Electron Beam Deflection

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Initial Velocity of Electron Beam

The electrons are emitted from a tungsten filament heated by an electrical circuit. To supply the heat, the tungsten will be placed at the cathode of the electron gun assembly which will be supplied with a voltage from a variable DC source. Depending on whether or not a transformer is in place, the voltage can vary from up to 120V to within the tens of kilovolts range. By scaling the voltage V by the charge of an electron (given below in Coulombs) q, we can set an upper bound to the energy that each electron will have.

$$E = qV$$
$$E = 1.60217663 \times 10^{-19} \cdot V$$

From here, using the fact that the rest mass of an electron is $9.1093837 \times 10^{-31}$ kilograms, we can calculate the velocity of each electron. However, note that when using a trial case of 50kV with the classic relation $E_K = \frac{1}{2}mv^2$ we get:

$$E_K = 8.01088315 \times 10^{-15} \quad \text{Joules}$$

$$v = \sqrt{\frac{2E_K}{m}}$$
$$v = 132620511$$
$$c = 299792458$$
$$\implies \frac{v}{c} = 0.442374409184 \ \frac{m}{s}$$

Since the electrons are travelling at a considerable fraction of the speed of light relativistic effects become noticeable. Let's recalculate the velocity of the electron while taking relativistic effects into account. The relativistic kinetic energy is:

$$E_k = \left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1\right) mc^2$$
$$\frac{E_k}{mc^2} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1$$
$$\left(\frac{1}{1+\frac{E_k}{mc^2}}\right)^2 = 1 - \frac{v^2}{c^2}$$
$$v = c\sqrt{1 - \left(\frac{1}{1+\frac{E_k}{mc^2}}\right)^2}$$

Using our previous value for kinetic energy, the velocity becomes:

$$v = 123720202.894$$
$$\frac{v}{c} = 0.41268617536$$

Lorentz Force On Electron Beam