In the following, p will be the parameter, x and y will be the clocks, a and b two natural numbers $\in \mathbb{N}$.

Tile forcing the following interval: $p \in (\frac{a}{2}, \infty)$.

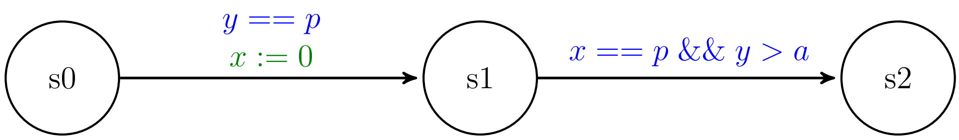

Fig. 1: Tile 1

Tile forcing the following interval: $p \in (0, \frac{b}{2})$.

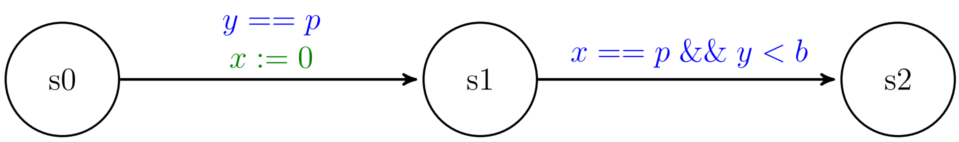

Fig. 2: Tile 2

The above tiles, namely Tile 1 and Tile 2, can be considered as the basic building blocks for constructing every other interval, thanks to the possibility of chaining them together, hence restricting the interval in which the parameter p will fall.

The following one has been obtained by concatenating the aforementioned tiles.

Tile forcing the following interval: $p \in (\frac{a}{2}, \frac{b}{2})$.

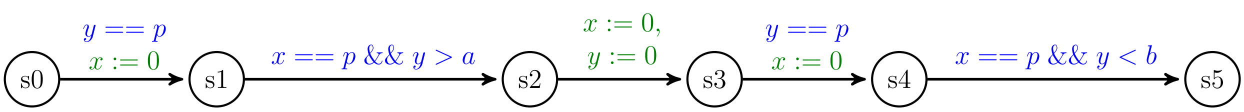

Fig. 3: Tile 3

Please note that Tile 3 can be written more concisely without using concatenation.

Tile forcing the following interval: $p \in (\frac{a}{2}, \frac{b}{2})$.

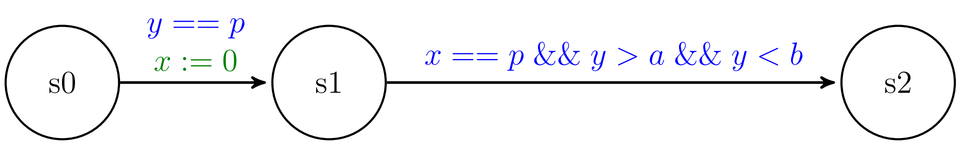

Fig. 4: Tile 4

The following tile has 2 output states, giving rise to the ability of choosing one or the other in an OR condition.

Tile forcing the following interval: $p \in (0, \frac{a}{2}) \vee p \in [\frac{b}{2}, \frac{b}{2}]$

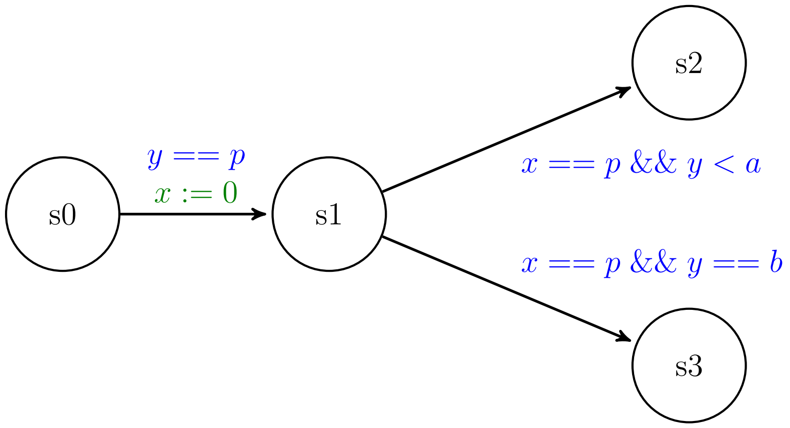

Fig. 5: Tile 5

Please note that in such cases, unless some other previous or subsequent tile forces the parameter in a specific interval, our approach is based on trying all the parameter values starting from $n/2$ (or $n/2 + \alpha$), so the OR tile isn't really non-deterministic in that sense. For example, testing Tile 5 in tChecker with values $a == 8$ and $b == 4$ the result yielded $p == 0.5$.

The following tile has several inputs and outputs, being the most general type of tile we can build (one with n inputs and m outputs).

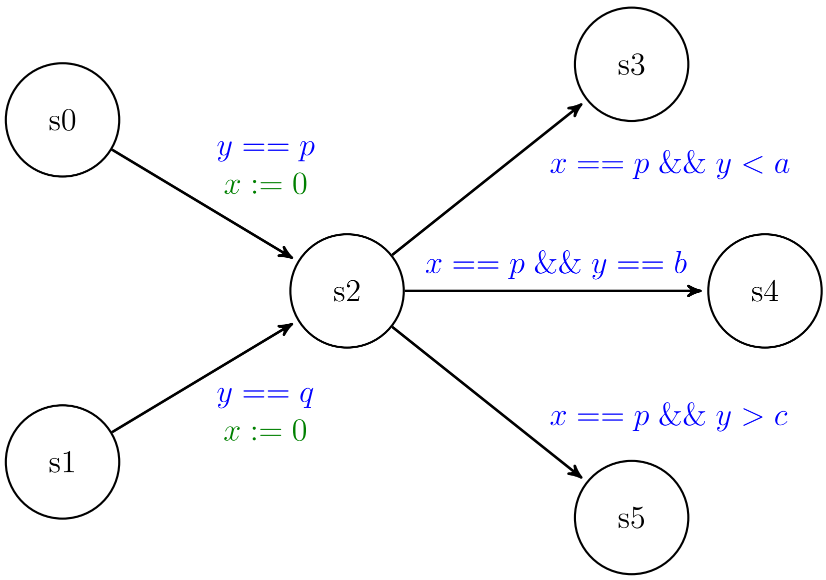

Fig. 6: Tile 6

Understanding whether having multiple input states is useful or not is still under study.

Up to now, we have seen intervals such as these ones: $(0, \frac{b}{2})$ or $(\frac{a}{2}, \frac{b}{2})$, where we can notice a 2 at the denominator of the constants delimiting the interval.
One question that may arise is: *Can such intervals be made arbitrarily small?* That is, is it possible to obtain intervals like $(\frac{a}{n}, \frac{b}{n})$, where $n \in \mathbb{N} - \{0\}$ is an arbitrarily-chosen constant?
Unfortunately, I don't believe this is actually possible without violating our working assumptions.

In the following we recap our base assumptions on the TAs we work with:

1. The grammar for clock constrains does not admit algebraic operations, that is, we cannot have a clock constraint like this: $x < a + b$;
2. TAs are actually nrt-TAs, i.e. a clock cannot be reset and tested in the same transition;
3. We have only two clocks and one parameter (although the constraint on the parameter is not relevant for these considerations).

We have to show that it is not possible to construct a tile that is capable of forcing an arbitrarily small interval without violating any of the aforementioned assumptions.
For the sake of simplicity with the drawings, we will show it for the case in which $n = 3$, but a generalization of these concepts is straightforward.

The first case study will try to preserve both nrt and two clocks assumptions. However, this would require illegal clock constraints syntax, as can be seen in the following figure.

Tile forcing the following interval: $p \in (0, \frac{a}{3})$.

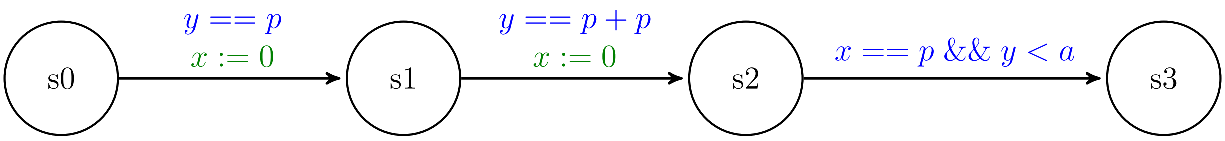

Fig. 7: Arbitrary interval Tile 1

Since here we let the first transition fire when y is exactly equal to p, the second transition to fire when y is exactly equal to $2p$ and the last transition to fire when x is exactly equal to p and y less than a, since y is never reset, in the last transition it must hold that $y == 3p$. But, since we violated condition (1) above, we cannot use this construction in our analysis.

Our second case study addresses the nrt nature of the TAs. It is possible to construct arbitrarily small intervals, if we get rid of the nrt property.

Tile forcing the following interval: $p \in (0, \frac{a}{3})$.

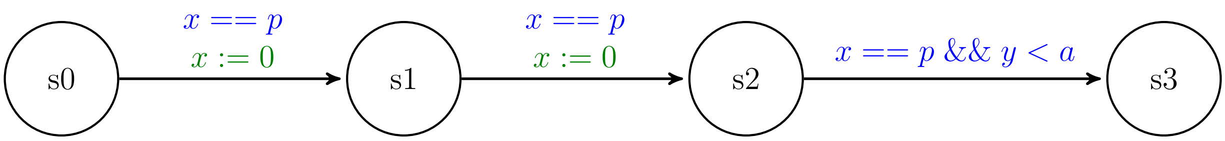

Fig. 8: Arbitrary interval Tile 2

Since in this case y is never reset, we can consider x as a sort of counter. At the end, in the last transition y will have a value corresponding to $3p$. The same considerations from the previous case also hold here. But, as mentioned at the beginning, condition (2) is violated, so we cannot use this construction in our analysis.

The last case affects the number of clocks: we can get a tile forcing an arbitrarily small interval, but at the expense of adding a new clock for each natural $n \in \mathbb{N} - \{0\}$ we want to use as the denominator.

Tile forcing the following interval: $p \in (0, \frac{a}{3})$.

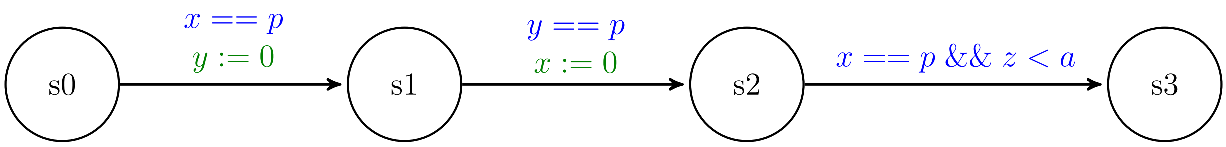

Fig. 9: Arbitrary interval Tile 3

Similar considerations can be done on why at the end z is equal to $3p$ (it should be trivial to notice at this stage). The tile is nrt, but having more than 2 clocks and thus violating condition (3), we cannot use this construction in our analysis.

In conclusion of these considerations, it is worth noticing that, in order to construct the aforementioned illegal tiles, the number of states grows linearly with respect to the natural $n \in \mathbb{N} - \{0\}$ we want to use as the denominator: $T(n) = \Theta(n)$.
The same growth also happens in the case in which we opt for adding more clocks.
Hence, attention should be put when combining them for constructing Tiled TAs, due to the growth of the states set and transitions sets (and anything related to this).