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A decision-support framework for residential heating decarbonisation policymaking

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ABSTRACT

The decarbonisation of residential heating systems has become increasingly important to meet the global goals of minimising carbon emissions and combating climate change. However, with rising energy costs, this can be a significant challenge for low-income households. This study presents a novel optimisation framework to aid the decarbonisation of residential heating in the United Kingdom by combining technology-related decision-support with policy decisions. The framework can recommend the optimal retrofit of low-carbon heating technologies and fabric improvement measures such as insulation upgrades for improving energy efficiency. Concurrently, the optimal financial contributions towards investment costs from grants supporting low-income households and social housing is determined. It also includes piecewise linearisations to capture the detailed operation of air source heat pumps, which are set to replace natural gas-based heating systems, and assesses the eligibility of each dwelling for grant funding. A large case study consisting of social housing stock in Woking, UK, has been used to test the framework. Three scenarios are used to assess the efficacy of existing technology and policy combinations to meet local emissions reduction targets, which are benchmarked against emissions from existing gas-based heating systems and insulation measures. Results highlight the limitations of existing UK grants, as these can only achieve an emissions reduction of 33.5% without incurring significant additional investment costs to the local council. The lack of support towards installing hot water tanks, which are required for the operation of heat pumps, is another major limitation in existing grants. A proposed scenario, which introduces a fictional grant with unlimited funding, sheds light on the much larger grant contributions expected to achieve an emissions reduction of 66.8%, which surpasses local targets. These results also suggest the need for operational support to cope with much higher energy bills, especially for low-income and/or fuel-poor households, due to the electrification of heating systems. Overall, the framework is a useful tool for local councils, policy makers, and other stakeholders to make informed decisions on the affordable decarbonisation of residential heating systems.

1. Introduction

Climate change and its effects pose an undeniable threat to natural ecosystems and human wellbeing. It is estimated that approximately 14% of UK greenhouse gas emissions, which contribute to climate change, are a result of residential heating [1]. Addressing emissions in this sector will be vital if carbon neutrality by 2050, part of Goal 13 of the United Nations Sustainable Development Goals, is to be achieved [2]. A 2019–20 English Housing Survey found that 86% of UK households use gas-fuelled central heating [3]. When coupled with low-carbon electricity, heat pumps form a promising alternative in a net-zero carbon future. Air-source heat pumps (ASHP), account for the majority of installations due to the low cost relative to their ground and water source counterparts [4]. As of 2019, heat pumps meet less than 3% of building heating needs [5]. In the UK, the range of the gas network has allowed traditional heating methods to stay economically attractive – thereby weakening the appeal of heat pumps. Consequently, various government grants and schemes are in place to fund the installation of ASHPs, as well as the retrofitting of insulation measures [4], to

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Nomenclature

Subscripts

- d Heating and insulation measures
- g Grant
- i Housing cluster
- j Insulation retrofit option
- k Hot water storage tank
- p Air-source heat pump
- s Season
- t Time interval
- *baseline* Baseline reference year
- *new* New reference year

Superscripts

- *amb* Ambient
- *annual* Annual
- *boiler* Boilers
- *CARBON* Carbon dioxide equivalent (CO_2e)
- *char* Charged
- *days* Days
- *disch* Discharged
- *EB* Electric boilers
- *ed* Eligibility of housing cluster
- *GB* Gas boilers
- *gr* Grant
- *grid* Electricity grid
- *HT* Heating and storage technology
- *HW* Hot water
- *ins* Insulation
- *Insl* Insulation retrofits
- *INV* Investment
- *loss* Loss
- *min* Minimum
- *OM* Operation and management of boiler
- *pump* Air-source heat pumps
- *room* Room
- *sec* Secondary
- *SH* Space heating
- *tank* Hot water storage tank
- *ws* Water supply

Parameters

- C^{GAS} Price of gas [£/kWh]
- $C^{ins}_{i,j}$ Cost of retrofitting insulation measure j at cluster i [£]
- C^{tax} Carbon tax [£/kg CO_2e]
- c_w Specific heat capacity of water [kWh/kg K]
- CA_i Cost adjustment factor for insulation pricing at cluster i [−]
- cap^{boiler}_b Heating capacity of boiler b [kW]
- $cap^{pump}_{t,p}$ Heating capacity of heat pump p at time t [kW]
- CC^{boiler}_b Capital cost of boiler b [£]
- CC^T_g Total budget for grant g [£]
- CC_g Funding limit per household of grant g [£]
- CC^{tank}_k Capital cost of hot water storage tank k [£]
- CC^{pump}_p Capital cost of heat pump p [£]
- $CO_2e^{annual}_{baseline}$ Baseline annual CO_2e emissions of system [CO_2e]
- $CoP^{pump}_{t,p}$ Coefficient of Performance of heat pump p at time t [−]
- CRF Capital recovery factor [−]
- $E^{load}_{i,t}$ Electricity load of cluster i at time t [kWh]
- $ELEC_b$ Binary indicator that takes a value of 1 if boiler b is electricity-based [0,1]
- GAS_b Binary indicator that takes a value of 1 if boiler b is gas-based [0,1]
- $H^{HW}_{i,t}$ Hot water load of cluster i at time t [kWh]
- $H^{SH}_{i,t}$ Space heating demand of cluster i at time t [kWh]
- H^{loss}_k Heat loss of hot water storage tank k [kW]
- $I^{gr}_{g,d}$ Parametric indicator reflecting eligibility of measure d for grant g [−]
- $I^{sec}_{g,d}$ Parametric indicator reflecting whether a measure d is eligible as a secondary measure under grant g [−]
- $I^{ed}_{i,g}$ Parametric indicator reflecting eligibility of housing cluster i for grant g [−]
- $I_{i,j}$ Binary indicator reflecting eligibility of cluster i for insulation measure j [0,1]
- $I_{i,j}$ Parametric indicator reflecting eligibility of cluster i for insulation measure j [−]
- IC^{boiler}_b Installation cost of boiler b [£]
- IC^{pump}_p Installation cost of heat pump p [£]
- N^{days} Number of days in season [days]
- N^{houses}_i Number of houses in cluster i [−]
- P^{grid}_t Grid electricity tariff at time t [£/kWh]
- $R_{i,j}$ Average demand reduction of an insulation measure j at time t [−]
- $R_{i,j}$ Average heating demand reduction of insulation measure j at housing cluster i [−]
- T_{amb} Ambient air temperature [° C]
- T^{min} Minimum hot water storage temperature [° C]
- T^{ws}_p Water supply temperature of heat pump p [° C]
- T^{room} Room temperature [° C]
- V_k Volume of hot water storage tank k [m^3]
- η_b Thermal efficiency of boiler b [−]
- n Number of years [−]
- r Interest rate [−]
- Δt Duration of time interval [h]
- ρ Density of water [kg/m^3]

Positive variables

- $AC^{INV,EB}$ Annualised investment cost of electric boilers [£]
- $AC^{INV,GB}$ Total annualised investment cost of gas boilers [£]
- $AC^{INV,Insl}$ Annualised investment cost of insulation retrofits [£]
- $AC^{INV,pump}$ Annualised investment cost of heat pumps [£]
- $AC^{INV,tank}$ Annualised investment cost of hot water storage tanks [£]
- C^{CARBON} Cost of carbon emissions [£]
- $C_{i,g,d}$ Grant contribution towards technology d
- $C^{OM,GB}$ Operating cost of gas boiler [£]
- C^{grid}_s Cost of grid electricity in season s [£]
- $C^{OM,GB}_s$ Operation and maintenance cost of gas boilers in seasons s [£]
- $CO_2e^{annual}_{new}$ New annual CO_2e emissions of system [CO_2e]
- $Cap^{HT}_{i,s}$ Capacity of heating and storage technology at housing cluster i in seasons s [−]
- $E^{grid}_{i,t}$ Electricity purchased from grid by cluster i at time t [kWh]
- $E^{boiler}_{i,t,b}$ Electricity load of electric boiler b at cluster i at time t [kWh]
- $E^{pump}_{i,t,p}$ Electricity load of heat pump p at cluster i at time t [kWh]
- $G^{boiler}_{i,t,b}$ Natural gas demand of boiler b at time t [kW]
- $H^{load}_{i,t}$ Heating load of cluster i at time i [kWh]
- $H^{boiler}_{i,t,b}$ Heat produced by boiler b at cluster i at time t [kW]
- $H^{char}_{i,t,k}$ Heat charged into hot water storage tank k at time t at cluster i [kWh]
- $H^{disch}_{i,t,k}$ Heat discharged by hot water storage tank across time t [kWh]

$H_{i,t,k}$	Effective heat in hot water storage tank k at time t at cluster i [° C]
$H^{pump}_{i,t,p,k}$	Heat produced by heat pump p into hot water storage tank k at time t at cluster i [kWh]
$T_{i,t,k}$	Temperature of water storage [° C]
TAC	Total annualised cost [£]

Binary variables

$A_{i,p}$	1 if heat pump p is selected by cluster i
$B_{i,b}$	1 if boiler b is selected by cluster i
$W_{i,j}$	1 if insulation measure j is selected by cluster i
$W_{i,j}$	1 if insulation measure j is selected by cluster i
$Y_{i,g,d}$	1 if grant g is used by cluster i to fund measure d
$Z_{i,k}$	1 if hot water storage tank k is selected by cluster i

List of abbreviations

ASHPs	Air-source heat pumps
BAU	Business-*as*-Usual
CAPEX	Capital expenditure
CHM	Cambridge Housing Model
CoP	Coefficient of Performance
DES	Distributed Energy Systems
ECO	Energy Company Obligation
EPC	Energy Performance Certificate
ER	CO_2e emissions reduction
GHG-GJS	Green Homes Grant – Green Jump Surrey
MILP	Mixed-integer linear programming
OPEX	Operating expenditure
PROP	Proposed
SHDF	Social Housing Decarbonisation Fund

encourage their uptake.

The integration and retrofit of low-carbon heating technologies in buildings, especially residential settings, have been increasingly investigated using mathematical optimisation models. These are commonly used to determine the designs and operational schedules of low-carbon and renewable heating technologies, as these can capture the large number of considerations and constraints relevant to various technologies, external networks, and stakeholders involved. Small-scale low-carbon and renewable heating technologies are increasingly studied in optimisation models for microgrids and Distributed Energy Systems (DES), which also consider technologies for meeting other energy needs, such as electricity and cooling. Focusing on design models, the key decision variables in these models, such as the capacities of heating technologies and their locations, are influenced by operational limitations such as weather and demand, socio-economic factors such as policies and tariffs, and other technical and environmental constraints [6]. Mixed-integer linear programming (MILP) formulations have been abundantly employed in literature to capture these considerations, and these have been preferred over nonlinear programming models which are complex to formulate and solve. In Ma et al. [7], a MILP model is created to find the optimal design and operational strategy of a DES, satisfying heat loads with a CHP or a ground source heat pump. The Coefficient of Performance (CoP) of the heat pump, a nonlinear temperature dependent variable, is treated as a constant parameter to maintain linearity in the formulation. This is also done by Morvaj et al. for air-to-water heat pumps [8]. Beck et al. [9] also consider a mixed-integer linear formulation to incorporate heat pumps and thermal storage, but considers CoP to be a function of Carnot quality number and fixed temperatures. Verhelst et al. [10] demonstrate that assuming a constant CoP for air-to-water heat pumps predicts higher electricity consumption and large unrealistic power fluctuations when compared to more detailed representations, which also impact the design and costs predicted by such models. Bloess et al. [11] emphasise that constant CoP values overlook the dependency on temperature lift, with large differences between air source and water supply temperatures leading to reductions in performance. To remedy this, piecewise linearisation methods [12], empirical linear [13,14] or nonlinear [15,16] formulae based on manufacturer datasheets may be used to simulate more realistic heat pump performance.

Building energy efficiency is often a prerequisite for the retrofit of low-carbon heating technologies such as heat pumps, as inefficient buildings can exacerbate operating costs and any carbon emissions associated with electricity from the grid. Existing government grants often require additional insulation measures to be retrofitted, to increase the energy efficiency of the building [17]. In the U.K., the energy efficiency of the building is graded within Energy Performance Certificates (EPC) [18]. MILP models for building insulation decisions have been explored extensively in literature, to determine insulation type [19,20], material thickness [21,22], or both [23–25]. Some studies have also focused on the assessment of environmental impacts through the means of insulation material life cycle assessment in Refs. [25–27]. Despite the necessity to consider the retrofit of insulation measures in combination with low-carbon heating, few studies have attempted this in MILP formulations. These include studies that have limited insulation measures to just one type, with a focus on optimising insulation thickness [22,23,28]. In some studies [19,20,24], heating technology decisions are

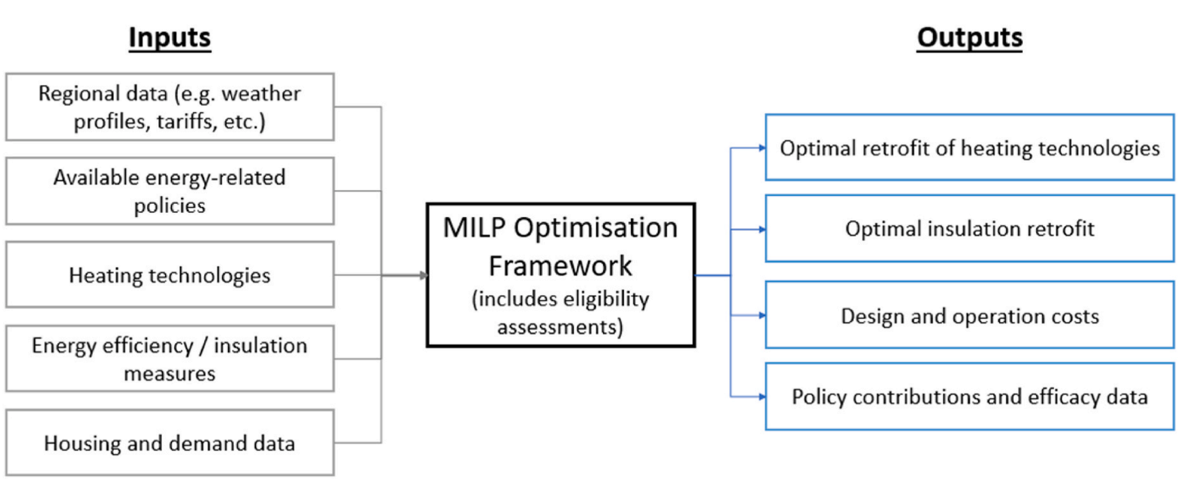


Fig. 1. Optimisation modelling framework.

limited to boiler fuel type, without considering other low-carbon heating technologies. These studies also do not incorporate an investment cost for the heating technologies, a parameter that may affect optimal insulation design. The lack of low-carbon heating technology decisions made simultaneously alongside insulation decisions in existing optimisation models is a significant limitation to be addressed.

As mentioned previously, energy policies and grants that provide financial support play a critical role in decarbonising and upgrading residential heating systems and improving heat retention, while making such systems and measures affordable. Many studies acknowledge the importance of considering energy policies and subsidies in optimisation frameworks [29]. However, there are few optimisation models that address them in detail, while encompassing low-carbon heating and energy decisions which these grants support. This is confirmed in an extensive literature review focusing on policy incorporation in microgrid frameworks [30]. Furthermore, much of the policies considered are related to the operation of technologies, as opposed to investment of technologies [30]. Policies associated with low-carbon electricity and heat generation, such as the Renewable Heat Incentive (RHI) and Feed-In-Tariff (FIT) schemes applicable to operational costs, have been previously considered in optimisation frameworks for designing such systems [31–33]. Renaldi et al. [34] consider the Renewable Heat Incentive [35] to reduce overall design costs, while incorporating heating pumps with a simplistic linear regression to estimate CoP. Calise et al. [36] consider a 50% capital investment subsidy when designing a Heating, Ventilation, and Air Conditioning (HVAC) system for an Italian residential building, tested on TRNSYS [37]. While this study does not consider any detailed constraints for the subsidies allocation to different technologies, it does shed light on the importance of such subsidies to make investments more affordable to consumers. Zwickel-Bernhard et al. [38] optimise the balance between subsidy payments to the property owner and the tenant in retrofitting wall insulation and installing renewable heating solutions. These grants take the form of initial investment contributions to the property owner and yearly energy bill support for the tenant. However, this multi-period model focuses heavily on the grant distribution trait of the problem, simplifying insulation and heating constraints. As existing studies have overlooked policies that support investment decisions for low-carbon heating and improving energy efficiency, most optimisation frameworks for designing low-carbon heating systems and supporting policy decisions related to these remain ill-suited for the purpose.

1.1. Contributions of this study

As illuminated previously in the literature review, there is a significant lack of optimisation frameworks that capture the decarbonisation of heating systems while including the impact of subsidies and specific grants directly. To address this gap, this study proposes a MILP framework, portrayed in Fig. 1, which encompasses low-carbon heating technologies, insulation measures for improving energy efficiency, and financial decisions influenced by policies that assist in the decarbonisation process. In detail, the work aims.

- To develop an optimisation model to explore the allocation and investment decisions for low-carbon heating technologies for retrofit in existing social housing stock through an MILP model that includes air-source heat pumps, electric boilers, and different common insulation options;
- To include, for the first time in an optimisation formulation, the eligibility and effects of the various policy and grants available for a given housing stock;
- To include heat pump performance details via a piecewise linearisation scheme;
- To reveal the impacts of different policies. The approach can determine economic impacts on different stakeholders, which policies are effective at encouraging technology uptake in terms of carbon emissions and cost reductions, and support decision-making on new policy development;
- To conduct a case study on Woking to test the proposed model and provide decision-making support on a real dataset.

The framework determines the optimal mix of low-carbon heating technologies, such as electric boilers and a variety of air-to-water heat pumps and employs piecewise linearisations obtained from manufacturer datasheets to capture their operation in detail. To support these decisions, the optimal retrofit of common insulation measures is also modelled, following an eligibility assessment of the characteristics and EPC of buildings. A generic formulation for policies and grants that enable the investment of low-carbon heating and insulation measures is also proposed, where the eligibility and availability of each technology and measure under each grant is assessed in detail prior to determining the overall investment decisions. The framework is tested using a real-world case study consisting of social housing in the Borough of Woking, UK Existing government grants for social housing, some with specific constraints and requirements, are also included. The framework is tested under the objective of minimising total annualised costs to the consumer or investor, which includes overall investment and operational costs with reductions from grant contributions, as well as a combined objective of minimising total costs alongside operational carbon emissions.

The framework can be used as a tool for evaluating the efficacy of the proposed decarbonisation methods and policies, as it is capable of evaluating and satisfying the prerequisites of policies while making technological decisions towards decarbonisation. Testing various new scenarios using the developed tool, prior to societal implementation, will allow decision-makers to gain further insights as to the potential technoeconomic performance of a proposed policy at its mathematical optimum. It can also be used by other stakeholders, such as local councils and homeowners, who wish the assess the investment of low-carbon heating technologies and energy efficiency improvements with support from existing grants. Overall, the framework and study facilitates the decarbonisation of existing heating systems.

1.2. Paper overview

The mathematical model which underpins the optimisation framework proposed in this study is presented in Section 2. This section describes the constraints and equations related to cost calculations, heating and storage technologies, insulation measures, and policies. The case study used to test the framework is detailed in Section 3, which includes parameters and considerations associated with the housing stock, technologies, grants, and the various scenarios used for model testing. Results for each of the scenarios are presented and extensively discussed in Section 4. The Conclusions in Section 5 summarise the key outcomes of this study, and the potential for future research.

2. Mathematical model

The mathematical formulations used in the framework are provided in this Section. This includes the objective function and general electricity and heating balances provided in Section 2.1, specific equations for the heating technologies in Section 2.2, constraints incorporating insulation measures in Section 2.3, and formulations capturing policy-related decision making in Section 2.4.

2.1. General equations

The objective function minimises Total Annualised Cost TAC for all households, after any grants have been paid toward the investment costs of heating technologies and fabric improvement measures. This includes any remaining investment costs of gas boilers $AC^{INV,GB}$, electric boilers

$AC^{INV,EB}$, heat pumps $AC^{INV,pump}$, hot water tanks $AC^{INV,tank}$, and insulation $AC^{INV,Insl}$. It also includes operational costs calculated for each season, such as the total seasonal cost of purchasing electricity C_s^{grid} and gas $C_s^{OM,GB}$ from external networks:

$$\min TAC = AC^{INV,GB} + AC^{INV,EB} + AC^{INV,pump} + AC^{INV,tank} + AC^{INV,Insl} + \sum_{s\in S}\left(C_s^{grid} + C_s^{OM,GB}\right) \tag{1}$$

The equations associated with each of the terms in this objective function are presented in the subsequent sub-sections. This objective function does not consider carbon emissions. An alternative multi-objective formulation which encompasses the impacts of carbon emissions using a carbon cost C^{CARBON} (further described by equation S(E4) – SE7 in Appendix B: Further calculations), is given below.

$$\min TAC = AC^{INV,GB} + AC^{INV,EB} + AC^{INV,pump} + AC^{INV,tank} + AC^{INV,Insl} + \sum_{s\in S}\left(C_s^{grid} + C_s^{OM,GB}\right) + C^{CARBON} \tag{2}$$

Note that the model has been implemented such that each season $s \in S$ can be solved independently, as previously presented by Ref. [39], where linking constraints ensure that key variables remain the same across each season. The capacity of each heating and storage technology considered in this study, indicated by HT for notational simplicity, must remain the same across each season for each cluster and technology:

$$If\ s > 1 Cap_{i,s}^{HT} = Cap_{i,s-1}^{HT} \forall i \in I, s \in S \tag{3}$$

The investment cost formulation for each technology is presented in each relevant subsection. The overall electrical and thermal power balances applicable to each season are presented below. Note that subscript s has been removed in the subsequent formulations for notational simplicity.

The satisfaction of electricity demand $E_{i,t}^{load}$ is described as below, where $E_{i,t}^{grid}$ is the power bought from the grid to satisfy the loads. The power consumption by heating technologies such as electric boilers $E_{i,t,b}^{boiler}$ and heat pumps $E_{i,t,p}^{pump}$ are also included in this constraint. Note that no electricity generating technologies are considered in this study, as the primary focus is heating.

$$\mathrm{E}_{\mathrm{i,t}}^{\mathrm{load}} + \sum_b E_{i,t,b}^{boiler} + \sum_p E_{i,t,p}^{pump} \le E_{i,t}^{grid} \forall i \in I, t \in T \tag{4}$$

Heat load $H_{i,t}^{load}$ is the summation of hot water $\mathrm{H}_{\mathrm{i,t}}^{\mathrm{HW}}$ and space heating $\mathrm{H}_{\mathrm{i,t}}^{\mathrm{SH}}$ demands. Energy efficiency improvement measures, such as insulation measures, are included in this formulation, which have the potential to reduce the space heating demand. To account for these, an average demand reduction $\mathrm{R}_{\mathrm{i,j}}$ per insulation measure $j \in J$ is applied. A binary variable $W_{i,j}$ is used to indicate selection of insulation measures, while $\mathrm{I}_{\mathrm{i,j}}$ is a parametric indicator for whether a cluster is eligible for the insulation type based on its characteristics (Note that these measures are further detailed in Section 2.3).

$$H_{i,t}^{load} = \mathrm{H}_{\mathrm{i,t}}^{\mathrm{HW}} + \mathrm{H}_{\mathrm{i,t}}^{\mathrm{SH}}\left(1 - \sum_j W_{i,j}\cdot \mathrm{R}_{\mathrm{i,j}}.\mathrm{I}_{\mathrm{i,j}}\right) \forall i \in I, t \in T \tag{5}$$

The heating load is met by the boiler's heat $H_{i,t,b}^{boiler}$ and heat discharge $H_{i,t,k}^{disch}$ of the hot water storage tank associated with a heat pump:

$$H_{i,t}^{load} \le \sum_b H_{i,t,b}^{boiler}\cdot\Delta t + \sum_k H_{i,t,k}^{disch} \forall i \in I, t \in T \tag{6}$$

The seasonal cost of buying electricity from the grid C^{grid} is calculated based on the electricity purchased, the associated tariff $\mathrm{P}_{\mathrm{t}}^{\mathrm{grid}}$ which may or may not be time-dependent, and the time interval Δt, and the number of days in each season $\mathrm{N}^{\mathrm{days}}$:

$$C^{grid} = \sum_{i,t} E_{i,t}^{grid}\cdot\Delta \mathrm{t}\cdot \mathrm{P}_{\mathrm{t}}^{\mathrm{grid}}\cdot \mathrm{N}^{\mathrm{days}} \tag{7}$$

2.2. Heating technologies

2.2.1. Boilers

There are several constraints common to both gas boilers and electric boilers, for which the combined $b \in B \in [GB, EB]$ has been utilised. A dwelling i is limited to one boiler through the use of a binary variable $B_{i,b}$. The boiler b can also only produce heat if $B_{i,b}$ is equal to 1 to indicate its installation at i.

$$\sum_b B_{i,b} \le 1 \forall i \in I \tag{8}$$

A Big-M constraint is used to enforce a logical constraint, where heat from the boiler $H_{i,t,b}^{boiler}$ should only be produced if the boiler b has been selected for the dwelling. In these constraints, M is a large value which acts as an upper bound while avoiding nonlinearity. In this case, M is assigned a value of 100 kW, large enough to exceed any reasonable domestic boiler output.

$$H_{i,t,b}^{boiler} \le \mathrm{M}\cdot B_{i,b} \forall i \in I, t \in T, b \in B \tag{9}$$

The boiler capacity $\mathrm{cap}_{\mathrm{b}}^{\mathrm{boiler}}$ is chosen to ensure that the heat demand at each timepoint is met. The heat produced by boiler b at each time point cannot exceed this capacity:

$$H_{i,t,b}^{boiler} \le \mathrm{cap}_{\mathrm{b}}^{\mathrm{boiler}}\cdot B_{i,b} \tag{10}$$

As the boiler can be either fuelled by gas or electricity, both must be considered when calculating the output. The fuel demands, $E_{i,t,b}^{boiler}$ or $G_{i,t,b}^{boiler}$ for electricity and gas, respectively, is multiplied by the respective efficiency η_{b}. The parameters $\mathrm{ELEC}_{\mathrm{b}} \in \{0,1\}$ and $\mathrm{GAS}_{\mathrm{b}} \in \{0,1\}$ are used to indicate the type of boiler (where $\mathrm{ELEC}_{\mathrm{b}} = 1$ if b is an electric boiler), such that the correct fuel type is used.

$$H_{i,t,b}^{boiler} = \left(E_{i,t,b}^{boiler}\cdot \mathrm{ELEC}_{\mathrm{b}} + G_{i,t,b}^{boiler}\cdot \mathrm{GAS}_{\mathrm{b}}\right)\cdot\Delta t\cdot\eta_{\mathrm{b}} \forall i \in I, t \in T, b \in B \tag{11}$$

The annualised boiler investment costs for gas boilers $AC^{INV,GB}$ and electric boilers $AC^{INV,EB}$ are calculated below, where $\mathrm{CC}_b^{\mathrm{boiler}}$ is the capital cost of boiler b (£), $\mathrm{IC}^{\mathrm{boiler}}$ is the installation cost in £ and $\mathrm{N}_{\mathrm{i}}^{\mathrm{houses}}$ defines the number of houses in cluster i. CRF is the capital recovery factor used to annualise the costs. Note that the grant contribution towards each boiler type is also considered, $C_{i,g,d\in GB}$ and $C_{i,g,d\in EB}$ (see Section 2.4 for further details).

$$AC^{INV,GB} = \sum_i\sum_b \mathrm{GAS}_{\mathrm{b}}\left(B_{i,b}\cdot\left(\mathrm{CC}_b^{\mathrm{boiler}} + \mathrm{IC}^{\mathrm{boiler}}\right) - \sum_g C_{i,g,d\in GB}\right)\cdot \mathrm{N}_{\mathrm{i}}^{\mathrm{houses}}\cdot\mathrm{CRF} \tag{12}$$

$$AC^{INV,EB} = \sum_i\sum_b \mathrm{ELEC}_{\mathrm{b}}\left(B_{i,b}\cdot\left(\mathrm{CC}_b^{\mathrm{boiler}} + \mathrm{IC}^{\mathrm{boiler}}\right) - \sum_g C_{i,g,d\in EB}\right)\cdot \mathrm{N}_{\mathrm{i}}^{\mathrm{houses}}\cdot\mathrm{CRF} \tag{13}$$

The CRF is calculated as shown below, considering the interest rate r and the number of years n, equivalent to the lifetime of the all the technologies considered.

$$\mathrm{CRF} = \frac{r(1+r)^n}{(1+r)^n - 1} \tag{14}$$

The operating cost of the boiler results from either gas or electricity usage. As the term $\sum_b E_{i,t,b}^{boiler}$ is already included in electricity balance formulation, only gas costs $C^{OM,GB}$ need to be calculated. $\mathrm{C}^{\mathrm{GAS}}$ is the fuel price (£/kWh).

$$C^{OM,GB} = \sum_{i,b} G^{boiler}_{i,t,b} \cdot \Delta t \cdot C^{GAS} \quad (15)$$

2.2.2. Heat pumps

When a heat pump $p \in P$ is installed in a dwelling, it is always coupled with a hot water storage tank [40]. These have subscript $k \in K$. Pump capacity $\mathrm{cap}^{\mathrm{pump}}_{\mathrm{t,p}}$ and Coefficient of Performance $\mathrm{CoP}_{\mathrm{t,p}}$ are based on nonlinear functions of the ambient temperature $\mathrm{T_{amb}}$ that are divided into six linear formulations through piecewise linearisation. The coefficients *a, b, c* and *d* vary per pump model. Performance data and the piecewise temperature intervals are sourced from a Mitsubishi Electric Renewable Heating Databook [41]. These formulations for capacities and CoPs are defined as Eqs. (16) and (17), respectively.

$$\text{if } \mathrm{T^{amb}_t} > 15\,^\circ\mathrm{C}, \mathrm{cap}^{\mathrm{pump}}_{\mathrm{t,p}} = \mathrm{a6_p} \cdot \left(\mathrm{T^{amb}_t} - 15\right) + \mathrm{b6_p} \forall i \in I, t \in T, p \in P \quad (16)$$

$$\text{if } 15\,^\circ\mathrm{C} \geq \mathrm{T^{amb}_t} > 12\,^\circ\mathrm{C}, \mathrm{cap}^{\mathrm{pump}}_{\mathrm{t,p}} = \mathrm{a5_p} \cdot \left(\mathrm{T^{amb}_t} - 12\right) + \mathrm{b5_p} \forall i \in I, t \in T, p \in P$$

$$\text{if } 12\,^\circ\mathrm{C} \geq \mathrm{T^{amb}_t} > 7\,^\circ\mathrm{C}, \mathrm{cap}^{\mathrm{pump}}_{\mathrm{t,p}} = \mathrm{a4_p} \cdot \left(\mathrm{T^{amb}_t} - 7\right) + \mathrm{b4_p} \forall i \in I, t \in T, p \in P$$

$$\text{if } 7\,^\circ\mathrm{C} \geq \mathrm{T^{amb}_t} > 2\,^\circ\mathrm{C}, \mathrm{cap}^{\mathrm{pump}}_{\mathrm{t,p}} = \mathrm{a3_p} \cdot \left(\mathrm{T^{amb}_t} - 2\right) + \mathrm{b3_p} \forall i \in I, t \in T, p \in P$$

$$\text{if } 2\,^\circ\mathrm{C} \geq \mathrm{T^{amb}_t} > -7\,^\circ\mathrm{C}, \mathrm{cap}^{\mathrm{pump}}_{\mathrm{t,p}} = \mathrm{a2_p} \cdot \left(\mathrm{T^{amb}_t} - (-7)\right) + \mathrm{b2_p} \forall i \in I, t \in T, p \in P$$

$$\text{if } -7\,^\circ\mathrm{C} \geq \mathrm{T^{amb}_t} > -10\,^\circ\mathrm{C}, \mathrm{cap}^{\mathrm{pump}}_{\mathrm{t,p}} = \mathrm{a1_p} \cdot \left(\mathrm{T^{amb}_t} - (-10)\right) + \mathrm{b1_p} \forall i \in I, t \in T, p \in P$$

$$\text{if } \mathrm{T^{amb}_i}\,^\circ\mathrm{C} \leq -10\,^\circ\mathrm{C}, \mathrm{cap}^{\mathrm{pump}}_{\mathrm{t,p}} = 0 \forall i \in I, t \in T, p \in P$$

$$\text{if } \mathrm{T^{amb}_t} > 15\,^\circ\mathrm{C}, \mathrm{CoP}^{\mathrm{pump}}_{\mathrm{t,p}} = c_6 \cdot \left(\mathrm{T^{amb}_t} - 15\right) + d_6 \forall i \in I, t \in T, p \in P \quad (17)$$

$$\text{if } 15\,^\circ\mathrm{C} \geq \mathrm{T^{amb}_t} > 12\,^\circ\mathrm{C}, \mathrm{CoP}^{\mathrm{pump}}_{\mathrm{t,p}} = c_5 \cdot \left(\mathrm{T^{amb}_t} - 12\right) + d_5 \forall i \in I, t \in T, p \in P$$

$$\text{if } 12\,^\circ\mathrm{C} \geq \mathrm{T^{amb}_t} > 7\,^\circ\mathrm{C}, \mathrm{CoP}^{\mathrm{pump}}_{\mathrm{t,p}} = c_4 \cdot \left(\mathrm{T^{amb}_t} - 7\right) + d_4 \forall i \in I, t \in T, p \in P$$

$$\text{if } 7\,^\circ\mathrm{C} \geq \mathrm{T^{amb}_t} > 2\,^\circ\mathrm{C}, \mathrm{CoP}^{\mathrm{pump}}_{\mathrm{t,p}} = c_3 \cdot \left(\mathrm{T^{amb}_t} - 2\right) + d_3 \forall i \in I, t \in T, p \in P$$

$$\text{if } 2\,^\circ\mathrm{C} \geq \mathrm{T^{amb}_t} > -7\,^\circ\mathrm{C}, \mathrm{CoP}^{\mathrm{pump}}_{\mathrm{t,p}} = c_2 \cdot \left(\mathrm{T^{amb}_t} - (-7)\right) + d_2 \forall i \in I, t \in T, p \in P$$

$$\text{if } -7\,^\circ\mathrm{C} \geq \mathrm{T^{amb}_t} > -10\,^\circ\mathrm{C}, \mathrm{CoP}^{\mathrm{pump}}_{\mathrm{t,p}} = c_1 \cdot \left(\mathrm{T^{amb}_t} - (-10)\right) + d_1 \forall i \in I, t \in T, p \in P$$

$$\text{if } \mathrm{T^{amb}_t} \leq -10\,^\circ\mathrm{C}, \mathrm{CoP}^{\mathrm{pump}}_{\mathrm{t,p}} = 0.0001 \forall i \in I, t \in T, p \in P$$

The binary variable $A_{i,p}$ indicates whether pump p is in use, with (18) constraining the system such that only one pump can be used at each house. The Big-M constraint (19) ensures that linearity can be maintained for the logical constraint where a pump p can only provide heat to tank k if the pump is in use.

$$\sum_{p} A_{i,p} \leq 1 \forall i \in I \quad (18)$$

$$H^{pump}_{i,t,p,k} \leq \mathrm{M} \cdot A_{i,p} \forall i \in I, t \in T, p \in P \quad (19)$$

A boiler and a heat pump cannot both be installed simultaneously at house i:

$$\sum_{i,p} A_{i,p} + \sum_{i,b} B_{i,b} \leq 1 \quad (20)$$

The electricity demand in kWh to run the pump is defined by (21), where $H^{pump}_{i,t,p,k}$ is the heat produced by the pump.

$$E^{pump}_{i,t,p} \leq \frac{\sum_k H^{pump}_{i,t,p,k}}{\mathrm{CoP}^{\mathrm{pump}}_{\mathrm{t,p}}} \forall i \in I, t \in T, p \in P \quad (21)$$

Heat produced by the pump in kWh cannot exceed its capacity at time t.

$$\sum_{k} H^{pump}_{i,t,p,k} \leq \mathrm{cap}^{\mathrm{pump}}_{\mathrm{t,p}} \cdot \Delta t \forall i \in I, t \in T, p \in P \quad (22)$$

The annualised pump investment cost $AC^{INV,pump}$ is calculated by (23), where CC^{pump}_{p} is the capital cost of pump p in £ and $\mathrm{IC}^{\mathrm{pump}}$ is the installation cost (£). It also takes the grant contribution towards ASHPs into account $C_{i,g,d \in P}$; see Section 2.4 for further details.

$$AC^{INV,pump} = \sum_{i,p}\left(\left(A_{i,p} \cdot \left(\mathrm{CC}^{pump}_{p} + \mathrm{IC}^{\mathrm{pump}}\right) - \sum_{g} C_{i,g,d \in P}\right) \cdot \mathrm{N^{houses}_i}\right) \cdot \mathrm{CRF} \quad (23)$$

2.2.3. Hot water storage tanks

Heat into the hot water tank $H^{char}_{i,t,k}$ at house i at time t is equal to the sum of all heat pump outputs:

$$H^{char}_{i,t,k} = \sum_{p} H^{pump}_{i,t,p,k} \ \forall i \in I, t \in T, k \in K \quad (24)$$

The binary variable $Z_{i,k}$ indicates whether tank $k \in K$ is in use, with (25) constraining the system such that only one tank can be used at each house. The Big-M constraint (26) ensures that heat from heat pumps $H^{char}_{i,t,k}$ can only flow into tank k if $Z_{i,k} = 1$.

$$\sum_{k} Z_{i,k} \leq 1 \forall i \in I, k \in K \quad (25)$$

$$H^{char}_{i,t,k} \leq \mathrm{M} \cdot Z_{i,k} \forall i \in I, t \in T, k \in K \quad (26)$$

For modelling purposes, the temperature in the tank $T_{i,t,k}$ can take on a value of zero if the tank is not used (i.e., when $Z_{i,k} = 0$). When $Z_{i,k} = 1$, the tank temperature must be maintained above a set minimum $\mathrm{T^{min}}$.

$$T_{i,t,k} \geq \mathrm{T^{min}} \cdot Z_{i,k} \forall i \in I, t \in T, k \in K \quad (27)$$

The temperature in the tank is equal to the effective heat $H_{i,t,k}$ (energy available for heating above room temperature $\mathrm{T^{room}}$) plus room temperature, set here as 20 °C. Any heat within the tank below room temperature cannot be used to heat the room in this system without infringing the Second Law of Thermodynamics. As a result, $H_{i,t,k}$ only concerns heat at temperatures above 20 °C. Multiplying $\mathrm{T^{room}}$ by the binary variable $Z_{i,k}$ sets the temperatures of unused tanks to 0. The specific heat capacity $\mathrm{c_w}$ of water is 0.00116 kW h/kgK. Water density and volume of the tank are ρ and $\mathrm{V_k}$, respectively.

$$T_{i,t,k} = \frac{H_{i,t,k}}{\mathrm{c_w} \rho \mathrm{V_k}} + \mathrm{T^{room}} \cdot Z_{i,k} \forall i \in I, t \in T, k \in K \quad (28)$$

Eq. (29) limits the temperature in the tank to the water supply temperature $\mathrm{T^{ws}_p}$ of the heat pump (for these pumps, 55 °C).

$$T_{i,t,k} \leq \mathrm{T^{ws}_p} \forall i \in I, t \in T, k \in K \quad (29)$$

Eqs. (30) and (31) return the effective heat $H_{i,t,k}$ (heat above room temperature) in the hot water tank at time t, which is equal to the heat at $(t-1)$ and the heat charged $H^{char}_{i,t,k}$ into the tank, minus the heat discharged $H^{disch}_{i,t,k}$ to the dwelling and heat losses $\mathrm{H^{loss}_k}$ in kW to the surroundings. In this model, it is assumed that the starting temperature of the tank is $\mathrm{T^{min}}$. Hence, the $H_{i,t-1,k}$ term for (31) equals the effective heat at this temperature when $t = 1$. Including $Z_{i,k}$ ensures that this only applies to the tank k being used.

$$if\ \mathrm{t} > 1, H_{i,t,k} = H_{i,t-1,k} + \left(H_{i,t,k}^{char} - \left(H_{i,t,k}^{disch} + \mathrm{H}_k^{\mathrm{loss}} \cdot \Delta \mathrm{t}\right)\right) \forall i \in I, t \in T, k \in K \tag{30}$$

$$if\ \mathrm{t} = 1, H_{i,t,k} = Z_{i,k} \cdot \left(\mathrm{T}^{\min} - \mathrm{T}^{\mathrm{room}}\right) \cdot (\rho \mathrm{c_w} \mathrm{V_k}) + \left(H_{i,t,k}^{char} - \left(H_{i,t,k}^{disch} + \mathrm{H}_k^{\mathrm{loss}} \cdot \Delta \mathrm{t}\right)\right) \forall i \in I, t \in T, k \in K \tag{31}$$

Eq. (32) links the heat at the start $t = 1$ and end $t = \mathrm{FT}$ of the day to ensure that there are no discrepancies between days in each season (as a representative day for each season is employed in this model):

$$H_{i,t=1,k} = H_{i,t=FT,k} \forall i \in I, k \in K \tag{32}$$

Eq. (33) calculates the annualised investment cost $AC^{INV,tank}$ of the tanks. Note that $C_{i,g,d\in K}$ represents the contribution towards the capital costs from grants $g \in G$, for technologies $d \in K$ which represents tanks; see Section 2.2.3.

$$AC^{INV,tank} = \sum_{i,k} \left(\left(Z_{i,k} \cdot \mathrm{CC}_k^{tank} - \sum_g C_{i,g,d\in K} \right) \cdot \mathrm{N}_\mathrm{i}^{\mathrm{houses}} \right) \cdot \mathrm{CRF} \tag{33}$$

2.3. Insulation measures

The average demand reduction $\mathrm{R_{i,j}}$ for each insulation measure, first presented in Eq. (5), is considered. The sum of percentage reductions from these insulation measures cannot exceed 1:

$$\sum_j W_{i,j} \cdot \mathrm{R_{i,j}} \cdot \mathrm{I_{i,j}} \le 1 \forall i \in I \tag{34}$$

The overall annualised cost of installing insulation measures $AC^{INV,insul}$ takes into account the capital cost of each measure $\mathrm{C}_{i,j}^{\mathrm{ins}}$. A cost adjustment factor $\mathrm{CA_i}$ is used to adjust the capital cost if the dwelling i is a bungalow, as the average values for $\mathrm{C}_{i,j}^{\mathrm{ins}}$ used are applicable to houses (values are provided in Appendix E). The grant contribution towards each insulation measure is also considered by $C_{i,g,j}$.

$$AC^{INV,insul} = \mathrm{CRF} \cdot \sum_i \sum_j W_{i,j} \cdot \mathrm{C}_{i,j}^{\mathrm{ins}} \cdot \mathrm{CA_i} - \sum_g C_{i,g,j} \tag{35}$$

As the insulation measures do not have a variable for capacity, a linking constraint for the seasonal binary variable $W_{i,j,s}$ is utilised to ensure that the same insulation measure is selected across seasons.

$$If\ s > 1 W_{i,j,s} = Cap_{i,j,s-1}^{DER} \forall i \in I, j \in J, s \in S$$

2.4. Policy formulation

The set $g \in G$ for policies and grants includes those contributing to capital costs. All heating technologies and insulation measures are now defined as set $d \in D = \{GB, EB, P, J\}$ (see Section 3.3 for the specific grants considered).

A parametric indicator $\mathrm{I}_{\mathrm{i,g,d}}^{\mathrm{gr}} \in \{0, 1\}$ is used for all eligible heating technologies and insulation measures to represent whether cluster i is eligible for measure d under grant g as a primary measure. Each dwelling also has to be assessed for its eligibility, based on characteristics such as EPC band. This is indicated by another parameter $\mathrm{I}_{i,g}^{\mathrm{ed}} \in \{0, 1\}$. Binary variable $Y_{i,g,d}$ indicates whether the grant g has been used for cluster i and measure d. If a cluster does not have an eligible primary measure for grant g, then the binary variable must be zero:

$$Y_{i,g,d} \le \mathrm{I}_{\mathrm{g,d}}^{\mathrm{gr}} \cdot \mathrm{I}_{i,g}^{\mathrm{ed}} \forall i \in I, d \in D, g \in G \tag{36}$$

A second parametric indicator $\mathrm{I}_{\mathrm{g,d}}^{\mathrm{sec}}$ is used to indicate whether a measure is eligible as a secondary measure. Secondary measures can only be installed if a primary measure, such as insulation, has been installed, as in (37). If no insulation measures can be installed, this constraint can be neglected for i. This applies to any policy where only one primary measure must be chosen before a secondary measure can be installed, for each cluster and each policy. The eligibility of primary and secondary insulation measures is defined by $\mathrm{I_{i,j}}$.

$$if \sum_j \mathrm{I_{i,j}} \ne 0\ then \sum_d \left(Y_{i,g,d} \left(1 - I_{g,d}^{sec} \right) \right) \ge Y_{i,g,d} \cdot I_{g,d}^{sec} \forall i \in I, d \in D, g \in G \tag{37}$$

Note that $Y_{i,g,d\in J}$ can only be 1 if insulation measure j is eligible for installation at dwelling i, as indicated by the insulation parameter $\mathrm{I_{i,j}}$.

$$\sum_g Y_{i,g,d\in J} \le \mathrm{I_{i,j}} \forall i \in I, d \in D, g \in G \tag{38}$$

The nonnegative grant contribution towards the capital cost of a technology or measure $C_{i,g,d}$ can only take a value greater than zero if the binary variable takes the value 1. This is presented as a big-M constraint.

$$C_{i,g,d} \le \mathrm{M} \cdot Y_{i,g,d} \forall i \in I, d \in D, g \in G \tag{39}$$

Parameter $\mathrm{CC_g}$ represents the maximum contribution offered by the grant for eligible measures for each household. The total grant contribution for each dwelling has to be less than or equal to this maximum cost reduction.

$$\sum_{d\in D} C_{i,g,d} \le \mathrm{CC_g} \forall i \in I, g \in G \tag{40}$$

Similar to the maximum grant contribution imposed on each household, there is also a maximum grant contribution available to the region or local area assessed, $\mathrm{CC}_\mathrm{g}^\mathrm{T}$. Therefore, the summation of the grant contributions for this local area, accounting for the total number of houses in each cluster $\mathrm{N}_\mathrm{i}^{\mathrm{houses}}$, cannot exceed $\mathrm{CC}_\mathrm{g}^\mathrm{T}$.

$$\sum_{i\in I, d\in D} C_{i,g,d} \cdot \mathrm{N}_\mathrm{i}^{\mathrm{houses}} \le \mathrm{CC}_\mathrm{g}^\mathrm{T}\ g \in G \tag{41}$$

The total grant contribution for each technology d must be less than or equal to the capital cost incurred by the technology d. Eq. (42) represents this for insulation, where the insulation capital cost $\mathrm{C}_{i,j}^{\mathrm{ins}}$, cost adjustment factor for bungalows $\mathrm{CA_i}$, and the binary variable $W_{i,j}$ indicating the selection of the measure, are all utilised. Eq. (43) represents this for ASHPs, where the total grant contribution is restricted by the binary selection $A_{i,p}$, capital cost of each pump CC_p^{pump}, and installation cost $\mathrm{IC}^{\mathrm{pump}}$. Eq. (44) constrains total grant contribution for boilers, where only gas boilers are considered (using the parameter $\mathrm{GAS_b}$) as existing grants do not fund electric boilers. The boiler binary selection $B_{i,b}$, capital cost $\mathrm{CC}_\mathrm{b}^{\mathrm{boiler}}$, and installation cost $\mathrm{IC}^{\mathrm{boiler}}$ is also included. The grant contribution for tanks are also considered, as shown in Eq. (45), with respect to its binary selection and capital cost.

$$\sum_g C_{i,g,d\in J} \le W_{i,j} \cdot \mathrm{C}_{i,j}^{\mathrm{ins}} \cdot \mathrm{CA_i} \forall i \in I, j \in J \tag{42}$$

$$\sum_g C_{i,g,d\in P} \le \sum_p A_{i,p} \cdot \left(\mathrm{CC}_p^{pump} + \mathrm{IC}^{pump} \right) \forall i \in I, p \in P \tag{43}$$

$$\sum_g C_{i,g,d\in GB} \le \sum_b B_{i,b} \cdot \mathrm{GAS_b} \left(\mathrm{CC}_b^{boiler} + \mathrm{IC}^{boiler} \right) \forall i \in I, b \in B \tag{44}$$

$$\sum_g C_{i,g,d\in K} \le \sum_p Z_{i,k} \cdot \mathrm{CC}_k^{Tank} \forall i \in I, k \in K \tag{45}$$

Only one grant is allowed to contribute towards each technology d, as this is a condition enforced by most grants:

$$\sum_{g\in G} Y_{i,g,d} \le 1 \tag{46}$$

The equations above outline all the policy constraints introduced into the model formulation. Additionally, a discounted VAT of 5% is applied to the to the capital expenses of all insulation measures as a pre-optimisation calculation [42].

3. Case study

3.1. Housing stock

The case study tested is a stock of 1658 dwellings in the borough of Woking, Surrey, England. The majority of grants only apply to low-income households, and thus the social housing sector was chosen. For such dwellings, retrofit investment costs are incurred by the housing association, Woking Borough Council, and utility bills are paid by the household residents.

Information on dwelling type (e.g., house), dwelling type detail (e.g., detached) and Energy Performance Certificate (EPC) score was provided by the Council. To reduce computational complexity, the dwellings were clustered such that each cluster represented a type, type detail and EPC band. Large groups, such as flats with EPC C, were further divided further. The 39 resultant clusters are displayed in Appendix C: Dwelling clusters.

Energy demands for these clusters were determined using the Cambridge Housing Model (CHM). This model uses data from the 2011 English Housing Survey to estimate emissions and energy use for 22 million dwellings in England and Scotland. The dwellings are sorted into 14,951 housing codes by region, tenure type, EPC rating and more than 100 further building physics parameters [44]. Monthly energy demands for the housing codes most representative of the clusters were aggregated into four seasons. Dividing by the number of days resulted in a representative day for each season and housing cluster. Hourly energy consumption behaviour is based on data from Morvaj et al. [45]. The CHM is also used to determine baseline emissions $\mathrm{CO_2e^{annual}_{baseline}}$, against which emissions reduction will be determined. A Surrey County target reduction of 61% by 2035 will be considered in the new scenarios and designs [46], indicated by $CO_2e^{annual}_{new}$, which is described below in Eq. (47).

$$\frac{\mathrm{CO_2e^{annual}_{baseline}} - CO_2e^{annual}_{new}}{\mathrm{CO_2e^{annual}_{baseline}}} \leq 0.61 \tag{47}$$

Consequently, a projected reduced carbon intensity of the electrical grid in 2035, taken as 15 $\mathrm{gCO_2e/kWh}$, as predicted in the Sixth Carbon Budget [47].

3.2. Design options

Six boilers and three ASHPs are considered as heating technology options, as presented in Table 1 and Table 2, respectively. Note that the overall system lifetime is considered to be 20 years, i.e., all installed technologies are assumed to be operational for a total period of 20 years. The coefficients used in the piecewise linearisation for $\mathrm{cap^{pump}_{t,p}}$ and $\mathrm{CoP^{pump}_{t,p}}$ are presented in Appendix F: ASHP coefficients. The capacity listed by Mitsubishi Electric is included in Table 2 [48]. To use an ASHP as a heating source, a hot water storage tank must also be installed, the options for which are presented in Table 3.

Optionally, the clusters are allowed to retrofit the insulation measures listed in Table 4. The average space heating demand reduction $\mathrm{R_{I,j}}$ is determined using the CHM. The eligibility of a cluster to install an insulation form is dependent on the wall, loft and window characteristics defined in the CHM. Loft insulation is considered to be retrofitted to 300 mm. As the existing loft insulation at the clusters vary in thickness, the effect of retrofitting is strongly cluster dependent. Hence, the $\mathrm{R_{i,loft}}$ differs per cluster – as defined in Appendix G: Loft insulation performance. Costs are sourced from Ref. [49] and, where applicable, a VAT discount [50] has been applied. The number of eligible insulation measures available for the total case study (1658 dwellings) is outlined in Table 5.

Table 1
Parameters for boilers.

Boiler	$\mathrm{cap^{boiler}_b}$ (kW)	$\mathrm{CC^{boiler}_b}$ (£)	η_b	Type
7E	7	1,030	1.000	Electric
11E	11	1,110	1.000	Electric
12E	12	1,439	1.000	Electric
24G	24	811	0.911	Gas
25G	25	744	0.891	Gas
30G	30	852	0.891	Gas

Table 2
Parameters for ASHPs.

ASHP	Listed capacity (kW)	CoP *at* $\mathrm{T^{amb}_t} = 2°\mathrm{C}$	CC^{pump}_p (£)	$\mathrm{T^{ws}_p}$ (°C)
HP50	5.0	1.98	2,333	55
HP60	6.0	2.45	3,053	55
HP85	8.5	2.30	3,577	55

Table 3
Parameters for hot water tanks.

Hot water tank	$\mathrm{V_k}$ (L)	CC^{Tank}_k (£)	$\mathrm{H^{loss}_k}$ (kW)
150 L	150	1,510	0.048
170 L	170	1,565	0.051

3.3. Grants and scenarios

The existing applicable grants are presented in Table 6. As Green Jump Surrey is a county addition to the Green Homes Grant, the two are combined into one grant, as in Table 7. Budget limitations for the Social Housing Decarbonisation Fund differ based on the dwelling's EPC rating. Hence, the fund is divided into the individual grants. This fund also has its own unique set of constraints (SE1 – SE3), outlined in Appendix A: SHDF specific constraints. Note that no grants apply to homes rated EPC A, B or C. Relevant assumptions for Table 7 are discussed in Appendix D: Policy assumptions. Primary and secondary measures are reflected by a 'P' and 'S', respectively. The limits $\mathrm{CC^T_{GHG\_GJS}}$, $\mathrm{CC^T_{PROP}}$ and $\mathrm{CC_{PROP}}$ are set to a value sufficiently high for the grant to effectively be unlimited.

Three scenarios are constructed based on these grants. The *No* Grant scenario omits any form of government contributions. The Business-*as*-Usual, or *BAU*, scenario includes the grants currently applicable to Woking Borough. These include GHG-GJS, ECO and SHDF. The *Proposed* scenario additionally includes the *PROP* grant, an unlimited grant that also funds hot water tanks – a key investment overlooked by existing strategy. Their emissions and annual energy bills are compared to a baseline defined by the case study's existing heating installations, as per the Cambridge Housing Model. These installations consist of electric boilers and gas boilers with efficiencies often lower than state-of-the-art options.

4. Results and discussion

4.1. No carbon tax

This section discusses the results of the optimal design, utilising Eq. (1) as the objective function without carbon costs, under the three scenarios. Additionally, Surrey's emissions target is tested by constraining the system to achieve an ER of at least 61% by 2035. The total annualised costs and associated carbon emissions reductions are presented in Table 8. Under the *Proposed* scenario, an ER of 66.8% is achieved, regardless of the 61% constraint. This is due to the availability of sufficient funding from the grants that enable the retrofit all EPC D/E/F dwellings with air-source heat pumps and insulation measures, decreasing carbon emissions beyond the 61% ER target.

4.1.1. Insulation

Fig. 2a reflects the number of insulation measures installed under the

Table 4
Parameters for Insulation measures.

Insulation form	$R_{i,j}$	Detached $C^{ins}_{i,j}$ (£)	Semidetached $C^{ins}_{i,j}$ (£)	End-terrace $C^{ins}_{i,j}$ (£)	Mid-terrace $C^{ins}_{i,j}$ (£)	Flat $C^{ins}_{i,j}$ (£)
Loft	See Appendix G	525	438	438	350	N/A
Cavity	0.261	534	416	416	341	N/A
Solid wall	0.435	4,189	3,281	3,281	2,188	N/A
Double glazing	0.066	7,100	5,950	5,950	4,450	3,000

Table 5
Number of eligible insulation measures available to case study.

	Loft	Cavity	Solid wall	Double glazing	Total
EPC A/B/C	434	165	0	0	599
EPC D/E/F	330	92	250	90	762
Total	764	257	250	90	1,361

Table 6
Existing case study grants.

Grant	Abbreviation	Issuing authority	Description
Green Homes Grant [51]	GHG	UK government	Supports the investment of low-carbon heating technologies and energy efficiency (insulation measures) for low-income households.
Green Jump Surrey [52]	GJS	Surrey County Council	Provides additional investment funds as a top-up for GHG
Energy Company Obligation [53]	ECO	Energy companies	Primarily supports the investment of energy efficiency (insulation) measures for low-income households
Social Housing Decarbonisation Fund [43]	SHDF	UK government	Supports the investment of low-carbon heating technologies and energy efficiency measures for social housing.

various scenarios with no ER constraint. *No* Grant results indicate that, for loft and cavity wall insulation, operating expenditure (OPEX) savings from retrofitting can outweigh its capital expenditure (CAPEX) over the course of the assumed 20-year timeline. All 257 eligible dwellings install cavity wall insulation, with 319 of 330 eligible energy inefficient households also retrofitting loft insulation. However, despite 434 EPC A/B/C dwellings being eligible for loft insulation, the model predicts that only 5 benefit financially from installing such measure. This is likely due to the demand reduction factor $R_{i,j}$ being lower for these households, as can be expected for these relatively energy efficient homes.

Section 3.2 states that solid wall insulation can reduce space heating demand significantly. However, with high investment costs starting at £2188, no dwellings are able to achieve sufficient OPEX savings for it to be financially advantageous within 20 years. In the *BAU* scenario, however, the SHDF grant provides sufficient funding for 100 of 250 eligible households to install solid wall insulation. Due to double glazing having the highest upfront cost and the lowest $R_{i,j}$ value, none of the 90 eligible dwellings install this measure under the *No* Grant and *BAU* scenarios. It is only retrofitted under the *Proposed* scenario, where the unlimited PROP budget is able to fully fund any insulation for homes with EPC D/E/F. Note that homes with EPC A/B/C are not eligible for any grants, and hence demonstrate no change across scenarios.

After the model is constrained such that carbon emissions must be reduced by 61% from baseline values, the trends identified in Fig. 2a do not apply. In the *No* Grant scenario, a significant increase in retrofitting the dwellings in the A/B/C group occurs, with an additional 268 dwellings installing loft insulation. Furthermore, 150 EPC D/E/F dwellings install the financially unattractive solid wall insulation to achieve Surrey's ER target. Under *BAU*, grants are available to EPC D/E/F households. As a result, an additional 100 solid wall insulation measures are installed and the focus is shifted towards improving the energy performance of the fuel poor dwellings. This trend continues under the *Proposed* scenario, where the additional PROP grant provides further funding. The costs to Woking Borough Council associated with these retrofits are outlined in Table 9. Naturally, there are operational cost savings associated with retrofitting insulation measures. These are discussed in Section 4.1.3.

4.1.2. *Heating*

As demonstrated by the *No* Grant results in Fig. 3a, heat pumps are not currently an economically attractive form of domestic heating without financial assistance towards their costs. The high investment costs of the ASHPs and accompanying hot water storage tanks result in a CAPEX significantly larger than that of gas boilers. Under *BAU,* funding is available to promote the installation of ASHPs. Storage tanks, costing at least £1510, remain unfunded, and therefore is a cost incurred to the consumer or council.

Table 8
Total annualised cost and ER for the optimal solutions.

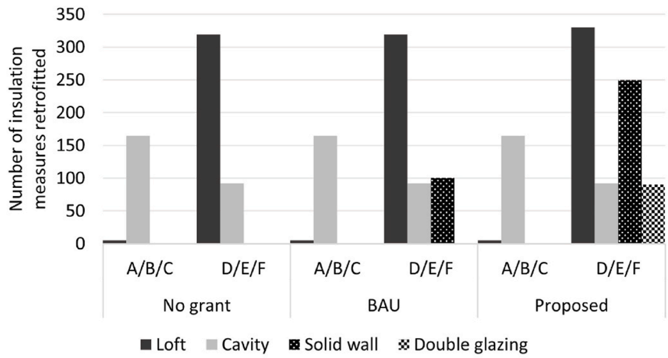
Scenario	Total annualised cost (£)	Emissions reduction ER
No Grant	1,982,675	31.3%
No Grant[a]	2,605,957	61.1%
BAU	1,955,946	33.5%
BAU[a]	2,558,625	61.2%
Proposed	1,873,085	66.8%

[a] System constrained to achieving at least 61% carbon emissions reduction from baseline values.

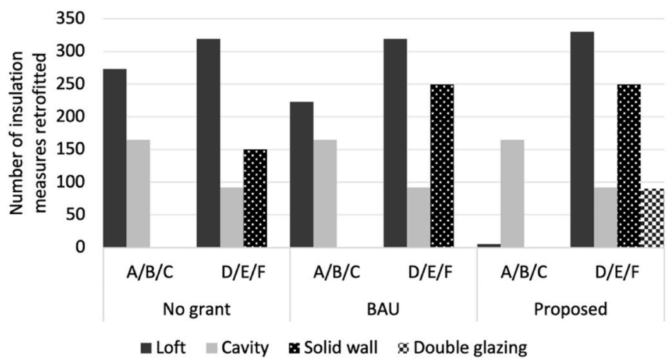
Table 7
Grant parameters.

Grant	Eligible EPC	Household Limit CC_g	Case Study Limit CC^T_g	Gas Boilers	Electric Boilers	ASHPs	Loft	Cavity	Solid wall	Double glazing	Hot water tanks
GHG-GJS	E,F,G	£15,000	£300,734	–	–	P	P	P	P	–	–
ECO	E,F,G	£10,000	N/A	–	–	–	P	P	P	P	–
SHDFd	D	£10,000	£402,073	–	–	S	P	P	P	P	–
SHDFe	E	£12,000	£482,488	–	–	S	P	P	P	P	–
PROP	D,E,F,G	N/A	N/A	–	S	S	P	P	P	P	S

Note: 'P' indicates Primary and 'S' indicates Secondary measures.



(a)



(b)

Fig. 2. Insulation retrofitting with (a) no ER target and (b) a 61% ER target.

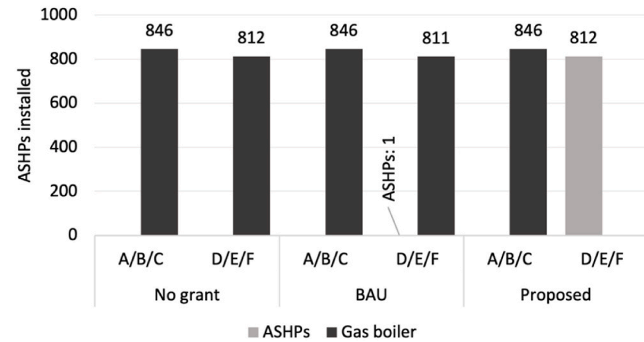
Table 9
Insulation cost changes (excluding grant contributions) under various grant scenarios.

Emissions reduction target	No grant	BAU	Proposed
No target	£233,155	£206,081	£204,439
61%	£906,390	£1,256,390	£204,439
Change	+£673,235	+£1,050,309	–

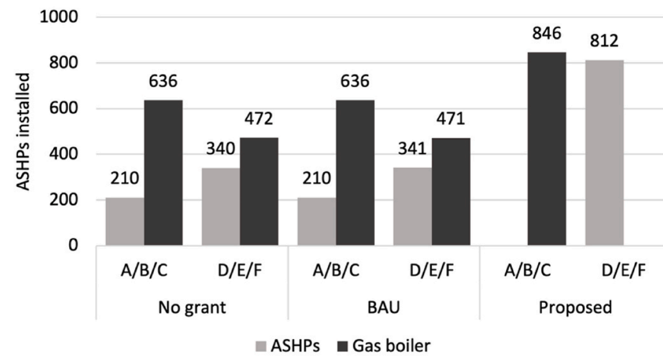
For the case study timeline, average predicted unit costs for electricity and gas are 23.1 and 4.7 p/kWh, respectively [58-60] (see Appendix E: Scalars). The OPEX savings from a gas boiler eventually outweigh the higher CAPEX costs, leading to ASHPs being unfavourable for all but one housing cluster: C10 (see Appendix C for all cluster codes). This single-household cluster has an EPC of E, and is therefore also able to fully fund the installation of an ASHP under the SHDF with £10,771 – a value for which other households are ineligible. Under the *Proposed* scenario, the PROP grant is formulated such that it covers the capital costs of the hot water tanks as well as the heat pumps reducing investment costs for the overall ASHP system by at least £1510. This leads to renewable heating becoming competitive with non-funded gas boilers. Electric boilers do not present themselves as the optimal heating source in any scenarios due to their inefficiency compared to an ASHP's CoP. These results suggest that financial assistance in the form of grants could play a decisive role in installing more residential heat pumps, especially to support the UK Government's goal of installing 600,000 heat pumps by 2028 [54].

The results presented in.

Fig. 3b reflect the heating technologies installed to achieve Surrey's 2035 emissions reduction target. Though gas boilers remain the most prevalent domestic heating source, retrofitting ASHPs is crucial to decreasing emissions by 61% from baseline values. The investment costs associated with the heat pump uptake levels are outlined in Table 10.



(a)



(b)

Fig. 3. Heating technologies with (a) no ER target and (b) a 61% ER target.

4.1.3. *Grant contributions*

The increase in costs discussed in the previous section highlights the financial barriers associated with adopting low carbon heating and the importance of an effective decarbonisation strategy. In order to reduce carbon emissions, it is vital grants are constructed such that the measures capable of achieving the highest emission reductions are prioritised. All of the grants in this case study, with the exception of GHG-GJS, employ a 'fabric first' approach – implying eligible insulation measures must be installed before ASHPs are funded. This promotes the improvement of home energy efficiency, thereby helping to reduce operating costs and thermal discomfort within the homes. A fabric first approach also promotes the reduction of heating demand prior to installing new heating technologies, thereby avoiding the installation of unnecessarily large heating system. However, BAU results in Table 11 indicate that, in order to achieve decarbonisation targets, the optimal distribution of grants shifts from funding insulation measures to funding ASHPs. It must be noted that these results are specific to this case study and its grant structures and cannot be considered all-encompassing. 'Fabric first' constraints are not broken here as insulation measures are installed at the relevant households, though not funded by external grants. Additionally funding these measures would lead to grant budgets being exceeded.

Similarly, Table 12 breaks down the grant contributions under the *Proposed* scenario – which achieves an emissions reduction of 66.8 without any emissions target enforced. No limit is set on the PROP grant, and thus no compromises have to be made in terms of budget allocation. This allows for much larger contributions towards minimising emissions, especially towards costly ASHP investments, reducing the financial burden placed on the Council to fund such measures.

4.1.4. *Energy bills*

Fuel poverty is linked to a household's disposable income after energy bills have been paid. Hence, decreasing this household energy bill is key. Fig. 4 outlines the average energy bill for those living in EPC D/E/F

Table 10
Heating investment costs (excluding grant contributions) under various grant scenarios.

Emissions reduction target	No grant		BAU		Proposed	
	ASHPs	Gas boilers	ASHPs	Gas boilers	ASHPs	Gas boilers
No target	£0	£4,591,542	£0	£4,588,831	£0	£2,336,097
61%	£5,240,037	£3,076,856	£4,837,964	£3,074,146	£0	£2,336,097
Change	+£5,240,037	-£1,514,686	+£4,837,964	-£1,514,685	–	–

Table 11
Total grant contributions and consumer investments under BAU with and without an ER target.

Emissions reduction target	ASHP grant funding	Remaining ASHP investment[a]	Insulation grant funding	Remaining Insulation investment[a]
No target	£10,771	£0	£402,073	£206,081
61%	£411,600	£4,837,963	£0	£1,256,390

[a] Note that these remaining costs are to be paid by the Council.

Table 12
Total grant contributions under the Proposed Scenario.

ASHP grant funding	Insulation grant funding	Hot water tanks	Electric boilers
£7,945,720	£1,388,662	£1,232,548	£0

homes under the various scenarios. Due to its high unit cost relative to natural gas, electricity consumption dominates the annual expenditure. This is particularly evident when the system is constrained to reducing baseline carbon emissions by 61%. This constraint promotes the use of ASHPs, an electrified form of heating. Though this heating method boasts a higher energy efficiency than boilers, the significant difference in costs between gas and electricity leads to higher total energy bills – despite the additional insulation. Under the *Proposed* scenario, the households have unlimited funding to install insulation measures that help reduce the overall energy demand, and consequently, electricity consumption costs. However, the annual energy bill remains 4.7% higher than the baseline value. This implies that, without OPEX support and under current tariffs, the decarbonisation of heating will lead to higher annual energy bills and can potentially exacerbate fuel poverty.

4.2. Including carbon tax

The following section presents results for the multi-objective optimisation model, where Eq. (2) is used as the objective function. Iteratively increasing the weighting of this carbon tax C^{tax} produces results that reflect the system design at various degrees of decarbonisation. For all scenarios, the first set of results has a C^{tax} value of zero, mirroring the results discussed in Section 4. With a projected grid carbon intensity of 0.015 $kgCO_2e/kWh$, the maximum achievable emission reduction from baseline values is 96.7%.

In all cases, a trade-off can be seen between retrofitting insulation measures and ASHPs. The former is cheaper, but has a significantly smaller impact on decarbonisation performance of the system. Under the *No* Grant scenario, increasing insulation from 581 to 1234 installations results in a minor increase in emission reductions – reflected by the change from 31.3% to 37.5% in Fig. 5a. As weighting on carbon emissions increases, the system is driven to decarbonise further by installing ASHPs, which have a greater impact on driving down carbon emissions when compared to insulation measures. This results in a compromise between insulation measures and ASHPs, where insulation investment is reduced to facilitate increased ASHP installation, especially in instances where the tax is not sufficiently high for the simultaneous increase in insulation investment and ASHP installation to be cost-effective. The additional ASHPs allow the system to increase emissions reduction from 37.5% to 51.5%. Another notable trend from Fig. 5 is the phasing out of gas boilers. Under all scenarios, the system achieves an ER of more than 90% without fully phasing out gas boilers. This is likely due to the gas boiler options presented in Table 1 boasting high thermal efficiencies (reflecting technological improvements as seen in reality), relative to existing installations. As the carbon tax weighting increases significantly more than other costs, these final boilers are replaced by their electric counterparts due to their relatively low investment cost, relative to ASHPs. The dwellings at which these are installed are those with the lowest heating demand. For these households, the operational savings from an ASHP do not outweigh the high associated investment costs. This implies that electric boilers may be a promising solution to the decarbonisation of small homes – provided they are powered by low-carbon electricity. If the source of electricity is not fully carbon neutral, however, then the efficient nature of ASHPs leads to them being

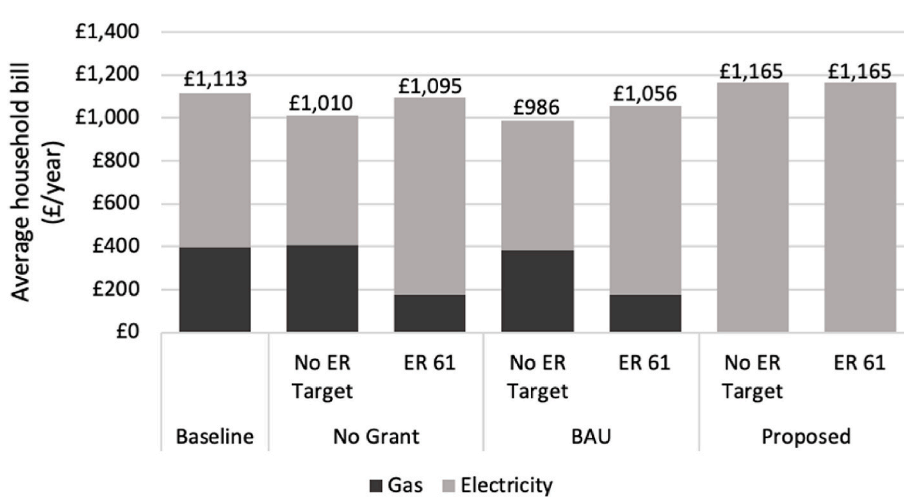


Fig. 4. Average annual energy bills for the fuel poor (based on Eqs. (7) and (15)).

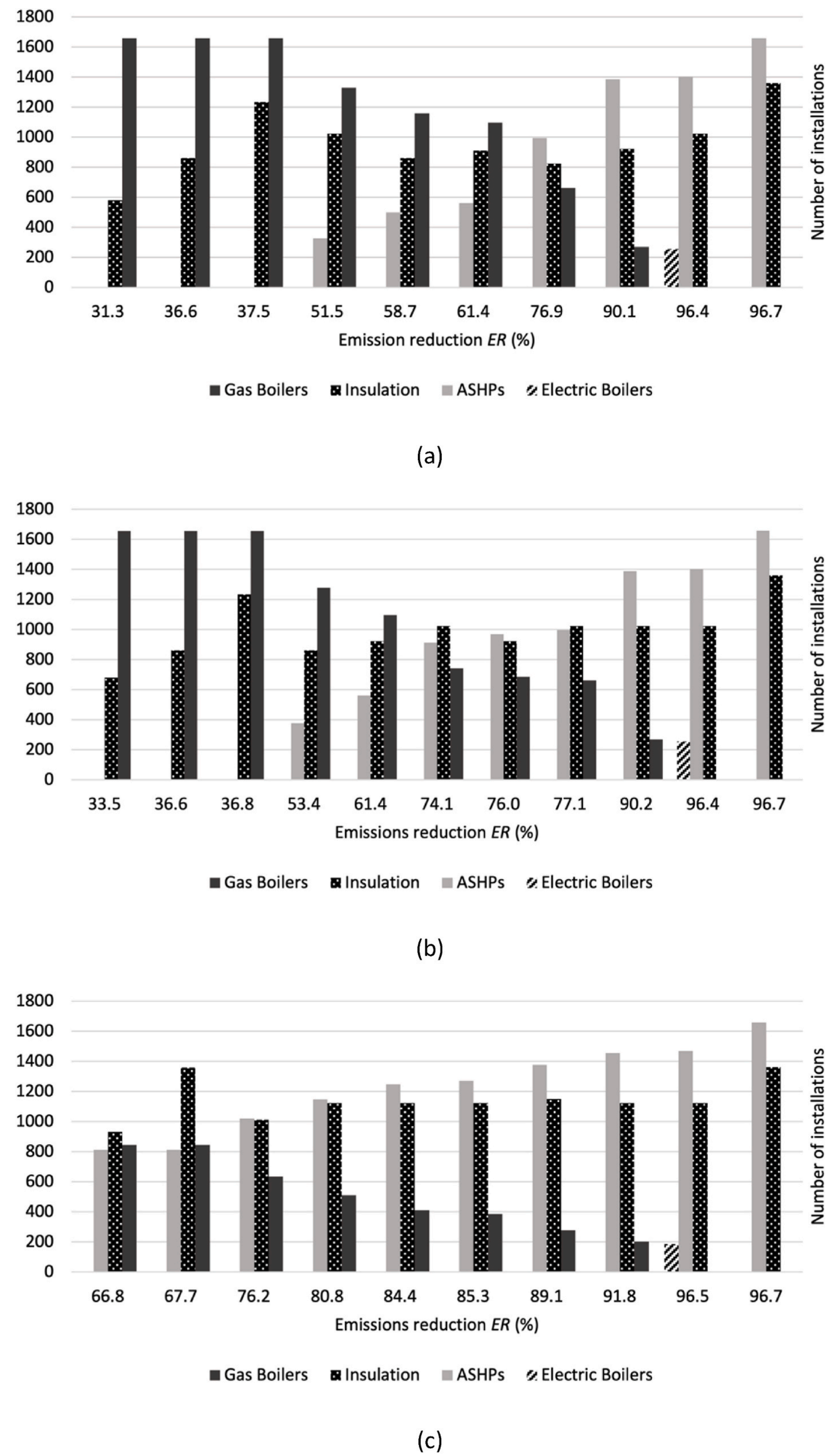


Fig. 5. Increasing carbon penalty results under the (a) No Grant (b) BAU and (c) Proposed Scenario.

the sole solution in this case study to minimising carbon emissions, as reflected by the results for ER = 96.7% in all scenarios.

4.3. Model limitations

The modelling framework proposed in this study tests the efficacy of residential heating and energy efficiency grants. In the case study tested, the *Proposed* scenario was used to highlight the importance of funding hot water storage tanks and the budget limitations of the current grant structure. In reality, such budget limitations are set due to the larger macroeconomic effects local authorities and governments may suffer. This modelling framework is unable to analyse such effects. The model also assumes that all installations are made in the beginning of the modelling period considered, and is unable to recommend the optimal year of installation. Such a model would require predictions on future grants and budgets, changes to technology measures such as declining costs and increased efficiencies, and potential changes to demand profiles. Furthermore, the thermal comfort and health benefits associated with improved quality of housing through insulation upgrades are not considered. Future work could include government expenditure and its contributions from different sources within the objective function and modelling framework, which would capture a more optimal scenario for both the private and public sectors.

5. Conclusions

This study aims to address the lack of optimisation frameworks to support decision-making in decarbonising residential heating systems. It does so by developing an MILP framework which captures key design, operational, and financial decision variables for low-carbon heat generation and storage, insulation measures to reduce energy demand, and requirements and contributions from policies associated with the former two. Additionally, the nonlinear nature of a heat pump's performance is represented through piecewise linearisation. Detailed policy requirements are integrated using generic formulations that can be applied to a range of capital investment grants. Consequently, the framework can be applied to a variety of case studies to aid stakeholders in their policy decision making process.

For this study, housing stock and policies specific to Woking Borough Council are used as a test case. Three scenarios are tested alongside Surrey County's carbon emission reduction target to examine the efficacy of the grants in place. The framework is used to quantitatively assess how effective various policy and technology combinations are in achieving practical objectives such as lowering operational costs or maximising carbon emissions reductions. Results for this case study illuminate that.

1. Utilising the existing grant structure contributes to 31.3% emission reduction, which does not meet the Council's target reduction of 61% by 2035.
2. A nearly 30-fold increase in investment on low-carbon technologies is required to achieve the 61% target.
3. The maximum emission reduction that can be achieved with current low-carbon heating technology (with an unlimited grant) at the projected carbon intensity of the electrical grid is 96.7%.
4. Capital investment in insulation measures can be outweighed by operational cost savings, i.e., the reduction in energy bills.
5. Without operational incentives, the electrification of heating would lead to an increase in utility costs incurred by the consumer, which could exacerbate fuel poverty.

Thus, the framework provides invaluable information to support decision-making, such as the limits of the system, feasible technological options, and policy contributions required to achieve decarbonisation targets.

Further research would benefit from setting realistic grant budgets under the Proposed scenario. Also, grant contribution data from Table 8 can be used to reformulate existing grants from BAU in a separate scenario. For this case study, the optimal grant allocation is shifted from retrofitting insulation measures to ASHPs, when Surrey's 61% emissions reduction target constraint is imposed. Hence, the 'fabric first' constraint of installing insulation before ASHPs could be removed to test the effects of this approach. Finally, rather than creating the additional grant, a scenario where existing grants also fund hot water storage tanks could be tested. Nevertheless, this work provides a flexible foundation upon which further case studies can developed, allowing policy makers to make informed decisions in decarbonisation strategy of the housing stock."

Credit author statement

Ishanki De Mel: Conceptualization, Methodology, Software, Writing – original draft, Writing – review & editing. **Floris Bierkens:** Data curation, Conceptualization, Methodology, Software, Writing – original draft, Writing – review & editing. **Xinyao Liu:** Methodology, Writing – review & editing. **Matthew Leach:** Conceptualization, Supervision, Writing – review & editing, Funding acquisition. **Mona Chitnis:** Supervision, Writing – review & editing, Funding acquisition. **Lirong Liu:** Conceptualization, Supervision, Writing – review & editing, Funding acquisition, Project administration. **Michael Short:** Conceptualization, Methodology, Software, Supervision, Writing – original draft, Writing – review & editing Funding acquisition.

Declaration of competing interest

The authors declare the following financial interests/personal relationships which may be considered as potential competing interests: This research was supported by the project 'Heat4All: Economics-informed optimisation model for future equitable decarbonised distributed heating systems' funded by the EPSRC Network + Decarbonisation of Heating and Cooling.

Data availability

Data will be made available on request.

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Appendix A. Supplementary data

Supplementary data to this article can be found online at https://doi.org/10.1016/j.energy.2023.126651.

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