

Maximum Initial Velocity of Electron Beam

The electrons are emitted from a tungsten filament heated by an electrical circuit. To supply the heat, the tungsten will be placed at the cathode of the electron gun assembly which will be supplied with a voltage from a variable DC source. Depending on whether or not a transformer is in place, the voltage can vary from up to 120V to within the tens of kilovolts range. By scaling the voltage V by the charge of an electron (given below in Coulombs) q , we can set an upper bound to the energy that each electron will have.

$$E = qV$$

$$E = 1.60217663 \times 10^{-19} \cdot V$$

Non-relativistic

From here, using the fact that the rest mass of an electron is $9.1093837 \times 10^{-31}$ kilograms, we can calculate the velocity of each electron. However, note that when using a trial case of 50kV with the classic relation $E_K = \frac{1}{2}mv^2$ we get:

$$E_K = 8.01088315 \times 10^{-15} \text{ Joules}$$

$$v = \sqrt{\frac{2E_K}{m}}$$

$$v = 132620511$$

$$c = 299792458$$

$$\Rightarrow \frac{v}{c} = 0.442374409184 \frac{m}{s}$$

Relativistic

Since the electrons are travelling at a considerable fraction of the speed of light relativistic effects become noticeable. Let's recalculate the velocity of the electron while taking relativistic effects into account. The relativistic kinetic energy is:

$$\begin{aligned} E_k &= \left(\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} - 1 \right) mc^2 \\ \frac{E_k}{mc^2} &= \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} - 1 \\ \left(\frac{1}{1 + \frac{E_k}{mc^2}} \right)^2 &= 1 - \frac{v^2}{c^2} \\ v &= c \sqrt{1 - \left(\frac{1}{1 + \frac{E_k}{mc^2}} \right)^2} \end{aligned}$$

Using our previous value for kinetic energy, the velocity becomes:

$$v = 123720202.894$$

$$\frac{v}{c} = 0.41268617536$$

Lorentz Force On Electron Beam

We want to determine what the force will be acting on the electron beam will be. We know that this will be the Lorentz force:

$$\mathbf{F} = q\mathbf{v} \times \mathbf{B}$$

However, for relativistic velocities, we will need to derive the relativistic Lorentz Force.

Equations of Motion