FINAL EXAMINATION
Report
Game Theory

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Abstract

It is a distance asynchronous individual written test to check that students understand and can apply main definitions, concepts and techniques presented on lectures (topics 3 and 4 presented on weeks 7-11 and 12-14). Individual data for tasks (according to birthdate 09.03.2000): $day = 09, month = 03, year = 2000$.

1 Task 1

1.1 Problem Description

Consider a game of two players (Alice and Bob) with the following payoff matrix $\begin{bmatrix} day & 24 & ye & 19 \\ month & 4 & ar & 61 \end{bmatrix}$. Rows of the matrix corresponds to strategies $A1$ and $A2$ of Alice, columns - to strategies $B1, B2, B3, B4$ of Bob. Firstly, characterize the game using terms and concepts introduced in the lecture notes. Then solve the game in mixed strategies.

1.2 Solution

Two player's game in normal form (topic 2, page 14th): Two players' game in the normal form is the 2-dimensional finite table (matrix) where:

- rows correspond to the strategies of the first player,
- columns correspond to the strategies of the second player,
- each row and each column produce a play and their intersection cell contains vector of individual payoffs.

Payoff (topic 2, page 16th): Let π be payoff function that maps every play $S = (s_A, s_B, \dots)$ of the game, where $s_A, s_B, \dots$ are strategies of players $A, B, \dots$ into vector of the payoffs in this play $\pi(S) = (\pi_A(S) : \pi_B(S) : \dots)$.

Zero-sum game in normal form (topic 2, page 26-27th): A game in the normal form is zero-sum if for every play S the overall sum $\sum_{X \text{ is a player}} \pi(X) = 0$. It also called matrix game, because payoffs can be represented by a matrix with the payoffs of the first player.

Solve the game in normal form (according to prof. Shilov): Solving the game in normal form means finding all its Nash equilibria - in pure and mixed strategies.

Solve the game (in normal form) in pure strategies (topic 2, page 36th): To solve in pure strategies a game (in the normal form) means to find all Nash equilibria of the game.

Nash equilibrium (topic 2, page 17th): Nash equilibrium is any play that is acceptable for all players.

Acceptable play (topic 2, page 17th): A play $S = (s_A, s_B, \ldots)$ is acceptable for a player X in $A, B, \ldots$ if $\pi_X(S) \geq \pi_X(S_{X:s'_X})$ for any legal strategy s'_X of the player X.

Notation (topic 2, page 16th): For any play $S = (s_A, s_B, \ldots)$ and any strategy s'_X of a player X in $A, B, \ldots$, let $S_{X:s'_X}$ be result of X playing s'_X instead of s_X in S (while all other players do not change their strategies).

Strict domination (topic 2, page 43th): For any player X in $(A, B, \ldots)$, for any two strategies s'_X and s''_X let us say that s'_X strictly dominates s''_X if $\pi_X(S_{X:s'_X}) > \pi_X(S_{X:s''_X})$ for every play S.

Elimination of dominated strategy (topic 2, page 44th): Let G be a game in the normal form of players $A, B, \ldots, X$ be a player in $A, B, \ldots$, with a strategy s_X dominated by some another strategy of the player. Then a game $G(X \backslash s_X)$ resulting from G by elimination (prohibition) of the strategy s_X for X has the same set of Nash equilibria as G.

Probabilities (topic 3, page 5th): Every pure strategy is a mixed strategy in which this pure strategy is chosen with probability 1, and all others – with probability 0.

Remark 1: A strictly dominated strategy is not used with positive probability in any mixed strategy equilibrium. This means that when we are looking for mixed strategy equilibria, we can eliminate from consideration all strictly dominated strategies (Slantchev 2008).

Remark 2: According to Remark 1, for the following tasks it means we can assume equivalence of elimination of all dominated mixed strategies for game in mixed strategies and elimination of strictly dominated strategies for game in pure strategies.

Thus it could be represented as zero-sum game of two players in the normal form. Let first player will be A, and the second is B. According to the birthdate matrix should be look like this:

$$\begin{bmatrix} 9:-9 & 24:-24 & 20:-20 & 19:-19 \\ 3:-3 & 4:-4 & 0:0 & 61:-61 \end{bmatrix}$$

At the beginning I need to eliminate the dominated strategies:

- strategy $B1$ dominates $B2$ because $\pi_B(A1, B1) > \pi_B(A1, B2) = -9 > -24$ and $\pi_B(A2, B1) > \pi_B(A2, B2) = -3 > -4$.
- strategy $B1$ dominates $B4$ because $\pi_B(A1, B1) > \pi_B(A1, B4) = -9 > -19$ and $\pi_B(A2, B1) > \pi_B(A2, B4) = -9 > -61$.

Hence I got the following matrix:

$$\begin{bmatrix} 9:-9 & 20:-20 \\ 3:-3 & 0:0 \end{bmatrix}$$

So, continue elimination:

- $A1$ dominates $A2$ since $\pi_A(A1, B1) = 9 > \pi_A(A2, B1) = 3$ and $\pi_A(A1, B2) = 20 > \pi_A(A2, B2) = 0$.

$$\begin{bmatrix} 9:-9 & 20:-20 \end{bmatrix}$$

Then, last step:

- $B1$ dominates $B2$ since $\pi_B(A1, B1) = -9 > \pi_B(A1, B2) = -20$.

$$\begin{bmatrix} 9:-9 \end{bmatrix}$$

Hence as I see there is Nash equilibrium $(A1, B1)$, meaning I can skip checking it in mixed strategies (according to the remark 1 and 2).

1.3 Answer

- The game could be characterized as zero-sum game in the normal form of two players.
- Solution of the game is Nash equilibria, that could be presented as play S in mixed strategies: $((1,0)(1,0,0,0))$. Where play S in mixed strategies stands for $((a_1,0)(b_1,0,0,0))$, where a_1 - probability for player A to choose strategy $A1$, b_1 - probability for player B to choose strategy $B1$.

2 Task 2

2.1 Problem Description

Consider a game of two players (Alice and Bob) with the following payoff matrix $\begin{bmatrix} day : 24 & ye : 19 \\ month : 4 & ar : 61 \end{bmatrix}$. Rows of the matrix corresponds to strategies $A1$ and $A2$ of Alice, columns - to strategies $B1$ and $B2$ of Bob. Firstly, characterize the game using terms and concepts introduced in the lecture notes. Then solve the game in mixed strategies.

2.2 Solution

As was mentioned in references 1.2 this game could be characterized as game in normal form of two players. Let's fill matrix according to birthdate:

$$\begin{bmatrix} 9 : 24 & 20 : 19 \\ 3 : 4 & 0 : 61 \end{bmatrix}$$

Define Alice as first player A, and the Bob as the second player B. Next, eliminate dominated strategies (according to 1.2):

- $A1$ dominates $A2$ since $\pi_A(A1,B1) = 9 > \pi_A(A2,B1) = 3$ and $\pi_A(A1,B2) = 20 > \pi_A(A2,B2) = 0$

I got

$$\begin{bmatrix} 9 : 24 & 20 : 19 \end{bmatrix}$$

- $B1$ dominates $B2$ since $\pi_B(A1,B1) = 24 > \pi_B(A1,B2) = 19$

Therefore the final matrix is

$$\begin{bmatrix} 9 : 24 \end{bmatrix}$$

Meaning I found Nash equilibrium, and it will be solution of the game in pure strategies (similar to 1.2).

2.3 Answer

- The game could be characterized as game of two players in normal form.
- Solution of the game is Nash equilibria, that could be presented as play S in mixed strategies: $((1,0)(1,0))$. Where play S in mixed strategies stands for $((a_1,0)(b_1,0))$, where a_1 - probability for player A to choose strategy $A1$, b_1 - probability for player B to choose strategy $B1$.

3 Task 3

3.1 Problem Description

Consider problem Rational Agents at the Marketplace (from lecture notes on topic 4). What are individual agents' beliefs, desires, and intentions in the model of the problem? Let agents A and B compete for a salesman, and the matrix of their game flip-or-bid game be

$$\begin{bmatrix} A\backslash B & bid & flip \\ bid & -ye : -ar & 0 : -month \\ flip & -day : 0 & -day : -month \end{bmatrix}$$

where $L_A = -day$ and $L_B = -month$ are individual (negative) losses in case of flip, $F_A = -ye$ and $F_B = -ar$ are individual (also negative) fins for simultaneous bidding. Characterize and solve flip-or-bid game.

3.2 Solution

This game could be characterized as game of two players in normal form (w.r.t. 1.2). Rows are strategies for player A, and columns are strategies for player B, thus organizing plays with individual payoffs. According to birthdate matrix is following:

$$\begin{bmatrix} -20 : 0 & 0 : -3 \\ -9 : 0 & -9 : -3 \end{bmatrix}$$

As in 1.2 and 2.2 eliminate dominated strategies to find all Nash equilibria:

- $B1$ dominates $B2$ since $\pi_B(A1, B1) = 0 > \pi_B(A1, B2) = -3$ and $\pi_B(A2, B1) = 0 > \pi(A2, B2) = -3$

Hence matrix will be

$$\begin{bmatrix} -20 : 0 \\ -9 : 0 \end{bmatrix}$$

Continue elimination:

- $A2$ dominates $A1$ since $\pi_A(A2, B1) = -9 > \pi_A(A1, B1) = -20$

So, it should look like this:

$$\begin{bmatrix} -9 : 0 \end{bmatrix}$$

Meaning that $(A2, B1)$ is Nash equilibrium (w.r.t. 1.2). This is solution in pure strategies, and could be presented as solution in mixed strategies as $((0, 1)(1, 0))$.

3.3 Answer

- Beliefs (w.r.t topic 4, page 7th, 17-21th):
  - All buyers are rational agents that can communicate, negotiate, make concessions, and flip individually and swap their salesmen pairwise (and only pairwise) in peer-to-peer manner. Every agent would like to communicate with any other will communicate eventually.
- Desires (w.r.t topic 4, page 7th):
  - Every buyer want to obtain exactly one cake with optimal price.
- Intentions (w.r.t topic 4, page 7th):
  - Every buyer can flip or bid salesman, so according to price.
- Game could be characterized as game in normal form of two players.
- Solution of the game is Nash equilibria, that could be presented as play in mixed strategies (as in 1.3 and 2.3): $((0, 1)(1, 0))$.

References

Slantchev, Branislav L. (May 2008). *Game Theory: Dominance, Nash Equilibrium, Symmetry*. URL: `http://slantchev.ucsd.edu/courses/gt/04-strategic-form.pdf`. (accessed: 13.12.2022).