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ASSESSING THE IMPACT OF MARKET DISTURBANCES USING INTERVENTION ANALYSIS*

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A recurring problem confronting marketing operatives at all levels is assessing the impact of market disturbances. The disturbances may be caused by marketing mix manipulations of the firm itself, by its competitors or by fundamental external changes in the environment in which the firm operates. This article discusses a technique, intervention analysis, by which the impact of such disturbances can be assessed. The technique is used to examine the effects of Proctor and Gamble's promotion of the American Dental Association endorsement of Crest on the market shares of Crest and Colgate dentifrice during the years 1958–1963.

1. Introduction

One of the most basic problems facing the marketing or product manager is how to use most efficiently the resources entrusted to his or her control. Ideally, in the absence of constraints, the manager would like to allocate dollars to a particular element of the marketing mix up to the point where the marginal revenue generated by the expenditure equals the cost of the effort. When constrained, the ideal allocation necessitates equating the marginal net contributions of each element of the mix ([3, pp. 187–203]; [9, pp. 362–418]).

Achieving the optimum allocation is a difficult task. Although the cost of any allocation is typically available, the revenues likely to be produced by a particular allocation are difficult to ascertain. In attempting to determine the potential impact on sales of, say, a contemplated change in product features or a change in the advertising appropriation or changes in any of the other elements in the marketing mix, the manager must recognize that the mix variables are often interrelated; some only operate when a minimum threshold is satisfied, most produce delayed effects, some possess economies of scale, and some are best when linked together sequentially [14]. Or, as one of the leading marketing authorities has stated, "Marketing decisions must be made in the context of insufficient information about processes that are dynamic, nonlinear, lagged, stochastic, interactive, and downright difficult [9, p. XI]."

This article discusses a technique, intervention analysis, which should prove helpful in assessing the impact of marketing mix manipulations, either by the firm itself or by its competitors. The technique is used to examine the effects of Proctor and Gamble's promotion of the American Dental Association endorsement of Crest on the market shares of Crest and Colgate dentifrice during the years 1958–1963.

The next section contains a brief description of the Crest-Colgate situation around the time of the American Dental Association endorsement of Crest. The models for intervention analysis are described in §3. The data and a definition of the market share variables are given in §4. The fitted models and the implied intervention effects are discussed in §5. The final section contains a summary of our results.

2. Background

Proctor and Gamble introduced Crest toothpaste nationally in 1956 at a time when Colgate-Palmolive's Colgate Dental Cream (the industry leader) enjoyed a substantial proportion of the market. For the next four years, Colgate retained its dominant

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position and Crest's market share remained relatively static. On August 1, 1960, however, the Council on Dental Therapeutics of the American Dental Association (ADA) endorsed Crest as an "important aid in any program of dental hygiene," and (as Shuchman and Riesz [12] have noted) Proctor and Gamble responded by reintroducing Crest in all major markets with an advertising campaign designed to capitalize on the ADA's unprecedented endorsement. During the ensuing period, Crest's market share climbed rapidly while Colgate's market share decreased markedly.

It appears as if the sudden and dramatic increase in Crest's market share during 1960 was due almost exclusively to the ADA endorsement of the fluoride additive.[1] This contention is based on the following observations:

(i) Crest's market share increased despite heavy competitive price and "dealing" activity (see Frank and Massey [7]).

(ii) The increase in Crest's market share cannot be explained by a saturation advertising campaign.[2] For example, although Crest's advertising budget increased from roughly 6.2 million dollars in 1960 to 10.5 million dollars in 1961, this was far below the 20 million dollar forecast made by several sources and below the initial introductory budget of 1956. In 1962, Crest's advertising budget declined somewhat to 9.2 million dollars and its market share continued to increase. Moreover, Colgate Dental Cream increased its advertising budget from 4.8 million in 1960 to about 9 million in 1961 yet saw its market share decline sharply.

(iii) The nature of the market share response—which is pictured in Figure 1 and describe in §5—points toward the influence of the endorsement rather than the influence of other variables. There is a substantial and almost immediate increase in level of Crest's market share at the time of the ADA endorsement and this increase is sustained throughout the observation period.

Brand managers are interested in the effect of an activity on aggregate market share. As pointed out in the introduction to this paper, we are concerned with assessing the impact of a particular promotional effort. Although the data[3] are old and have been extensively studied (see, for example, [4], [8], [10] and [11]), no one appears to have *directly* estimated the size and nature of the impact of the ADA endorsement.

3. Models

We model the variables of interest using time series methodology developed by Box and Jenkins and Box and Tiao (see [1] and [2]). In particular, the models are selected from the class of dynamic models

$$y_t = \sum_{i=1}^{k} Y_{ti} + N_t = \sum_{i=1}^{k} (\omega_i(B)/\delta_i(B))X_{ti} + (\theta(B)/\Phi(B))a_t, \qquad t = 1, 2, \ldots, n, \quad (1)$$

where the final output, y_t, is represented as the sum of *transferred* inputs, $Y_{t1}, \ldots, Y_{tk}$ and a noise, N_t.

Although the models in (1) may appear somewhat forbidding, in fact, they are capable of representing a wide variety of behaviors. A specific model can be identified from the data and parameters can be estimated using (in general) a nonlinear

[1] The gain in market penetration was certainly helped by the fact that Proctor and Gamble was able to add fluoride to Crest without impairing the taste. We do not explore the *specific* effects of taste on market share in this paper.

[2] The advertising budget figures which follow are taken from Shuchman and Riesz [12].

[3] The data are taken from the Market Research Corporation of America's (MRCA) panel of household purchasing records for the period January 1958 to April 1963. Dentifrices were deleted from MRCA's panel in April 1963.

estimation algorithm. The "dynamics" of the system are represented by the linear difference equations

$$\delta_i(B)Y_{ti} = \omega_i(B)X_{ti}, \qquad i = 1, 2, \ldots, k, \tag{2}$$

where $\delta_i(B)$ and $\omega_i(B)$ are polynomials of finite degrees in the backward shift operator B. (For example: $\delta_i(B)Y_{ti} = (1 - \delta_1 B - \delta_2 B^2)Y_{ti} = Y_{ti} - \delta_1 Y_{t-1,\,i} - \delta_2 Y_{t-2,\,i}$.) The roots of $\omega_i(B) = 0$ and $\delta_i(B) = 0$ are ordinarily such that the Y_{ti}'s do not depend overwhelmingly on values of X_{ti}'s in the remote past. The noise, N_t, is represented by the mixed autoregressive-moving average process

$$\Phi(B)N_t = \theta(B)a_t, \tag{3}$$

where, again, the autoregressive polynomial, $\Phi(B)$, and the moving average polynomial, $\theta(B)$, are polynomials in B of finite degress. In many applications the polynomial operator $\Phi(B)$ is factored as

$$\Phi(B) = (1 - B)^d \phi(B) \qquad (0 < d < \infty) \tag{4}$$

where the roots of $\phi(B) = 0$ lie outside the unit circle. This representation is useful for modeling certain kinds of nonstationary series. In particular, the dth difference of the process is taken to be stationary.

Finally we assume:

(i) $\ldots, a_{t-1}, a_t, a_{t+1}, \ldots$ is a sequence of independently distributed normal random variables each with mean zero and finite variance σ^2.

(ii) Each input X_{ti} is independent of N_t.

The X_{ti}'s are ordinarily exogeneous time series whose influence on y_t has to be taken into account. However, one or more of the inputs may be indicator variables representing the occurrence of unique events ("interventions") at particular points in time which are likely to affect the output. In this paper, the Proctor and Gamble announcement of the ADA endorsement of Crest is regarded as an intervention.

Some indication of the types of intervention effects that can be represented within the dynamic model framework is provided by supposing there is a single transfer Y_t to the output from the input X_t. In addition, let X_t be a "step" indicator where

$$X_t = S_t^{(T)} = \begin{cases} 0, & t < T, \\ 1, & t \geqslant T. \end{cases} \tag{5}$$

The appropriate transfer function is then

$$Y_t = \frac{\omega(B)}{\delta(B)} S_t^{(T)}.$$

If an event produces an output step change of unknown magnitude, ω, in the next period, the "dynamics" is given by $Y_t = \omega B S_t^{(T)}$. If the event is expected to produce an exponentially increasing (first order) response beginning with the next period, the dynamics can be represented by

$$Y_t = \frac{\omega B}{(1 - \delta B)} S_t^{(T)}$$

with $\delta < 1$. The steady state gain in this system is $\omega/(1 - \delta)$.

More examples of simple dynamic models for characterizing intervention effects are available in [2]. We note that differencing a step input produces the "pulse" input $P_t^{(T)}$ where

$$P_t^{(T)} = \begin{cases} 1, & t = T, \\ 0, & t \neq T. \end{cases} \tag{6}$$

Intervention extending over several time periods can often be represented by a series of pulses.

In the discussion which follows, we shall be primarily concerned with models which can be obtained from those given in (1) by setting $k = 2$, $\omega_i(B) = \omega_i$, $\delta_i(B) = (1 - B)$, $\theta(B) = (1 - \theta_1 B)$ and $\Phi(B) = (1 - B)$. Thus,

$$y_t = \frac{\omega_1}{(1 - B)} X_{t1} + \frac{\omega_2}{(1 - B)} X_{t2} + \frac{(1 - \theta_1 B)}{(1 - B)} a_t \tag{7}$$

or equivalently,

$$(1 - B) y_t = \omega_1 X_{t1} + \omega_2 X_{t2} + (1 - \theta_1 B) a_t, \qquad t = 1, 2, \ldots, n. \tag{8}$$

The general problems of model specification, parameter estimation, and diagnostic checking for models given by (1) are discussed in [1] and [13], and will not be considered in detail in this paper.

We point out that standard statistical procedures like a simple trend analysis or a comparison of market share averages from the "before ADA endorsement" and "after ADA endorsement" periods will, in this case, indicate the approximate magnitude of the ADA endorsement effect. However, these techniques will not (specifically) indicate the *rate of increase* in Crest market share due to the intervention nor the *nature* of the intervention effect. Moreover, any formal analysis based upon a student's t test or a nonparametric procedure rests heavily upon the assumption of independent observations. In our case, the data are in the form of a time series in which successive observations are serially *dependent* and, in addition, nonstationary. A model like (8) takes the dependence and nonstationarity into account.

4. The Data

The data are generated from 276 consecutive weeks of dentifrice purchasing history for families who were members of the Market Research Corporation of America's consumer panel during the years 1958–1963. Members of the panel were to record each purchase in the product class, giving the date of purchase, the brand, the number of units, the package size, the price, the store where purchased, and the type of deal involved, if any.

Our results are based on the purchasing history of "faithful respondents," that is, families who reported for at least 80% of the reporting periods. In addition, we consider only those purchases consisting of tubes of toothpaste weighing 5 oz. or more.

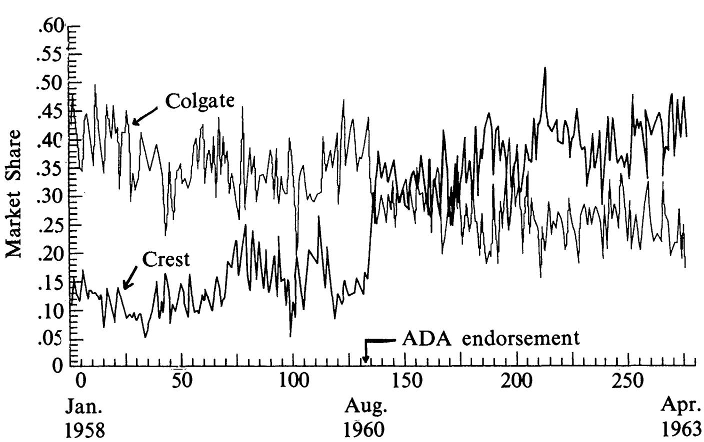

FIGURE 1. Market Share Series.

Following Frank and Massey [7], we define the market share variable, y_t, for brand j in week t as:

$$y_{jt} = \frac{\text{no. of lbs. of brand } j \text{ purchased in week } t}{\text{total no. of lbs. of dentifrice (of any kind) purchased in week } t}\,.$$

The market share time series are plotted in Figure 1. A visual examination of the data indicates:

(1) There is a general upward drift in the Crest market share series over the study period with a dramatic shift in "level" at the time (given by the arrow in the diagram) of the ADA endorsement.

(2) The general increase in the market share of Crest is accompanied by a rather large decrease in the market share of Colgate with a particularly sudden drop at the time of the ADA endorsement. This behavior implies a substantial number of Colgate purchasers were switching to Crest.

(3) With a combined market share consistently in the neighborhood of 40–60%, the Crest and Colgate brands dominate the market throughout the study period.

5. Model Building

In general, appropriate time series models can be tentatively identified by matching sample autocorrelation patterns with the theoretical autocorrelation patterns produced by members of the class of models under consideration. Least squares estimates of the model parameters are then obtained using readily available computer algorithms.[4] Finally, the residuals (the differences between the observations and the corresponding "fitted" values) can be examined for model adequacy using diagnostic techniques designed to check on the assumptions of randomness, normality, and zero mean. A complete discussion of this three-stage model building strategy is available in [1]. We point out that our results were obtained using this strategy.

The fitted market share models and the corresponding residual root mean square are displayed in the first two rows of Table 1. The indicator variables X_{t1} and X_{t2}, defined by

$$X_{t1} = P_t^{(135)} = \begin{cases} 1, & t = 135, \\ 0, & t \neq 135, \end{cases}$$

$$X_{t2} = P_t^{(136)} = \begin{cases} 1, & t = 136, \\ 0, & t \neq 136, \end{cases} \tag{9}$$

were included in the models to allow for dramatic changes in the local level of the series during the two-week period following the ADA endorsement of Crest.[5] Thus, the coefficients of X_{t1} and X_{t2} may be interpreted as measures of the effects on market share of Proctor and Gamble's decision to heavily advertise the ADA endorsement. Introducing the dummy variables into the model for the *differenced* series means that

[4] The models are, in general, nonlinear in the parameters so that an iterative "search" algorithm (see, for example, [5, Chapter 10]) is often required to locate the least squares values. We note that the least squares estimates are intuitively appealing but they are not, as one might suspect if the a_t's are assumed to be normally distributed, the maximum likelihood estimates. However, for the models suggested by our data, the least squares estimators are not likely to be appreciably different from the maximum likelihood estimators—the latter having desirable statistical properties for large samples.

[5] Other ways to allow for the intervention effect were considered. However, an initial examination of the differenced data indicated that the market share adjustment to the intervention was essentially accomplished over two consecutive weeks and that there was no *obvious* relationship between the two single week adjustments. Consequently, we decided to incorporate two indicator variables with multiplicate coefficients, rather than, say, a single indicator variable with more complicated dynamics to capture the intervention effect.

TABLE 1

Fitted Models and Corresponding Residual Root Mean Square for Dentifrice Data
(The numbers in parentheses beneath the coefficients are approximate standard errors.)

Series	Model	Residual Root Mean Square
Colgate market share:	$y_{1t}-y_{1t-1}=-\underset{(0.047)}{0.052}X_{t1}-\underset{(0.048)}{0.061}X_{t2}+a_t-\underset{(0.037)}{0.809}a_{t-1}$	$s=0.0466$
Crest market share:	$y_{2t}-y_{2t-1}=\underset{(0.045)}{0.065}X_{t1}+\underset{(0.045)}{0.112}X_{t2}+a_t-\underset{(0.039)}{0.779}a_{t-1}$	$s=0.0437$
Colgate market share as a function of Crest market share	$y_{1t}-y_{1t-1}=-\underset{(0.054)}{0.488}(y_{2t}-y_{2t-1})+a_t-\underset{(0.034)}{0.842}a_{t-1}$	$s=0.0420$

an increment equal to the *sum* of the dummy variable coefficients is absorbed into the level of the original series from week $t=136$ onwards.

To see this, consider the model in the first row of Table 1 with $t=135$. Using the definitions in (9), we can write

$$\begin{aligned}
y_{1,135} &= y_{1,134}-0.052X_{135,1}-0.061X_{135,2}+a_{135}-0.809a_{134}\\
&= y_{1,134}-0.052+a_{135}-0.809a_{134},\\
y_{1,136} &= y_{1,135}-0.052X_{136,1}-0.061X_{136,2}+a_{136}-0.809a_{135}\\
&= (y_{1,134}-0.052+a_{135}-0.809a_{134})-0.061+a_{136}-0.809a_{135}\\
&= y_{1,134}-(0.052+0.061)+a_{136}+0.191a_{135}-0.809a_{134}.
\end{aligned}$$

In general

$$y_{1,t}=y_{1,134}-(0.052+0.061)+a_t+0.191(a_{t-1}+\cdots+a_{135})-0.809a_{134},\qquad t\geqslant 136. \tag{10}$$

For Colgate, the sum, $-(0.052+0.061)=-0.113$ is permanently incorporated into the level of the series. Notice, however, that as t increases beyond $t=136$, the noise terms $a_t, a_{t-1},\ldots,a_{134}$ play an increasingly larger role in determining the market share value. This occurs because the original series is nonstationary and highly autocorrelated as evidenced by the large coefficient attached to the a_{t-1} term[6] in the model in Table 1.

Even though the estimated dummy variable coefficients displayed in Table 1 are (in most cases) small when compared with their standard errors, approximate tests[7] demonstrate that the true coefficients are not simultaneously zero. We prefer to keep both dummy variables in the models, even though, from a purely statistical point of view, only one may be justified. Retaining both variables captures the rather obvious shift in the levels of the market shares for Colgate and Crest that occurred over the *two-week* period beginning August 1, 1960.

Our analysis reveals that the impact of Proctor and Gamble's promotion of the

[6] The autocorrelation ("inertia") in the market share series can be demonstrated by expressing the market share in week t entirely in terms of previous weekly market shares and a single shock term a_t. Disregarding the dummy variable terms, the market share of Colgate at time t is given by

$$y_{1t}=0.19y_{1t-1}+0.81(0.19)y_{1t-2}+(0.81)^2(0.19)y_{1t-3}+\cdots+a_t.$$

Colgate market share can be viewed as essentially a geometric weighted average of the market shares in previous weeks where the weights decay rather slowly. A similar result holds for Crest.

[7] The test is based on a linearization of the time series models in the neighborhood of the least squares values. (Draper and Smith [5] have a description of the test procedure for linear regression models.)

ADA endorsement was substantial and immediate. It took only a matter of weeks for the full effect of the promotion to manifest itself in the market share variables. Moreover, the sums of the magnitudes of the dummy variable coefficients in the market share models indicate that the market share of Crest roughly doubled (a jump in the market share level of about 0.18) in the two weeks following the ADA endorsement while Colgate's market share decreased by roughly 25% (a decline in the market share level of about 0.11 as we pointed out following equation (10)).

The decrease in Colgate market share due to the ADA endorsement of Crest occurred in roughly equal increments over the two-week period. On the other hand, the increase in Crest market share during the second week of the period was about twice as large as the increase during the first week. We interpret this behavior to mean that Crest was attracting more new customers from users of brands other than Colgate during the second week following the ADA endorsement than during the first week.

Our models are very simple and yet, their ability to describe the behavior of the observed series is impressive. Figure 2 displays the fitted Crest market share series superimposed on the original series. The outcome for the remaining series is analogous.

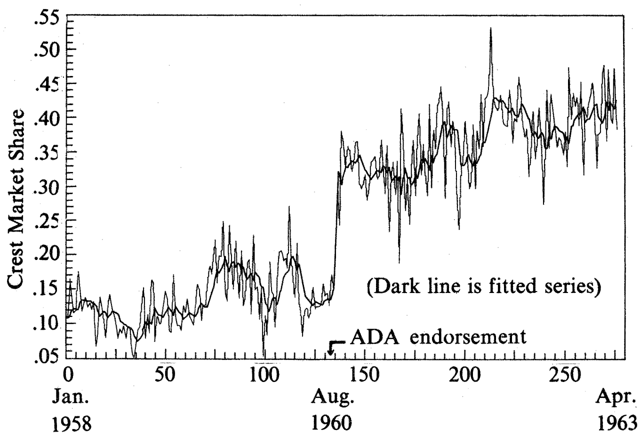

FIGURE 2. Original and Fitted Crest Market Share Series.

We have related the market shares at time t to their respective values at times $t-1$, $t-2, \ldots$. If we concentrate, for example, on the market share of Colgate, one would expect that some of the variation in Colgate market share could be explained by movement in the market share of Crest (recall the sum of the market shares of Colgate, Crest, and "the rest" is always unity). Consequently, the market share of Crest was used as input to the Colgate market share. The result of this modeling effort is displayed in the third row of Table 1.

As expected, there is a strong and immediate relationship between the market shares of Colgate and Crest. An increase in the market share of Crest is accompanied by a rather large decrease in the market share of Colgate, indicating (for our data) that a substantial proportion of Colgate purchasers were switching to Crest.[8] For example, an increase in the market share of Crest of, say, 0.20 is accompanied by a decrease in the market share of Colgate of about 0.10. We note that the dummy variables, X_{t1} and X_{t2}, are not required in the model relating the two market shares

[8] It is possible to regard Crest market share as the dependent variable and Colgate market share as the "explanatory" variable. The subsequent model is very similar to the one given in Table 1, with the roles of the market share variables interchanged. We prefer to regard Crest market share as input and Colgate market share as output because Colgate was the industry leader at this time and Crest was the "newcomer."

since the effects of the ADA endorsement on the Colgate and Crest market shares work in opposite directions and cancel one another out.

6. Conclusion

We have demonstrated through the use of intervention analysis that external events, if properly exploited, can have a substantial and immediate positive impact on brand market share. This can happen despite the vigorous efforts of competitors to counteract the impact. In our example, the market share of Crest toothpaste (roughly) doubled over the two-week period following the ADA endorsement of Crest as an "important aid in any program of dental hygiene." We note that Crest dentifrice was fully distributed at the time of the ADA endorsement so that the dramatic increase in Crest market share cannot be attributed to its sudden availability. In fact, an examination of the nature of the toothpaste market during the time period studied indicates that Proctor and Gamble's release of the ADA endorsement is the only major event that can be justifiably associated with the sudden increase in Crest market share in August, 1960.[9]

It must be emphasized that our inferences are based on data generated by the purchasing history of several thousand Chicago area families who were members of MRCA's consumer panel. Generalizations beyond this frame of reference may be inappropriate. Yet, it seems reasonable to conjecture that, for dentifrices, the Chicago market is indicative of the rest of the country and, consequently, that the *nature* of the relationships given in Table 1 held for the national market during the time period under investigation.

Intervention analysis appears to be a useful technique for assessing the impact of many marketing mix manipulations. For example, it can be used to adjust market share for the introduction of a new product, price controls, and different packaging. The technique is attractive because it provides a framework within which the *nature* of the impact can be modeled. Finally, intervention analysis is attractive from a strictly model building point of view because:

(1) The lag structure (dynamics) linking the dependent variable to a given set of independent variables can be identified from the data and is not determined arbitrarily as common practice frequently dictates.

(2) The appropriate noise structure is easily identified.

(3) The final model will, in general, be a parsimonious representation of the data.

(4) Comprehensive tests of model adequacy are available.[10]

[9] We have attributed Crest's sudden change in market share to the ADA endorsement *and its promotion*. This promotion was associated with increased advertising expenditures but these increased expenditures were far less than originally predicted. That is, the endorsement itself was heavily responsible for the increase in Crest market share. We did not explicitly examine the nature of the interaction (if any) between the ADA endorsement and advertising expenditures.

[10] The data used in this study were kindly furnished by the Market Research Corporation of America. This work was partially supported by a grant from the Graduate School of Business, University of Wisconsin.

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