Project of the course
Complex Systems and Network Science

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Abstract. This is an experiment base work where we try to understand the behavior of a social system like a city for example combining together the Axelrod's model of cultural dissemination and the Schelling's model of self-segregation. The idea is to show how the agents behave in different circumstances dealing with agents of different cultural background and how their own {in}tolerance level influences this behavior.

Keywords: culture dissemination, self-segregation, homophily, tolerance, cultural traits, cultural code, state of equilibrium, cultural clusters.

1 Introduction

The modern societies of our times can be viewed as mixture of different cultures. Of course the notion of "culture" is very broad and in time there have been many definitions of this concept. In this work we are going to think about the culture as a set of attributes that are subject of social influence. As Axelrod[1] put it in his famous article: "culture is what social influence influences...". In this sense peoples that belong in the same culture share similar fundamental characteristics like language, religion, origin etc. Imagine a big modern city like New York, London or Paris where the inhabitants belong in a large spectrum of cultural background, one of the questions that rise spontaneously is: How does the individuals belonging to different cultures interact with each other and how their culture influence other cultures? In order to ask this very important questions we have first to consider the previous works on this subjects. In the field of social science there are many distinguished authors that with their work have contributed to the better understanding of the social phenomena. We are going to focus on the work of two of this authors, namely Thomas Schelling[3] and Robert Axelrod. The former model deals with the phenomenon of segregation. Schelling wanted to understand the social dynamics behind the extreme separation of populations based on specific criteria such as, income, race, origin and even sex or age. On real example of the segregation based on income is the distribution of the inhabitants in the city of New York. This type of segregation is called "residential segregation". The example shows that if we consider the income as our basic criteria there is extreme intolerance between individuals. In other words rich peoples demand to be surrounded by similar rich folks. Let's have a closer look to the Schelling's model. Basically the model setup is simple, the world consists of a 2-dimensional grid and each individual occupy a cell on the grid. The model assumes that there is a percentage of empty cells, this percentage is controlled by a parameter on the model. Each cell is surrounded by 8 neighbor cells. The main idea of the model is that each individual wants that a specific fraction (threshold) of his neighbors to be similar to himself. If this condition is not fulfilled the individual is allowed to move to another free cell. For example one individual is happy if 5 of his 8 neighbors are similar to him, otherwise the individual become unhappy and in the next run of the model moves to another free cell of the grid if there is one. The model stops when there are no more moves, in other words when all the individuals are happy. One thing to keep in mind in this model is that even a happy individual may become unhappy in the next run of the model and this may happened for two reasons: First, if a neighbor leaves (moves out), this action results to a structural change on the grid and in the same time changes the value of the threshold of the neighbor cells. One can see this as a chain reaction involving other individuals on the grid. The second reason for an individual to become unhappy is the exactly opposite to the first one, in other words if a new individual moves in. The astonishing fact about this model is that it describes a very counterintuitive behavior, in other words even though the individual per se may be very tolerant the overall behavior results on a very segregated society. The later model, proposed by Axelrod in 1997 analyses the phenomenon of cultural dissemination. In his own words "... people tend to become more alike in their beliefs, attitudes and behavior when they interact, why do not all such differences eventually disappear?". The question that Axelrod tries to ask in his model is exactly this: Why the process of becoming similar (cultural assimilation) stop just before its completion? This process of becoming similar is very important in the society and can explain many important observations such as state formation, succession conflicts, political and ideological cleavages, integration of individual moving in new societies and much more. Just how we did with the first model we are going to say a few words about this model too. Before jump into the model lets introduce three important premises. First we are using ABMs (Agent-Based Models), second, there is no central authority to control the behavior of the agents and third the agents have an adaptive rather than rational behavior. We claim that the same rules holds in the Schelling's model too. Now, in the Axelrod's model the culture is treated as a vector of variables (F) and each variable has a range of possible values, called cultural traits. An example of the vector F may be, F = [language, religion, race] and some possible values for the variable language can be, language = {english, italian, albanian}. The individuals lives in static world, (no movement allowed) represented as μ x μ grid where μ = (int) Sqrt(N) and N is the total number of individuals (agents). Each individual has normally 4 or 8 neighbors (corners and borders excluded). We compute the cultural similarity as the number of identical traits over the length of the vector F (total number of characteristics taken in consideration). As you can imagine bigger is this number, better chances are that this individuals with interact with each other exchanging cultural aspects. This phenomenon is called homophily in the literature. Some interesting results that this models provide are the following: First, even though at the beginning of the simulation the agents are very different, eventually after some time they are going to be more similar to each other resulting in some few stable regions (cultural communities or clusters) that will remain unchanged until the end of the simulation. This number of stable regions is very small, with an average value of 3 even if we trick all the other parameters this average value remain stable. Second, the number of neighbors is important. In larger neighborhoods we observe the creation of fewer cultural regions. So here we can say that the size of the neighborhood (number of linked neighbors) matters. Third, the total number of individuals (the size of the grid) is not a very important parameter in the creation of the cultural regions. This are some of the insights of the Axelrod's model of cultural dissemination. In the following pages we are going to discuss a new model that combine together both models and we are going to observe and analyze the trends and the behaviors of the agents.

2 Proposed Model

In the previous chapter we spent some time describing the two models in question. It was more an historical overview rather than a formal description, and helped us to create the context in which we are going to work later on. In this sense we are going to need some of the concepts and the results of this two original models in order to construct a new model that combine both of them. One way to see this is to say, that we are going to enrich the static model of Axelrod with a dynamic component provided by the Schelling's model. Some distinguish work have been done on this subject by Garcia-Lazaro et al.[2]. The result show a broad range of behaviors that can be observe when you change some of the key parameters of the system, like the number of empty cells for example. Some of this result can be anticipated by simple reasoning but other results are bizarre and hard to anticipated at the start of the simulation.

In our model, we expect to observe similar behaviors, to be able to lay down hypothesis and to proof them with the experiments. The following sections describe the specification and the implementation of the model.

2.1 Model specification

This model describes a small community, like a small city. If we stick with this metaphor we can say that each city has a number of houses called sites in our model. The number of the sites (N) is an important parameter of the model. Each house is part of a larger system called neighbourhood. The size of the neighbourhood depends on the specific geographic layout of the city. Each house may be free, nobody lives there or may be occupied. The probability of a house to be free is controlled by the variable (e), this results into a total of Ne free houses. On the other side the number of occupied houses can be computed as $N(1 - e)$. All inhabitants of the city (the individuals that occupy a house) are characterized by a *cultural code* that we express with the letter σ. The *cultural code* per se is a set of F attributes, where each attribute is call a *cultural trait*. We consider at maximum q traits, where q is a number chosen in the process of setup. A practical example may be the following.

The city of Bologna is inhabited by individuals with different cultural backgrounds. If we pick two individuals, one for each culture, i and j, a possible cultural code distribution may be:

σ(i) = {italian, catholic, white } and *σ(j) = {french, muslim, black }*

In this example the set of attributes is F = {language, religion, colour} and q = { 2 }.
The parameters F e q are also very important for our model, the first represent the *complexity* of the *cultural code* and the second the different cultures resulting on the population. High numbers of F a q results to a rich cultural environment.
To recap, at this point we have generate N sites, occupy $N(1 - e)$ of them and assign to each individual a cultural code using a set of attributes F and a multiplicity q for each attribute. The next step is to execute the model.
At each step of the simulation, each individual i choose randomly one of his neighbours, say j and computes the *cultural overlap* ω_{ij} that represent the number of similar traits that they have in common. More formally:

$$\omega_{i,j} = \frac{1}{F} \sum_{k=1}^{F} \delta(\sigma_k(i), \sigma_k(j)) \qquad (1)$$

Here δ represents the Kronecker's function that produce 1 if two numbers are identical and 0 otherwise.

$$\delta_{i,j} = \begin{cases} 1, i = j \\ 0, i \neq j \end{cases} \qquad (2)$$

Once each individual has compute the *cultural similarity* with each of his neighbours the next step consists on the trait exchange between neighbours. More in detail, individual i randomly choose one of the F traits of his neighbour j, say the trait k and copy it in his *cultural code* with a probability given by the value ω_{ij}. Formally we express this operation of value exchange with the following formula:

$$\sigma_k(i) = \sigma_k(j) \qquad (3)$$

In simple words we are saying that the probability of one individual to copy a trait of another individual is higher if their *cultural overlap* is higher. This is something that we encounter in the everyday life. For example people that speak the same language are prone to interact with each other. We refer to this phenomena as *homophily* that is the attraction of individual based on similar cultural traits.
Now, the probability that this imitation does not occur is equal to $(1 - \omega_{ij})$. If this is the case the individual i will compute the average of the cultural overlap with all his neighbours. Mathematically this shown by the following formula.

$$\varpi_i = \frac{1}{k_i} \sum_{j=1}^{k_i} \omega_{i,j} \qquad (4)$$

Here ϖ_i is called *average cultural overlap* and is computed considering all the k neighbours of i. The average cultural overlap express the similarity of an individual with all his neighbourhood. We want to stress that the neighbourhood consist only of occupied sites, free sites are not considered because they have 0 contribute to the whole sum.
Each individual has a threshold value called *tolerance T.* If the value of ϖ_i is smaller than the value of T the individual decides to move to another free site if there is one

available. One can think that the value *T* in a more restrictive sense represent a notion of intolerance of the individuals against their neighbourhood.
The model stops when there are no more interactions between the individuals. By interactions here we mean moves and imitations. When we reach this phase we say that the system has reached an equilibrium.

2.2 Model implementation

Before going into the details of the implementation we want to spent some time talking about the tools that we are going to use. The experiments are executed using a programmable agent based model environment called NetLogo version 6.0.3. This tool provide a high level programming language and permits to create fast and easy simulations.
In our implementation of the previous model the setup of the variables is the following:
The total number of sites (*N*), including the free sites is 900. This number chosen because it has a perfect square root of 30. We use this numbers in the construction of the lattice topology.
The vector *F* that represents the *cultural code (cc)* is defined in the interval [3, 9]. This choice is justified because is more realistic, in the sense that a *cultural code* of only one trait is not interested and doesn't allow to study the full complexity of the communities.
Cultural traits (ct) are represented as a number that can take values in the range from 0 to 9. As you can see this number is large enough to model a modern multicultural society, and this is done on purpose.
The probability of empty sites (*e*) is another very important parameter in the model. We expect to see very interesting behaviors tweaking this variable.
The *tolerance (T)* is implemented like a slider in the interface and can get any value from 0 to 100%.
All this parameters are accessible in the phase of setup of the model in the form of sliders in the NetLogo's interface.
Now, for this model to execute we have to create a network, previously we use the metaphor of city. As we know a network is made of nodes connected by edges, that in our model represents sites that are linked together to create neighborhoods. For this purpose we use 4 types of networks, namely:
A Random Network created ad hoc that doesn't allow isolated sites. In case there is a site that has no neighbors we explicitly ask for a new link to be added from the site to another random site independently on its status (free or occupied).
A Preferential Attachment network that is another random network that allow a better visualization of the sites.
And last but not least two versions of Lattice networks, one in which each site has 4 neighbors and another where each site has 8 neighbors. In order to better understand the behavior of the model we have provide visual aids like monitors and plots in the NetLogo's interface. Figure 1 shows the interface of the model. Figure 1 shows the interface of the model. On the right side of the interface is located the "world" when

one can see the model in action. In this model the free sites are represented with the shape of a *green star* and the occupied sites with shape of a *circle*.

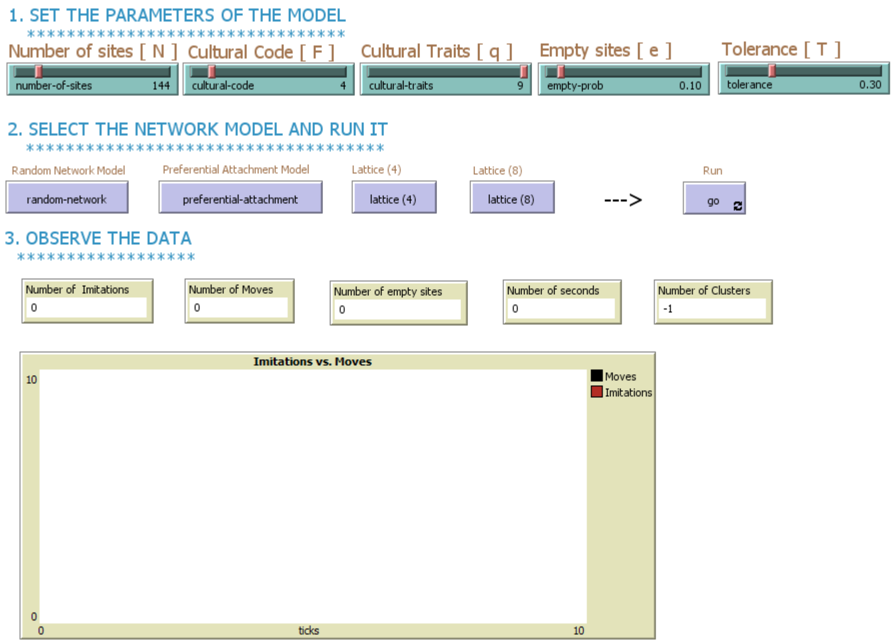


Fig 1. Starting interface

3 Model evaluation

Considering the number of parameters and network topologies we have created, result near impracticable to execute experiments for each possible combination of this variables. In order to get some real, meaningful results we have to consider ranges of parameters instead of single values. We are going to apply the following protocol for the experiments.

0. Before the actual execution of an experiment we will try to anticipate the result and then compare it with the actual output.
1. We are going to create intervals of values and within this intervals we are going to pick the values for the experiments. For example if we consider the parameter *T*, we will divide it in 3 intervals, [0, 30], [31, 60], [61, 100].
2. After having decide the variables and the network topology we are going to run multiple times the simulation for each setup in order to be sure that the result is real and not just a model anomaly.
3. The results of each experiment will be shown using tables, plots and screenshots.

The first topology that we are going to consider is the Random Network. In this implementation it's not easy to separate the clusters in a clear, visible manner but we are going to bypass this limitation using a variable that count then number of clusters.

3.1 Random Network

In our implementation of the Random Network, each node has at least one link to another random node. This is done in order to avoid isolated nodes at the start of the simulation. The following portion of code do exactly this:

```
ask sites [create-link-with one-of other sites
if (count link-neighbors)  = 0 [
ask sites [create-link-with one-of other sites]
]]
```

3.1.1 Experiment 1

Setup: $N = 500; cc = 6; ct = 9; e = 5\%; T = 10\%$.

In this first configuration, given the number of possible populations and the low probabilities of free sites and tolerance we are expecting to see a number of well defined clusters to form. The following table show the average results of 5 executions of the model.

No: Run	No: Imitations	No: Moves	No: Free Sites	No: Clusters
1	476248	467	22	17
2	67403	490	23	46
3	289788	458	26	24
4	142223	509	23	30
5	664517	438	26	22
				27,8

Table 1

The results show an even distribution of the number of well distinct populations (clusters) that are created. As one can observe on the table there are two runs where the result exceeds by far the average and we think that this is caused by the randomness of the model. For example in the second run we observe the formation of 46 clusters and in the same time the number of imitations is limited considering the other executions. The best explanation for this is to say that most individuals are trying to maintain their original cultural code and avoid imitations. Not all of them will succeed in this enterprise. Now, to get a clue of how the total number of sites influence the number of clusters we are going to run the model with the maximum value of sites available *($N = 900$)*. In this new setup we expect to see the increase of the number of clusters being formed with the increase of the total number of the sites, something that is quite normal. Having set the empty probability as a function of the total number of sites, this is traduced to a larger number of empty sites for the agents to move to. The table 2 show the data obtained running the model to this slightly modified configuration.

Setup: *N = 900; cc = 6; ct = 9; e = 5%; T = 10%.*

No: Run	No: Imitations	No: Moves	No: Free Sites	No: Clusters
1	464263	873	42	43
2	863321	800	43	41
3	671396	796	48	41
4	870031	841	49	36
5	302922	849	53	52
				42,6

Table 2

The experiments show that if we increase the number of sites from 500 to 900, the number of imitations, the number of the moves and the number of the clusters increase. This increment seems to obey a linear function. For completeness we execute the model even for small values of *N*. In the case of *N = 100* we observed that the average number of separated clusters was 12 and the average number of moves was 85. If we try to further reduce the number of total sites, we see that the average number of clusters being form fluctuate between 5 to 10. We can think that really small number of sites, less than 10 are not realistic for the purpose of this model and we can skip this results. The conclusion of this experiment shows that the total number of sites is closely related with the number of clusters. If we vary this parameter and keep the others fixed we will see that the number of different clusters of populations will change following the trend of the first parameter. The same behavior is preserved even if we vary both the number of sites and the cultural code. The experiments show that if we set *N*, *cc* and *ct* to the maximum values, the number of formed clusters is lower as an absolute value but the trend is the same as the previous runs. The insight that we get here is that for large number of sites, for large and different populations, if the number of free sites is limited and the tolerance is low the number of cluster that can be formed is relatively large but stable. This value is varies from 20 to 40. From another point of view this means that the individuals try to preserve their own culture.

3.1.2 Experiment 2

Setup: *N = 900; cc = 9; ct = 9; e =* [*5%; 10%; 15%; 30%*]*; T = 15%.*

The objective of the second experiment is to explore the influence of the parameter *T* and *e* on the overall behavior of the model. In this set of runs we are going to keep the parameters *N*, *cc*, *ct* and *T* fixed and vary *e*. The behavior of the model is very predictable, in the sense that a low value of *e* means less possibilities for the agents to move. This is traduced to a larger number of imitations and movements. The number of the clusters being formed increase as we increase the number of free sites. The data show that for low percentages of empty sites the average number of imitations is very large and it take a large amount of time for the model to reach a stable configuration. As we increase gradually the values of *e* the number of clusters increase but the number of imitations, moves and time to reach the equilibrium decrease. For values of *e* larger than 30% the average number of formed clusters remain stable, around 70. For the purpose of a more realistic model we are not going to consider values that exceed 30%.

Free sites	Average number of Imitations	Average number of Moves	Average number of Clusters
5%	1452064	1958	31
10%	867603	1907	48
15%	433864	1604	60
25%	175604	1863	74
30%	141524	1865	71

Table 3

This data seem to confirm our intuition that if the intolerance is low and there are enough empty sites the people will move less and imitate less because they are going to keep their cultural code intact.

3.1.3 Experiment 3

Setup: $N = 900; cc = 9; ct = 9; e = 20\%; T =[\ 15\%; 30\%; 45\%; 65\%;80\%\ ]$.

In this experiment we are interested to understand the connection between the parameter *T*, that in fact represent the intolerance of an individual against his neighborhood and the other parameters. In the following tests we are going to vary the values of the T parameter and observe the behavior of the model. Intuitively when the intolerance is low the number of the clusters being formed will be high, on the other side if we increase the percentage of intolerance the number of different clusters should reduce. Table 4 show the data obtained from the tests executed for the different values of *T*.

Tolerance (T)	Average number of Imitations	Average number of Moves	Average number of Clusters
15%	422867	1844	60
30%	71495	51337	90
45%	412260	1313698	81
65%	825774	2284688	60
80%	1119471	2771416	28

Table 4

As you can see the number of the different clusters first grow, reaching the peak at 30% and then decrease rapidly. On the other hand the number of the imitations and moves only increase reaching very high values. We can say that when the individuals are intolerant they will keep moving to other free sites and imitate neighbors until the different groups of individuals will separate themselves from the others. In order to obtain a more precise understanding of this phenomena we are going to confront this results with the results observed when we run the model in different network topologies.

3.2 Preferential Attachment

Preferential Attachment is a well-known algorithm that obey the heavy-tail low. The idea here is that the newly created nodes (agents, in our case) what to be linked with an existing node that has may other nodes liked to it. This is an algorithmic implementation of the "rich-get-richer" empiric observation.

3.2.1 Experiment 1

Setup: $N = [100, 300, 500, 900]; cc = 9; ct = [3, 6, 9]; e = 20\%; T = 10\%$

In this set of experiments we are going to keep fixed the value of *cc*, *T e* and vary *N* and *ct*. The data should prove that if we change the total number of sites and the number of different populations, the number of different clusters will change following the same trend. In other words if we increase the number of sites and the number of populations the number of clusters will increase in the same fashion.

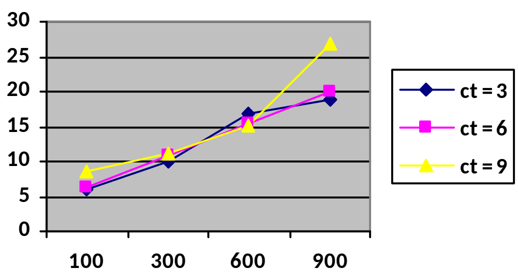
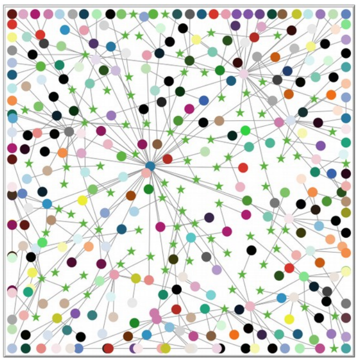
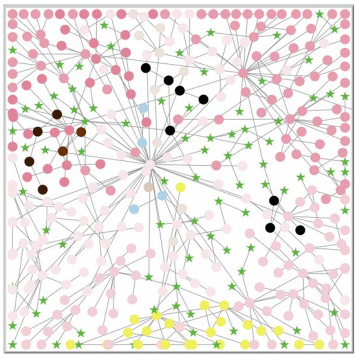


Fig 2. Linear increase of the number of clusters

The chart above show the relation between *N*, *ct* and the number of different clusters generated. We have plot *N* in the x axis, the number of clusters in the y axis and vary *ct*. The figure 3 above show the model before and after the execution.

The test shown in the figures above involved 400 sites and as one can observe the number different clusters formed during the process are well defined. In this run of the model we had 11 different clusters of populations. Concluding this experiment we can say that the number of clusters formed in the process is linear to the total number of sites and the total number of all possible different populations existing at the beginning of the experiment.

3.2.2 Experiment 2

Setup: N = [*900*]*; cc = 9; ct* = 6*; e* =[*10%; 15%; 30%*]*; T* =[*10%*]

In this experiment we are interested to investigate the behavior of the model when the number of free sites and the intolerance changes. Intuitively if the number of free sites increase the model should reach the equilibrium faster and the number of clusters of populations should increase slightly in a linear fashion. The tests show that when we vary the number of empty sites keeping fixed the intolerance the number of the clusters increase. For values of *e* between 10% and 15% the number of clusters is stable, with an average of 10 clusters of different populations. For values of *e* 30% the number clusters triples, reaching an average value of 31. In order to be more realistic we should take values of *e* smaller than 30%. A good approximation here may be 5% that reflect better the reality in most cities. The tests show that if we reduce the probability of empty sites the number of clusters being formed decrease dramatically. If we assume a value of *e* equal to 5% the number of observed clusters is only 5. The figure below show this case.

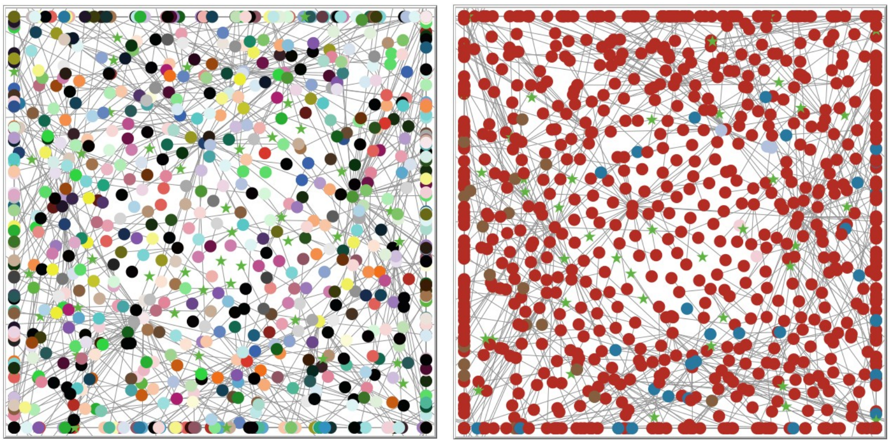


Fig 4. Before and after the execution of the model with e = 5%

What happens if we further reduce the probability of empty sites, let's say 1%? We assume that the model will take much more time to reach an equilibrium and that the number of clusters will smaller. Indeed the data obtained by the tests show that the number of clusters remain stable at the value of 4. At this point we can conclude that the probability of empty sites influence the total number of clusters being formed in the process for e given set of values of the other parameters. From now on we are going to consider the value of e to be 5%.

3.2.3 Experiment 3

Setup: N = [*900*]; cc = *9*; ct = 6; e =[*5%*]; T =[*10%, 30%, 50%, 65%, 80%*]

The purpose of this experiment is to investigate the way in which the model behave when we change the tolerance parameter. The expectances are that the number of clusters will increase. Another thing that we observe on this tests is the number of moves that an individual perform increase rapidly from hundreds to millions and this is easy to explain. As the intolerance increase above the threshold the individuals are obligated to move to other free sites and from there the process start again leading to other moves. The table 5 shows the data in an analytic form.

Tolerance (T)	Test 1	Test 2	Test 3	Test 4	Test 5	Average Clusters
10%	6	3	7	9	8	6.6
30%	44	36	32	36	39	37.4
50%	43	41	33	47	29	38.6
65%	37	31	35	11	41	31
80%	4	17	18	10	3	10.4
90%	3	2	3	3	3	2.8

Table 5

Studying the data on the table one can reach the conclusion that the number of clusters increase rapidly until a certain value of *T*, close to 50% and from then on start to decrease. We observe that for values of *T* close to 90% the number of clusters is very small, reaching an average of 2.8 clusters but still heterogeneous. Even in this extreme case of intolerance the system doesn't converge into an unique cluster of individuals with the same cultural code.

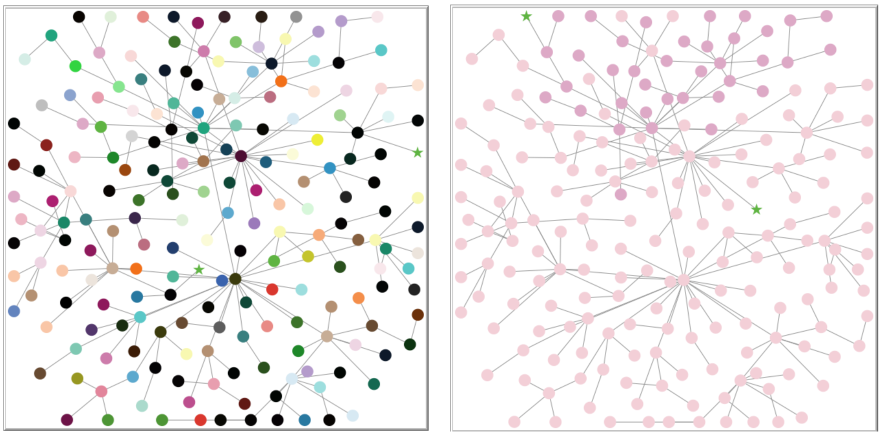


Fig 5. Before and after the execution with *N = 200, cc = 9, ct = 6, e = 5%, T = 90%*

The best we can hope for is to have 2 different well separated clusters, one of them is very big the second is smaller. Even thought the clusters are well separated a close look into their cultural code reveal many similarities, something that you can see on the color scheme too.

3.3 Lattice

The lattice topology is a special case of network topology where the nodes are placed on the intersection of the lines of a two-dimensional grid. This lines represent the edges. Depending on the implementation a node may have 4 or 8 neighbors. In this work we are going to consider both of them. We are going to implement the lattice as a rectangle grid where the size of each dimension is the exact square root of the number of nodes. This is the reason why the maximum value of *N* is 900 (30x30).

3.3.1 Experiment 1 - 4 neighbor lattice

Considering the results of the previous topologies, we hope to see the same behavior in the lattice. First we are going to use a rectangle lattice where each node is linked with other 4 nodes. We expect to see a small number of clusters at the end of the execution. Indeed after more than 15 million of imitations and 128 moves the system reach an equilibrium consisting of only two clusters of population. In this test we used the following configuration: *N = 900; cc = 9; ct =* 6*; e = 5%; T = 10%*
Using the above setup, in the second run the system reached a state of equilibrium consisting of on unique cluster. The figure 6 show the result of the test.

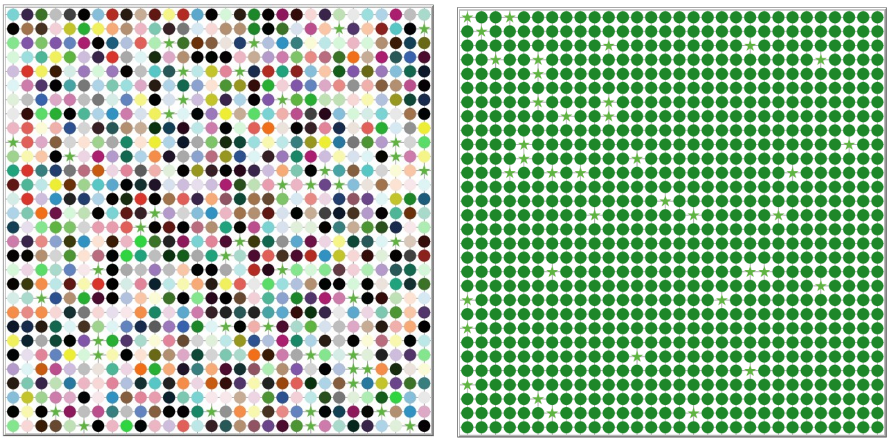


Fig 5. Before and after the execution with $N = 900, cc = 9, ct = 6, e = 5\%, T = 10\%$

The next step is to observe how the changes on T influence the overall behavior of the system. It looks like in the case of a lattice topology it is easier for the model to converge into a unique cluster, some kind of globalized cultural code. Other tests have shown that this characteristic remain even when we increase the probability of empty sites. Now we wish to explore this data in order to investigate the way in which the general behavior of the model changes when we change the T and e values. The chart in the figure 6 show the way in which the average number of the clusters changes as we change the two parameters.

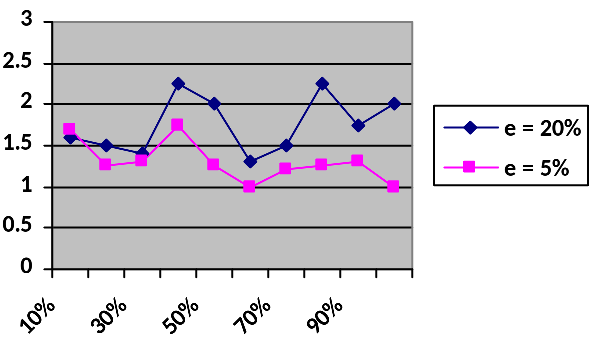


Fig 6. Rectangular lattice

3.3.2 Experiment 1 - 8 neighbor lattice

In the following experiment we are going to use an octagonal lattice. In this configuration each node has 8 neighbors. The fact that each node is connected with the double number of neighbors should mean that would be easy to reach an equilibrium faster containing an unique population that has a global cultural code.

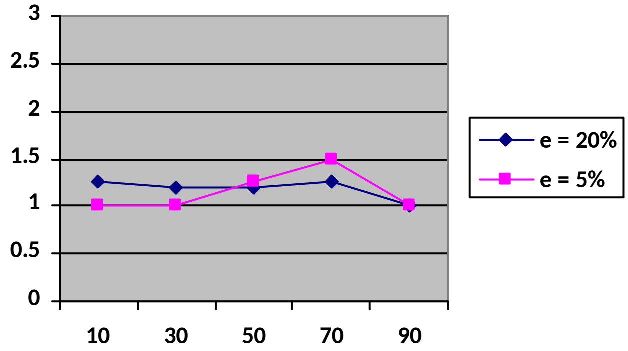


Fig 7. Octagonal lattice

4 Conclusions

The idea of this work was to put together Axelrod's model of cultural dissemination and Schelling's model of self-segregation. In order to observe the behavior of the agents in different settings we used 4 types of networks, each of them with different characteristics and behavior. We used the data obtained by the experiments to prove the intuitive behaviors of the agents. The results vary slightly from one network to another but this is due the particularities of the networks. The macro-behavior is the same at all the models. The idea is that even though the tolerance level is low the whole system behave in a closely intolerant fashion leading to a large number of moves and imitations. At the we can observe different clusters of populations forming, in some cases with the right values of the *T* parameter this lead to a global cultural code. This fact is easy to view in the case of the lattice topology.

References

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