Heuristic

The heuristic for node n is defined as follows:

$$h(n) = \frac{\text{Number of non-green nodes}}{\text{b+1}}.$$

where b is the maximum degree of the graph.

Proof of Consistency and admissibility

Suppose we have the following configuration:

```
      n
     / \
    /   \
  n'------G_n'
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I have decided that the path cost for informed search algorithms is simply the number of clicks so far. Thus, the cost of going from state n to n' using action a is: $c(n, a, n') = 1$
To prove the triangle inequality, we can see that the worst case scenario is when action a on node n causes the most number of red/black nodes to turn green. The maximum number of red/black nodes that can turn green in one action is equal to $b + 1$ where b is the maximum degree of the graph.
Suppose $r(n)$ returns the number of non-green nodes in state n; So in the worst case scenario $r(n') = r(n) - (b + 1)$ then we have:

$$\begin{aligned}
h(n) &\leq c(n, a, n') + h(n') \rightarrow \\
\frac{r(n)}{b+1} &\leq c(n, a, n') + \frac{r(n')}{b+1} \rightarrow \\
\frac{r(n)}{b+1} &\leq 1 + \frac{r(n) - (b+1)}{b+1} \rightarrow \\
\frac{r(n)}{b+1} &\leq 1 + \frac{r(n)}{b+1} - \frac{b+1}{b+1} \rightarrow \\
\frac{r(n)}{b+1} &\leq 1 + \frac{r(n)}{b+1} - 1 \rightarrow \\
\frac{r(n)}{b+1} &\leq \frac{r(n)}{b+1} \text{ This is a true statement, thus the initial hypothesis is correct..}
\end{aligned}$$

Since the heuristic function is consistent, it must be admissible as well.