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CONSTRAINT ON SEESAW MODEL PARAMETERS WITH ELECTROWEAK VACUUM STABILITY

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Within the standard model, the electroweak vacuum is metastable. We study how heavy right-handed neutrinos in seesaw model have impact on the stability through their loop effect for the Higgs potential. Requiring the lifetime of the electroweak vacuum is longer than the age of the Universe, the constraint on parameters such as their masses and the strength of the Yukawa couplings is obtained.

Keywords: vacuum stability, lifetime of vacuum, neutrino physics.

INTRODUCTION

So far, there have been many studies on whether an electroweak (EW) vacuum is stable or not. Due to precise measurements of the Higgs and top masses, within the Standard Model (SM), the EW vacuum is shown to be metastable [1, 2]. This means that although the EW vacuum is unstable and may decay to the true vacuum, the lifetime of the EW vacuum is longer than the age of the Universe. If the EW vacuum in the model is unstable, such as in the SM, the calculation of the lifetime is important to analyze whether the model can guarantee the stability of the EW vacuum or not.

Coleman and Callan [3, 4] first derived the decay rate for unstable vacuum in quantum theory. Isidori *et al.* [5] estimated the lifetime of the EW vacuum up to the one-loop level within the SM, and they constrained on the Higgs mass by requiring that the lifetime be longer than the age of the Universe. Actually, the Higgs mass discovered in 2012 indicates that the EW vacuum is metastable rather than absolutely stable [1, 2]. The ultimate goal of our study is to estimate the lifetime of the EW vacuum for a new model beyond the SM and constrain on the new parameters by the requirement for the EW vacuum to be stable.

To discuss the lifetime of the EW vacuum, following [5, 6], we write the lifetime τ as

$$\frac{\tau}{T_U} = \frac{R_M^4}{T_U^4} \exp(S_0[\phi_b] + \Delta S), \tag{1}$$

where T_U is the age of the Universe (13.8 billion years), $S_0[\phi_b]$ is the tree level action with bounce solution ϕ_b, which is the solution to the Euclidean equation of motion for action $S_0[\phi_b]$, R_M is the bounce size maximizing the decay rate of unstable vacuum, and ΔS is the loop correction from SM particles and new particles beyond the SM. The Euclidean equation of motion for the action $S_0[\phi_b]$ can be solved analytically with boundary conditions $\lim_{r\to\infty} \phi_b(r) = 0$ and $\phi_b'(0) = 0$, so that we can obtain the bounce solution ϕ_b and the tree level action $S_0[\phi_b]$,

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$$\phi_b(r) = \sqrt{\frac{2}{|\lambda|}}\frac{2R}{r^2+R^2}, \tag{2}$$

$$S_0[\phi_b] = 8\pi^2/3|\lambda|. \tag{3}$$

where we have used the Higgs potential $V(\phi) = \frac{\lambda}{4}\phi^4$ and the Higgs quartic coupling λ. As follows from Eqs. (1) and (3), the lifetime at the tree level is mainly determined by the running of the Higgs quartic coupling $\lambda(\mu)$.

We study the stability of the EW vacuum in the seesaw model which can explain the measurements of neutrino oscillations (tiny neutrino masses) by introducing right-handed neutrinos to the SM. One-generation neutrino Lagrangian has the following form:

$$\mathcal{L} = i\overline{\nu_{\mathrm{L}}}\gamma^{\mu}\partial_{\mu}\nu_{\mathrm{L}} + i\overline{N_{\mathrm{R}}}\gamma^{\mu}\partial_{\mu}N_{\mathrm{R}} - y_{\nu}\overline{l_{\mathrm{L}}}\tilde{\phi}N_{\mathrm{R}} - \frac{M_{\mathrm{R}}}{2}\overline{N_{\mathrm{R}}^{c}}N_{\mathrm{R}} + \mathrm{h.c.}, \tag{4}$$

where ν_{L} and N_{R} are left- and right-handed neutrinos, respectively; l_{L} is the $SU(2)$ left-handed lepton doublet and ϕ is the $SU(2)$ Higgs doublet; y_{ν} is the neutrino-Higgs Yukawa coupling, and M_{R} is the majorana neutrino mass. Through the neutrino-Higgs Yukawa interaction, neutrino can contribute to the running of Higgs self-coupling λ, which determines the lifetime of the EW vacuum at tree-level, and to the one-loop correction of action ΔS in Eq. (1). We would like to consider three-generation neutrinos in the seesaw model, but the derivation for the loop correction of one-generation neutrino is not yet complete. So we will show interim results for the loop correction of one-generation neutrino.

LOOP CORRECTIONS TO THE LIFETIME OF THE ELECTROWEAK VACUUM IN THE SEE SAW MODEL

In this section, we derive loop corrections of Higgs and top to the lifetime of the EW vacuum and show interim results for the loop correction of one-generation neutrino. We can write the loop corrections of Higgs, top, and neutrino as follows:

$$\Delta S_{\mathrm{H}} = \frac{1}{2}\ln\left(\frac{\det' S''_{\mathrm{H}}[\phi_b]}{\det S''_{\mathrm{H}}[0]}\frac{1}{R_M^{10}}\right) - \ln J_{\mathrm{trans}} - \ln J_{\mathrm{dil}}, \tag{5}$$

$$\Delta S_t = -\frac{3}{2}\ln\left(\frac{\det S''_t[\phi_b]}{\det S''_t[0]}\right), \tag{6}$$

$$\Delta S_{\nu} = -\frac{1}{2}\ln\left(\frac{\det S''_{\nu}[\phi_b]}{\det S''_{\nu 0}[0]}\right). \tag{7}$$

where $\det S''$ denotes the determinant of double functional derivative of S and the apostrophe of $\det' S''_{\mathrm{H}}$ denotes the extraction of zero modes due to translational symmetry and scale invariant. We note that R_M^{-10} in the first term of Higgs correction ΔS_{H} compensates for the dimensions extracted due to the zero modes, and $-\ln J_{\mathrm{trans}}$ and $-\ln J_{\mathrm{dil}}$ denote the contributions of zero modes from translational symmetry and scale invariant, respectively,

$$-\ln J_{\text{trans}} = -\ln\left(\frac{S^2[\phi_b]}{4\pi^2}\right) = -\ln\left(\frac{16\pi^2}{9|\lambda|^2}\right), \tag{8}$$

$$-\ln J_{\text{dil}} = -\frac{1}{2}\ln\left(\frac{1}{2\pi}\int d^4x\left(\frac{\partial\phi_b}{\partial R}\right)^2\right) = -\frac{1}{2}\ln\left(\frac{8\pi}{|\lambda|}\ln\left(\frac{1}{Rv}\right)^2\right), \tag{9}$$

where Rv is cut off for infrared divergence.

Next, we explain briefly how to calculate the ratio of determinants in ΔS. Here $\det S''[\phi_b]$ is the product of eigenvalues λ_i for $S''[\phi_b]\psi = [-\partial^2 + W(r)]\psi = \lambda_i\psi$. We can obtain the product of its eigenvalues λ_i by using the Gel'fand–Yaglom method. Namely, we only solve the differential equation $[-\partial^2 + W(r)]\psi = 0$ with boundary conditions $\lim_{r\to 0} u_j(r) = 0$ and $\lim_{r\to 0} u'_j(r) = 1$, where $u(r)$ stands for the radial function of ψ and j is the angular momentum. Thanks to the Gel'fand–Yaglom method, $\lim_{r\to\infty} u_j(r)$ becomes the product of eigenvalues λ_i. Thus,

$$\frac{\det S''[\phi_b]}{\det S''[0]} = \lim_{r\to\infty}\frac{u_j(r)}{u_{j0}(r)} = \lim_{r\to\infty}\rho_j(r), \tag{10}$$

where $\rho_j(r) = u_j(r)/u_{j0}(r)$. We can conclude that the calculation of the ratio of determinants in ΔS is reduced to solving the differential equation for $\rho_j(r)$.

Next, we summarize the procedure of renormalization following [5, 6]. Since ΔS calculated up to one-loop level is divergent, we need to implement the renormalization for loop contribution to be finite. The finite loop correction ΔS_r is defined as

$$\Delta S_r = \Delta S - \Delta S_{\text{ct}} = \Delta S - \Delta S^{[2]} + \Delta S^{[2]} - \Delta S_{\text{ct}}, \tag{11}$$

where ΔS_{ct} is the counter term for canceling divergence from the loop correction, and we have inserted $\Delta S^{[2]}$ so that we can calculate $\Delta S - \Delta S^{[2]}$ and $\Delta S^{[2]} - \Delta S_{\text{ct}}$, respectively. Here $\Delta S^{[2]}$ is the expansion of ΔS up to second order with respect to W:

$$\Delta S^{[2]} = \frac{1}{2}[\ln\mathrm{SDet}(-\partial^2 + W) - \ln\mathrm{SDet}(-\partial^2)]_{\mathcal{O}(W^2)} = \frac{1}{2}\mathrm{STr}[(-\partial^2)^{-1}W] - \frac{1}{4}\mathrm{STr}[(-\partial^2)^{-1}W(-\partial^2)^{-1}W], \tag{12}$$

where SDet is super determinant and $\mathrm{STr} = \mathrm{Tr}$ or $\mathrm{STr} = -2\mathrm{Tr}$ for boson or fermion, respectively. The first expression in Eq. (12) is the perturbative version of ΔS up to the second order in terms of W, so we can solve $\Delta S^{[2]}$ by pertubative expansion $\rho_j(r) = 1 + \rho_j^{(1)}(r) + \rho_j^{(2)}(r)$. The perturbative expansion up to the second order is sufficient for $\Delta S - \Delta S^{[2]}$ to be finite for summing angular momentum j. On the other hand, the second expression in Eq. (12) can be evaluated by integration over momentum space. We adopt $\overline{\mathrm{MS}}$ scheme and eliminate the divergence of evaluated $\Delta S^{[2]}$ by using the counter term ΔS_{ct}. We can finally obtain the finite correction $\Delta S^{[2]} - \Delta S_{\text{ct}}$.

Higgs quantum correction

We estimate the renormalized Higgs loop correction $\Delta S_{\mathrm{H}r}$. In order to calculate ΔS_{H}, we derive the differential equation for $\rho_j(r)$,

$$\rho''_j(r) + \frac{4j+3}{r}\rho'_j(r) = V''(\phi_b(r))\rho_j(r)\,. \tag{13}$$

We can analytically solve this equation and then get ΔS_{H}:

$$\Delta S_{\mathrm{H}} = \frac{1}{2}\sum_{j>1/2}(2j+1)^2 \lim_{r\to\infty} \ln \rho_j(r)\,, \tag{14}$$

where $(2j+1)^2$ is degeneracy for fixed j. To calculate $\Delta S_{\mathrm{H}}^{[2]}$, we substitute $\rho_j(r) = 1 + \rho_j^{(1)} + \rho_j^{(2)}$ into Eq. (13) and obtain the differential equation in each order

$$\rho_{j''}^{(1)}(r) + \frac{4j+3}{r}\rho_{j'}^{(1)}(r) = V''(\phi_b(r))\,, \tag{15}$$

$$\rho_{j''}^{(2)}(r) + \frac{4j+3}{r}\rho_{j'}^{(2)}(r) = V''(\phi_b(r))\rho_j^{(1)}(r)\,. \tag{16}$$

Solving numerically these differential equations, we can compute $\rho_j^{(1)}(r)$ and $\rho_j^{(2)}(r)$. Thus, we can estimate numerically $\Delta S_{\mathrm{H}} - \Delta S_{\mathrm{H}}^{[2]}$:

$$\Delta S_{\mathrm{H}} - \Delta S_{\mathrm{H}}^{[2]} = \frac{1}{2}\sum_{j>1/2}(2j+1)^2 \lim_{r\to\infty}\ln\rho_j(r) - \frac{1}{2}\sum_{j=0}(2j+1)^2 \lim_{r\to\infty}\left(\rho_j^{(1)}(r) + \rho_j^{(2)}(r) - \frac{1}{2}\rho_j^{(1)}(r)^2\right) = 12.3\,. \tag{17}$$

As mentioned above, since $\Delta S_{\mathrm{H}}^{[2]}$ can be integrated over the momentum space, we find

$$\Delta S_{\mathrm{H}}^{[2]} - \Delta S_{\mathrm{Hct}} = -\frac{5}{2} - 3L\,, \tag{18}$$

where $L = \ln(R\mu e^{\gamma_{\mathrm{E}}}/2)$ and γ_{E} is the Euler constant.

Top quantum correction

Similarly to the case of Higgs fluctuation, we need to derive the differential equation in terms of $\rho_j(r)$ to estimate ΔS_t. In this case, there are two radial function α_{1J} and α_{2J} corresponding to $\rho_j(r)$ in the Higgs case. These differential equations are

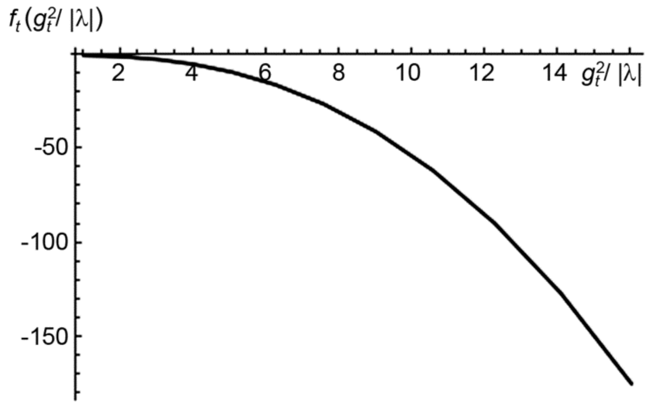

Fig. 1. Numerical result for $\Delta S_t - \Delta S_t^{[2]} = f_t(g_t^2/|\lambda|)$.

$$\begin{bmatrix} -\frac{d^2}{dr^2}+\frac{J(J-1)}{r^2}+\frac{g_t^2}{2}\phi_b^2(r) & \frac{g_t}{\sqrt{2}}\phi_b'(r) \\ \frac{g_t}{\sqrt{2}}\phi_b'(r) & -\frac{d^2}{dr^2}+\frac{J(J+1)}{r^2}+\frac{g_t^2}{2}\phi_b^2(r) \end{bmatrix} \begin{bmatrix} \alpha_{1J}(r) \\ \alpha_{2J}(r) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \tag{19}$$

where g_t is top-Higgs Yukawa coupling. In the limit of $g_t \to 0$, we can easily find the solution $\alpha_{01J} = r^J$, $\alpha_{02J} = r^{J+1}$ of these equation. We define $\rho_{iJ}(r) = \alpha_{iJ}(r)/\alpha_{0iJ}(r)$ $(i=1,2)$ and then derive the following differential equations:

$$\rho_{1J}''(r) + \frac{2J}{r}\rho_{1J}'(r) = \frac{g_t^2}{2}\phi_b^2(r)\rho_{1J} + \frac{g_t}{\sqrt{2}}\phi_b'(r)\rho_{2J}(r)r, \tag{20}$$

$$\rho_{2J}''(r) + \frac{2(J+1)}{r}\rho_{2J}'(r) = \frac{g_t^2}{2}\phi_b^2(r)\rho_{2J} + \frac{g_t}{\sqrt{2}}\phi_b'(r)\frac{\rho_{2J}(r)}{r}. \tag{21}$$

In this case, we must numerically solve $\rho_{1J}(r)$ and $\rho_{2J}(r)$. Using perturbative expansions of $\rho_{1J}(r)$ and $\rho_{2J}(r)$, we can also calculate $\Delta S_t^{[2]}$. According to these calculation, we estimate $\Delta S_t - \Delta S_t^{[2]} = f_t(g_t^2/|\lambda|)$, and Fig. 1 shows the dependence of f_t on $g_t^2/|\lambda|$.

The evaluation of $\Delta S^{[2]}$ is similar to the Higgs case

$$\Delta S_t^{[2]} - \Delta S_{tct} = \frac{g_t^2}{|\lambda|}\left(\frac{13}{6}+2L\right) + \frac{g_t^4}{|\lambda|^2}\left(\frac{5}{6}+L\right), \tag{22}$$

where $L = \ln(R\mu e^{\gamma_E}/2)$ and γ_E is Euler constant.

Neutrino quantum correction

In this part, we show interim results for neutrino loop correction. The determinant of $S''_{\nu}[\phi_b]$ is written as

$$\mathrm{Det}S''_{\nu}[\phi_b] = \mathrm{Det}\left[\begin{pmatrix} \gamma_\mu \partial_\mu & \frac{y_\nu}{\sqrt{2}}\phi_b \\ \frac{y_\nu}{\sqrt{2}}\phi_b & \gamma_\mu \partial_\mu + M_N \end{pmatrix}\begin{pmatrix} \gamma_\mu \partial_\mu & \frac{y_\nu}{\sqrt{2}}\phi_b \\ \frac{y_\nu}{\sqrt{2}}\phi_b & \gamma_\mu \partial_\mu + M_N \end{pmatrix}^\dagger\right]. \tag{23}$$

The expression corresponding $[-\partial^2 + W(r)]\psi = 0$ becomes

$$\left(-\partial^2 + \frac{y_\nu^2}{2}\phi_b^2\right)u + \frac{y_\nu M_N}{\sqrt{2}}\left(\phi_b + \frac{\gamma_\mu \hat{x}_\mu}{M_N}\phi'_b\right)v = 0\,, \tag{24}$$

$$\frac{y_\nu M_N}{\sqrt{2}}\left(\phi_b + \frac{\gamma_\mu \hat{x}_\mu}{M_N}\phi'_b\right)u + \left(-\partial^2 + M_N^2 + \frac{y_\nu^2}{2}\phi_b^2\right)v = 0\,, \tag{25}$$

where $\psi(r,\theta) = \begin{pmatrix} u(r,\theta) \\ v(r,\theta) \end{pmatrix}$ and θ denotes the angular dependence (three polar angles). Since we can expand u and v as eigen functions of L^2 in $-\partial^2 = -\frac{d^2}{dr^2} - \frac{3}{r}\frac{d}{dr} + \frac{L^2}{r^2}$, we will get the differential equations for the radial functions of u and v.

SUMMARY

We have shown the computations of the Higgs and top loop correction to the lifetime of the EW vacuum and interim results of the neutrino loop correction. In order to survey the vacuum stability of the seesaw model, we need to estimate loop corrections of neutrino and gauge bosons. The complete calculation of loop correction is important to reduce the uncertainty of renormalization scale dependence [7]. We will continue to calculate the loop correction of neutrino, and this calculation will help the analysis of the vacuum stability in the seesaw model.

The lifetime of the EW vacuum in the seesaw model with three degenerate neutrinos and constrain on the neutrino-Higgs Yukawa coupling and right handed neutrino mass was computed in [8]. We plan to generalize this analysis to the case of non-degenerate neutrinos and to use the measurements of neutrino oscillations as inputs computing the lifetime of the EW vacuum. It is interesting to study how the measurements of neutrino oscillation reflect the EW vacuum stability.

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