Residential segregation and cultural dissemination: An Axelrod-Schelling model.

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In the Axelrod's model of cultural dissemination, we consider mobility of cultural agents through the introduction of a density of empty sites and the possibility that agents in a dissimilar neighborhood can move to them if their mean cultural similarity with the neighborhood is below some threshold. While for low values of the density of empty sites the mobility enhances the convergence to a global culture, for high enough values of it the dynamics can lead to the coexistence of disconnected domains of different cultures. In this regime, the increase of initial cultural diversity paradoxically increases the convergence to a dominant culture. Further increase of diversity leads to fragmentation of the dominant culture into domains, forever changing in shape and number, as an effect of the never ending eroding activity of cultural minorities.

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The use of agent-based models (ABM) [1] in the study of social phenomena provides useful insights about the fundamental causal mechanisms at work in social systems. The large-scale (macroscopic) effects of simple forms of (microscopic) social interaction are very often surprising and generally hard to anticipate, as vividly demonstrated by one of the earliest examples of ABM, the Schelling [2, 3] model of urban segregation, that shows how residential segregation can emerge from individual choices, even if people have fairly tolerant preferences regarding the share of like persons in a residential neighborhood.

To gain insights on the question of why cultural differences between individuals and groups persist despite tendencies to become more alike as a consequence of social interactions, Axelrod [4] proposed an ABM for the dissemination of culture, that has subsequently played a prominent role in the investigation of cultural dynamics. Questions concerning the establishment, spread and sustainability of cultures, as well as on the "pros and cons" of cultural globalization versus the preservation and coexistence of cultural diversity, are of central importance both from a fundamental and practical point of view in today's world.

The Axelrod model implements the idea that social influence is "homophilic", *i.e. the likelihood that a cultural feature will spread from an individual to another depends on how many other features they may have already in common* [4]. The resulting dynamics converges to a global monocultural macroscopic state when initial cultural diversity is below a critical value, while above it homophilic social influence is unable to inforce cultural homogeneity, and multicultural patterns persist asymptotically. This change of behavior has been characterized [5, 6, 7, 8] as a non-equilibrium phase transition. Subsequent studies have analyzed the effects on this transition of different lattice or network structures [9, 10], the presence of different types of noise ("cultural drift") [11, 12], as well as the consideration of external fields (influential media) [13] and global or local non-uniform couplings [14]. Up to now, no investigation of the effects of agent mobility on cultural transmission has been carried out, with the exception of [15], where individuals move following the gradient of a "sugar" landscape (that they consume) and interact culturally with agents in their neighborhood, i.e., mobility is not culturally driven.

In this paper we incorporate into the Axelrod cultural dynamics the possibility that agents living in a culturally dissimilar environment can move to other available places, much in the spirit of the Schelling model of segregation. This requires the introduction of a density of empty sites h in the discrete space (lattice) where agents live. As anticipated by [15] the expectations are that the agents mobility should enhance the convergence to cultural globalization, in the extent that it acts as a sort of global coupling between agents. It turns out that these expectations are clearly confirmed when the density h of empty sites is low enough so that the set of occupied sites percolates the lattice: The transition value depends linearly with the number of agents, so that in an infinite system (thermodynamical limit) only global cultural states are possible. However, for large enough values of h, new phenomena appear associated to the mixed Axelrod-Schelling social dynamics, including a new multicultural fragmented phase at very low values of the initial cultural diversity, a (seemingly first order) transition to cultural globalization triggered by mobility, and the fragmentation of the dominant culture into separated domains that change continuously as the result of erosive processes caused by the mobility of cultural minorities.

In the Axelrod model of cultural dissemination, a culture is modelled as a vector of F integer variables $\{\sigma_f\}$ ($f = 1, ..., F$), called cultural *features*, that can assume q values, $\sigma_f = 0, 1, ...q-1$, the possible *traits* allowed per feature. At each elementary dynamical step, the culture $\{\sigma_f(i)\}$ of an individual i randomly chosen is allowed to change (social influence) by imitation of an uncommon feature's trait of a randomly chosen neighbor j, with a probability proportional to

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the cultural overlap ω_{ij} between both agents, defined as the proportion of shared cultural features,

$$\omega_{ij} = \frac{1}{F}\sum_{f=1}^{F}\delta_{\sigma_f(i),\sigma_f(j)}, \tag{1}$$

where $\delta_{x,y}$ stands for the Kronecker's delta which is 1 if $x = y$ and 0 otherwise. Note that in the Axelrod dynamics the mean cultural overlap $\bar{\omega}_i$ of an agent i with its k_i neighbors, defined as

$$\bar{\omega}_i = \frac{1}{k_i}\sum_{j=1}^{k_i}\omega_{ij} \quad , \tag{2}$$

not always increases after an interaction takes place with a neighboring agent: indeed, it will decrease if the feature whose trait has been changed was previously shared with at least two other neighbors.

To incorporate the mobility of cultural agents into the Axelrod model, two new parameters are introduced, say the density of empty sites h, and a threshold T ($0 \le T \le 1$), that can be called *intolerance*. After each elementary step of the Axelrod dynamics, we perform the following action: If imitation has not occurred and $\omega_{ij} \neq 1$, we compute the mean overlap (2) and if $\bar{\omega}_i < T$, then the agent i moves to an empty site that is randomly chosen. Finally, in the event that the agent i randomly chosen is isolated (only empty sites in its neighborhood), then it moves directly to an empty site.

We define the mobility m_i of an agent i as the probability that it moves in one elementary dynamical step (provided it has been chosen):

$$m_i = (1 - \bar{\omega}_i)\ \Theta(T - \bar{\omega}_i) \quad , \tag{3}$$

where $\Theta(x)$ is the Heaviside step function, that takes the value 1 if $x > 0$, and 0 if $x \le 0$. For an isolated agent, that moves with certainty, one may convene that its mean cultural overlap is zero, so that expression (3) applies as well. The average mobility m of a configuration is the average of the mobility of the agents:

$$m = \frac{1}{N}\sum_{i=1}^{N} m_i \quad , \tag{4}$$

where N is the total number of cultural agents. We will consider below two-dimensional square lattices of linear size L, so that $N = (1-h)L^2$, periodic boundary conditions, and von Neumann neighborhoods, so that the number k_i of neighbors of an agent i is $0 \le k_i \le 4$. We fix the number of cultural features to $F = 10$, and vary the parameters q, h and T, as well as the linear size L of the lattice.

For the initial conditions for the cultural dynamics, N cultural agents are randomly distributed in the $L \times L$ sites of the square lattice, and randomly assigned a culture. The simulation is stopped when the number n_a of active links (*i.e.*, links such that $0 < \omega_{ij} < 1$) vanishes. The results shown below are obtained by averaging over a large number (typically $5 \cdot 10^2\ -\ 10^4$) different initial conditions.

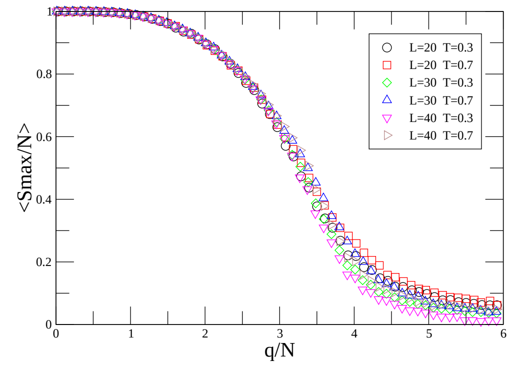

FIG. 1: Order parameter $\langle S_{max}\rangle/N$ versus scaled initial cultural diversity q/N for a very small density of empty sites $h = 0.05$ and different values of the intolerance $T = 0.3,\ 0.7$, and of the lattice linear size $L = 20,\ 30,\ 40$, as indicated in the inset.

The usual order parameter for the Axelrod model is $\langle S_{\max}\rangle/N$, where $\langle S_{\max}\rangle$ is the average number of agents of the dominant (most abundant) culture. Large values (close to unity) of the order parameter are the signature of cultural globalization. In Fig. 1, we plot the order parameter versus the initial cultural diversity scaled to the population size, q/N, for a small value of the density of empty sites $h = 0.05$, and different values of the intolerance T and of the linear size L. We observe the collapse in a single curve of the graphs corresponding to different lattice sizes and, moreover, that the results are rather insensitive to the intolerance values.

For a fixed value of the initial cultural diversity q, the larger the size N of the population is, the more likely an agent can share a cultural feature with someone else in the population. Hence, as mobility allows contacts with virtually anybody, the increase of the population size enhances the tendency towards cultural globalization, and the monocultural (ordered) phase extends up to higher values of the parameter q. The critical value q_c of the transition between consensus and a disordered multicultural phase diverges with the system size $q_c \sim N$, so that in the thermodynamical limit only global cultural states are possible for a small density h of empty sites.

We will focuss hereafter on larger values of the density h of empty sites, a regime where the cultural dynamics shows strikingly different features. At very low values of the initial cultural diversity q (so that cultural convergence is strongly favored), the asymptotic states are characterized by low values of the order parameter $\langle S_{\max}\rangle/N$. The reason for the absence of cultural globalization in this regime is the formation of disconnected monocultural domains, a fact that requires values of the density $1-h$ of cultural agents (at least) close to (or below) the site percolation threshold value for the square lattice (0.593). This new kind of macroscopic multicultural state is thus of a very different nature from the multicultural phase of the original Axelrod model ($h = 0$). The values of the order parameter in this *fragmented* phase, represented in Fig. 2a as a function of q/N with $h = 0.5$ and $T = 0.7$ and for several values of L, decrease with increasing lattice size, and the expectation is that the order parameter vanishes in the thermo-

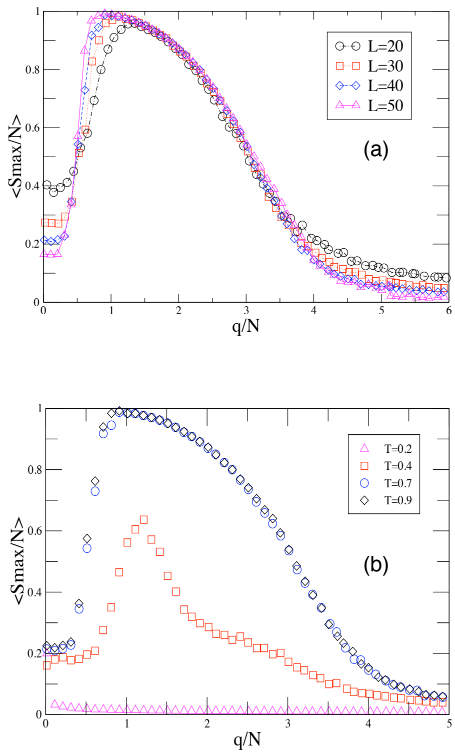

FIG. 2: (color online) Order parameter $\langle S_{max} \rangle / N$ versus scaled initial cultural diversity q/N for an intermediate value of the density of empty sites $h = 0.5$. Panel (a) corresponds to a high value of the intolerance $T = 0.7$, and different lattice linear sizes $L = 20, 30, 40, 50$, while in panel (b) $L = 40$, and different values of the intolerance $T = 0.2, 0.4, 0.7, 0.9$ are used. See the text for further details.

dynamical limit, because the largest cluster size below percolation should be independent of the lattice size.

The increase of q from the very small values that correspond to the fragmented multicultural phase has the seemingly paradoxical effect of increasing the order parameter $\langle S_{\max} \rangle / N$ values, *i.e.*, the increase of the initial cultural disorder promotes cultural globalization. To understand this peculiar behavior, one must consider the effect of the increase of q in the initial mobility of the agents. One expects that the higher the value of q is, the lower the initial values of the cultural overlap ω_{ij} among agents are, and then the higher the initial mobility of agents should be. Under conditions of high mobility, the processes of local cultural convergence are slower than the typical time scales for mobility, so that the agents can easily move before full local consensus can be achieved, propagating their common features, and enhancing the social influence among different clusters. In other words, the attainment of different local consensus in disconnected domains is much less likely to occur, and one should expect the coarsening of a dominant culture domain that reaches a higher size. A straightforward prediction of this argument is that one should

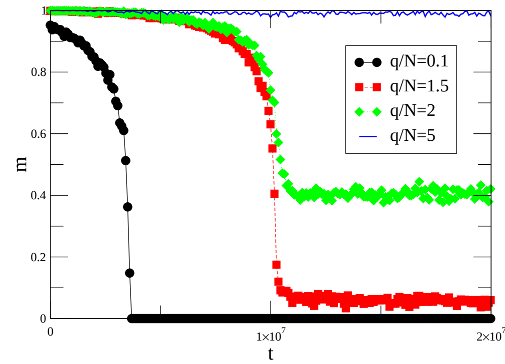

FIG. 3: (color online) Average mobility m versus time t for $h = 0.5$, $L = 30$, $T = 0.7$ and different values of the scaled initial culture diversity q/N as indicated. Unlike the other figures, in this case each curve represents the results of a single realization. See the text for further details.

observe higher values of $\langle S_{\max} \rangle / N$ for higher values of the intolerance T, because agents mobility is an increasing function of this parameter (see eq. (3)). The numerical results shown in Fig. 2b for different values of T and $h = 0.5$ nicely confirm this prediction, in support of the consistency of the previous argument. Interestingly, for high values of the intolerance T, an almost full degree of cultural globalization is reached, as indicated by the values $\langle S_{\max} \rangle / N \simeq 1$ of the order parameter. In those final states almost all agents belong to a single connected monocultural cluster. On the contrary, for very low values of T when mobility is not enhanced, multiculturalism prevails for the whole range of q values.

To characterize the passage from the multicultural fragmented phase to global consensus with increasing initial cultural diversity, we have computed the histograms of the values of $S_{\max}/N$ at values of q where the order parameter increases. The histograms display the bimodal characteristics of a first-order transition. In a fraction of realizations, the transient mobility is able to spread social influence among the clusters so that global consensus is finally reached. This fraction increases with q, to the expense of the fraction of realizations where fragmented multiculturality is reached.

Further increase of the initial cultural diversity q enhances the likelihood of agents sharing no cultural feature with anybody else in the finite population. The presence of these culturally "alien" agents decreases the value of the order parameter and the increase of their number with q is concomitant with the transition to multiculturality in the original Axelrod model (as well as here, for finite populations). We see in Fig. 2b that the increase of the intolerance parameter T shifts this transition to higher values of q/N, in agreement with the enhancement of the convergence to globalization that T produces via mobility, as discussed above. Each alien agent has, at all times, a mobility $m_i = 1$, and the average mobility cannot decrease in time to zero value when they appear. In other words, the asymptotic states of the cultural dynamics are no longer characterized by $m = 0$. The time evolution of the average mobility m for particular realizations at $h = 0.5$,

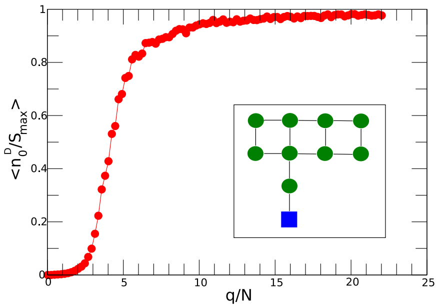

FIG. 4: (color online) As a quantitative measure of this erosion phenomenon we plot here the stationary value of the averaged fraction n_0^D/S_{max} of isolated individuals of the dominant culture versus q/N, for $h = 0.5$, $T = 0.7$, and $L = 30$. The inset shows an illustrative configuration where erosion can take place.

$T = 0.7$, $L = 30$ and different values of q/N is shown in Fig 3. The value of q/N beyond which the stationary average mobility is larger than zero signals the appearance of these alien cultural agents.

In addition, the restless character of the alien agents has an important effect on the geometry of the dominant culture, namely its *erosion*. As an illustrative example, let us consider the situation represented in the inset of Fig. 4, in which an agent i of the dominant culture is placed at the frontier of a cluster, having a single neighbor of his kind, and assume that an alien agent j has moved recently to one of the empty neighboring sites of i. When agent i is chosen for an elementary dynamical step, there is a probability $1/2$ of choosing agent j for an imitation trial. As $\omega_{ij} = 0$, and then $\bar{\omega}_i = 1/2$, the agent i will move from there to a randomly chosen empty site whenever the intolerance parameter is $T > 1/2$. We see that, for this particular situation, the erosion of the dominant culture cluster will occur with probability one half.

Note that the erosion of the dominant culture cluster does not change the size $S_{\max}$ of the dominant culture. It simply breaks it up into separate domains, some of them consisting of single (isolated) individuals. These isolated members of the dominant culture will eventually adhere to domains, to be at a later time again exposed to erosion, and so on. Therefore the shape and number of domains of the dominant culture (as well as that of the other ones), fluctuate forever. The number n_0^D of isolated dominant culture agents reaches a stationary value that results from the balance between erosive and adhesive processes. To quantify the strength of the eroding activity of cultural minorities we show in Fig. 4 the stationary value of the averaged fraction $\langle \frac{n_0^D}{S_{max}} \rangle$ of isolated individuals of the dominant culture versus the scaled initial cultural diversity, for $h = 0.5$, $T = 0.7$, and $L = 30$. Soon after the transition from the fragmented multicultural phase to globalization occurs, erosion increases dramatically, largely contributing to the large values of the stationary mobility m that characterize the multicultural states in the model here introduced.

In summary, the introduction of agents mobility through this segregation mechanism into the Axelrod cultural dynamics leads to an enhancement of the convergence to cultural globalization for small densities of empty sites, while for larger densities a new type of multicultural fragmented phase appears at low values of the initial cultural diversity q, followed by a new transition to globalization for increasing values of q that is triggered by the increase in the initial mobility. Moreover, in the genuine Axelrod transition from global consensus to polarization, the shape and number of cultural domains are here dynamically fluctuating by the competitive balance of erosive and adhesive processes associated to the agents mobility.

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