NA 568 - Winter 2022

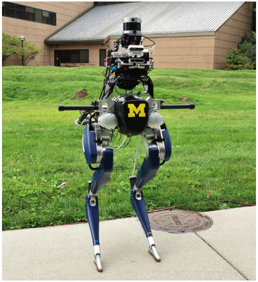
Introduction

Maani Ghaffari

January 6, 2022

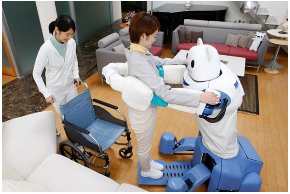
UNIVERSITY OF MICHIGAN

Robotics Systems: Why and How?



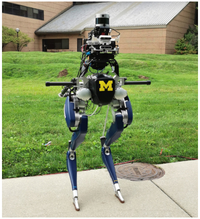
Robotics Systems: Why and How?

- Serve society;
- Improve quality of life;
- Dignified life and fulfilling one's potential;
- Aligned with the general goals of science and engineering;
- What can an embodied intelligent moving machine do?
  This is unique to robotics.



Robotics Systems: Why and How?

- Navigate and traverse a disaster site;
- Delivery and transportation;
- Collaborate in teams with other robots, or with humans for search and rescue or scientific exploration and discovery;
- Assistive robots for homecare; companions, teachers, or indoor and outdoor tour guides;
- Entertainment;
- Pandemics response!?
- ...



Environmental Monitoring and Information Gathering

Can we predict or prevent natural disasters and environmental problems better?

Left: Forest fires near Sydney, Australia, Oct. 23, 2013. Right: International Space Station image of flooding in Cambodia, Nov. 1, 2013.

Credits: NASA

Environmental Monitoring and Information Gathering

- Remote sensing enables data collection for disaster response efforts.

Left: Forest fires near Sydney, Australia, Oct. 23, 2013. Right: International Space Station image of flooding in Cambodia, Nov. 1, 2013.

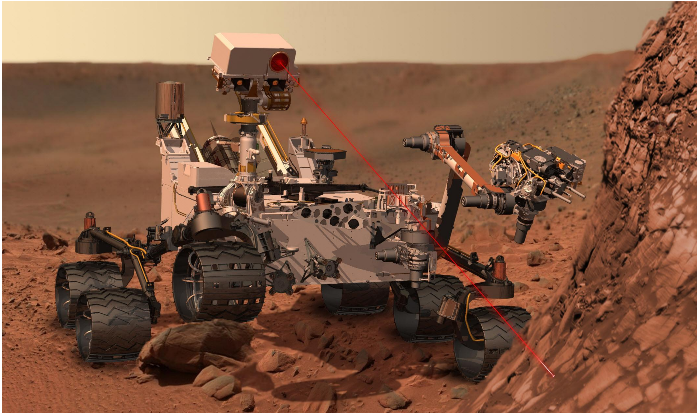
Credits: NASA

Environmental Monitoring and Information Gathering

- In many scientific discoveries, remote sensing alone is not sufficient for testing hypotheses.

Credits: (left) https://tinyurl.com/rw4pvad (middle) https://tinyurl.com/qr2bp8b (right) https://tinyurl.com/stvr4nx

- Robotic systems are enabling scientific technology for directly sampling and analyzing surface and subsurface compositions.



Credits: NASA JPL

Contact-Aided Invariant EKF on Mini Cheetah

https://www.youtube.com/watch?v=oVbP-Y8xT_E

Robot Museum Docent with UMMA

https://vimeo.com/385119483/61359f6f1c

- Inclusive experience, no matter what background.

- Learning about people through art and helping them to learn about themselves.

- Human-level perception and autonomy?

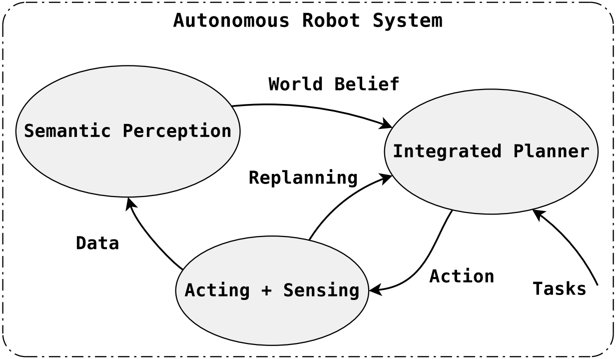
Social Navigation - Follow Social Norm

Keep a good distance when talking to different people

Lead a group of people

Social Navigation - Follow Social Norm

Do not disturb people


Autonomous Robot System
Semantic Perception
World Belief
Integrated Planner
Replanning
Data
Acting + Sensing
Action
Tasks


fetch
CULTURE CLASH
Robots at the Museum of Art
UNIVERSITY OF MICHIGAN
MUSEUM OF ART
ROBOTICS

Course Objective

- Teaching essential mathematics and algorithms behind the current robotics research
- Prepare you for research and development in robotics
- Solving real robotic problems

Logistics and Collaboration Policy

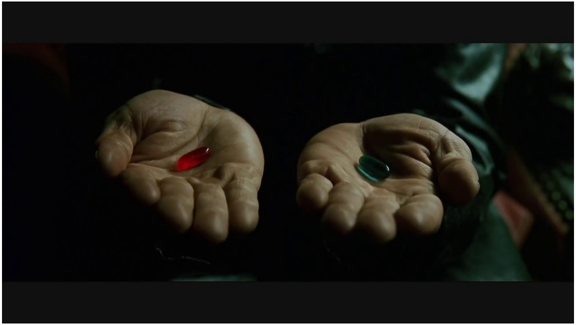
- All official course info on Canvas
- Join Piazza from Canvas
- Your assignments must represent original work.
  - Can talk with other students at a conceptual level all you want.
  - CANNOT share non-trivial code or solutions.
- It is critical that you encounter unexpected challenges!
  - Solving these problems is where you'll learn the material!

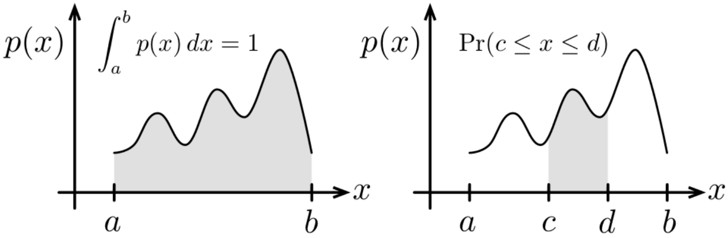
Assignments

Late submissions will not be accepted. However, given the broad range of students in this course (grad students, full-time engineers, remote students, etc.), and to be as inclusive as possible, we let the Gradescope submission open for the entire weekend when the due date is on Friday.

- 7 Homeworks (solving problems and coding)
- For Homeworks 1-6, the lowest grade will be automatically dropped for everyone.
- Final Project

Choose





Courtesy: T. Barfoot

Joint and Conditional Distribution

Let X and Y be two random variables.

- The joint distribution of X and Y is:
  $p(x,y) = p(X = x$ and $Y = y)$;

- If X and Y are independent then $p(x,y) = p(x)p(y)$

- The conditional probability of X given Y is:
  $p(x|y) = \frac{p(x,y)}{p(y)} \qquad p(y) > 0.$

- Given $p(x,y)$, the marginal distribution of X can be computed by summing (integration) over Y.

- The law of total probability is its variant which uses the conditional probability definition

$$p(x) = \sum_{y \in \mathcal{Y}} p(x,y) = \sum_{y \in \mathcal{Y}} p(x|y)p(y)$$

  and for continuous random variables is

$$p(x) = \int_{y \in \mathcal{Y}} p(x,y)dy = \int_{y \in \mathcal{Y}} p(x|y)p(y)dy$$

Bayes' Rule

- $p(x,y) = p(x|y)p(y) = P(y|x)p(x)$
-
$$p(\text{hypothesis}|\text{data}) = \frac{p(\text{data}|\text{hypothesis})p(\text{hypothesis})}{p(\text{data})}$$
$$p(x|y) = \frac{p(y|x)p(x)}{p(y)} = \frac{p(y|x)p(x)}{\sum_{x\in\mathcal{X}} p(y|x)p(x)}$$
-
$$\text{Posterior} = \frac{\text{Likelihood} \times \text{Prior}}{\text{Evidence (Marginal Likelihood)}}$$

Example

An autonomous car is approaching a traffic light which can be either green, yellow, or red. The car is programmed to be conservative and thus it will stop if it detects a yellow or red light; otherwise it will continue driving. Previous tests have demonstrated that due to sensor imperfections, the car will drive through (without stopping) 10% of yellow lights, 95% of green lights, and 1% of red lights. The traffic light is on a continuous cycle (30 seconds green, 5 seconds yellow, 25 seconds red). You are riding in the car and are busy working on your Mobile Robotics project (i.e., not watching the road, light, etc.). You feel the car stop as it approaches the traffic light described above. What is the probability that the traffic light was yellow when the vehicle sensed it?

Answer

Let S represent the event that the vehicle stopped, G the event that the light was green, Y that it was yellow, R that is was red.

- *Given:* $P(S|Y) = 0.90, P(S|G) = 0.05, P(S|R) = 0.99, P(Y) = 5/60, P(R) = 25/60, P(G) = 30/60$
- *Find:* $P(Y|S)$

Answer

Let S represent the event that the vehicle stopped, G the event that the light was green, Y that it was yellow, R that is was red.

- *Given:* $P(S|Y) = 0.90, P(S|G) = 0.05, P(S|R) = 0.99, P(Y) = 5/60, P(R) = 25/60, P(G) = 30/60$
- *Find:* $P(Y|S)$

$$P(Y|S) = \frac{P(S|Y)P(Y)}{P(S)}$$

$$P(Y|S) = \frac{P(S|Y)P(Y)}{P(S|Y)P(Y) + P(S|R)P(R) + P(S|G)P(G)}$$

$$P(Y|S) = \frac{0.90(5/60)}{0.90(5/60) + 0.99(25/60) + 0.05(30/60)} = 14.63\%$$

Bayes' Rule with Prior Knowledge

- Given three random variables X, Y, and Z, Bayes' rule relates the prior probability distribution, $p(x|z)$, and the likelihood function, $p(y|x,z)$, as follows.

$$p(x|y,z) = \frac{p(y|x,z)p(x|z)}{p(y|z)}$$

- Given Z, if X and Y are **conditionally independent** then $p(x,y|z) = p(x|z)p(y|z)$

Example

Height and vocabulary are not independent; but they are conditionally independent if you add age.

https://en.wikipedia.org/wiki/Conditional_independence#Examples

The univariate (one-dimensional) *Gaussian (or normal) distribution* with mean μ and variance σ^2 has the following Probability Density Function (PDF).

$$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp(-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2})$$

We often write $x \sim \mathcal{N}(\mu, \sigma^2)$ or $\mathcal{N}(x; \mu, \sigma^2)$ to imply that x follows a Gaussian distribution with mean $\mu = \mathbb{E}[x]$ and variance $\sigma^2 = \mathbb{V}[x]$.

The multivariate Gaussian (normal) distribution of an n-dimensional random vector $x \sim \mathcal{N}(\mu, \Sigma)$, with mean $\mu = \mathbb{E}[x]$ and covariance $\Sigma = \mathrm{Cov}[x] = \mathbb{E}[(x-\mu)(x-\mu)^\mathsf{T}]$ is

$$p(x) = (2\pi)^{-\frac{n}{2}} |\Sigma|^{-\frac{1}{2}} \exp(-\frac{1}{2}(x-\mu)^\mathsf{T}\Sigma^{-1}(x-\mu))$$

Let $x = \mathrm{vec}(x_1, x_2)$ and $x \sim \mathcal{N}(\mu, \Sigma)$ where

$$\mu = \begin{bmatrix} 0.0 \\ 0.5 \end{bmatrix}, \Sigma = \begin{bmatrix} 0.8 & 0.3 \\ 0.3 & 1.0 \end{bmatrix}$$

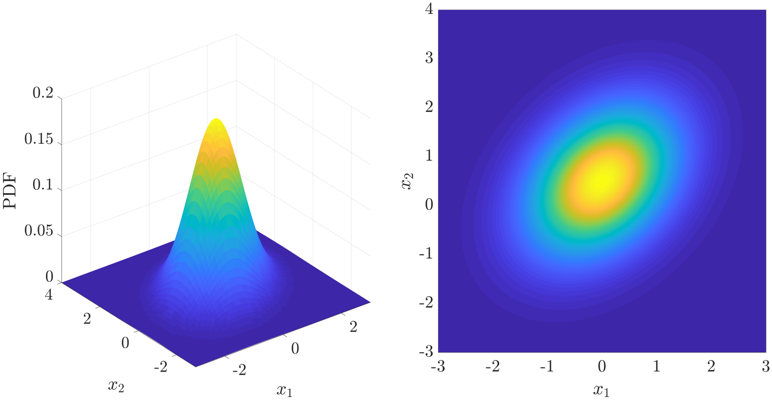


Figure: Left, two-dimensional PDF; right, top view of the first plot.

Marginalization and Conditioning of Normal Distribution

Let x and y be jointly Gaussian random vectors

$$\begin{bmatrix} x \\ y \end{bmatrix} \sim \mathcal{N}(\begin{bmatrix} \mu_x \\ \mu_y \end{bmatrix}, \begin{bmatrix} A & C \\ C^\mathsf{T} & B \end{bmatrix})$$

then the marginal distribution of x is

$$x \sim \mathcal{N}(\mu_x, A)$$

and the conditional distribution of x given y is

$$x|y \sim \mathcal{N}(\mu_x + CB^{-1}(y - \mu_y), A - CB^{-1}C^\mathsf{T})$$

Let $x = \mathrm{vec}(x_1, x_2)$ and $x \sim \mathcal{N}(\mu, \Sigma)$ where

$$\mu = \begin{bmatrix} 0.0 \\ 0.5 \end{bmatrix}, \Sigma = \begin{bmatrix} 0.8 & 0.3 \\ 0.3 & 1.0 \end{bmatrix}$$

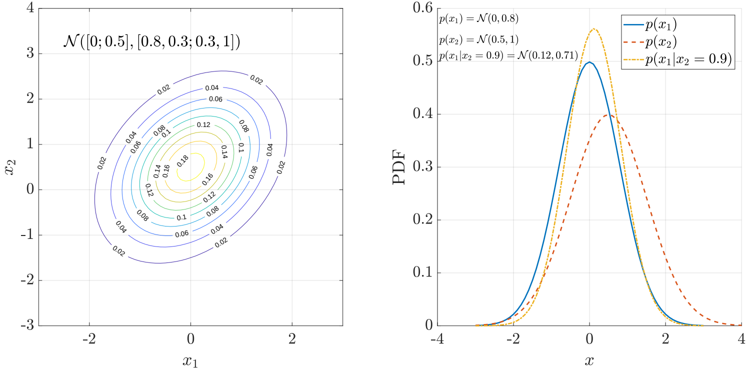


Figure: Left, the contour plot of the PDF; right, the marginals and the conditional distribution of $p(x_1|x_2 = 0.9)$.

Affine Transformation of a Multivariate Gaussian

Suppose $x \sim \mathcal{N}(\mu, \Sigma)$ and $y = Ax + b$.

Then $y \sim \mathcal{N}(A\mu + b, A\Sigma A^{\mathsf{T}})$.

$$\mathbb{E}[y] = \mathbb{E}[Ax + b] = A\mathbb{E}[x] + b = A\mu + b$$

$$\begin{aligned}
&\mathrm{Cov}[y] = \mathbb{E}[(y - \mathbb{E}[y])(y - \mathbb{E}[y])^{\mathsf{T}}] \\
&= \mathbb{E}[(Ax - A\mu)(Ax - A\mu)^{\mathsf{T}}] = A\mathbb{E}[(x - \mu)(x - \mu)^{\mathsf{T}}]A^{\mathsf{T}} \\
&= A\Sigma A^{\mathsf{T}}
\end{aligned}$$

- Given:
  - Stream of observations $z_{1:t}$ and action data $u_{1:t}$
  - Sensor/measurement model $p(z_t|x_t)$
  - Action/motion/transition model $p(x_t|x_{t-1},u_t)$
- Wanted:
  - The state x_t of dynamical system
  - The posterior of state is called belief $bel(x_t) = p(x_t|z_{1:t}, u_{1:t})$

Algorithm 1 `Bayes-filter`

Require: Belief $bel(x_{t-1}) = p(x_{t-1}|z_{1:t-1}, u_{1:t-1})$, action u_t, measurement z_t;

1: **for** all state variables **do**

2: $\quad \overline{bel}(x_t) = \int p(x_t|x_{t-1}, u_t) bel(x_{t-1}) \mathrm{d}x_{t-1}$ // Predict using action/control input u_t

3: $\quad bel(x_t) = \eta p(z_t|x_t)\overline{bel}(x_t)$ // Update using perceptual data z_t

4: **return** $bel(x_t)$

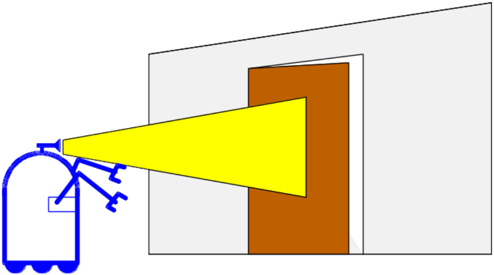
Linear:

- Kalman Filter: unimodal linear filter
- Information Filter: unimodal linear filter

Nonlinear:

- Extended Kalman Filter: unimodal nonlinear filter with Gaussian noise assumption
- Extended Information Filter: unimodal nonlinear filter with Gaussian noise assumption
- Particle Filter: multimodal nonlinear filter

- Suppose a robot obtains measurement z, e.g., using its camera;
- What is $p(\mathrm{open}|z)$?



- $p(\text{open}|z)$ is **diagnostic**.
- $p(z|\text{open})$ is **causal**.
- Often causal knowledge is easier to obtain.
- Bayes rule allows us to use causal knowledge:

$$p(\text{open}|z) = \frac{p(z|\text{open})p(\text{open})}{p(z)}$$

Sensor model (likelihood):

- $p(z = \text{sense\_open}|\text{open}) = 0.6$
- $p(z = \text{sense\_open}|\neg\text{open}) = 0.3$

Prior knowledge (non-informative in this case):

- $p(\text{open}) = p(\neg\text{open}) = 0.5$

Update/Correction:

$$p(\text{open}|z) = \frac{p(z|\text{open})p(\text{open})}{p(z|\text{open})p(\text{open}) + p(z|\neg\text{open})p(\neg\text{open})}$$

$$p(\text{open}|z = \text{sense\_open}) = \frac{0.6 \times 0.5}{0.6 \times 0.5 + 0.3 \times 0.5} = 0.6667$$

Remark

z raises the probability that the door is open.

- Suppose our robot obtains another observation z_2.
- How can we integrate this new information?
- More generally, how can we estimate $p(x|z_1,\ldots,z_n)$?

$$p(x|z_1, \ldots, z_n) = \frac{p(z_n|x, z_1, \ldots, z_{n-1})p(x|z_1, \ldots, z_{n-1})}{p(z_n|z_1, \ldots, z_{n-1})}$$

Assumption (Markov Assumption)

z_n *is* **independent** *of* $z_1, \ldots, z_{n-1}$ **if** *we* **know** x.

$$p(x|z_1, \ldots, z_n) = \frac{p(z_n|x, z_1, \ldots, z_{n-1})p(x|z_1, \ldots, z_{n-1})}{p(z_n|z_1, \ldots, z_{n-1})}$$

Assumption (Markov Assumption)

z_n *is* **independent** *of* $z_1, \ldots, z_{n-1}$ **if** *we* **know** x.

or equivalently we can state:

Assumption (Markov Property)

*The Markov property states that "***the future is independent of the past if the present is known.***" A stochastic process that has this property is called a* **Markov process***.*

$$p(x|z_1,\ldots,z_n) = \frac{p(z_n|x, z_1,\ldots,z_{n-1})p(x|z_1,\ldots,z_{n-1})}{p(z_n|z_1,\ldots,z_{n-1})}$$

Assumption (Markov Assumption)

z_n *is* **independent** *of* $z_1,\ldots,z_{n-1}$ **if** *we* **know** x*.*

$$p(x|z_1,\ldots,z_n) = \frac{p(z_n|x)p(x|z_1,\ldots,z_{n-1})}{p(z_n|z_1,\ldots,z_{n-1})}$$

$$= \eta_n \; p(z_n|x)p(x|z_1,\ldots,z_{n-1}) = \eta_{1:n} \; \prod_{i=1}^{n} p(z_i|x)p(x)$$

where $\eta_{1:n} := \eta_1\eta_2\cdots\eta_n$.

Readings

- Probabilistic Robotics: Ch. 1 and 2, Understand Example 2.4.2
- State Estimation for Robotics: Ch. 2
- Lecture note 1
- Bar-Shalom Ch. 1.3-1.6

Next Time

- Kalman Filtering
- Readings:
  - Probabilistic Robotics: Ch. 3
  - State Estimation for Robotics: Ch. 3
  - Lecture note 2
  - Bar-Shalom Ch. 2 and 5