Electric fields and point charges

Physics 225 – Background wiki

ELECTRIC FIELD FOR A SINGLE POINT CHARGE

The electric field ($\vec{E}$) is defined by the electric force ($\vec{F}$) it exerts on a test charge, divided by the magnitude of the test charge (q):

$$\vec{E} \equiv \frac{\vec{F}_{on\ q}}{q}. \tag{2.1}$$

By this definition, the units of electric field are the unit of force divided by the unit of charge, or Newtons per Coulomb (N/C).

Electric fields point away from positive source charges and towards negative source charges. The electric field of a **point charge** (the force per unit charge the source charge would exert on another test charge) can be adapted from Coulomb's Law and has the form

$$\vec{E} = \frac{Q}{4\pi\epsilon_o r^2}\hat{r}, \tag{2.2}$$

where Q is the source charge, r is the distance in meters from the source charge to the test point (where you are interested in determining the field), $\hat{r}$ is a unit vector that points from the source charge to the test point, and $\epsilon_0 = 8.85 \times 10^{-12}\ \frac{C^2}{N\cdot} m^2$ is the permittivity of free space.

ELECTRIC FIELD FROM A DIPOLE

An electric dipole (Figure 2.1) consists of equal positive and negative charges, at a fixed distance d from each other.

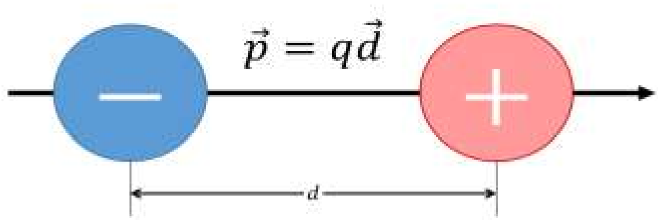

Figure 2.1. An illustration of an electric dipole consisting of two charges having equal magnitude but opposite charge separated by a distance d. We define the electric dipole moment to be a

vector quantity $\vec{p}$ that points in the direction from the negative charge to the positive charge. In this figure, the origin (r = 0) is at the center point between the charges.

We define a vector quantity known as the **electric dipole moment,** $\vec{p}$, in terms of the charge magnitude (both charges in a dipole have the same magnitude, but opposite charge) and separation distance as

$$\vec{p} = q\vec{d}, \tag{2.3}$$

where $\vec{d}$ is the distance vector that points from the negative to the positive charge.

Along the axis of the dipole—the axis illustrated in Figure 2.1—the electric field is given by

$$\vec{E}_{dipole} = \frac{\vec{p}}{4\pi\epsilon_0}\frac{2r}{\left(r-\frac{d}{2}\right)^2\left(r+\frac{d}{2}\right)^2} \tag{2.4}$$

While Equation 2.4 is a bit complicated, things become clearer when we look at test points where $r \gg d$ (that is, you look at the dipole from far away compared to the separation). In this limit, we can consider d to be negligible and Equation 2.4 simplifies to

$$\vec{E}_{dipole} \approx \frac{\vec{p}}{4\pi\epsilon_0}\frac{2}{r^3}. \tag{2.5}$$

Looking at the bisector that lies halfway between the two charges, it can be shown that at a distance $r \gg d$ the electric field becomes

$$\vec{E}_{bisector} \approx -\frac{\vec{p}}{4\pi\epsilon_0}\frac{1}{r^3}. \tag{2.6}$$

Equation 2.6 is an appropriate approximation of the electric field along the bisector of a dipole when looking at it from far away.