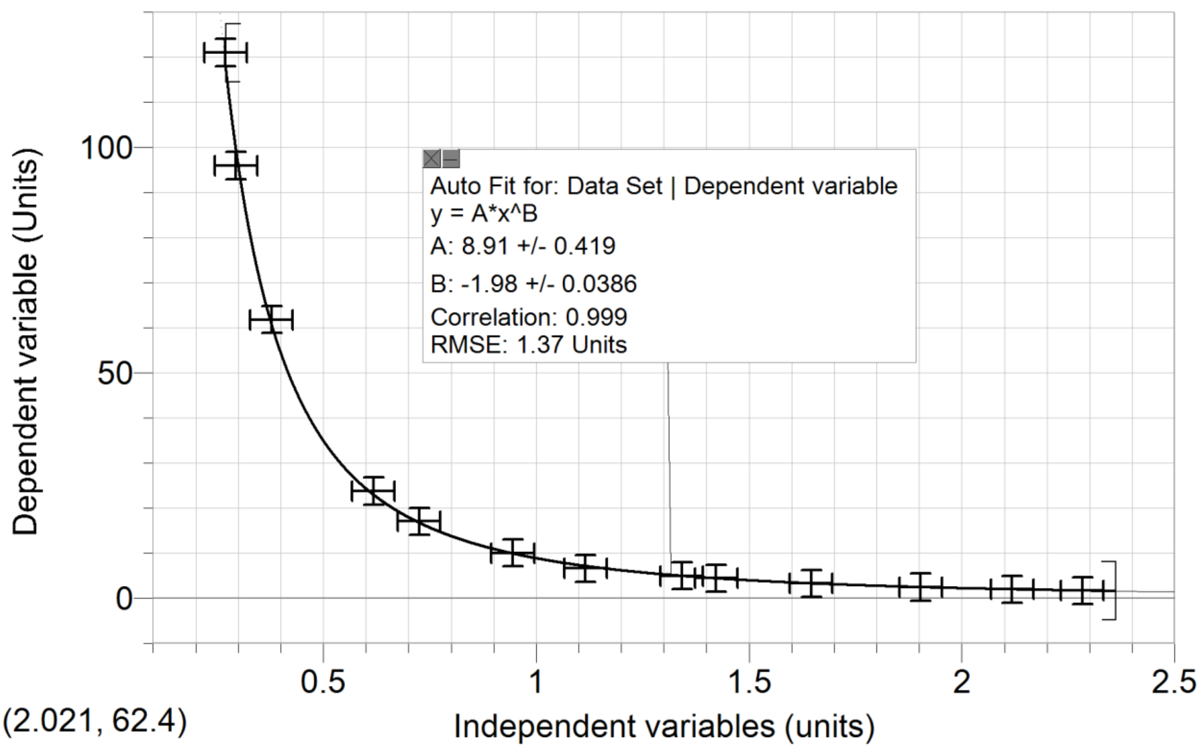

Figure 2: Graph with error bars and a fit with uncertainty.

Uncertainty shows up on the graph in two ways:

1. The data points have vertical and horizontal error bars, showing the uncertainty of each measurement.
2. The fit has uncertainty in calculated values. For example, the power, B, is -1.98 ± 0.04 (no units).