SURVIVAL TOOLBOX - Maths for Biology

Computational Methods in Ecology and Evolution
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Trigonometric relationships

1. $\sin(x+\frac{\pi}{2}) = \cos x$; $\cos(x+\frac{\pi}{2}) = -\sin x$; $\sin(x-\frac{\pi}{2}) = -\cos x$; $\cos(x-\frac{\pi}{2}) = \sin x$;
2. $$\sin^2 x + \cos^2 x = 1.$$
3. $$\sin(x+y) = \sin x \cos y + \sin y \cos x$$
   (Particularly important is $\sin 2x = 2\sin x \cos x$).
4. $$\cos(x+y) = \cos x \cos y - \sin x \sin y$$
   (Again, when $x = y$ we obtain $\cos(2x) = \cos^2 x - \sin^2 x$).
5. $$\tan(x+y) = \frac{\tan x + \tan y}{\text{1-tanxtany}}$$
   (For $x = y$ leads to $\tan 2x = \frac{2\tan x}{1-\tan^2 x}$).
6. $$\sin x + \sin y = 2\sin\frac{x+y}{2}\cos\frac{x-y}{2}.$$
7. $$\cos x + \cos y = 2\cos\frac{x+y}{2}\cos\frac{x-y}{2}.$$
8. $$\cos x - \cos y = -2\sin\frac{x+y}{2}\sin\frac{x-y}{2}.$$

9.
$$\sin^2 x = \frac{1 - \cos 2x}{2}$$
and
$$\cos^2 x = \frac{1 + \cos 2x}{2}.$$

10.
$$\sin^2 x = \frac{\tan^2 x}{1 + \tan^2 x}$$
and
$$\cos^2 x = \frac{1}{1+\tan^2 x}.$$

11.
$$\sin x = \frac{2\tan(x/2)}{1 + \tan^2(x/2)}$$
and
$$\cos x = \frac{1 - \tan(x/2)}{1+\tan^2(x/2)}.$$

Logarithmic relationships

1. For all $x, y \in \mathbb{R}$
$$\log_2 xy = \log_a x + \log_a y$$
and
$$\log_a x^y = y \log_a x.$$

2. For all $x, y \in \mathbb{R}$ with $y \neq 0$
$$\log_a \frac{x}{y} = \log_a x - \log_a y.$$

3. For all $x, y \in \mathbb{R}$ with $y \neq 0$
$$\log_b x = \frac{\log_a x}{\log_a b}$$
and
$$a^x = b^{x \log_b a}.$$

These relationships are particularly important for the neperian logarithm:

4.
$$\log_b x = \frac{\ln x}{\ln b}$$
and
$$a^x = e^{x \ln a}.$$

Infinitesimal equivalences

The following are equivalent infinitesimal when $x \to 0$:

1. $$\sin x \approx x \approx \arcsin x.$$

2. $$\tan x \approx x \approx \arctan x.$$

3. $$1 - \cos x \approx \frac{x^2}{2}.$$

4. $$\log(1 + x) \approx x.$$

5. $$a^x - 1 \approx x \log a$$

   for $a > 0, a \neq 1$.