Instituto Tecnológico y de Estudios Superiores de Monterrey

Análisis de Sistemas de Imagenología (Grupo 201 y 202)

Actividad 3.
Nuclear Medicine

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Actividad 3. Nuclear Medicine

1. Radioactivity

a. How can the half-life of a radioactive isotope be calculated?

The half-life can be calculated by taking two measurements of Counts Per Second (CPS_1, CPS_2) with known times (t_1, t_2), and from equation (1) obtain an expression which allows us to calculate the half-life, by ignoring the A_o:

$$CPS(t) = A_o e^{-\lambda t} \qquad (1)$$

b. Give a realistic value of the half-life for some radioactive tracers.

Below, two tables with half-life and application values for some radioactive tracers are added. Note that the second one corresponds to those used in medical applications.

Radionuclide	Half-Life	Application(s)
Arsenic-74*	17.9 d	A positron-emitting chemical analogue of phosphorus
Barium-128*	2.4 d	Parent in the generator system for producing the positron emitting ^{128}Cs, a potassium analogue
Berylium-7*	53.37 d	Berylliosis studies
Bromine-77	57 h	Radioimmunotherapy
Bromine-82	35.3 h	Used in metabolic studies and studies of estrogen receptor content
*Carbon-11**	20.3 min	*Positron emitter for metabolism imaging*
Cobalt-57*	270 d	Calibration of imaging instruments
Copper-62	9.8 min	Heart perfusion
Copper-64	12.8 h	Used as a clinical diagnostic agent for cancer and metabolic disorders
Copper-67	58.5 h	Radioimmunotherapy
Chromium-51	27.8 d	Used to assess red blood cell survival
Fluorine-18	109.7 min	*Positron emitter used in glucose analogue uptake and neuroreceptor imaging*
Gallium-68	68 min	Required in calibrating PET tomographs. Potential antibody level
Germanium-68*	287 d	Parent in the generator system for producing the positron emitting ^{68}Ga
Indium-111*	2.8 d	Radioimmunotherapy

TABLE 14.1 (continued) Radionuclides Used in Biomedicine

Radionuclide	Half-Life	Application(s)
Tungsten-178*	21.5 d	Parent in generator system for producing ^{178}Ta, short lived scanning agent
Tungsten-188	69 d	Decays to rhenium-188 for treatment of cancer and rheumatoid arthritis
Vanadium-48*	16.0 d	Nutrition and environmental studies
Xenon-122*	20 h	Parent in the generator system for producing the positron emitting ^{122}I
Xenon-127*	36.4 d	Used in lung ventilation studies
Xenon-133	5.3 d	Used in lung ventilation and perfusion studies
Yttrium-88*	106.6 d	Radioimmunotherapy
Yttrium-90	64 h	Used to radiolabel various molecules as cancer therapeutic agents
Zinc-62*	9.13 h	Parent in the generator system for producing the positron emitting ^{62}Cu
Zirconium-89*	78.4 h	Radioimmunotherapy, positron emitter

c. Which recommendations would you give to the patient and his/her environment?

The recommendations that we would give to the patient are:

1. Avoiding close contact with others
2. Limiting activities that could increase the heart rate or increase sweating
3. Drinking plenty of fluids to help flush out the tracer from the body.

4. Dispose of bodily fluids properly: The patient should dispose of any bodily fluids, such as urine, feces, or saliva, in a manner recommended by the medical staff. This may include flushing the toilet twice, washing hands thoroughly after handling bodily fluids, and using separate towels or linens.
5. Limit contact with pets.
6. Keep a safe distance from others: The patient should keep a safe distance from others, particularly pregnant women and children, for the first few hours after the injection.
7.

2. **What is the problem when using filtered backprojection in nuclear medicine imaging?**

While filtered backprojection is a useful technique, it can lead to certain image artifacts that can affect the accuracy of the reconstructed image; for example, it doesn't take into account the attenuation coefficient of gamma rays, which gives it a 0 value.

3. **Explain how the two images in Figure B.30 were acquired. What is the difference between them and why?**

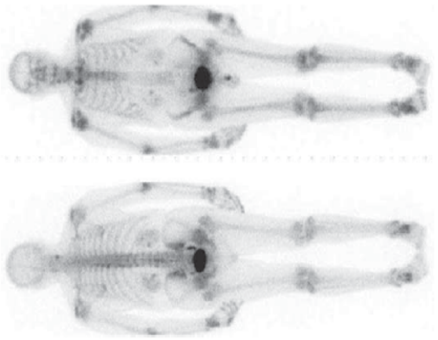

Figure B.30

The difference lies in the positioning of the person, one image was taken from behind and another from the front. The attenuation coming from the spine is defined more in one image than in the other and vice versa with the sternum, this due to the positioning of the detector placed on the back or in front of the person.

4. **A colleague in a PET center would like to know whether they should put on a lead apron to protect themselves against the irradiation from the positron emitters. We know that the mass density of lead is $11.35\ g/cm^3$ and that its linear attenuation coefficient for this kind of y-rays is $1.75\ cm^{-1}$.**
   a. **An apron that absorbs ¾ of the irradiation would provide satisfactory protection. What is the thickness of lead (in cm) required to obtain a transmission of 25% (i.e., ¾ is absorbed)? Assume a perpendicular incidence of the radiation with the apron.**

$$e^{-\mu\Delta x} = \frac{1}{4} \quad \Rightarrow \quad ln\left(e^{-\mu\Delta x}\right) = ln\left(\frac{1}{4}\right) \quad \Rightarrow \quad -\mu\Delta x = ln\left(\frac{1}{4}\right) \quad \Rightarrow \quad \Delta x = -\frac{ln\left(\frac{1}{4}\right)}{\mu}$$

$$\Delta x = -\frac{ln\left(\frac{1}{4}\right)}{1.75\ cm^{-1}} = 0.79\ldots\ cm \sim 0.8\ cm$$

b. What is the weight of this lead apron with a transmission of 25% if about 1.5 m? (flexible, but lead containing) material is needed? Neglect the other material components in the apron.

$$v = A\Delta x = (1.5\,m^2)(0.8\,cm) = (1.5\,m^2)(100\,cm)^2(0.8\,cm) = 12,000\,cm^3$$

$$m = \rho v = (11.35\,g/cm^3)(12,000\,cm^3) = 1362000\,g = 136.2\,kg$$

c. What is your advice with respect to the question of putting on a lead apron? Assume that 10 kg is the maximum bearable weight for an apron.

A lead apron is typically used to shield against ionizing radiation, such as X-rays and gamma rays, but it is not effective in shielding against radiation from positron emitters used in nuclear medicine imaging. This is because positron emitters, such as those used in PET imaging, release positrons (positively charged electrons) that quickly interact with electrons in the surrounding tissue, producing annihilation photons that exit the body in all directions. These annihilation photons are highly penetrating and cannot be effectively shielded by a lead apron.

5. Given is a positron emitting point source at position x = x* in a homogeneously attenuating medium (center x = 0, -L ≤ x ≤ L) with attenuation coefficient μ (Figure B.31). Detector 1 has radius R_1, and detector 2 has radius $R_2 = \frac{R_1}{2}$. The detectors count all the incoming photons (i.e., the absorption efficiency is 100%). Counter A counts all photons independent of the detector, while counter B counts only the coincidences. Because D >> L, D + L ≈ D. If μ = 0, detector 1 would count N photons per time unit.

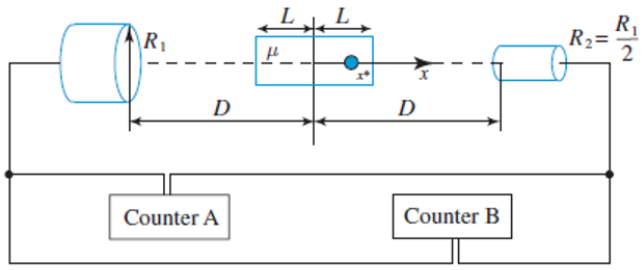

Figure B.31

a. Calculate the average number of photons per time unit measured by counter A as a function of μ, x, and N. Calculate the standard deviation for repeated measurements.

$$\lambda = Ne^{-\mu(L+x)} + \frac{N}{4}e^{-\mu(L-x)} \qquad \sigma = \sqrt{\lambda}$$

b. Repeat these calculations for counter B.

$$\lambda = \frac{N(1-e^{-2\mu L})}{2\mu} \qquad \sigma = \sqrt{\frac{N(1-e^{-2\mu L})}{2\mu}}$$

6. **How does a gamma camera react on a simultaneous (i.e., within a time window ΔT) hit of two photons of 140 keV each if the energy window is [260 keV, 300 keV]? What is the probability of a simultaneous (i.e., within a time window ΔT) hit of two photons as a function of the activity A ((i.e., average number of photons per time unit) and the time resolution ΔT?**

When a gamma camera detects two photons simultaneously, it cannot determine the location of the annihilation event (i.e., where the positron and electron annihilated). Therefore, the event is not used for image reconstruction and is considered a coincidence event. If the energy window of the gamma camera is set to [260 keV, 300 keV], a simultaneous hit of two 140 keV photons would not be detected as a valid event. This is because the total energy deposited by the two photons is 280 keV, which is outside the energy window. The probability of a simultaneous hit of two photons within a time window T can be calculated using the Poisson distribution. Assuming that the arrival of photons follows a Poisson process with an average rate of A (i.e., activity), the probability of detecting k photons within a time window T is given by:

$$P(k) = (AT)^k * \exp(-AT) / k!$$

The probability of detecting two photons within the time window T is therefore:

$$P(2) = (AT)^2 * \exp(-AT) / 2! = (AT)^2 * \exp(-AT)$$

The time resolution T of a gamma camera is typically in the range of a few nanoseconds to several hundred picoseconds. The probability of a simultaneous hit of two photons therefore decreases rapidly with decreasing time resolution. For example, assuming an activity of 10,000 photons per second (i.e., A = 10,000), the probability of a simultaneous hit of two photons within a time window of 1 ns is:

$$P(2) = (10{,}000 * 1e\text{-}9)^2 * \exp(-10{,}000 * 1e\text{-}9) = 1e\text{-}20$$

This is an extremely low probability and demonstrates the importance of time resolution in gamma cameras for accurate detection of coincidences.

9. **How does the Compton scatter influence the spatial resolution in SPECT and PET respectively?**

In SPECT imaging, Compton scatter can result in blurring of the reconstructed image and reduced spatial resolution. This is because the scattered photons can create additional signals in the detector, which can interfere with the primary signal from the radioisotope. This can cause the image to appear blurry or fuzzy, making it difficult to distinguish small features or lesions.

In PET imaging, Compton scatter can also result in reduced spatial resolution, but the effect is generally less pronounced than in SPECT. This is because PET systems are designed to detect coincident photons, which can help to reduce the impact of Compton scatter. However, even in PET imaging, Compton scatter can still cause image blurring and reduced spatial resolution, particularly in regions of high activity or in areas with high-density tissues.

12. Given are a square detector with collimator with known geometry, and a point source at distance *x* (Figure B.36).

a. Calculate the sensitivity of the point source at distance x.

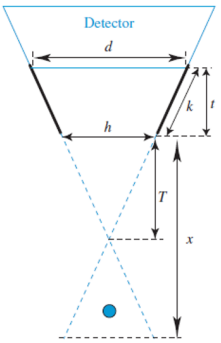

Figure B.36

$$Sensitivity = \frac{\pi\frac{h}{2}}{\frac{4}{3}\pi x^2}$$

b. Calculate the FWHM of the point spread function at distance x. Perform the calculations for both x ≤ T and x ≥ T.

$$A_e = \frac{4}{3}\pi x^2$$

20. Given a point source with activity P = 1 mCi at a distance L = 30 cm from a cube with size $h = 5\,cm$ and attenuation coefficient $\mu = 0.1\,cm^{-1}$ (Figure B.43). The density of the cube is 1 kg/1, and the half-life of the tracer is $T_{\frac{1}{2}} = 2h$. Calculate the absorbed dose (in mGy) of the cube after several days. Note that $1\,eB = 1.602x10^{-19}\,J$ and $1\,mCi = 3.7x10^{7}\,Bq$.

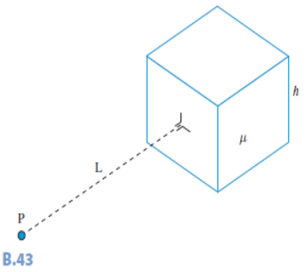

Figure B.43

$$E = hv = 511\,keV \Rightarrow (511\,keV)(1.6x10^{-19}\,J)(10^{3}) = 3.7x10^{7}\,J/photon$$

$$A = N\lambda = \frac{ln(2)}{t_{\frac{1}{2}}}N = 3.7x10^{7}\,J/photon \Rightarrow N = \frac{3.7x10^{7}\,J/photon}{\frac{ln(2)}{2\,h}} = 106,759,433\,photons$$

$$Absorbed\ Dose = (5\,cm)(0.1\,cm^{-1})(106,759,433\,photons) = 53,378,716\,photons$$