Instituto Tecnológico y de Estudios Superiores de Monterrey

Análisis de Sistemas de Imagenología (Grupo 201 y 202)

Actividad 2.
CT

Profesor

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Actividad 2. CT

1. **Linear absorption coefficient.**
   a. **Although the linear absorption coefficient μ depends on the energy, this dependence is not taken into account in filtered backprojection. Explain.**

Filtered backprojection is an algorithm used in computed tomography (CT) to reconstruct an image from the measured projections obtained from the CT scanner. The algorithm does not explicitly take into account the energy dependence of the linear absorption coefficient of a material because the frequency-based filter used in the algorithm is designed to compensate for it. The filter attenuates the high-frequency components of the measured projections, which correspond to the high-energy photons that are more likely to be absorbed by the material being imaged. Therefore, by attenuating the high-frequency components, the filter effectively compensates for the energy dependence of the X-ray beam.

   b. **What is the effect of this approximation on the image quality?**

Images look darker in the center, since attenuation is less in this area.

2. **Given an image consisting of two small bars, and projection $p(r, \theta)$ at angle θ (Figure B.2). An enlargement of this projection at angle θ look as in Figure B.3 and can mathematically be written as**

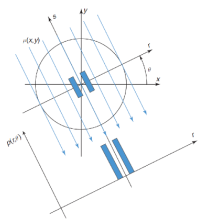
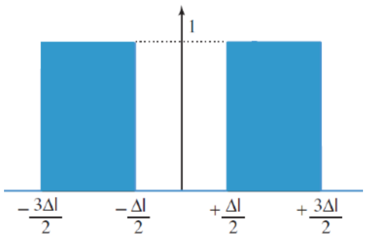


Figure B.2



$$p(r) = \Pi\left(\frac{r}{3\Delta l}\right) - \Pi\left(\frac{r}{\Delta l}\right).$$

   a. **Calculate the Fourier transform of this projection.**

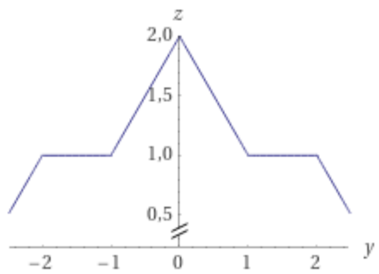
$$\frac{3\, sinc(\frac{3\omega}{2})}{\sqrt{2\pi}} - \frac{sinc(\frac{3\omega}{2})}{\sqrt{2\pi}}$$

   b. **The projection is now measured with an X-ray beam with width $\Delta s = 3\Delta l$. Assume a rectangular slice sensitivity profile (SSP).**
      i. **Calculate the resulting Fourier transform of this measured projection.**

$$\left(\frac{3\, sinc(\frac{3\omega}{2})}{\sqrt{2\pi}} - \frac{sinc(\frac{3\omega}{2})}{\sqrt{2\pi}}\right)\left(\frac{3\, sinc(\frac{3\omega}{2})}{\sqrt{2\pi}}\right) = \frac{9\, sinc^2(\frac{3\omega}{2})}{2\pi} - \frac{3\ sinc^2(\frac{3\omega}{2})}{2\pi}$$

$$R = \frac{3\, sinc^2\left(\frac{3\omega}{2}\right)(3-1)}{2\pi}$$

ii. **Draw the measured projection (as a function of r).**



iii. **What is the maximal size of Δs you would recommend?**

The maximal size of Δs that is recommended is $2\Delta l$.

iv. **What is the maximal sampling distance Δr you would recommend?**

The maximal sampling distance of Δr that is recommended is Δl.

3.

a. **Filtered backprojection (FBP) uses the ramp filter $|k|$ cut off at k_{max} (Figure B. 4).**

b. **Prove that the inverse Fourier transform of this function is**

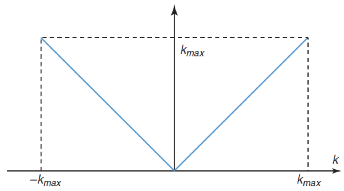


Figure B.4

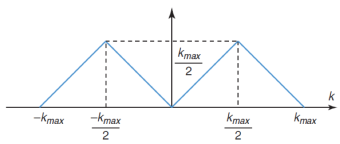


Figure B.5

$$q(r) = \frac{k_{max}\sin(2\pi k_{max} r)}{\pi r} - \frac{1-\cos(2\pi k_{max} r)}{2\pi^2 r^2}$$

knowing that this filter (i.e., the ramp filter $|k|$ cut off at k_{max}) can be written as the difference of a block and a triangle. Start by writing the exact mathematical expression of this block and triangle.

A harp cut off at k_{max} is often avoided by suppressing the highest spatial frequencies. Assume the filter function as shown in Figure B. 5 instead of the ramp filter. Calculate the inverse Fourier transform of this function and compare it to $q(r)$.

$$R = \sqrt{\frac{2}{\pi}}\sin(\omega) - \frac{sinc^2\left(\frac{\omega}{2}\right)}{\sqrt{2\pi}}$$

4. **Multislice helical CT with 64 detector rows, 180º interpolation, 4 rotations per second, and scan length 40 cm. In section 3.3.3.1 we have assumed a rectangular slice sensitivity profile (SSP) of width Δz. In practice, however, the sensitivity will be higher in the center of the slice profile. Let us assume a triangular SSP $\Lambda\left(\frac{z}{\Delta z}\right)$ with $\Delta z = 0.6\ mm$ in the center of the FOV.**
    a. **What is the effect on the resolution as compared to a rectangular SSP of width $\Delta z = 0.5\ mm$. Explain.**

They have practically the same effect, since frequencies greater than 5 are suppressed by half the signal.

b. **Calculate the maximal value of the table feed TF to avoid aliasing?**

Since the maximal frequency expected is 5 cycles per millimeter, based on the Nyquist Theorem, the sampling frequency has to be 2 times the previous established frequency, being equal to 10 cycles per millimeter, which means that the gap between cycles is equal to 0.1 mm (0.1 mm per cycle).

c. **Calculate the scan time. Note: the Fourier transform of $\Lambda\left(\frac{z}{\Delta z}\right)$ is**

$$\frac{\Delta z}{2} sinc^2\left(\pi \frac{\Delta z}{2} k\right).$$

$$\frac{40\ cm}{0.1\ mm} = \frac{400\ mm}{0.1\ mm} = 4000 \quad \Rightarrow \quad \frac{4000}{64\ detectors} = 62.5\ s$$

5. **Given are two different tissues *a* and *b*. Two different detector sizes are used (Figure B.6). In the first case the detector is twice as large as in the second case.**
    a. **Calculate the linear attenuation coefficients μ_a, μ_b, and μ_{a+b} from the input intensity I_i and the output intensities I_o, I_{oa} and I_{ob}.**

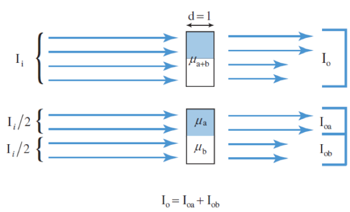


Figure B.6

$$\mu_a = ln\left(\frac{I_i}{2I_{oa}}\right); \quad \mu_b = ln\left(\frac{I_i}{2I_{ob}}\right); \quad \mu_{a+b} = ln\left(\frac{I_i}{I_o}\right)$$

b. **Show that μ_{a+b} is always an underestimate of the mean linear attenuation $(\mu_a + \mu_b)/2$.**

To show that a+b is always an underestimate of the mean linear attenuation (a+b)/2, we can use the relationship between the linear attenuation coefficients a and b and the sum of the attenuation coefficients a+b, as follows:

$$\frac{a+b}{2} = \frac{ln\left(\frac{I_i}{I_{oa}}\right)+ln\left(\frac{I_i}{I_{ob}}\right)}{2} = \frac{ln\left(\frac{I_i}{I_{oa}}\frac{I_i}{I_{ob}}\right)}{2} = \frac{ln\left(\frac{I_i^2}{I_{oa}I_{ob}}\right)}{2}$$

On the other hand, we have:

$$ln\left(\frac{I_i}{I_o}\right) = ln\left(\frac{I_i}{I_{oa}}\right) + ln\left(\frac{I_i}{I_{ob}}\right) = ln\left(\frac{I_i}{I_{oa}}\frac{I_i}{I_{ob}}\right)$$

Substituting this into the equation for (a+b)/2, we get:

$$\frac{a+b}{2} = \frac{ln\left(\frac{I_i^2}{I_{oa}I_{ob}}\right)}{2} = \frac{ln\left(\frac{I_i}{I_o}\right)}{2}$$

Therefore, we can see that (a+b)/2 is equal to half of the natural logarithm of the ratio of the input intensity Ii to the output intensity Io. This represents the mean linear attenuation of the object being imaged. From the given equations for a, b, and a+b, we can see that a+b is simply the sum of a and b. Substituting these into the equation for (a+b)/2, we get:

$$\frac{a+b}{2} = \frac{ln\left(\frac{I_i}{I_{oa}}\right)+ln\left(\frac{I_i}{I_{ob}}\right)}{2} = \frac{ln\left(\frac{I_i}{I_{oa}}\frac{I_i}{I_{ob}}\right)}{2} = \frac{ln\left(\frac{I_i^2}{I_{oa}I_{ob}}\right)}{2}$$

Comparing this to the expression for a+b, we can see that a+b is equal to twice the natural logarithm of the product of the output intensities Ioa and Iob divided by the input intensity I_i.

Therefore, we can conclude that a+b is always an underestimate of the mean linear attenuation (a+b)/2, since

$$\frac{ln\left(\frac{I_i^2}{I_{oa}I_{ob}}\right)}{2}$$

is always less than or equal to

$$ln\left(\frac{I_{oa}I_{ob}}{I_i^2}\right),$$

which is twice the natural logarithm of the product of the output intensities Ioa and Iob divided by the input intensity Ii. This means that the true mean linear attenuation of the object being imaged is always greater than or equal to the value estimated from the sum of the attenuation coefficients.

c. **What is the influence of this underestimate on a reconstructed CT image? Explain.**

An underestimate of the linear attenuation coefficients in CT can have a significant impact on the reconstructed CT image. This is because the CT image is generated by computing the line integrals of the linear attenuation coefficients along a large number of projection rays, and any error or bias in the estimated attenuation coefficients will be amplified in the reconstruction process. An underestimate of the attenuation coefficient can lead to a reduction in the measured attenuation values, loss of contrast in the reconstructed image, and image artifacts that can obscure important details in the image and affect the diagnostic accuracy. It is important to accurately estimate the linear attenuation coefficients in CT through careful calibration and quality assurance procedures, as well as the use of appropriate correction algorithms and techniques to achieve high-quality and diagnostically useful images.

6. **Assume multislice scanning without table motion. The detector width in the center of the FOV is 0.5 mm. Assume a block-shaped SSP in the axial direction (z-direction, see Figure B.7).**

    a. **Draw schematically the Fourier transform in the z-direction of the measured (sampled) projections.**

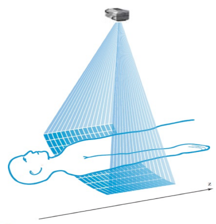
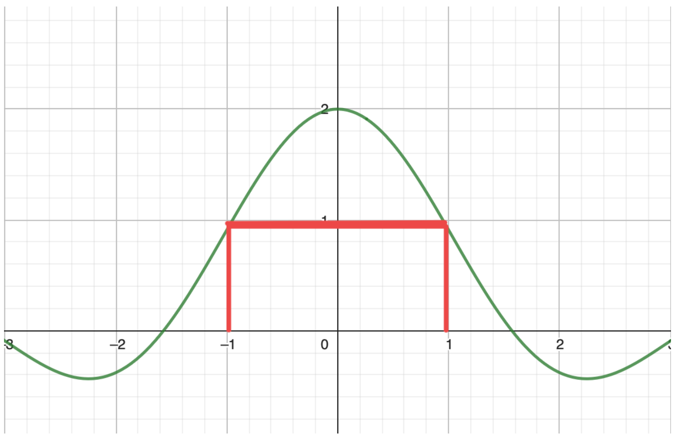


Figure B.7



b. **What is the maximum useful frequency of the sampled signal in the z-direction?**

$$f_s = \frac{1}{0.5\ mm} = 2\ cycles/mm \quad \Rightarrow \quad f_N > 2f_{max} \quad \Rightarrow \quad f_{max} = 1\ cycle/mm$$

c. **What is the minimal distance δ in the z-direction between small details to be distinguishable? (Represent neighboring details by a sinusoidal function).**

$$d = \frac{1}{2\Delta f} = \frac{1}{(2)(0.5\ cycles/mm)} = 1\ mm$$

7. **How can the resolution of a CT image be improved without increasing the dose?**

The only way possible to improve the resolution without increasing the dose is to increase the number of detectors, for example, from 512 to 1024 detectors.

How can the SNR of a CT image be improved without increasing the dose?

The only physical way possible to improve the SNR without increasing the dose is to decrease the number of detectors, for example, from 1024 to 512. But, there is another solution that has nothing to do with hardware but software, it is the backprojection algorithm which is able to improve the SNR by applying a low pass filter, suppressing high frequencies just with software.