

# SURVIVAL TOOLBOX - Maths for Biology

Computational Methods in Ecology and Evolution  
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## Trigonometric relationships

1.  $\sin(x + \frac{\pi}{2}) = \cos x$ ;  $\cos(x + \frac{\pi}{2}) = -\sin x$ ;  $\sin(x - \frac{\pi}{2}) = -\cos x$ ;  $\cos(x - \frac{\pi}{2}) = \sin x$ ;

2.

$$\sin^2 x + \cos^2 x = 1.$$

3.

$$\sin(x + y) = \sin x \cos y + \sin y \cos x$$

(Particularly important is  $\sin 2x = 2 \sin x \cos x$ ).

4.

$$\cos(x + y) = \cos x \cos y - \sin x \sin y$$

(Again, when  $x = y$  we obtain  $\cos(2x) = \cos^2 x - \sin^2 x$ ).

5.

$$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

(For  $x = y$  leads to  $\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$ ).

6.

$$\sin x + \sin y = 2 \sin \frac{x + y}{2} \cos \frac{x - y}{2}.$$

7.

$$\cos x + \cos y = 2 \cos \frac{x + y}{2} \cos \frac{x - y}{2}.$$

8.

$$\cos x - \cos y = -2 \sin \frac{x + y}{2} \sin \frac{x - y}{2}.$$

9.

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$

and

$$\cos^2 x = \frac{1 + \cos 2x}{2}.$$

10.

$$\sin^2 x = \frac{\tan^2 x}{1 + \tan^2 x}$$

and

$$\cos^2 x = \frac{1}{1 + \tan^2 x}.$$

11.

$$\sin x = \frac{2 \tan(x/2)}{1 + \tan^2(x/2)}$$

and

$$\cos x = \frac{1 - \tan(x/2)}{1 + \tan^2(x/2)}.$$

## Logarithmic relationships

1. For all  $x, y \in \mathbb{R}$

$$\log_2 xy = \log_a x + \log_a y$$

and

$$\log_a x^y = y \log_a x.$$

2. For all  $x, y \in \mathbb{R}$  with  $y \neq 0$

$$\log_a \frac{x}{y} = \log_a x - \log_a y.$$

3. For all  $x, y \in \mathbb{R}$  with  $y \neq 0$

$$\log_b x = \frac{\log_a x}{\log_a b}$$

and

$$a^x = b^{x \log_b a}.$$

These relationships are particularly important for the neperian logarithm:

4.

$$\log_b x = \frac{\ln x}{\ln b}$$

and

$$a^x = e^{x \ln a}.$$

## Infinitesimal equivalences

The following are equivalent infinitesimal when  $x \rightarrow 0$  :

1.

$$\sin x \approx x \approx \arcsin x.$$

2.

$$\tan x \approx x \approx \arctan x.$$

3.

$$1 - \cos x \approx \frac{x^2}{2}.$$

4.

$$\log(1 + x) \approx x.$$

5.

$$a^x - 1 \approx x \log a$$

for  $a > 0, a \neq 1$ .