NA 568 - Winter 2022

Localization

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Slides: Courtesy of Ryan Eustice

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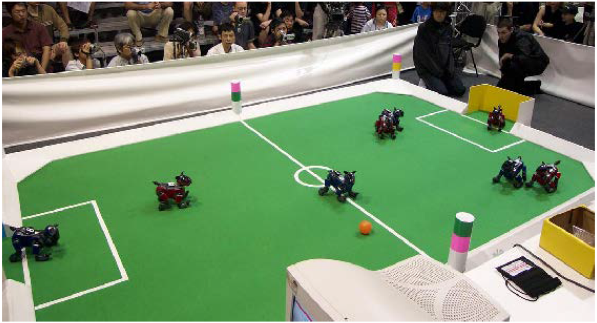
Localization

"Using sensory information to locate the robot in its environment is the most fundamental problem to providing a mobile robot with autonomous capabilities." [Cox '91]

- **Given**
  - Map of the environment.
  - Sequence of sensor measurements.
- **Wanted**
  - Estimate of the robot's position.
- **Problem classes**
  - Position tracking
  - Global localization
  - Kidnapped robot problem (recovery)

Landmark-based Localization



EKF_localization (μ_{t-1}, Σ_{t-1}, $\mathbf{u}_t$, $\mathbf{z}_t$, $\boldsymbol{m}$):

Prediction:

$$G_t = \frac{\partial g(\mathbf{u}_t, \mu_{t-1})}{\partial \mathbf{x}_{t-1}} = \begin{pmatrix} \frac{\partial x'}{\partial \mu_{t-1,x}} & \frac{\partial x'}{\partial \mu_{t-1,y}} & \frac{\partial x'}{\partial \mu_{t-1,\theta}} \\ \frac{\partial y'}{\partial \mu_{t-1,x}} & \frac{\partial y'}{\partial \mu_{t-1,y}} & \frac{\partial y'}{\partial \mu_{t-1,\theta}} \\ \frac{\partial \theta'}{\partial \mu_{t-1,x}} & \frac{\partial \theta'}{\partial \mu_{t-1,y}} & \frac{\partial \theta'}{\partial \mu_{t-1,\theta}} \end{pmatrix}$$

Jacobian of g w.r.t location

$$V_t = \frac{\partial g(\mathbf{u}_t, \mu_{t-1})}{\partial \mathbf{u}_t} = \begin{pmatrix} \frac{\partial x'}{\partial v_t} & \frac{\partial x'}{\partial \omega_t} \\ \frac{\partial y'}{\partial v_t} & \frac{\partial y'}{\partial \omega_t} \\ \frac{\partial \theta'}{\partial v_t} & \frac{\partial \theta'}{\partial \omega_t} \end{pmatrix}$$

Jacobian of g w.r.t control

$$M_t = \begin{pmatrix} \alpha_1 v_t^2 + \alpha_2 \omega_t^2 & 0 \\ 0 & \alpha_3 v_t^2 + \alpha_4 \omega_t^2 \end{pmatrix}$$

Motion noise

$$\bar{\mu}_t = g(\mathbf{u}_t, \mu_{t-1})$$

Predicted mean

$$\bar{\Sigma}_t = G_t \Sigma_{t-1} G_t^T + V_t M_t V_t^T$$

Predicted covariance

EKF_localization ($\mu_{t-1}, \Sigma_{t-1}, \boldsymbol{u}_t, \boldsymbol{z}_t, \boldsymbol{m}$):

Correction:

$$\hat{\mathbf{z}}_t = \begin{pmatrix} \sqrt{(m_x - \bar{\mu}_{t,x})^2 + (m_y - \bar{\mu}_{t,y})^2} \\ \operatorname{atan2}(m_y - \bar{\mu}_{t,y}, m_x - \bar{\mu}_{t,x}) - \bar{\mu}_{t,\theta} \end{pmatrix}$$ Predicted measurement mean

$$H_t = \frac{\partial h(\bar{\mu}_t, \mathbf{m})}{\partial \mathbf{x}_t} = \begin{pmatrix} \frac{\partial r_t}{\partial \bar{\mu}_{t,x}} & \frac{\partial r_t}{\partial \bar{\mu}_{t,y}} & \frac{\partial r_t}{\partial \bar{\mu}_{t,\theta}} \\ \frac{\partial \phi_t}{\partial \bar{\mu}_{t,x}} & \frac{\partial \phi_t}{\partial \bar{\mu}_{t,y}} & \frac{\partial \phi_t}{\partial \bar{\mu}_{t,\theta}} \end{pmatrix}$$ Jacobian of h w.r.t location

$$Q_t = \begin{pmatrix} \sigma_r^2 & 0 \\ 0 & \sigma_\phi^2 \end{pmatrix}$$ Measurement covariance

$$S_t = H_t \bar{\Sigma}_t H_t^T + Q_t$$ Innovation covariance

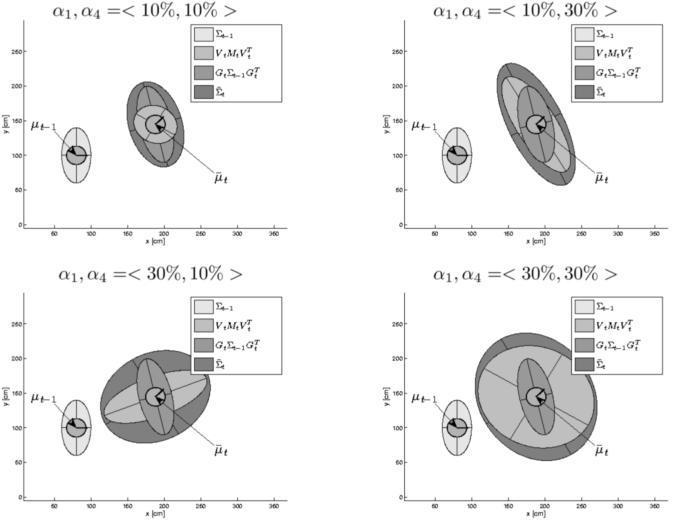
$$K_t = \bar{\Sigma}_t H_t^T S_t^{-1}$$ Kalman gain

$$\mu_t = \bar{\mu}_t + K_t(\mathbf{z}_t - \hat{\mathbf{z}}_t)$$ Updated mean

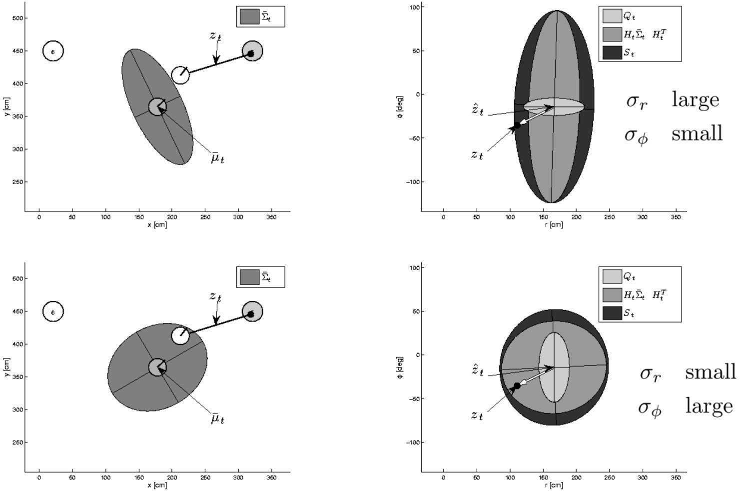
$$\Sigma_t = (I - K_t H_t)\bar{\Sigma}_t$$ Updated covariance

EKF Motion Prediction

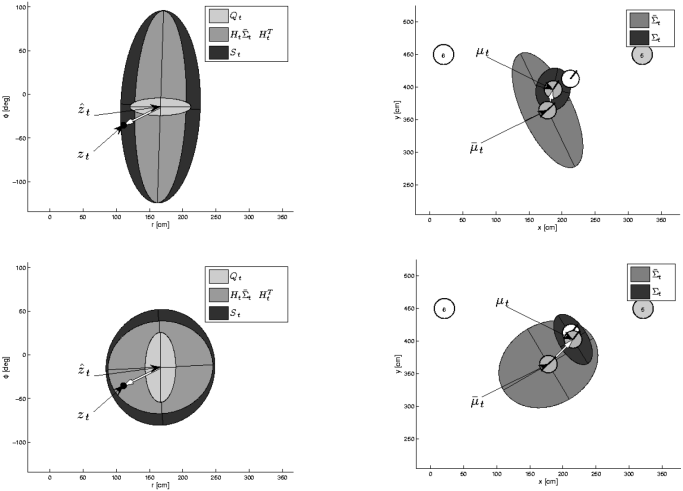
$$M_t = \begin{bmatrix} \alpha_1 v_t^2 + \alpha_2 w_t^2 & 0 \\ 0 & \alpha_3 v_t^2 + \alpha_4 w_t^2 \end{bmatrix}$$



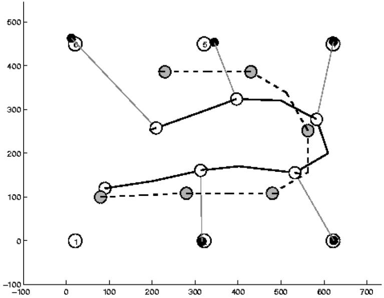
EKF Observation Prediction

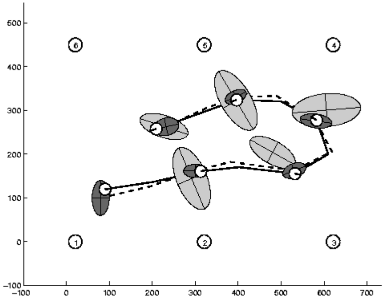


EKF Correction Step

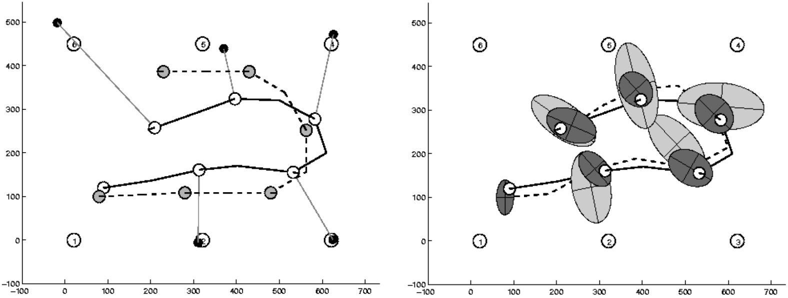


Estimation Sequence (1): Accurate Landmark Sensor

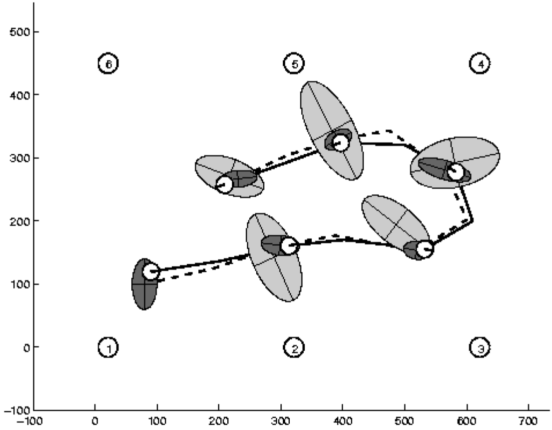




Estimation Sequence (2): Less Accurate Landmark Sensor



Comparison to GroundTruth



UKF_localization (μ_{t-1}, Σ_{t-1}, $\mathbf{u}_t$, $\mathbf{z}_t$, **m**):

Prediction:

$$M_t = \begin{pmatrix} \alpha_1 v_t^2 + \alpha_2 \omega_t^2 & 0 \\ 0 & \alpha_3 v_t^2 + \alpha_4 \omega_t^2 \end{pmatrix}$$ Motion noise

$$Q_t = \begin{pmatrix} \sigma_r^2 & 0 \\ 0 & \sigma_r^2 \end{pmatrix}$$ Measurement noise

$$\mu_{t-1}^a = \begin{pmatrix} \mu_{t-1}^T & (0\,0)^T & (0\,0)^T \end{pmatrix}$$ Augmented state mean

$$\Sigma_{t-1}^a = \begin{pmatrix} \Sigma_{t-1} & 0 & 0 \\ 0 & M_t & 0 \\ 0 & 0 & Q_t \end{pmatrix}$$ Augmented covariance

$$\chi_{t-1}^a = \begin{pmatrix} \mu_{t-1}^a & \mu_{t-1}^a + \gamma\sqrt{\Sigma_{t-1}^a} & \mu_{t-1}^a - \gamma\sqrt{\Sigma_{t-1}^a} \end{pmatrix}$$ Sigma points

$$\bar{\chi}_t^x = g(\mathbf{u}_t + \chi_t^u, \chi_{t-1}^x)$$ Prediction of sigma points

$$\bar{\mu}_t = \sum_{i=0}^{2L} w_m^i \, \bar{\chi}_{i,t}^x$$ Predicted mean

$$\bar{\Sigma}_t = \sum_{i=0}^{2L} w_c^i \left(\bar{\chi}_{i,t}^x - \bar{\mu}_t\right)\left(\bar{\chi}_{i,t}^x - \bar{\mu}_t\right)^T$$ Predicted covariance

UKF_localization (μ_{t-1}, Σ_{t-1}, $\mathbf{u}_t$, $\mathbf{z}_t$, $\boldsymbol{m}$):

Correction:

$$\bar{Z}_t = h\left(\bar{\chi}_t^x\right) + \chi_t^z$$ Measurement sigma points

$$\hat{\mathbf{z}}_t = \sum_{i=0}^{2L} w_m^i \, \bar{Z}_{i,t}$$ Predicted measurement mean

$$S_t = \sum_{i=0}^{2L} w_c^i \left(\bar{Z}_{i,t} - \hat{\mathbf{z}}_t\right)\left(\bar{Z}_{i,t} - \hat{\mathbf{z}}_t\right)^T$$ Pred. measurement covariance

$$\Sigma_t^{x,z} = \sum_{i=0}^{2L} w_c^i \left(\bar{\chi}_{i,t}^x - \bar{\mu}_t\right)\left(\bar{Z}_{i,t} - \hat{\mathbf{z}}_t\right)^T$$ Cross-covariance

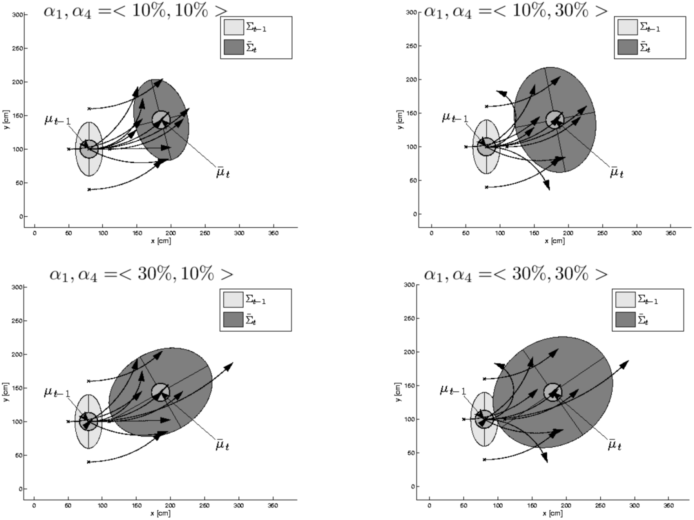
$$K_t = \Sigma_t^{x,z} S_t^{-1}$$ Kalman gain

$$\mu_t = \bar{\mu}_t + K_t(\mathbf{z}_t - \hat{\mathbf{z}}_t)$$ Updated mean

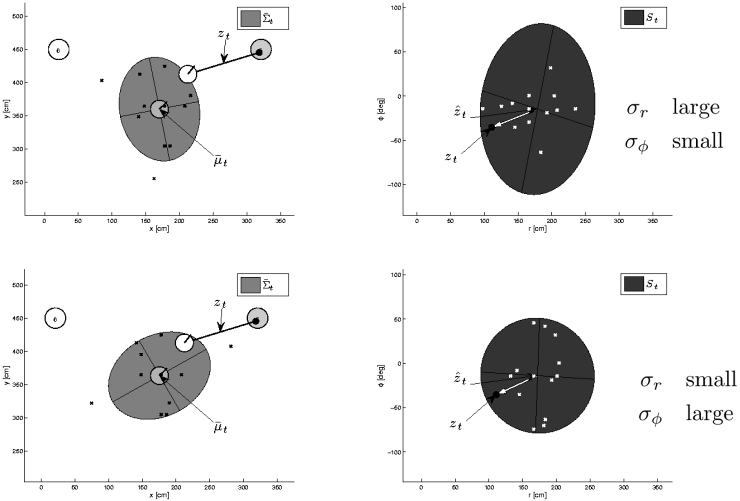
$$\Sigma_t = \bar{\Sigma}_t - K_t S_t K_t^T$$ Updated covariance

UKF Motion Prediction

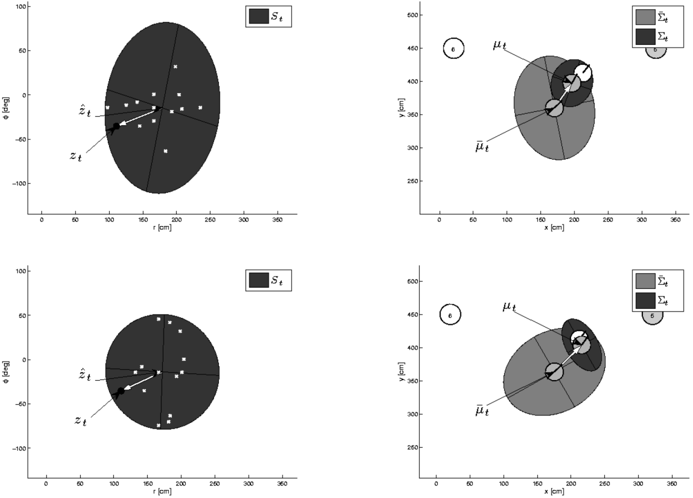
$$M_t = \begin{bmatrix} \alpha_1 v_t^2 + \alpha_2 w_t^2 & 0 \\ 0 & \alpha_3 v_t^2 + \alpha_4 w_t^2 \end{bmatrix}$$



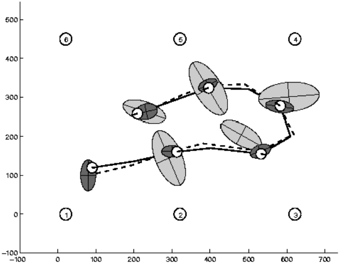
UKF Observation Prediction



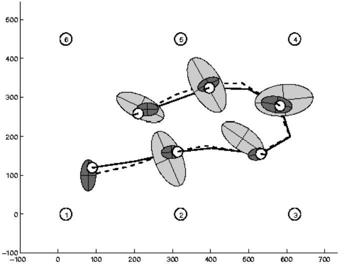
UKF Correction Step



Estimation Sequence

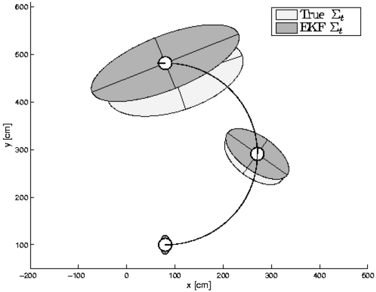


EKF

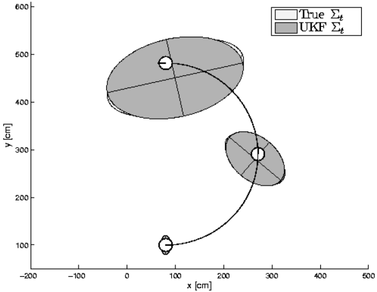


UKF

Prediction Quality

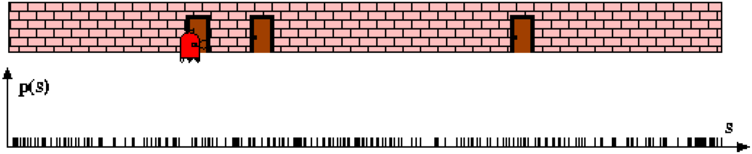


EKF



UKF

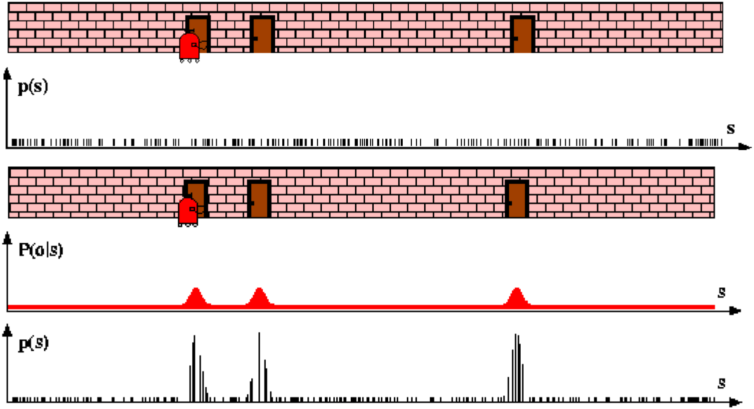
Particle Filters



Sensor Information: Importance Sampling

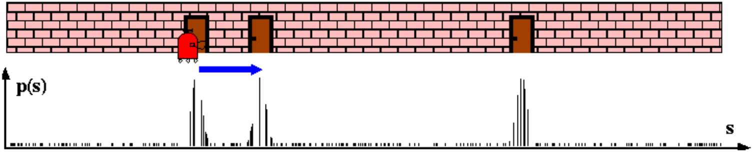
$$Bel(\mathbf{x}) \quad \leftarrow \quad \alpha\, p(\mathbf{z}\,|\,\mathbf{x})\, Bel^{-}(\mathbf{x})$$

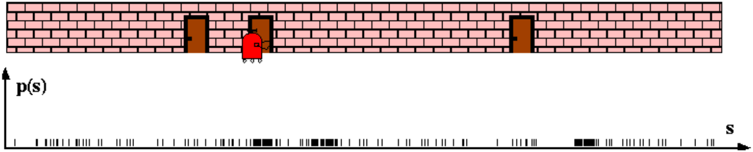
$$w \quad \leftarrow \quad \frac{\alpha\, p(\mathbf{z}\,|\,\mathbf{x})\, Bel^{-}(\mathbf{x})}{Bel^{-}(\mathbf{x})} \quad = \quad \alpha\, p(\mathbf{z}\,|\,\mathbf{x})$$



Robot Motion

$$Bel^{-}(\mathbf{x}) \quad \leftarrow \quad \int p(\mathbf{x} \mid \mathbf{u}, \mathbf{x}')\, Bel(\mathbf{x}')\ \mathrm{d}\mathbf{x}'$$





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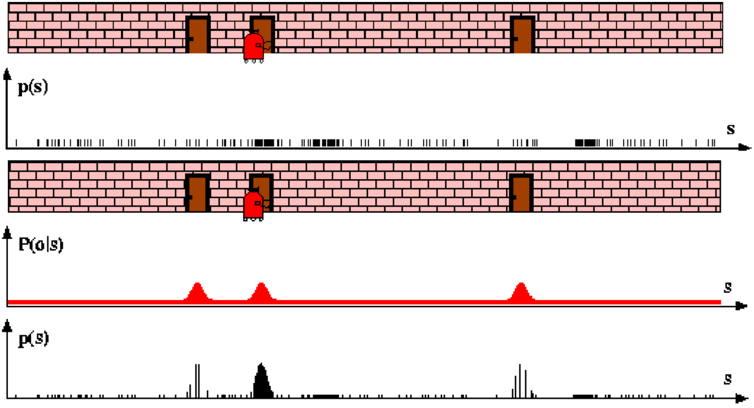

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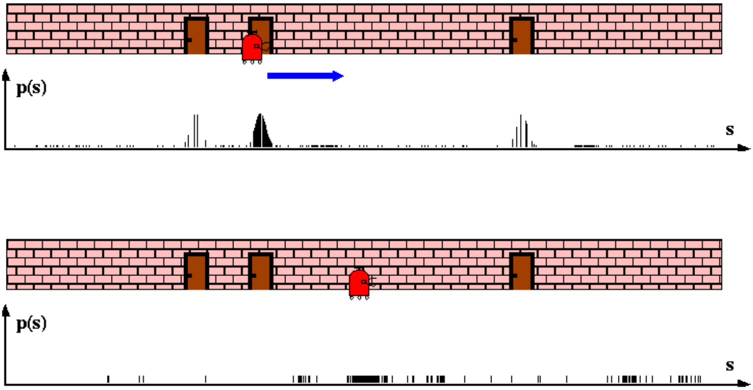
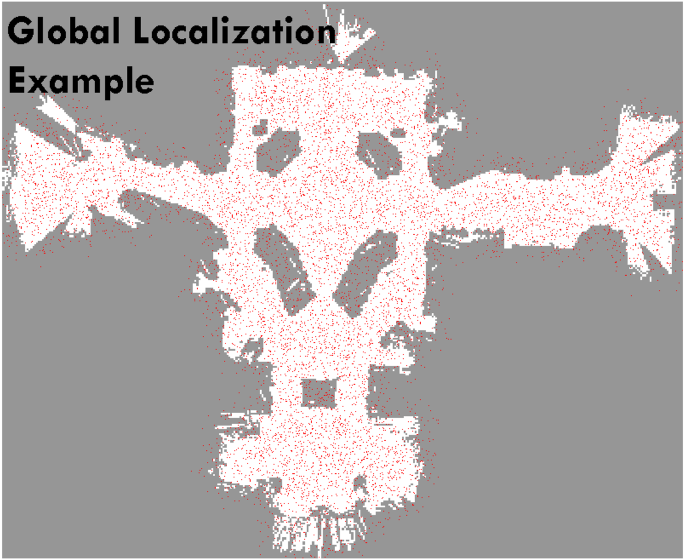
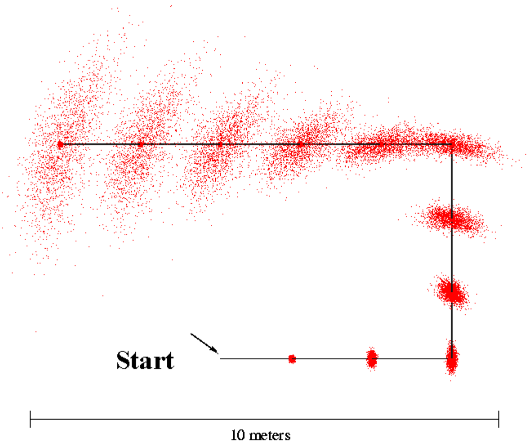
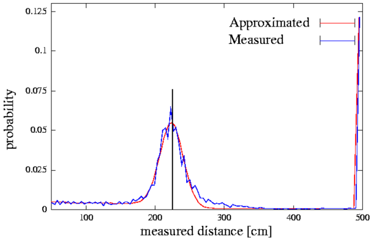



Global Localization
Example

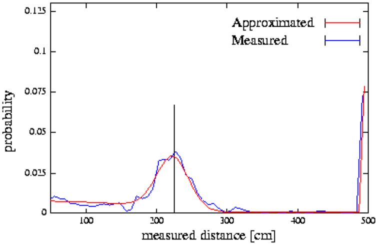
Motion Model Reminder



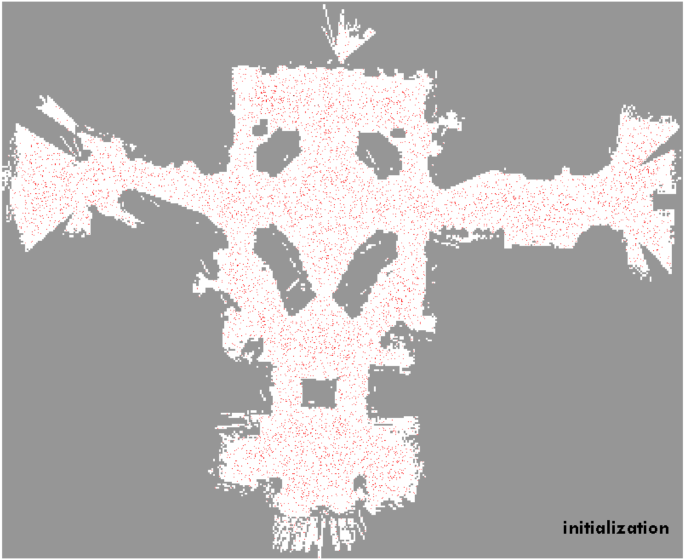
Proximity Sensor Model Reminder



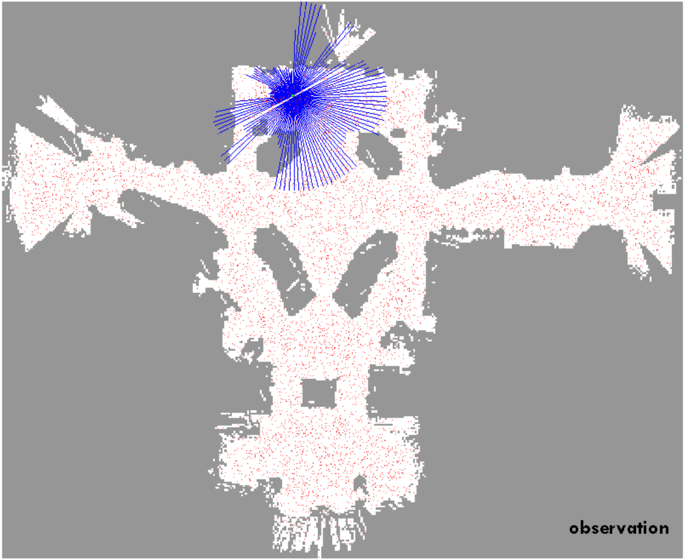
Laser sensor



Sonar sensor



Courtesy: Thrun, Burgard, Fox



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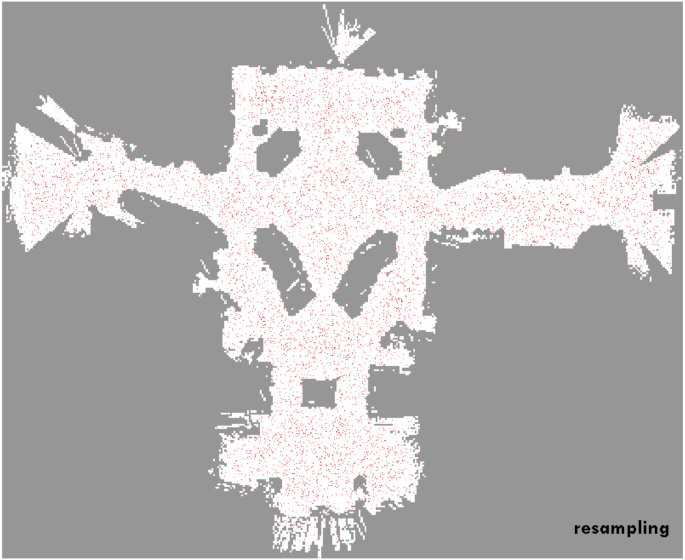

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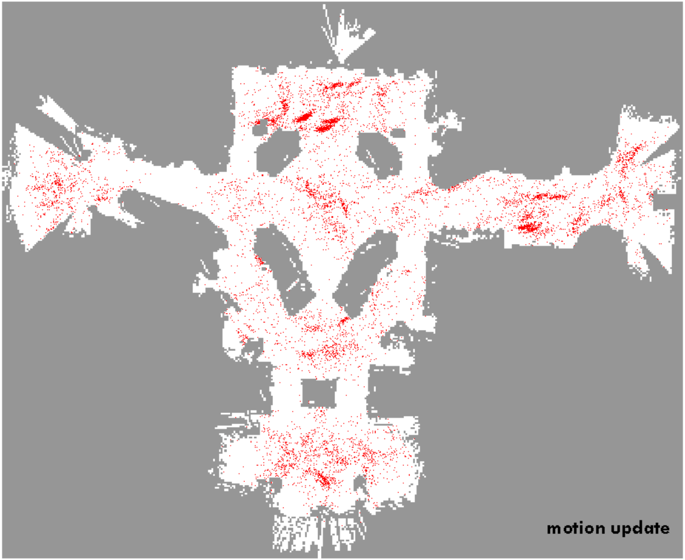

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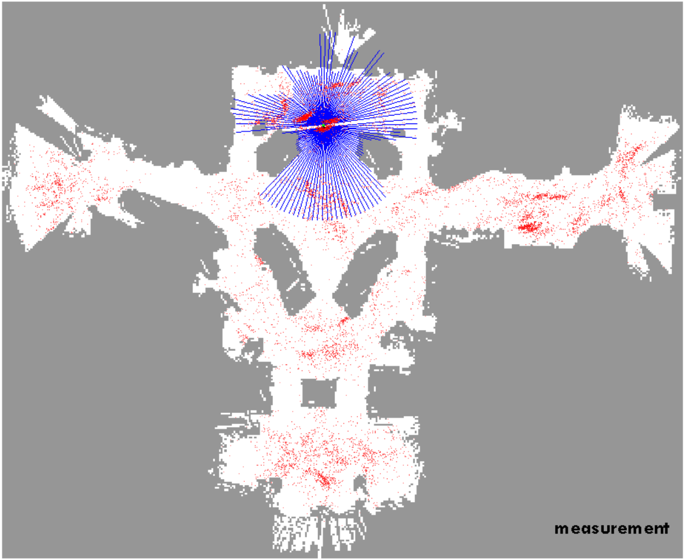

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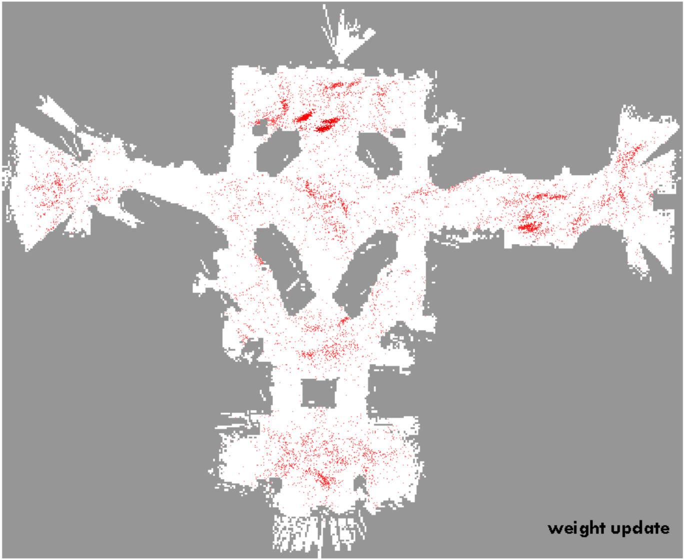

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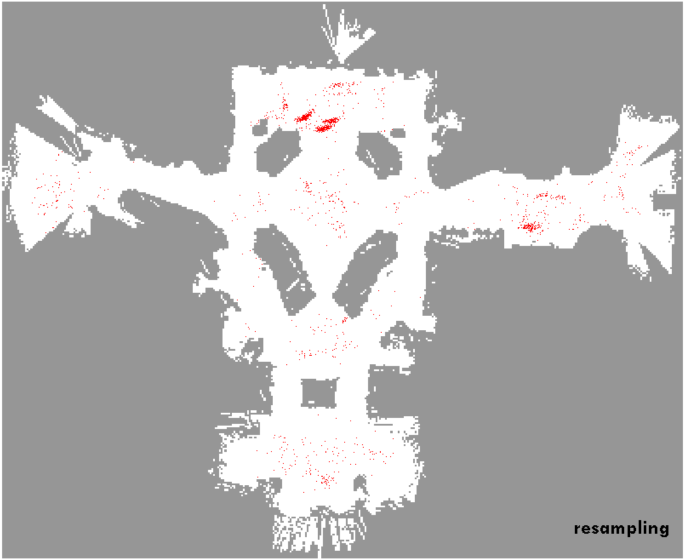

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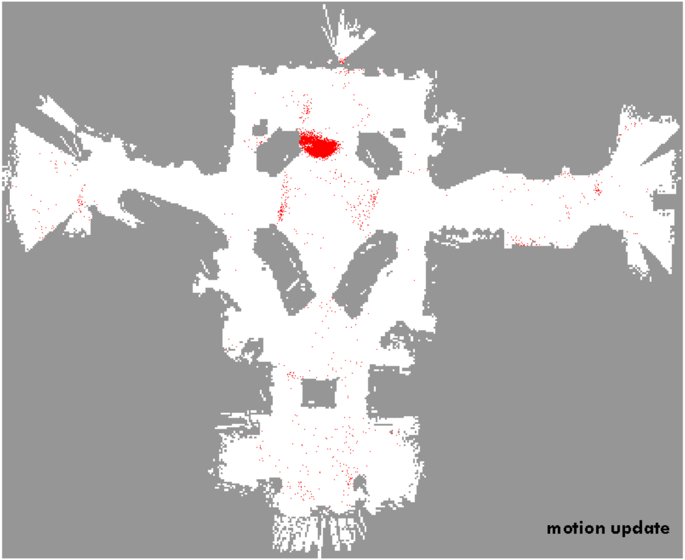

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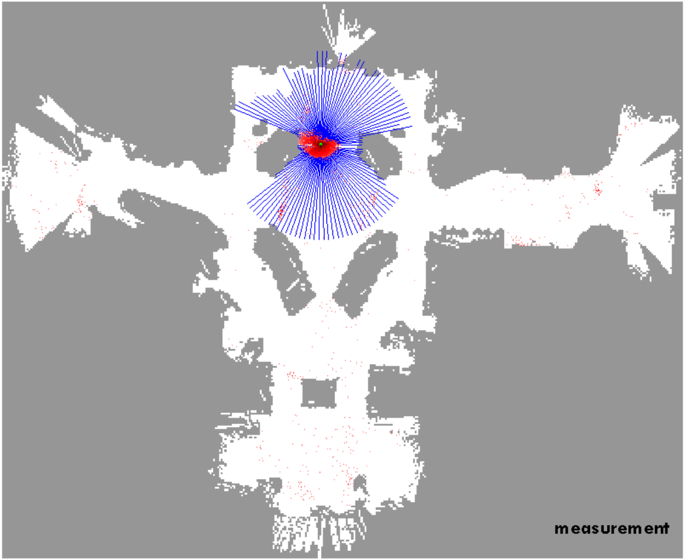

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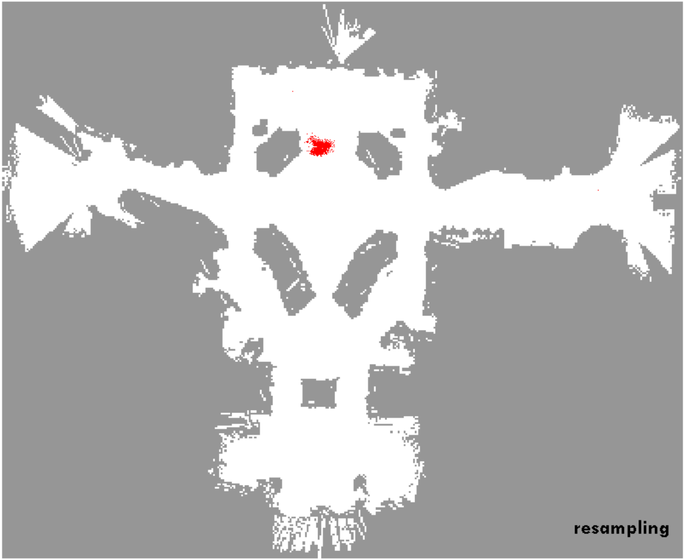

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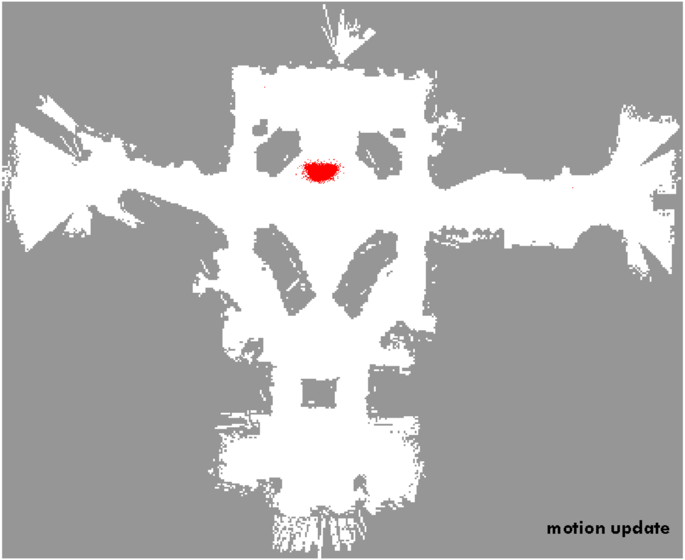

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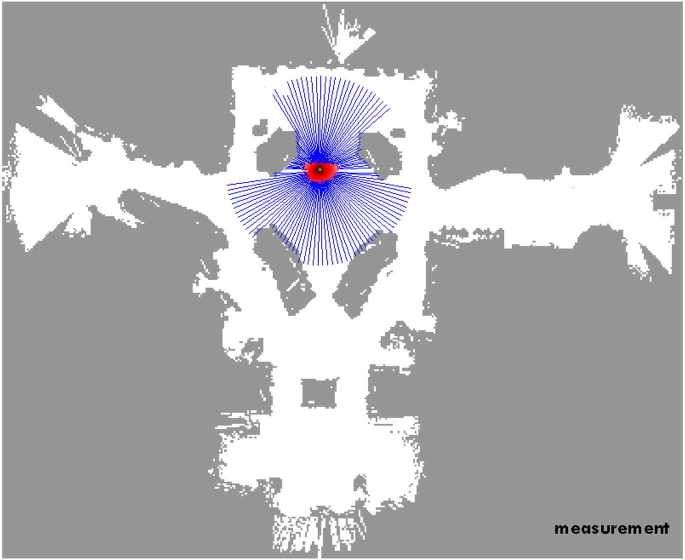

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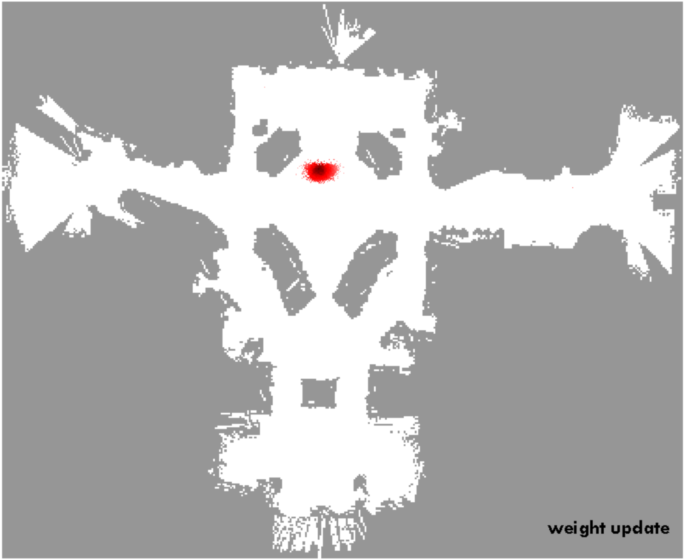

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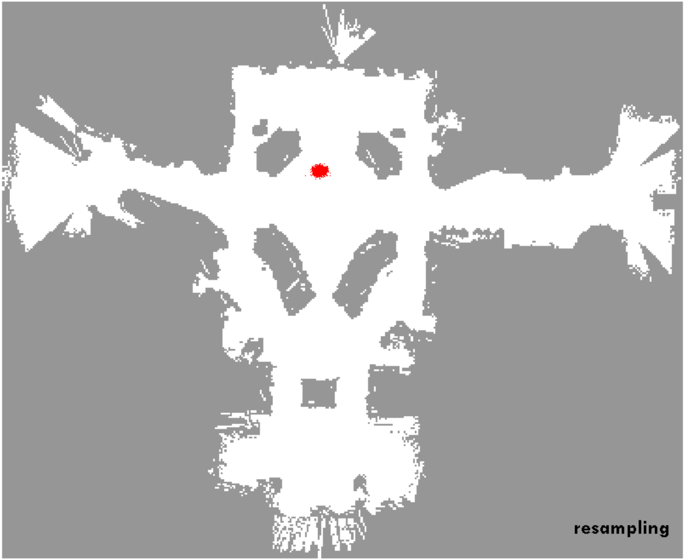

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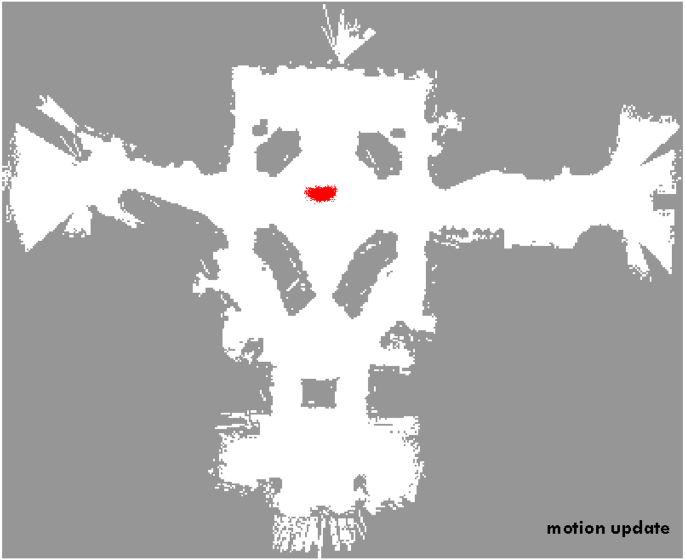

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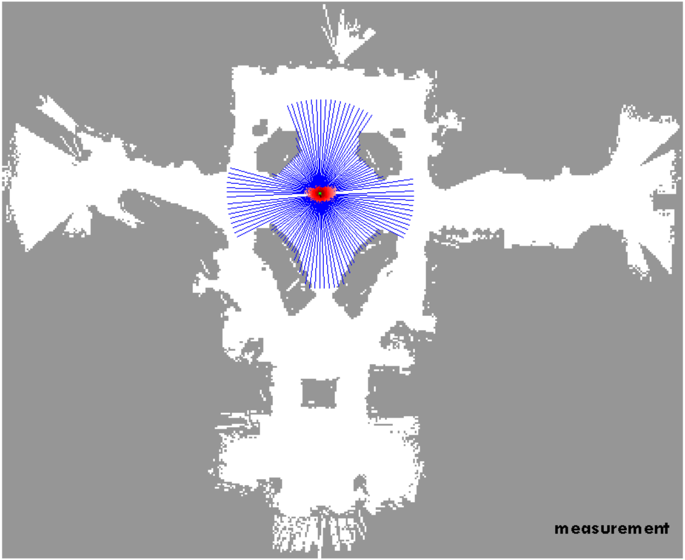

Courtesy: Thrun, Burgard, Fox

Importance Sampling with Resampling: Landmark Detection Example

Summary – Particle Filters

- Particle filters are non-parametric, recursive Bayes filters
- Posterior is represented by a set of weighted samples
- Proposal to draw the samples for $t+1$
- Weight to account for the differences between the proposal and the target
- Work well in low-dimensional spaces

Summary – PF Localization

- Particles are propagated according to the motion model
- They are weighted according to the likelihood of the observation
- Called: Monte-Carlo localization (MCL)
- MCL is the gold standard for mobile robot localization today

Readings

- Probabilistic Robotics: Ch. 7 and 8