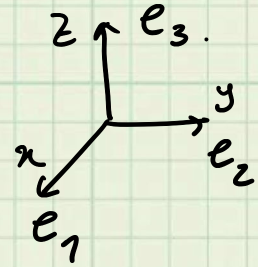
$$SO(3) = \left\{ A \in \mathbb{R}^{3\times 3} \;\middle|\; A^T = -A \right\}$$

$$\omega = \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix} \in \mathbb{R}^3$$



wedge notation

$$\omega^\wedge = \begin{bmatrix} 0 & \omega_3 & -\omega_2 \\ -\omega_3 & 0 & \omega_1 \\ \omega_2 & -\omega_1 & 0 \end{bmatrix} \in SO(3)$$

vee:

$$S = \omega^\wedge \quad , \quad S^\vee = (\omega^\wedge)^\vee = \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix}$$

$$G_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} \qquad e_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$G_2 = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \qquad e_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$G_3 = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad e_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\omega^\wedge = \omega_1 G_1 + \omega_2 G_2 + \omega_3 G_3, \qquad \omega = \omega_1 e_1 + \omega_2 e_2 + \omega_3 e_3$$

Cross product & Lie Bracket

$e_1 \times e_2 = e_3$

$e_2 \times e_1 = -e_3$

$[G_1, G_2] := G_1 G_2 - G_2 G_1 = G_3$

$[G_2, G_1] = -G_3$

$\dot{R} R^T = \omega^{s\wedge}$

$R^T \dot{R} = \omega^{b\wedge}$

for any $R \in SO(3)$

$R^T R = I \;,\; R R^T = I$

$$\frac{d}{dt}(R^T R) = \dot{R}^T R + R^T \dot{R} = 0$$

$$= (R^T \dot{R})^T = -R^T \dot{R} \in so(3)$$

$$A^T = -A$$

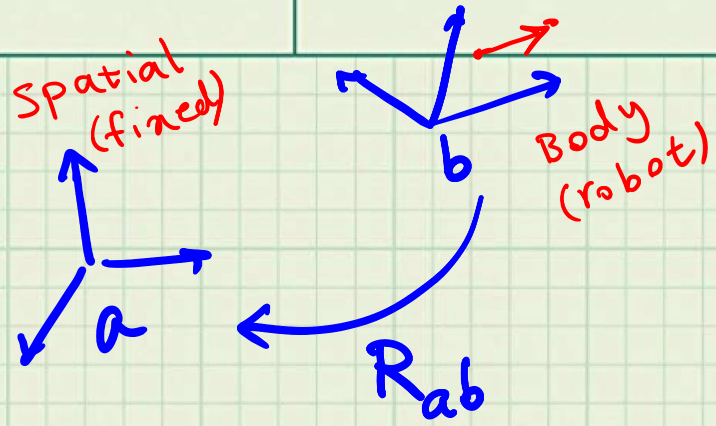
$$\frac{d}{dt}(R R^T) = \dot{R} R^T + R \dot{R}^T$$

$$= \dot{R} R^T = -(\dot{R} R^T) \in so(3)$$

$$R^T \dot{R} = \hat{\omega}$$

$$R R^T \dot{R} = R \hat{\omega}$$

$$\dot{R} = R \hat{\omega}$$



$$\omega^s = R_{sb} \omega^b$$

$$P_a = R_{ab} P_b$$

$$\boxed{\dot{R} = R \, \hat{\omega^b}}$$

$$V_b = P_b - q_b$$

$$\frac{\dot{R}}{R} = \hat{\omega^b}$$

$$R V_b = R p_b - R q_b = p_a - q_a$$

$$R_{k+1} = R_k \exp(\hat{\omega^b} \Delta t)$$

$$\boxed{V_a = R V_b}$$

$$R_{k+1} = R_k \Delta R_k$$

$$\dot{x} = a x$$

$$\int \frac{\dot{x}}{x} dt = \int a \, dt$$

$$\int_0^{\Delta t} a \, dt = a t \Big|_0^{\Delta t} = a \Delta t$$

$$\log |x| = \int a(t) \, dt$$

$$x = c e^{\int a(t) \, dt}$$

$$x(0) = x_0$$

$$\dot{R}R^T = \omega^{?\wedge}$$

$$AB \neq BA$$

$$\dot{R} = \omega^{?\wedge} R$$

$$\boxed{R\omega^\wedge R^T = (R\underline{\omega})^\wedge}$$

$$\dot{R} = RR^T\omega^{?\wedge}R = R(R^T\omega^{?\wedge}R)$$

$$R \to R^T$$

$$\boxed{R^T\omega^\wedge R = (R^T\omega)^\wedge}$$

$$\left\{\begin{array}{l} \dot{R} = R(\underline{R^T\omega^?})^\wedge \\ \dot{R} = R\,\underline{\omega}^{b\wedge} \end{array}\right.$$

$$\Rightarrow R^T\omega^? = \omega^b \Rightarrow \boxed{\omega^s = R_{sb}\,\omega^b}$$

$$\dot{R} = \omega^{s\wedge} R$$

$$R_{K+1} = \exp(\omega^{s\wedge}\Delta t)\,R_K$$

prove $R\omega^\wedge R^T = (R\omega)^\wedge$, $\omega^\wedge \in so(3)$, $\omega \in \mathbb{R}^3$

$a \in \mathbb{R}^3$

$a, b \in \mathbb{R}$

$(R\omega)^\wedge a = R\omega \times a$ (I)

$$\underline{a \times b = a^\wedge b}$$

$$Ra \times Rb = R(a \times b)$$

$$= R\omega \times RR^T a$$

$$= R(\omega \times R^T a)$$

$$= R(\omega^\wedge R^T a) = R\omega^\wedge R^T a$$

$$\Rightarrow (R\omega)^\wedge = R\omega^\wedge R^T \quad \text{(II)}$$

What about Rot. & tran.

$\omega \in \mathbb{R}^3, \; v \in \mathbb{R}^3$

$$R\omega^b = \omega^s$$

?

$$\text{matrix} \cdot \begin{bmatrix} \omega^b \\ v^b \end{bmatrix} =$$

6×1

$\swarrow$

twist ξ

$$\dot{p}_a = \dot{p}_b + \hat{\omega}(p_a - p_b)$$

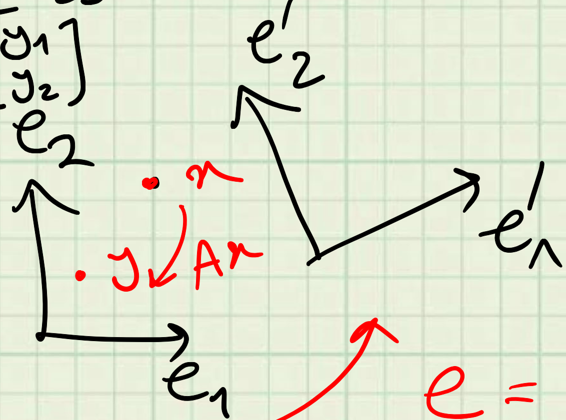
$$v_a = v_b + \omega \times r$$

Conjugation & matrix Similarity

$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$$y = Ax \qquad y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

$$y' = Tx'$$



$$e = P^{-1}e'$$
$$e' = Pe$$

Suppose $y = P^{-1}y'$

$x = P^{-1}x'$

$$y = Ax \Rightarrow P^{-1}y' = AP^{-1}x' \Rightarrow y' = PAP^{-1}x'$$

$$T = PAP^{-1}$$