EXERCISES (iii) - Maths for Biology

Computational Methods in Ecology and Evolution
Imperial College London
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Exercises Integrals

Immediate integrals   Calculate:

$$a) \quad \int \sqrt[m]{x^n}\,dx$$
$$b) \quad \int \tan^2 x\,dx$$

Integral methods (by substitution)   Obtain by substitution the integrals:

$$a) \quad \int \frac{x^3}{2+x^8}\,dx$$
$$b) \quad \int \frac{1}{x \log x}\,dx$$

Integral methods (by parts)   Obtain by parts the integrals:

$$a) \quad \int x^3 \log(x)\,dx$$
$$b) \quad \int e^x \cos(x)\,dx$$

Integral methods (rational functions)   Obtain the following integrals (remember the factorization techniques for rational functions and apply them to obtain immediate integrals):

$$a) \quad \int \frac{7x-6}{x^2+x-6}\,dx$$
$$b) \quad \int \frac{x^2+1}{x^4-x^2}\,dx$$

Applications. Molecular motor work   A molecular motor has an interesting behaviour when it is pulled with some force ($F > 0$ (pN=picoNewtons)) with optical tweezers moving it in a single dimension. It looks that, pulling it in this way, it is able to accumulate potential energy that starts releasing at some point (that we will arbitrarily fix as $x = 0$) performing work against the tweezers ($F < 0$) and still moving along an interval Δx (nanometres). Then it is needed to apply again a force to keep it moving ($F > 0$). This process is well approximated with the force function $F(x) = (x - 1)(x + 2)$. Plot this function and try to understand what is described in the problem. Considering that the molecular motor "walks" from $x = -3$ to $x = 2$, calculate the net work of the process in that pathway and discuss the meaning.

Optional exercises

Functions with no antiderivative   There are some integrals that cannot be solved by analytical methods. Surprisingly, one of this functions is $f(x) = e^{-x^2}$, that you should recognize as it rules most of the statistics that we use, because it is the well-known Gaussian function! Another useful function with no antiderivative is the sinc function $g(x) = \frac{\sin x}{x}$ ($x \neq 0$) that in $x = 0$ is defined as $g(0) = 1$. Look for information about the error function ($\text{erf}(x)$) and the sine integral function ($\text{Si}(x)$) and briefly explain its use and relation with $f(x)$ and $g(x)$. Which is the value of the integral of $f(x)$ in $\mathbb{R}$? Now check the definition of normal distribution and its relation with the Gaussian function. Clue: Wikipedia.

Applications. Hysteresis   In the following paper [1] we read the following:

"An important issue in cell biology is to understand how cells respond in a decisive manner (digital) to the graded (analog) input of increasing amounts of receptor stimulation. Employing synergistic in silico and in vitro methods, we find that signaling in a population of lymphocytes is digital in character; i.e., a bimodal response emerges as stimulus is increased past a threshold. Digital signaling in individual cells requires SOS-mediated Ras activation. A further unanticipated characteristic of Ras activation via SOS is hysteresis in the dose-response curve; i.e., the response to the same stimulus dose depends upon whether the prevailing level of stimulus is achieved by increasing or decreasing the stimulus from its previous value. Our results suggest that bimodal responses and hysteresis also provide a mechanism for short-term molecular "memory", making it easier to activate membrane-proximal signaling in previously stimulated cells. This may enable T lymphocytes to integrate signals from serial encounters with rare antigen-bearing cells."

Although the paper talks about hysteresis systematically and they do simulations and experiments to show that it exists, you will not find a quantification of hysteresis (I mean, a value associated to physical dimensions). Thus, I am still wondering what hysteresis is meant (more precisely) and I would like to ask you here for some help to understand the paper. So, imagine that you only have the data shown in Fig. 6A, and explain how would you proceed to provide an analytical estimation (a number and dimensions obtained from any analytical function that you think would fit well the data) of the hysteresis of the system. Note that you should not do it, you just need to say how would you do it (which function would you propose, etc.)

Applications. The power of termites   We would like to analyse the metabolism of termites and we thought that it would be cool to start with a crazy top-down estimation. Termites build impressive cathedral mounds and, thus, knowing that N individuals were involved in its construction and the physical properties of the mound, we can give an estimation of the work done for each individual on average. The data we have from the cathedral are its density ($\rho = 20 kg/m^3$), its height ($h = 12.5m$) and we will model it as a pyramid with a square base whose side on the floor has length $l = 2.25m$. With these data you should be able to estimate the total work. Clues: As it is a pyramid, the length of the side of its section (which is a square) is smaller at higher heights. Look for the function relating l and h using triangles similarity. Then think a little bit in physics, and estimate the weight of a dh slice of the pyramid... and you should be able to solve it integrating!

Moments   Go to Wikipedia article "Moment (mathematics)" and check the section "Significance of the moments". Then go to the article "Power law distribution" and check the section "Power law probability distribution" and try to understand the problems that these distributions have in the determination of the moments.

References

[1] J. Das, M. Ho, J. Zikherman, C. Govern, M. Yang, A. Weiss, A. K. Chakraborty, and J. P. Roose, "Digital signaling and hysteresis characterize ras activation in lymphoid cells," *Cell*, vol. 136, no. 2, pp. 337–351, 2009.