Prospect Analysis of L$n + \square$

Jay Lee

June 2, 2022

L0 + $\square$

e	::=	$\square$	*hole*
	|	n	*integer*
	|	x	*variable*
	|	e `+` e	*addition*
	|	`-`e	*negation*
	|	`if` e e e	*conditional*
	|	`let` x e e	*let binding*

L1 + $\square$

e	::=	$\square$	*hole*
	|	n	*integer*
	|	x	*variable*
	|	`(`e`,` e`)`	*pair*
	|	e`.1`	*first projection*
	|	e`.2`	*second projection*
	|	e `+` e	*addition*
	|	`-`e	*negation*
	|	`case` x `(`y`,` z`)` e e	*match*
	|	`if` e e e	*conditional*
	|	`let` x e e	*let binding*

Shape Analysis

Contexts	Γ	$\in$	$Id \xrightarrow{\text{fin}} Types \times Paths$
	Γ	::=	$\emptyset \mid \Gamma + x : \langle \tau, p \rangle$
Types	τ	::=	$\iota \mid \tau * \tau$
Paths	p	::=	$\cdot$
		|	`PPair(`p`,`p`)`
		|	`PAdd(`p`,`p`)`
		|	`PCaseP(`p`,`p`)`
		|	`PCaseN(`p`,`p`)`
		|	`PIfTru(`p`,`p`)`
		|	`PIfFls(`p`,`p`)`
		|	`PLet(`p`,`p`)`

$$\frac{}{\Gamma \vdash \Box : \langle \tau, \cdot \rangle}$$

$$\frac{}{\Gamma \vdash n : \langle \iota, \cdot \rangle}$$

$$\frac{\Gamma \vdash e_1 : \langle \tau_1, p_1 \rangle \qquad \Gamma \vdash e_2 : \langle \tau_2, p_2 \rangle}{\Gamma \vdash \texttt{(}e_1\texttt{, } e_2\texttt{)} : \langle \tau_1 * \tau_2, \texttt{PPair(}p_1\texttt{,}p_2\texttt{)} \rangle}$$

$$\frac{\Gamma \vdash e : \langle \tau_1 * \tau_2, p \rangle}{\Gamma \vdash e\texttt{.1} : \langle \tau_1, p \rangle}$$

$$\frac{\Gamma \vdash e : \langle \tau_1 * \tau_2, p \rangle}{\Gamma \vdash e\texttt{.2} : \langle \tau_2, p \rangle}$$

$$\frac{\Gamma \vdash e_1 : \langle \iota, p_1 \rangle \qquad \Gamma \vdash e_2 : \langle \iota, p_2 \rangle}{\Gamma \vdash e_1 \texttt{ + } e_2 : \langle \iota, \texttt{PAdd(}p_1\texttt{,}p_2\texttt{)} \rangle}$$

$$\frac{\Gamma \vdash e : \langle \iota, p \rangle}{\Gamma \vdash \texttt{-}e : \langle \iota, p \rangle}$$

Assume `x` in a `case` clause is an expression; otherwise, simply set its path to $\cdot$.

$$\frac{\Gamma \vdash x : \langle \tau_y * \tau_z, p_x \rangle \qquad \Gamma + y : \tau_y + z : \tau_z \vdash e_1 : \langle \tau_1, p_1 \rangle}{\Gamma \vdash \texttt{case } x \texttt{ (}y\texttt{, } z\texttt{) } e_1 \texttt{ } e_2 : \langle \tau_1, \texttt{PCaseP(}p_x\texttt{,}p_1\texttt{)} \rangle}$$

$$\frac{\Gamma \vdash x : \langle \iota, p_x \rangle \qquad \Gamma \vdash e_2 : \langle \tau_2, p_2 \rangle}{\Gamma \vdash \texttt{case } x \texttt{ (}y\texttt{, } z\texttt{) } e_1 \texttt{ } e_2 : \langle \tau_2, \texttt{PCaseN(}p_x\texttt{, } p_2\texttt{)} \rangle}$$

$$\frac{\Gamma \vdash e_P : \langle \iota, p_P \rangle \qquad \Gamma \vdash e_T : \langle \tau_T, p_T \rangle}{\Gamma \vdash \texttt{if } e_P \texttt{ } e_T \texttt{ } e_F : \langle \tau_T, \texttt{PIfTru(}p_P\texttt{, } p_T\texttt{)} \rangle} \, e \neq 0$$

$$\frac{\Gamma \vdash e_P : \langle \iota, p_P \rangle \qquad \Gamma \vdash e_F : \langle \tau_F, p_F \rangle}{\Gamma \vdash \texttt{if } e_P \texttt{ } e_T \texttt{ } e_F : \langle \tau_F, \texttt{PIfFls(}p_P\texttt{, } p_F\texttt{)} \rangle} \, e = 0$$

$$\frac{\Gamma \vdash e_x : \langle \tau_x, p_x \rangle \qquad \Gamma + x : \tau_x \vdash e_B : \langle \tau_B, p_B \rangle}{\Gamma \vdash \texttt{let } x \texttt{ } e_x \texttt{ } e_B : \langle \tau_B, \texttt{PLet(}p_x\texttt{,}p_B\texttt{)} \rangle}$$

L2 + □

e	::=	$\Box$	*hole*
	|	n	*integer*
	|	x	*variable*
	|	`(`e`, `e`)`	*pair*
	|	e`.1`	*first projection*
	|	e`.2`	*second projection*
	|	e `+` e	*addition*
	|	`-`e	*negation*
	|	`case` x `(`y`, `z`)` e e	*match*
	|	`if` e e e	*conditional*
	|	`let` x e e	*let binding*
	|	`repeat` f x e e	*let f(x) = e in f(e)*
	|	f e	*application*

L3 + □

e	::=	$\square$	*hole*
	|	n	*integer*
	|	x	*variable*
	|	`(`e`,` e`)`	*pair*
	|	e`.1`	*first projection*
	|	e`.2`	*second projection*
	|	e `+` e	*addition*
	|	`-`e	*negation*
	|	`case` x `(`y`,` z`)` e e	*match*
	|	`if` e e e	*conditional*
	|	`let` x e e	*let binding*
	|	$\lambda x.e$	*abstraction*
	|	`rec` f $\lambda x.e$	*recursion*
	|	f e	*application*