Natural Language Processing

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Part 1: Probability models of Language

The Language Modeling problem

Setup

- Assume a (finite) vocabulary of words:
  $$\mathcal{V} = \{\textit{killer}, \textit{crazy}, \textit{clown}\}$$

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- Use $\mathcal{V}$ to construct an infinite set of *sentences*
$$\begin{aligned}\mathcal{V}^+ = \ \{ & \\ & \text{clown, killer clown, crazy clown,} \\ & \text{crazy killer clown, killer crazy clown,} \\ & \ldots \\ \} & \end{aligned}$$

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- Assume a (finite) vocabulary of words:
  $$\mathcal{V} = \{\textit{killer}, \textit{crazy}, \textit{clown}\}$$
- Use $\mathcal{V}$ to construct an infinite set of *sentences*
  $$\mathcal{V}^+ = \{$$
  clown, killer clown, crazy clown,
  crazy killer clown, killer crazy clown,
  ...
  $$\}$$
- A *sentence* is **defined** as each $s \in \mathcal{V}^+$

The Language Modeling problem

Data

Given a training data set of example sentences $s \in \mathcal{V}^+$

The Language Modeling problem

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Language Modeling problem

Estimate a probability model:

$$\sum_{s \in \mathcal{V}^+} p(s) = 1.0$$

- p(clown) = 1e-5
- p(killer) = 1e-6
- p(killer clown) = 1e-12
- p(crazy killer clown) = 1e-21
- p(crazy killer clown killer) = 1e-110
- p(crazy clown killer killer) = 1e-127

Why do we want to do this?

Scoring Hypotheses in Speech Recognition

From acoustic signal to candidate transcriptions

Hypothesis	Score
the station signs are in deep in english	-14732
the stations signs are in deep in english	-14735
the station signs are in deep into english	-14739
the station 's signs are in deep in english	-14740
the station signs are in deep in the english	-14741
the station signs are indeed in english	-14757
the station 's signs are indeed in english	-14760
the station signs are indians in english	-14790
the station signs are indian in english	-14799
the stations signs are indians in english	-14807
the stations signs are indians and english	-14815

Scoring Hypotheses in Machine Translation

From source language to target language candidates

Hypothesis	Score
we must also discuss a vision .	-29.63
we must also discuss on a vision .	-31.58
it is also discuss a vision .	-31.96
we must discuss on greater vision .	-36.09
⋮	⋮

Scoring Hypotheses in Decryption

Character substitutions on ciphertext to plaintext candidates

Hypothesis	Score
Heopaj, zk ukq swjp pk gjks w oaynap?	-93
Urbcnw, mx hxd fjwc cx twxf j bnlanc?	-92
Wtdepy, oz jzf hlye ez vyzh l dpncpe?	-91
Mjtufo, ep zpv xbou up lopx b tfdsfu?	-89
Nkuvgp, fq aqw ycpv vq mpqy c ugetgv?	-87
Gdnozi, yj tjp rvio oj fijr v nzxmzo?	-86
Czjkve, uf pfl nrek kf befn r jvtivk?	-85
Yvfgra, qb lbh jnag gb xabj n frperg?	-84
Zwghsb, rc mci kobh hc ybck o gsqfsh?	-83
Byijud, te oek mqdj je adem q iushuj?	-77
Jgqrcl, bm wms uylr rm ilmu y qcapcr?	-76
Listen, do you want to know a secret?	-25

Scoring Hypotheses in Spelling Correction

Substitute spelling variants to generate hypotheses

Hypothesis	Score
… stellar and versatile **acress** whose combination of sass and glamour has defined her …	-18920
… stellar and versatile **acres** whose combination of sass and glamour has defined her …	-10209
… stellar and versatile **actress** whose combination of sass and glamour has defined her …	-9801

T9 to English

Grover, King, & Kushler. 1998.
Reduced keyboard disambiguating computer. US Patent 5,818,437

Sequence of numbers to English

Input	Hypothesis	Score
46 04663	GO HOOD	-24
46 04663	GO HOME	-10
843 0746453 06678 07678527 0243373 0460843 096753	?	?

Probability models of language

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- **And** the model should be equal to $\sum_{s \in \mathcal{V}^+} P(s)$.

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- Write down a new model over $\mathcal{V}^+$ such that $P(s \mid \ell)$ is in the model
- **And** the model should be equal to $\sum_{s \in \mathcal{V}^+} P(s)$.
- Write down the model

$$\sum_{s \in \mathcal{V}^+} P(s) = \ldots$$

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Part 2: *n*-grams for Language Modeling

Language models

n-grams for Language Modeling

Handling Unknown Tokens

Smoothing n-gram Models

Interpolation: Jelinek-Mercer Smoothing

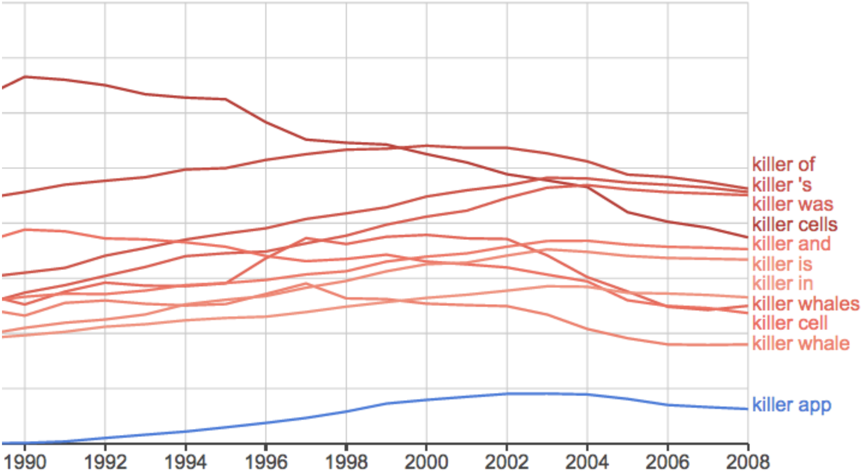
Backoff Smoothing with Discounting

Evaluating Language Models

Event Space for n-gram Models

n-gram Models

Google *n*-gram viewer



Learning Language Models

- Directly count using a training data set of sentences: $w_1, \ldots, w_n$:

$$p(w_1, \ldots, w_n) = \frac{c(w_1, \ldots, w_n)}{N}$$

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- What are the chances you will see a sentence: `crazy killer clown crazy killer`?

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- Problem: does not generalize to new sentences unseen in the training data.
- What are the chances you will see a sentence: `crazy killer clown crazy killer`?
- In NLP applications we often need to assign non-zero probability to previously unseen sentences.

Learning Language Models

Apply the Chain Rule: the unigram model

$$\begin{aligned} p(w_1, \ldots, w_n) &\approx p(w_1)p(w_2)\ldots p(w_n) \\ &= \prod_i p(w_i) \end{aligned}$$

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\end{aligned}
$$

Big problem with a unigram language model

p(the the the the the the the) $>$ p(we must also discuss a vision .)

Learning Language Models

Apply the Chain Rule: the bigram model

$$\begin{aligned} p(w_1, \ldots, w_n) &\approx p(w_1)p(w_2 \mid w_1) \ldots p(w_n \mid w_{n-1}) \\ &= p(w_1) \prod_{i=2}^{n} p(w_i \mid w_{i-1}) \end{aligned}$$

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\end{aligned}
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Better than unigram

p(the the the the the the the) $<$ p(we must also discuss a vision .)

Learning Language Models

Apply the Chain Rule: the trigram model

$$
\begin{aligned}
&p(w_1, \ldots, w_n) \approx \\
&\qquad p(w_1)p(w_2 \mid w_1)p(w_3 \mid w_1, w_2) \ldots p(w_n \mid w_{n-2}, w_{n-1}) \\
&\qquad p(w_1)p(w_2 \mid w_1) \prod_{i=3}^{n} p(w_i \mid w_{i-2}, w_{i-1})
\end{aligned}
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Learning Language Models

Apply the Chain Rule: the trigram model

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\begin{aligned}
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& \qquad p(w_1)p(w_2 \mid w_1) \prod_{i=3}^{n} p(w_i \mid w_{i-2}, w_{i-1})
\end{aligned}
$$

Better than bigram, but . . .

p(we must also discuss a vision .) might be zero because we have not seen p(discuss | must also)

Maximum Likelihood Estimate

Using training data to learn a trigram model

- Let $c(u, v, w)$ be the count of the trigram u, v, w, e.g. $c(crazy, killer, clown)$. $P(u, v, w) = \frac{c(u,v,w)}{\sum_{u,v,w} c(u,v,w)}$

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- For any u, v, w we can compute the conditional probability of generating w given u, v:

$$p(w \mid u, v) = \frac{c(u, v, w)}{c(u, v)}$$

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- Let $c(u, v)$ be the count of the bigram u, v, e.g. $c(crazy, killer)$. $P(u, v) = \frac{c(u,v)}{\sum_{u,v} c(u,v)}$
- For any u, v, w we can compute the conditional probability of generating w given u, v:
$$p(w \mid u, v) = \frac{c(u, v, w)}{c(u, v)}$$
- For example:
$$p(clown \mid crazy, killer) = \frac{c(crazy, killer, clown)}{c(crazy, killer)}$$

Number of Parameters

How many probabilities in each *n*-gram model

- Assume $\mathcal{V} = \{killer, crazy, clown, UNK\}$

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How many unigram probabilities: $P(x)$ for $x \in \mathcal{V}$?

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How many bigram probabilities: $P(y|x)$ for $x, y \in \mathcal{V}$?

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How many bigram probabilities: $P(y|x)$ for $x, y \in \mathcal{V}$?

$4^2 = 16$

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- Assume $\mathcal{V} = \{killer, crazy, clown, UNK\}$

Question

How many trigram probabilities: $P(z|x, y)$ for $x, y, z \in \mathcal{V}$?

$4^3 = 64$

Number of Parameters

Question

- Assume $|\mathcal{V}| = 50{,}000$ (a realistic vocabulary size for English)
- What is the minimum size of training data in tokens?
  - If you wanted to observe all unigrams at least once.
  - If you wanted to observe all trigrams at least once.

Some trigrams should be zero since they do not occur in the language, $P(the \mid the, the)$.
But others are simply unobserved in the training data, $P(idea \mid colourless, green)$.

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125,000,000,000,000 (125 Ttokens)

Some trigrams should be zero since they do not occur in the language, $P(\textit{the} \mid \textit{the}, \textit{the})$.
But others are simply unobserved in the training data, $P(\textit{idea} \mid \textit{colourless}, \textit{green})$.

Handling tokens in test corpus unseen in training corpus

Assume closed vocabulary

In some situations we can make this assumption, e.g. our vocabulary is ASCII characters

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Interpolate with unknown words distribution

We will call this *smoothing*. We combine the *n*-gram probability with a distribution over unknown words

$$P_{\mathrm{unk}}(w) = \frac{1}{V_{\mathrm{all}}}$$

V_{all} is an estimate of the vocabulary size including unknown words.

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$$P_{\mathrm{unk}}(w) = \frac{1}{V_{\mathrm{all}}}$$

V_{all} is an estimate of the vocabulary size including unknown words.

Add an `<unk>` word

Modify the training data L by changing words that appear only once to the `<unk>` token. Since this probability can be an over-estimate we multiply it with a probability $P_{\mathrm{unk}}(\cdot)$.

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Part 3: Smoothing Probability Models

Interpolation: Jelinek-Mercer Smoothing

$$P_{ML}(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i)}{c(w_{i-1})}$$

- $P_{JM}(w_i \mid w_{i-1}) = \lambda P_{ML}(w_i \mid w_{i-1}) + (1 - \lambda) P_{ML}(w_i)$
  where, $0 \leq \lambda \leq 1$

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- Jelinek and Mercer (1980) describe an elegant form of this **interpolation**:

$$P_{JM}(n\text{gram}) = \lambda P_{ML}(n\text{gram}) + (1-\lambda) P_{JM}(n-1\text{gram})$$

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$$P_{JM}(n\text{gram}) = \lambda P_{ML}(n\text{gram}) + (1 - \lambda) P_{JM}(n - 1\text{gram})$$

- What about $P_{JM}(w_i)$?
  For missing unigrams: $P_{JM}(w_i) = \lambda P_{ML}(w_i) + (1 - \lambda)\frac{\delta}{V}$
  $0 < \delta \leq 1$

Interpolation: Finding λ

$$P_{JM}(n\text{gram}) = \lambda P_{ML}(n\text{gram}) + (1-\lambda) P_{JM}(n-1\text{gram})$$

- Deleted Interpolation (Jelinek, Mercer)
  compute λ values to minimize cross-entropy on **held-out** data which is deleted from the initial set of training data

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- Deleted Interpolation (Jelinek, Mercer)
  compute λ values to minimize cross-entropy on **held-out** data which is deleted from the initial set of training data
- Improved JM smoothing, a separate λ for each w_{i-1}:

$$P_{JM}(w_i \mid w_{i-1}) = \lambda(w_{i-1})P_{ML}(w_i \mid w_{i-1}) + (1-\lambda(w_{i-1}))P_{ML}(w_i)$$

Backoff Smoothing with Discounting

- Absolute Discounting (aka *abs*) (Ney, Essen, Kneser)

$$P_{abs}(y \mid x) = \begin{cases} \frac{c(xy)-D}{c(x)} & \text{if } c(xy) > 0 \\ \alpha(x)P(y) & \text{otherwise} \end{cases}$$

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- where $\alpha(x)$ is chosen to make sure that $P_{abs}(y \mid x)$ is a proper probability

$$\alpha(x) = 1 - \sum_{y} \frac{c(xy) - D}{c(x)}$$

Backoff Smoothing with Discounting

x	$c(x)$	$c(x) - D$	$\frac{c(x)-D}{c(the)}$
the	48		
the,dog	15	14.5	14.5/48
the,woman	11	10.5	10.4/48
the,man	10	9.5	9.5/48
the,park	5	4.5	4.5/48
the,job	2	1.5	1.5/48
the,telescope	1	0.5	0.5/48
the,manual	1	0.5	0.5/48
the,afternoon	1	0.5	0.5/48
the,country	1	0.5	0.5/48
the,street	1	0.5	0.5/48
TOTAL			0.8958
the,UNK	0		0.1042

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Part 4: Evaluating Language Models

Evaluating Language Models

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- Let $T = s_0, \ldots, s_m$ be a test corpus with sentences s_i

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- T is assumed to be separate from the training data used to train our language model $P(s)$
- What is $P(T)$?

Evaluating Language Models: Independence assumption

- $T = s_0, \ldots, s_m$ is the text corpus with sentences s_0 through s_m

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- $P(T) = P(s_0) \cdot P(s_1) \cdot P(s_2) \cdot \ldots \cdot P(s_m) = \prod_{i=0}^{m} P(s_i)$

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- $P(s_i) = P(w_0^{(i)}, \ldots, w_{n_i}^{(i)})$ – which can be any n-gram language model
- A language model is better if the value of $P(T)$ is higher for unseen sentences T, we want to maximize:

$$P(T) = \prod_{i=0}^{m} P(s_i)$$

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- M is the total number of tokens in the test corpus T:

$$M = \sum_{i=0}^{m} \text{length}(s_i)$$

Evaluating Language Models: Computing the Average

- However, T can be any arbitrary size
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- Instead of the probability for a given T we can compute the *average* probability.
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$$M = \sum_{i=0}^{m} \text{length}(s_i)$$

- The average *log* probability of the test corpus T is:

$$\frac{1}{M} \log_2 \prod_{i=0}^{m} P(s_i) = \frac{1}{M} \sum_{i=0}^{m} \log_2 P(s_i)$$

Evaluating Language Models: Perplexity

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Evaluating Language Models: Perplexity

- The average *log* probability of the test corpus T is:

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- Note that ℓ is a negative number
- We evaluate a language model using *Perplexity* which is $2^{-\ell}$

Evaluating Language Models

Question

Show that:

$$2^{-\frac{1}{M}\log_2 \prod_{i=0}^{m} P(s_i)} = \frac{1}{\sqrt[M]{\prod_{i=0}^{m} P(s_i)}}$$

Evaluating Language Models

Question

What happens to $2^{-\ell}$ if any n-gram probability for computing $P(T)$ is zero?

Evaluating Language Models: Perplexity

Progress on the 1B Word Benchmark

Model	Params	Perplexity	Citation
unigram	775K	955	Chelba+ 2013
bigram	1B	137	Chelba+ 2013
trigram	1B	74	Chelba+ 2013
interpolated 5-gram	1.76B	67.6	Chelba+ 2013
10skip-gram+SNM	33B	52.9	Shazeer+ 2014
RNN-256 + 9-grams	20B	58.3	Chelba+ 2013
RNN-1024 + 9-grams	20B	51.3	Chelba+ 2013
Big LSTM+CNN	1.04B	30	Jozefowicz+ 2016
10 LSTMs+10skip-SNM	43B	23.7	Jozefowicz+ 2016
GPT2	1.54B	42.16	Radford+ 2019
Transformer XL	1.04B	21.8	Dai+ 2019
OmniNet	100M	21.5	Tay+ 2021

Natural Language Processing

Anoop Sarkar
anoopsarkar.github.io/nlp-class

Simon Fraser University

Part 5: Event space in Language Models

Trigram Models

- The trigram model:
  $P(w_1, w_2, \ldots, w_n) =$
  $P(w_1) \times P(w_2 \mid w_1) \times P(w_3 \mid w_1, w_2) \times P(w_4 \mid w_2, w_3) \times$
  $\ldots P(w_i \mid w_{i-2}, w_{i-1}) \ldots \times P(w_n \mid w_{n-2}, \ldots, w_{n-1})$

Trigram Models

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  $\ldots P(w_i \mid w_{i-2}, w_{i-1}) \ldots \times P(w_n \mid w_{n-2}, \ldots, w_{n-1})$
- Notice that the length of the sentence n is variable
- What is the event space?

The `stop` symbol

- Let $\mathcal{V} = \{a, b\}$ and the language L be $\mathcal{V}^*$

The `stop` symbol

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- So strings in this language L are:

$$\begin{array}{rl} a \text{ stop} & 0.5 \\ b \text{ stop} & 0.5 \\ aa \text{ stop} & 0.5^2 \\ bb \text{ stop} & 0.5^2 \\ & \vdots \end{array}$$

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- The sum over all strings in L should be equal to 1:

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- But $P(a) + P(b) + P(aa) + P(bb) = 1.5$!!

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The stop symbol

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  We need to model variable length sequences
- Add an explicit probability for the stopsymbol:

$$P(a) = P(b) = 0.25$$

$$P(\texttt{stop}) = 0.5$$

The stop symbol

- What went wrong?
  We need to model variable length sequences
- Add an explicit probability for the stopsymbol:

$$P(a) = P(b) = 0.25$$

$$P(\text{stop}) = 0.5$$

- $P(\text{stop}) = 0.5$, $P(a \text{ stop}) = P(b \text{ stop}) = 0.25 \times 0.5 = 0.125$, $P(aa \text{ stop}) = 0.25^2 \times 0.5 = 0.03125$ (now the sum is no longer greater than one)

The `stop` symbol

- With this new `stop` symbol we can show that $\sum_w P(w) = 1$
  Notice that the probability of any sequence of length n is $0.25^n \times 0.5$
  Also there are 2^n sequences of length n

$$\sum_w P(w) =$$

$$\sum_{n=0}^{\infty} 2^n \times 0.25^n \times 0.5$$

$$\sum_{n=0}^{\infty} 0.5^n \times 0.5 = \sum_{n=0}^{\infty} 0.5^{n+1}$$

$$\sum_{n=1}^{\infty} 0.5^n = 1$$

The $\mathtt{stop}$ symbol

- With this new $\mathtt{stop}$ symbol we can show that $\sum_w P(w) = 1$
  Using $p_s = P(\mathtt{stop})$ the probability of any sequence of length n is $p(n) = p(w_1, \ldots, w_{n-1}) \times p_s(w_n)$

$$\begin{aligned}\sum_w P(w) &= \sum_{n=0}^{\infty} p(n) \sum_{w_1,\ldots,w_n} p(w_1, \ldots, w_n) \\ &= \sum_{n=0}^{\infty} p(n) \sum_{w_1,\ldots,w_n} \prod_{i=0}^{n} p(w_i)\end{aligned}$$

$$\sum_{w_1,\ldots,w_n} \prod_i p(w_i) =$$

$$\sum_{w_1} \sum_{w_2} \ldots \sum_{w_n} p(w_1) p(w_2) \ldots p(w_n) = 1$$

The `stop` symbol

$$\sum_{w_1} \sum_{w_2} \ldots \sum_{w_n} p(w_1) p(w_2) \ldots p(w_n) = 1$$

$$\begin{aligned}
\sum_{n=0}^{\infty} p(n) &= \sum_{n=0}^{\infty} p_s (1 - p_s)^n \\
&= p_s \sum_{n=0}^{\infty} (1 - p_s)^n \\
&= p_s \frac{1}{1 - (1 - p_s)} = p_s \frac{1}{p_s} = 1
\end{aligned}$$

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