

Team notebook

Pontificia Universidad Catolica del Peru - Treevial

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1 Data Structures

1.1 2D Segment Tree

```

// Iterative, 2d, commutative segment tree.
// Read into t[n,2*n]*[m,2*m], call build().
typedef int T;
const int MAX = 1e3;
const T OpId = 0; // Identity element
// Associative, commutative operation
T Op(T val1, T val2){
    return val1 + val2;
}
T t[2*MAX][2*MAX];
int n, m;

void build(){
    for( int i = n-1 ; i > 0 ; i-- )
        for ( int j = 2*m ; j >= m ; j-- )
            t[i][j] = Op(t[i<<1][j],
                        t[i<<1|1][j]);
    for ( int i = 2*n ; i > 0 ; i-- )
        for ( int j = m-1 ; j > 0 ; j-- )
            t[i][j] = Op(t[i][j<<1],
                        t[i][j<<1|1])
}

void modify( int x, int y ){
    int q = y;
    while (q >= 1)
        t[x][q] = Op(t[x][q<<1],
                    t[x][q<<1|1]);
}

void modify( int x , int y , T val ){
    t[x+=n][y+=m] = val;
    modify(x,y);
    while (x >= 1) {
        t[x][y] = Op(t[x<<1][y],
                    t[x<<1|1][y]);
        modify(x, y);
    }
}

```

```

T gety(int x, int yl, int yr){
    T ans = OpId;
    for ( yl += m, yr += m ; yl < yr ; yl >=
        1, yr >= 1){
        if (yl&1) ans = Op(ans, t[x][yl++]);
        if (yr&1) ans = Op(ans, t[x][--yr]);
    }
    return ans;
}

T get(int xl, int xr, int yl, int yr){ //
    [xl,xr]*[yl,yr)
    T ans = OpId;
    for ( xl += n, xr += n ; xl < xr ; xl >=
        1, xr >= 1){
        if (xl&1) ans = Op(ans, gety(xl++,
            yl, yr));
        if (xr&1) ans = Op(ans, gety(--xr,
            yl, yr));
    }
    return ans;
}

```

1.2 Block Cut Tree

```

/* TAP Police Query */

#include <sys/resource.h>
#define N 1010000
#define LGMAX 25

/* Start Block Cut Tree */

int P[N][LGMAX];
vector<ii> st;
int n,m,gid,art[N],num[N],low[N],par[N],idBct[N];
vector<int> adj[N];
vector<int> bct[N]; // bct = block cut tree
vector< pair<ii,int> > bridges;
vector< vector<ii> > bcc;

```

```

inline ii edge(int &a, int &b){ return
    ii(min(a,b),max(a,b)); }

void Tarjan(int u, bool root = false){
    num[u] = low[u] = gid++;
    int child = 0;
    for(int v : adj[u]) if(v != par[u]){
        if(num[v] == -1){
            child++;
            par[v] = u;
            st.push_back(make_pair(u,v));
            Tarjan(v);
            low[u] = min(low[u],low[v]);
            if(low[v] > num[u])
                bridges.push_back(make_pair(edge(u,v),0));
            if((root and child > 1) or (!root and
                low[v] >= num[u])){
                // punto de articulacion
                art[u] = 1;
                vector<ii> tmp;
                while(st.back() != make_pair(u,v))
                    tmp.push_back(st.back()),
                    st.pop_back();
                tmp.push_back(st.back()),
                st.pop_back();
                bcc.push_back(tmp);
            }
        }else if(num[v] < num[u]){
            low[u] = min(low[u],num[v]);
            st.push_back(make_pair(u,v));
        }
    }
}

void BuildBlockCut(){
    int cnt = 0;
    for(vector<ii> &vec : bcc){
        int aux = cnt++;
        if(vec.size() == 1){
            (*lower_bound(bridges.begin(),bridges.end(),
                make_pair(edge(vec[0].first,vec[0].second),-1)))
                = aux;
        }
    }
}

```

```

set<int> s;
for(ii &p : vec){
    int a = p.first, b = p.second;
    if(art[a]){
        if(idBct[a] == -1){
            s.insert(a);
            idBct[a] = cnt++;
            bct[aux].push_back(idBct[a]);
            bct[idBct[a]].push_back(aux);
        }else if(!s.count(a)){
            s.insert(a);
            bct[aux].push_back(idBct[a]);
            bct[idBct[a]].push_back(aux);
        }
    }else idBct[a] = aux;
    if(art[b]){
        if(idBct[b] == -1){
            s.insert(b);
            idBct[b] = cnt++;
            bct[aux].push_back(idBct[b]);
            bct[idBct[b]].push_back(aux);
        }else if(!s.count(b)){
            s.insert(b);
            bct[aux].push_back(idBct[b]);
            bct[idBct[b]].push_back(aux);
        }
    }else idBct[b] = aux;
}

void Go(){
    // Tarjan
    gid = 0;
    for(int i = 0; i < n; ++i)
        art[i] = 0, par[i] = num[i] = low[i] = -1;

    for(int i = 0; i < n; ++i) if(num[i] == -1){
        Tarjan(i,true);
        if(st.size() > 0) bcc.push_back(st);
        st.clear();
    }

    sort(bridges.begin(),bridges.end());
}

```

```

// construir block-cut tree
memset(idBct,-1,sizeof(idBct));
BuildBlockCut();
}

/* End Block Cut Tree */

int tin[N],tout[N],dep[N];

bool ancestor(int a, int b){
    return tin[b] >= tin[a] and tout[b] <= tout[a];
}

int __lca(int a, int b){
    if(dep[a] > dep[b]) swap(a,b);
    for(int j = LGMAX-1; j >= 0; --j)
        if(dep[b] - (1<<j) >= dep[a])
            b = P[b][j];
    if(a == b) return a;
    for(int j = LGMAX-1; j >= 0; --j)
        if(P[a][j] != P[b][j])
            a = P[a][j], b = P[b][j];
    return P[a][0];
}

void dfs(int u, int p = -1, int d = 0){
    tin[u] = gid++;
    P[u][0] = p;
    dep[u] = d;
    for(int j = 1; j < LGMAX; ++j)
        if(P[u][j-1] != -1)
            P[u][j] = P[P[u][j-1]][j-1];
    for(int v : bct[u]) if(v != p)
        dfs(v,u,d+1);
    tout[u] = gid++;
}

bool isBridge(int a, int b){
    vector< pair<ii,int> >::iterator it =
        lower_bound(bridges.begin(),
            bridges.end(),make_pair(edge(a,b),-1));
    if(it == bridges.end()) return false;
    return (*it).first == edge(a,b);
}

```

```

bool isLeaf(int a, int b){
    return adj[a].size() == 1 or adj[b].size() ==
        1;
}

bool onPath(int a, int c, int b){
    return ancestor(a,c) and ancestor(c,b);
}

int getBridgeId(int a, int b){
    return (*lower_bound(bridges.begin(),
        bridges.end(),make_pair(edge(a,b),-1))).second;
}

int main(){
    int q,t,a,b,c,d;
    scanf("%d %d",&n,&m);
    for(int i = 0; i < m; ++i){
        scanf("%d %d",&a,&b);
        // indexar en cero
        a--, b--;
        adj[a].push_back(b);
        adj[b].push_back(a);
    }

    Go();

    gid = 0;
    dfs(0);
    /* Responder queries */
    scanf("%d",&q);
    while(q--){
        scanf("%d",&t);
        if(t == 1){
            scanf("%d %d %d %d",&a,&b,&c,&d);
            a--; b--; c--; d--;
            if(isBridge(c,d)){
                if(isLeaf(c,d)){
                    if( (adj[c].size() == 1) and (a == c
                        or b == c) )
                        printf("ne\n");
                    else if( (adj[d].size() == 1) and (a
                        == d or b == d) )

```

```

                        printf("ne\n");
                    else printf("da\n");
                }else{
                    int u = idBct[a], v = idBct[b];
                    int w = getBridgeId(c,d), lca =
                        __lca(u,v);
                    if(onPath(lca,w,u) or
                        onPath(lca,w,v)) printf("ne\n");
                    else printf("da\n");
                }
            }else printf("da\n");
        }else{
            scanf("%d %d %d",&a,&b,&c);
            a--; b--; c--;
            if(art[c]){
                int u = idBct[a], v = idBct[b], w =
                    idBct[c], lca = __lca(u,v);
                if(onPath(lca,w,u) or onPath(lca,w,v))
                    printf("ne\n");
                else printf("da\n");
            }else printf("da\n");
        }
    }

    return 0;
}

```

1.3 Dynamic Hash

```

/* Dynamic Hash
*/

ll len1[N], len2[N];

struct Hash{
    vi dp1, dp2;
    string s;
    ll ans1, ans2, n;
    inline void build(ll id){
        if(!id) ans1=0, ans2=0;
        n=sz(s);
        FER(i, id, n) dp1.pb(0), dp2.pb(0);

```

```

        FER(i,id,sz(s)){
            if(s[i]>='a' && s[i]<='z'){
                ans1=(ans1*bas)%mod1;
                ans1+=(s[i]-'a'+1);
                ans2=(ans2*bas)%mod2;
                ans2+=(s[i]-'a'+1);
                if(ans1>=mod1) ans1-=mod1;
                if(ans2>=mod2) ans2-=mod2;
                dp1[i]=ans1, dp2[i]=ans2;
            }
            else if(s[i]>='A' && s[i]<='Z'){
                ans1=(ans1*bas)%mod1;
                ans1+=(s[i]-'A'+27);
                ans2=(ans2*bas)%mod2;
                ans2+=(s[i]-'A'+27);
                if(ans1>=mod1) ans1-=mod1;
                if(ans2>=mod2) ans2-=mod2;
                dp1[i]=ans1, dp2[i]=ans2;
            }
            else{
                ans1=(ans1*bas)%mod1;
                ans1+=(s[i]-'0'+55);
                ans2=(ans2*bas)%mod2;
                ans2+=(s[i]-'0'+5);
                if(ans1>=mod1) ans1-=mod1;
                if(ans2>=mod2) ans2-=mod2;
                dp1[i]=ans1, dp2[i]=ans2;
            }
        }
    }
}

inline ii Query(ll l, ll r){
    ll ans1=dp1[r-1], ans2=dp2[r-1];
    if(l){
        ll add1=(dp1[l-1]*len1[r-1])%mod1;
        ll add2=(dp2[l-1]*len2[r-1])%mod2;
        ans1-=add1, ans2-=add2;
        if(ans1<0) ans1+=mod1;
        if(ans2<0) ans2+=mod2;
        return {ans1, ans2};
    }
    return {ans1, ans2};
}
};

```

```
int main(){
    fastio;
    ll t1=1, t2=1;
    FER(i,0,N) {
        len1[i]=t1;
        len2[i]=t2;
        t1=(t1*bas)%mod1;
        t2=(t2*bas)%mod2;
    }
    return 0;
}
```

1.4 Dynamic Segment Tree Beats

/* Dynamic Segment Tree Beats */

```
struct T{
    ll fmx, fmn, smx, smn;
    T(){}
    T(ll fmx, ll fmn, ll smx, ll smn):
        fmx(fmx), fmn(fmn), smx(smx), smn(smn){}

    inline T Op(T a, T b){
        if(a.fmx < b.fmx){
            a.smx=a.fmx;
            a.fmx=b.fmx;
        }
        if(a.fmx == b.fmx){
            a.smx=max(a.smx, b.smx);
        }
        else a.smx=max(a.smx, b.fmx);
        if(a.fmn > b.fmn){
            a.smn=a.fmn;
            a.fmn=b.fmn;
        }
        if(a.fmn == b.fmn){
            a.smn=min(a.smn, b.smn);
        }
        else a.smn=min(a.smn, b.fmn);
        return a;
    }
    inline void Umax(ll x){
```

```
        if(fmx==fmn) fmx=x;
        if(smx==fmn) smx=x;
        fmn=x;
    }
    inline void Umin(ll x){
        if(fmn==fmx) fmn=x;
        if(smn==fmx) smn=x;
        fmx=x;
    }
};

struct Dinamic_Segment_Tree_Beats{
    ll n, nodes, root;
    T lazy[N];
    ll L[N], R[N];

    inline void updpro(T val, ll id, ll l, ll r){
        if(val.fmx<lazy[id].fmx){
            lazy[id].Umin(val.fmx);
        }
        if(val.fmn>lazy[id].fmn){
            lazy[id].Umax(val.fmn);
        }
    }

    inline void newLeaf(T val, ll id, ll l, ll r){
        lazy[id]=val;
    }

    inline void proh(ll id, ll l, ll r){
        ll mid=(l+r)>>1;
        if(l+1==r) return ;
        if(!L[id]){
            L[id]=nodes++;
            newLeaf(lazy[id], L[id], l, mid);
        }
        else updpro(lazy[id], L[id], l, mid);
        if(!R[id]){
            R[id]=nodes++;
            newLeaf(lazy[id], R[id], mid, r);
        }
        else updpro(lazy[id], R[id], mid, r);
    }
}
```

```
inline void UpdMax(ll x, ll y, ll val, ll &id,
    ll l, ll r){
    if(!id){
        id=nodes;
        L[id]=R[id]=0;
        nodes++;
    }
    proh(id, l, r);
    if(x>=r || y<=l || lazy[id].fmn>=val) return;
    if(x<=l && r<=y && lazy[id].smn>val &&
        lazy[id].fmx<val) {
        lazy[id].Umax(val);
        return;
    }
    ll mid=(l+r)>>1;
    if(x<mid) UpdMax(x, y, val, L[id], l, mid);
    if(y>mid) UpdMax(x, y, val, R[id], mid, r);
    lazy[id]=lazy[id].Op(lazy[L[id]],
        lazy[R[id]]);
}

inline ll Sol(ll x, ll y, ll id, ll l, ll r){
    proh(id, l, r);
    if(x>=r || y<=l) return -INF;
    if(x<=l && r<=y) return lazy[id].fmx;
    ll mid=(l+r)>>1, left, right;
    left=Sol(x, y, L[id], l, mid);
    right=Sol(x, y, R[id], mid, r);
    return max(left, right);
}

inline void build() {
    nodes=1, root=0;
    FER(i, 1, 2){
        lazy[i].fmx=lazy[i].smx=-INF;
        lazy[i].fmn=lazy[i].smn=INF;
        lazy[i].fmx=lazy[i].fmn=-1;
    }
}

inline void updMax(ll x, ll y, ll val) {
    UpdMax(x, y, val, root, 1, n);}
```

```

    inline ll que(ll x, ll y) { return Sol(x, y,
        root, 1, n);}
}st;

int main(){
    fastio;
    ll n, q; cin>>n>>q;
    st.n=n+1;
    st.build();
    st.updMax(1, st.n, 0);
    ll l, r, x, t;
    FER(i, 0, q){
        cin>>t;
        if(t==1){
            cin>>l>>r>>x;
            st.updMax(1, r+1, x);
        }
        else {
            cin>>l>>r;
            x=st.que(1, r+1);
            cout<<x<<endl;
        }
    }
    return 0;
}

```

1.5 Heavy Light Decomposition

// works over a segment tree st

```

struct HLDES{
    ll n, gid;
    ll tsz[1<<17], rt[1<<17], id[1<<17],
        idb[1<<17], d[1<<17], p[1<<17],
        val[1<<17];
    vii adj[1<<17];
    vi v[1<<17];
    inline void dfs(ll u, ll pp, ll depth, ll w){
        tsz[u]=1, d[u]=depth, p[u]=pp, val[u]=w;
        for(auto xd: adj[u]) if(xd.ff!=pp)
            dfs(xd.ff, u, depth+1, xd.ss),
            tsz[u]+=tsz[xd.ff];
    }
}

```

```

}
inline void go(ll u, ll root){
    rt[u]=root, id[u]=gid++;
    ll w=-1, bc=0;
    for(auto xd: adj[u]) if(xd.ff!=p[u] and
        tsz[xd.ff]>w) w=tsz[xd.ff], bc=xd.ff;
    if(bc) go(bc, root);
    for(auto xd: adj[u]) if(xd.ff!=p[u] and
        xd.ff!=bc) go(xd.ff, xd.ff);
}

inline void goback(){
    FER(i, 0, n) v[rt[i]].pb(i);
    FER(i, 0, n) if(sz(v[i])) for(auto xd:
        v[i]){
            idb[xd]=id[i]+sz(v[i])-(d[xd]-d[i])-1;
        }
}

inline void upd(ll a, ll b){
    ll l, r;
    while(rt[a]!=rt[b]){
        if(d[rt[a]]>d[rt[b]]) swap(a, b);
        l=id[rt[b]], r=id[b]+1;
        st.upd(1, r, 1);
        b=p[rt[b]];
    }
    if(d[a]>d[b]) swap(a, b);
    l=id[a]+1, r=id[b]+1;
    st.upd(1, r, 1);
}

inline void build(){
    st.n=n, gid=0;
    dfs(0, -1, 0, 0);
    go(0, 0);
    goback();
    FER(i, 0, n) st.ar[id[i]]=val[i];
    st.build();
}
}hld;

```

1.6 LCA Tree

```

vi adj[1<<17];
ll p[1<<17][18], be[1<<17], en[1<<17], pos;

inline void dfs(ll u, ll pp){
    p[u][0]=pp, be[u]=pos++;
    FER(i, 1, 18) if(p[u][i-1]!=-1)
        p[u][i]=p[p[u][i-1]][i-1];
    for(auto xd: adj[u]) if(xd!=pp) dfs(xd, u);
    en[u]=pos++;
}

inline bool test(ll a, ll b){
    if(a==-1) return true;
    return be[a]<=be[b] && en[a]>=en[b];
}

inline ll lca(ll a, ll b){
    if(test(a, b)) return a;
    if(test(b, a)) return b;
    IFR(i, 17, 0) if(!test(p[a][i], b)) a=p[a][i];
    return p[a][0];
}

inline void build() {fill(p, -1), dfs(0, -1);}

```

1.7 Lazy Segment Tree

```

struct ST{
    ll n, lazy[1<<19], ar[1<<19];
    ii tree[1<<19];

    inline ii Op(ii &val1, ii &val2) { return
        val1.ff>val2.ff? val1: val2;}
    inline void updpro(ll laz, ll id, ll l, ll
        r) {
        if(laz) {
            tree[id].ff+=laz;
            lazy[id]+=laz;
        }
    }
}

```

```

}
inline void proh(ll id, ll l, ll r){
    ll mid=(l+r)>>1;
    updpro(lazy[id], id<<1, l, mid);
    updpro(lazy[id], id<<1|1, mid, r);
    lazy[id]=0;
}
inline void Upd(ll x, ll y, ll val, ll id,
    ll l, ll r) {
    if(x>=r or y<=l) return ;
    if(x<=l and r<=y) {
        updpro(val, id, l, r);
        return;
    }
    proh(id, l, r) ;
    ll mid=(l+r)>>1;
    Upd(x, y, val, id<<1, l, mid);
    Upd(x, y, val, id<<1|1, mid, r);
    tree[id]=Op(tree[id<<1],
        tree[id<<1|1]);
}
inline ii Que(ll x, ll y, ll id, ll l, ll
    r) {
    if(x>=r or y<=l) return ii{-INF,
        -1};
    if(x<=l and r<=y) return tree[id];
    proh(id, l, r);
    ll mid=(l+r)>>1;
    ii left, right;
    left=Que(x, y, id<<1, l, mid);
    right=Que(x, y, id<<1|1, mid, r);
    return Op(left, right);
}
inline void Build(ll id, ll l, ll r) {
    if(l+1==r) {
        tree[id]=ii{ar[l], l};
        return ;
    }
    ll mid=(l+r)>>1;
    Build(id<<1, l, mid),
        Build(id<<1|1, mid, r) ;
    tree[id]=Op(tree[id<<1],
        tree[id<<1|1]);
}

```

```

inline void build() { fill(lazy, 0),
    Build(1, 0, n);}
inline void upd(ll x, ll y, ll val) {
    Upd(x, y, val, 1, 0, n);}
inline ii que(ll x, ll y) { return Que(x,
    y, 1, 0, n);}
}st;

```

1.8 Link Cut Tree

```

//Link cut tree

const int N = 1e5 + 2;

struct Node {
    Node *left, *right, *parent;
    bool revert;
    Node() : left(0), right(0), parent(0),
        revert(false) {}
    bool isRoot() {
        return parent == NULL ||
            (parent->left != this && parent->right
                != this);
    }
    void push() {
        if (revert) {
            revert = false;
            Node *t = left;
            left = right;
            right = t;
            if (left != NULL) left->revert =
                !left->revert;
            if (right != NULL) right->revert =
                !right->revert;
        }
    }
};

struct LinkCutTree{
    Node nos[N];

    LinkCutTree(){

```

```

        FER(i,0,N) nos[i] = Node();
    }

    void connect(Node *ch, Node *p, bool
        isLeftChild) {
        if (ch != NULL) ch->parent = p;
        if (isLeftChild) p->left = ch;
        else p->right = ch;
    }

    void rotate(Node *x){
        Node* p = x->parent;
        Node* g = p->parent;
        bool isRoot = p->isRoot();
        bool leftChild = x == p->left;

        connect(leftChild ? x->right : x->left, p,
            leftChild);
        connect(p, x, !leftChild);
        if (!isRoot) connect(x, g, p == g->left);
        else x->parent = g;
    }

    void splay(Node *x){
        while (!x->isRoot()) {
            Node *p = x->parent;
            Node *g = p->parent;
            if (!p->isRoot()) g->push();
            p->push();
            x->push();
            if (!p->isRoot()) {
                rotate((x == p->left) == (p ==
                    g->left) ? p : x);
            }
            rotate(x);
        }
        x->push();
    }

    Node *expose(Node *x) {
        Node *last = NULL, *y;
        for (y = x; y != NULL; y = y->parent) {
            splay(y);
            y->left = last;

```

```

        last = y;
    }
    splay(x);
    return last;
}

void makeRoot(Node *x) {
    expose(x);
    x->revert = !x->revert;
}

bool connected(Node *x, Node *y) {
    if (x == y) return true;
    expose(x);
    expose(y);
    return x->parent != NULL;
}

bool link(Node *x, Node *y) {
    if (connected(x, y)) return false;
    makeRoot(x);
    x->parent = y;
    return true;
}

bool cut(Node *x, Node *y) {
    makeRoot(x);
    expose(y);
    if (y->right != x || x->left != NULL ||
        x->right != NULL)
        return false;
    y->right->parent = NULL;
    y->right = NULL;
    return true;
}
};

```

1.9 Persistent Lazy Segment Tree Beats

```

struct T{

```

```

    ll fmx, fmn, smx, smn, cmx, cmn;
    ll sum, lval, add;
    T(){}
    T(ll fmx, ll fmn, ll smx, ll smn, ll cmx, ll
        cmn, ll sum):
        fmx(fmx), fmn(fmn), smx(smx), smn(smn),
        cmx(cmx), cmn(cmn), sum(sum){}
    inline void clear(){
        lval=INF, add=0;
    }
    inline T Op(T a, T b){
        if(a.fmx<b.fmx){
            a.cmx=0;
            a.smx=a.fmx;
            a.fmx=b.fmx;
        }
        if(a.fmx==b.fmx){
            a.cmx+=b.cmx;
            a.smx=max(a.smx, b.smx);
        }
        else a.smx=max(a.smx, b.fmx);
        if(a.fmn>b.fmn){
            a.cmn=0;
            a.smn=a.fmn;
            a.fmn=b.fmn;
        }
        if(a.fmn==b.fmn){
            a.cmn+=b.cmn;
            a.smn=min(a.smn, b.smn);
        }
        else a.smn=min(a.smn, b.fmn);
        a.sum+=b.sum;
        a.clear();
        return a;
    }
    inline void UMax(ll x){
        sum+=(x-fmn)*cmn;
        if(fmx==fmn) fmx=x;
        if(smx==fmn) smx=x;
        fmn=x;
        if(lval!=INF && lval<x) lval=x;
    }
    inline void Umin(ll x){
        sum+=(x-fmx)*cmx;

```

```

        if(fmn==fmx) fmn=x;
        if(smn==fmx) smn=x;
        fmx=x;
        if(lval!=INF && lval>x) lval=x;
    }
};

struct PLSTB{
    ll n;
    vi L, R, ar;
    vector<T> lazy;
    vector<bool> IsLazy;
    inline void updpro(T val , ll id, ll l, ll r){
        if(val.fmx<lazy[id].fmx){
            lazy[id].Umin(val.fmx);
        }
        if(val.fmn>lazy[id].fmn){
            lazy[id].UMax(val.fmn);
        }
    }
    inline void upddadd(ll val, ll id, ll l, ll r){
        lazy[id].fmx+=val, lazy[id].fmn+=val;
        if(lazy[id].smx!=-INF) lazy[id].smx+=val;
        if(lazy[id].smn!=-INF) lazy[id].smn+=val;
        lazy[id].sum+=val*(r-l);
        if(lazy[id].lval!=INF) lazy[id].lval+=val;
        else lazy[id].add+=val;
    }
    inline void updput(ll val ,ll id, ll l, ll r){
        lazy[id].fmx=val, lazy[id].fmn=val;
        lazy[id].smx=-INF, lazy[id].smn=INF;
        lazy[id].cmx=r-l;
        lazy[id].cmn=r-l;
        lazy[id].sum=val*(r-l);
        lazy[id].lval=val;
        lazy[id].add=0LL;
    }
    inline ll newLazyCre(ll id, ll l, ll r){
        L.pb(L[id]), R.pb(R[id]);

```



```

    lazy.pb(lazy[id]), IsLazy.pb(true);
    return sz(lazy)-1;
}

inline ll newParent(ll l, ll r){
    L.pb(l), R.pb(r), IsLazy.pb(false);
    lazy.pb(lazy[0].Op(lazy[l], lazy[r]));
    return sz(lazy)-1;
}

inline ll newLeaf(ll val){
    ll p=sz(lazy);
    lazy.pb(T{val, val, -INF, INF, 1, 1, val});
    IsLazy.pb(false), L.pb(0), R.pb(0);
    lazy[p].clear();
    return p;
}

inline void proh(ll id, ll l, ll r){
    if(IsLazy[id]){
        if(l>=r) return;
        ll mid=(l+r)>>1;
        L[id]=newLazyCre(L[id], l, mid);
        R[id]=newLazyCre(R[id], mid, r);
        if(lazy[id].lval!=INF){
            updput(lazy[id].lval, L[id], l,
                mid);
            updput(lazy[id].lval, R[id], mid,
                r);
            lazy[id].lval=INF;
            IsLazy[id]=false;
            return;
        }
        if(lazy[id].add){
            updadd(lazy[id].add, L[id], l, mid);
            updadd(lazy[id].add, R[id], mid, r);
            lazy[id].add=0LL;
        }
        updpro(lazy[id], L[id], l, mid);
        updpro(lazy[id], R[id], mid, r);
        IsLazy[id]=false;
    }
}

```

```

inline ll UpdMin(ll x, ll y, ll val, ll id,
    ll l, ll r){
    if(x>=r or y<=l or lazy[id].fmx<=val)
        return id;
    if(x<=l and r<=y and lazy[id].smx<val and
        lazy[id].fmx>val){
        ll idx=newLazyCre(id, l, r);
        lazy[idx].Umin(val);
        return idx;
    }
    proh(id, l, r);
    ll mid=(l+r)>>1, left, right;
    left=UpdMin(x, y, val, L[id], l, mid);
    right=UpdMin(x, y, val, R[id], mid, r);
    return newParent(left, right);
}

inline ll UpdMax(ll x, ll y, ll val, ll id,
    ll l, ll r){
    if(x>=r or y<=l or lazy[id].fmn>=val)
        return id;
    if(x<=l and r<=y and lazy[id].smn > val
        and lazy[id].fmn < val){
        ll idx=newLazyCre(id, l, r);
        lazy[idx].UMax(val);
        return idx;
    }
    proh(id, l, r);
    ll mid=(l+r)>>1, left, right;
    left=UpdMax(x, y, val, L[id], l, mid);
    right=UpdMax(x, y, val, R[id], mid, r);
    return newParent(left, right);
}

inline ll UpdQue(ll x, ll y, ll val, ll id,
    ll l, ll r){
    if(x>=r or y<=l) return id;
    if(x<=l and r<=y){
        ll idx=newLazyCre(id, l, r);
        updadd(val, idx, l, r);
        return idx;
    }
    proh(id, l, r);
    ll mid=(l+r)>>1, left, right;

```

```

    left=UpdQue(x, y, val, L[id], l, mid);
    right=UpdQue(x, y, val, R[id], mid, r);
    return newParent(left, right);
}

inline ll UpdPut(ll x, ll y, ll val, ll id,
    ll l, ll r){
    if(x>=r or y<=l) return id;
    if(x<=l and r<=y) {
        ll idx=newLazyCre(id, l, r);
        updput(val, idx, l, r);
        return idx;
    }
    proh(id, l, r);
    ll mid=(l+r)>>1, left, right;
    left=UpdPut(x, y, val, L[id], l, mid);
    right=UpdPut(x, y, val, R[id], mid, r);
    return newParent(left, right);
}

inline ll Query(ll x, ll y, ll id, ll l, ll
    r){
    if(x>=r or y<=l) return 0LL;
    if(x<=l and r<=y) return lazy[id].sum;
    proh(id, l, r);
    ll mid=(l+r)>>1, left, right;
    left=Query(x, y, L[id], l, mid);
    right=Query(x, y, R[id], mid, r);
    return left+right;
}

inline ll Build(ll l, ll r){
    ll mid=(l+r)>>1;
    if(l+1==r) return newLeaf(ar[l]);
    ll left=Build(l, mid), right=Build(mid, r);
    return newParent(left, right);
}

inline ll que(ll x, ll y, ll root) { return
    Query(x, y, root, 0, n);}
inline ll updmax(ll x, ll y, ll val, ll root)
    { return UpdMax(x, y, val, root, 0, n);}
inline ll updmin(ll x, ll y, ll val, ll root)
    { return UpdMin(x, y, val, root, 0, n);}

```

```

inline ll updA(ll x, ll y, ll val, ll root) {
    return UpdQue(x, y, val, root, 0, n);}
inline ll updP(ll x, ll y, ll val, ll root) {
    return UpdPut(x, y, val, root, 0, n);}
inline ll build() { n=sz(ar); return Build(0,
n);}

}st;

int main(){
    fastio;
    ll n, q; cin>>n>>q;
    FER(i,0,n) {
        ll x; cin>>x;
        st.ar.pb(x);
    }
    ll root=st.build();
    ll l, r, x, t;
    FER(i,0,q){
        cin>>t>>l>>r; l--;
        if(t==1){
            cin>>x;
            root=st.updmin(l, r, x, root);
        }
        else if(t==2){
            cin>>x;
            root=st.updmax(l, r, x, root);
        }
        else if(t==3){
            cin>>x;
            root=st.updA(l, r, x, root);
        }
        else if(t==4){
            cin>>x;
            root=st.updP(l, r, x, root);
        }
        else{
            x=st.que(l, r, root);
            cout<<x<<endl;
        }
    }
    return 0;
}

```

1.10 Persistent Lazy Segment Tree

// Persistent Lazy Segment Tree for a hash value
// All implementation are based on The clasical
problem

```

struct T{
    ll a, b, t, s;
    T(){
        T(ll a, ll b, ll t, ll s): a(a), b(b), t(t),
            s(s){}
        inline void clear(){
            a=b=t=s=0;
        }
    };

struct PLST{
    ll n;
    vi L, R, lazy, ar;
    vector<bool> IsLazy;
    vector<T> tree;

    inline T Op(T val1, T val2){
        T ty;
        ty.t=val1.t+val2.t;
        ty.s=val1.s+val2.s;
        ty.a=(val1.a*dp1[val2.t])%mod1;
        ty.a+=val2.a;
        ty.b=(val1.b*dp2[val2.t])%mod2;
        ty.b+=val2.b;
        if(ty.a>=mod1) ty.a-=mod1;
        if(ty.b>=mod2) ty.b-=mod2;
        return ty;
    }

    inline ll newLazy(ll id, ll add, ll l, ll r){
        L.pb(L[id]), R.pb(R[id]), lazy.pb(add),
            IsLazy.pb(true);
        ll p=sz(lazy)-1;
        if(add==INF){

```

```

            tree.pb(tree[id]);
            lazy[p]=lazy[id];
            IsLazy[p]=false;
            return p;
        }
        T cur;
        cur.t=r-l, cur.s=(r-l)*add;
        if(add==0) cur.a=ln1[r-l], cur.b=ln2[r-l];
        if(add==1) cur.a=ln1[r-l],
            cur.b=ln2[r-l];
        tree.pb(cur);
        return p;
    }

    inline ll newParent(ll l, ll r){
        L.pb(l), R.pb(r), tree.pb(Op(tree[l],
            tree[r]));
        lazy.pb(INF), IsLazy.pb(false);
        return sz(tree)-1;
    }

    inline ll newLeaf(T val){
        L.pb(0), R.pb(0), tree.pb(val);
        lazy.pb(INF), IsLazy.pb(false);
        return sz(tree)-1;
    }

    inline void proh(ll id, ll l, ll r){
        if(IsLazy[id]){
            if(l>=r) return;
            ll mid=(l+r)>>1;
            L[id]=newLazy(L[id], lazy[id], l, mid);
            R[id]=newLazy(R[id], lazy[id], mid, r);
            IsLazy[id]=false;
            lazy[id]=INF;
            return;
        }
    }

    inline ll updR(ll x, ll y, ll add, ll id, ll
        l, ll r){
        if(x>=r || y<=l) return id;
        if(x<=l && r<=y) return newLazy(id, add,
            l, r);

```

```

    proh(id, l, r);
    ll mid=(l+r)>>1;
    ll left=updR(x, y, add, L[id], l, mid),
        right=updR(x, y, add, R[id], mid, r);
    return newParent(left, right);
}

inline T Query(ll x, ll y, ll id, ll l, ll r){
    if(x>=r || y<=l) return T(OLL, OLL, OLL, OLL);
    if(x<=l && r<=y) return tree[id];
    proh(id, l, r);
    ll mid=(l+r)>>1;
    T left=Query(x, y, L[id], l, mid);
    T right=Query(x, y, R[id], mid, r);
    return Op(left, right);
}

inline ll Build(ll l, ll r){
    ll mid=(l+r)>>1;
    if(l+1==r) return newLeaf(T(ar[l], ar[l], 1, 0));
    ll left=Build(l, mid), right=Build(mid, r);
    return newParent(left, right);
}

inline ll que(ll l, ll r, ll timer) {
    T cur=Query(l, r, timer, 0, n);
    return cur.s;
}

inline ll find(ll a, ll p, ll l, ll r){
    proh(a, l, r);
    if(l>p) return -1;
    if(l+1==r) return tree[a].s==tree[a].t? -1: l;
    ll mid=(l+r)>>1;
    ll right=find(R[a], p, mid, r);
    if(right!=-1) return right;
    return find(L[a], p, l, mid);
}

inline ll upd(ll x, ll y, ll add, ll timer) {
    return updR(x, y, add, timer, 0, n);}

```

```

    inline ll build() { return Build(0, n);}
}st;

```

1.11 Persistent Segment Tree

```

struct PST{
    ll n;
    vi L, R, tree, ar;
    inline ll Op(ll &val1, ll &val2) { return val1+val2;}
    inline ll newParent(ll l, ll r){
        L.pb(l), R.pb(r), tree.pb(Op(tree[l], tree[r]));
        return sz(tree)-1;
    }

    inline ll newLeaf(ll val){
        L.pb(0), R.pb(0), tree.pb(val);
        return sz(tree)-1;
    }

    inline ll updR(ll p, ll add, ll id, ll l, ll r){
        r){
            if(l+1==r) return newLeaf(Op(tree[id], add));
            ll mid=(l+r)>>1;
            if(p<mid) return newParent(updR(p, add, L[id], l, mid), R[id]);
            return newParent(L[id], updR(p, add, R[id], mid, r));
        }

    inline ll Query(ll x, ll y, ll id, ll l, ll r){
        r){
            if(x>=r or y<=l) return OLL;
            if(x<=l and r<=y) return tree[id];
            ll mid=(l+r)>>1, left, right;
            left=Query(x, y, L[id], l, mid);
            right=Query(x, y, R[id], mid, r);
            return Op(left, right);
        }
    }
}

```

```

    inline ll Build(ll l, ll r){
        ll mid=(l+r)>>1;
        if(r-l<2) return newLeaf(ar[l]);
        ll left=Build(l, mid), right=Build(mid, r);
        return newParent(left, right);
    }

    inline ll que(ll l, ll r, ll timer) { return Query(l, r, timer, 0, n);}
    inline ll upd(ll p, ll add, ll timer) { return updR(p, add, timer, 0, n);}
    inline ll build() { n=sz(ar); return Build(0, n);}
}st;

```

1.12 Persistent Trie

```

/* Persistent Trie */
struct Ptrie{
    ll nod, m[1<<23][2], ind[1<<24];
    inline ll AddTrie(ll last, ll i, ll val, ll pos){
        pos){
            ll rt=nod++;
            if(i<0) return rt;
            ll r=(val>>i)&1;
            m[rt][r^1]=m[last][r^1];
            m[rt][r]=AddTrie(m[last][r], i-1, val, pos);
            ind[m[rt][r]]=pos;
            return rt;
        }

    inline ll QueryMax(ll id, ll val, ll pos){
        ll ans=0;
        IFR(i,19,0){
            ll r=(val>>i)&1;
            if(m[id][r^1]) {
                ll curnod=m[id][r^1];
                ll index=ind[curnod];
                if(index>=pos) id=m[id][r^1],
                    ans+=((r^1)<<i);
                else id=m[id][r], ans+=(r<<i);
            }
        }
    }
}

```

```

        else {
            id=m[id][r], ans+=(r<<i);
        }
    }
    return ans ^ val;
}
}st;

```

```

int main(){
    st.nod=1
    st.AddTrie(), st.QueryMax();
}

```

1.13 SQRTREE

```

ll mo;
struct T{
    ll l, r, cur;
    T(){
        T(ll l, ll r, ll cur): l(l), r(r), cur(cur){}
    };
    struct SqrtTree{
        ll n, lg, indexsz;
        vector<T> v;
        vi clz, layers, onlayer;
        vector<vector<T>> pref, suf, between;
        inline T Op(T &val1, T &val2){
            T ty;
            ty.l=val1.l;
            ty.r=val2.r;
            ty.cur=((val1.cur*val2.cur)%mo+mo)%mo;
            return ty;
        }
        inline ll Get(ll n){
            ll ans=0;
            while((1<<ans)<n) ans++;
            return ans;
        }
        inline void preprocess(vector<T> &a){
            n=sz(a), lg=Get(n), v=a;
            clz.resize(1<<lg), onlayer.resize(lg+1);

```

```

            FER(i, 0, sz(clz)) clz[i]=(i==0)? 0 :
                clz[i]>>1+1;
            ll tl=lg;
            while(tl>1){
                onlayer[tl]=sz(layers);
                layers.pb(tl);
                tl=(tl+1)>>1;
            }
            IFR(i, lg-1, 0) onlayer[i]=max(onlayer[i],
                onlayer[i+1]);
            ll mid=max(OLL, sz(layers)-1);
            ll t=(lg+1)>>1, m=1<<t;
            indexsz=(n+m-1)>>t;
            v.resize(n+indexsz);
            pref.assign(sz(layers), vector<T>(n+indexsz));
            suf.assign(sz(layers), vector<T>(n+indexsz));
            between.assign(mid, vector<T>((1<<lg)+m));
        }
        inline void BuildBlock(ll id, ll l, ll r){
            FER(i, l, r) pref[id][i]=(i==l)? v[i]:
                Op(pref[id][i-1], v[i]);
            IFR(i, r-1, l) suf[id][i]=(i==r-1)? v[i]:
                Op(v[i], suf[id][i+1]);
        }
        inline void BuildBetween(ll id, ll l, ll r, ll
            idx){
            ll t=(layers[id]+1)>>1, m=layers[id]>>1;
            ll ta1=1<<t, len=(r-l+ta1-1)>>t;
            FER(i, 0, len){
                T ans;
                FER(j, i, len){
                    T add=suf[id][l+(j<<t)];
                    ans=(i==j)? add: Op(ans, add);
                    between[id-1][idx+l+(i<<m)+j]=ans;
                }
            }
        }
        inline void BuildZero(){
            ll m=(lg+1)>>1;
            FER(i, 0, indexsz) v[n+i]=suf[0][i<<m];
            build(1, n, n+indexsz, (1<<lg)-n);
        }
        inline void updZero(ll id){

```

```

            ll m=(lg+1)>>1;
            v[n+id]=suf[0][id<<m];
            update(1, n, n+indexsz, (1<<lg)-n, n+id);
        }

        inline void build(ll id, ll l, ll r, ll idx){
            if(id>=sz(layers)) return;
            ll m=1<<((layers[id]+1)>>1);
            for(ll left=l; left<r; left+=m){
                ll right=min(left+m, r);
                BuildBlock(id, left, right);
                build(id+1, left, right, idx);
            }
            id? BuildBetween(id, l, r, idx) : BuildZero();
        }
        inline void update(ll id, ll l, ll r, ll idx,
            ll p){
            if(id>=sz(layers)) return ;
            ll m=(layers[id]+1)>>1, t=1<<m;
            ll bid=(p-1)>>m;
            ll left=l+(bid<<m), right=min(l+t, r);
            BuildBlock(id, left, right);
            id? BuildBetween(id, l, r, idx): updZero(bid);
            update(id+1, left, right, idx, p);
        }
        inline T query(ll l, ll r, ll idx, ll b){
            if(l==r) return v[l];
            if(l+1==r) return Op(v[l], v[r]);
            ll id=onlayer[clz[(l-b) ^ (r-b)]];
            ll t=(layers[id]+1)>>1, m=layers[id]>>1;
            ll in=((l-b)>>layers[id])<<layers[id]+b;
            ll left=((l-in)>>t)+1, right=((r-in)>>t)-1;
            T ans=suf[id][l];
            if(left<=right){
                ll idd=idx+in+(left<<m)+right;
                T add=(id==0)? query(n+left, n+right,
                    (1<<lg)-n, n):between[id-1][idd];
                ans=Op(ans, add);
            }
            ans=Op(ans, pref[id][r]);
            return ans;
        }
        inline T que(ll l, ll r){
            return query(l, r, 0, 0);

```

```

}
inline void upd(ll p, T val){
    v[p]=val;
    update(0, 0, n, 0, p);
}
inline void build(){
    build(0, 0, n, 0);
}
};
ll queries[N];
int main(){
    fastio;
    ll t; cin>>t;
    FER(ikl, 0, t){
        SQRTEE sqrtree;
        ll n, q; cin>>n>>mo>>q;
        vector<T> a;
        FER(i, 0, n){
            T x;
            cin>>x.cur;
            x.l=i, x.r=i;
            a.pb(x);
        }
        sqrtree.preprocess(a);
        sqrtree.build();
        ll q1=(q>>6)+2;
        ll l, r;
        FER(i,0,q1) cin>>queries[i];
        ll ja=0;
        FER(i, 0, q){
            (i%(1<<6))?(l=(l+ja)%n, r=(r+ja)%n):
            (l=(queries[i>>6]+ja)%n,
            r=(queries[(i>>6)+1]+ja)%n);
            if(l>r) swap(l, r);
            ja=(sqrtree.que(l, r).cur+1)%mo;
        }
        cout<<ja<<"\n";
    }
    return 0;
}

```

1.14 Segment Tree

```

//Segment Tree
struct SegmentTree{
    int n, t[2 * N];
    int OpId = 0;
    SegmentTree() {}
    int Op(int &u, int &v){ return u + v; }
    void build(){ IFR(i, n - 1, 1) t[i]=Op(t[i <<
        1], t[i << 1 | 1]); }
    void modify(int p, int val){ for(t[p+=n] = val
        ; p >= 1;) t[p] = Op(t[p << 1], t[p << 1 |
        1]); }
    int query(int l, int r){
        int ans1, ansr;
        ans1 = ansr = OpId;
        for(l += n, r += n; l < r; l >= 1, r >= 1){
            if(l&1) ans1 = Op(ans1, t[l++]);
            if(r&1) ansr = Op(t[--r], ansr);
        }
        return Op(ans1, ansr);
    }
};

```

1.15 Splay Tree

```

//-----
// Lazy splay tree propagation
// make -> n, preprocess() y build()
// Be careful about the lazy push values
// there isn't need to use an Op function (this
// may cause TL)
struct Node{
    Node *child[2], *p;
    bool t;
    ll val, ta, sum, add, l, r, mid;
    inline void clear(){
        ta=-INF;
    }
};

```

```

};
struct SplayTree{
    Node *nil, *root;
    ll ar[1<<18], n;
    inline void preprocess(){
        nil=new Node();
        nil->child[0] = nil->child[1] = nil->p=nil;
        nil->val=nil->ta=nil->sum=0;
        nil->t=false;
        nil->l=-INF;
        nil->r=-INF;
        nil->mid=-INF;
        nil->add=0;
        root=nil;
    }
    inline void upd(Node &x){
        x->ta=x->child[0]->ta + x->child[1]->ta+1;
        x->sum=x->child[0]->sum+x->child[1]->sum+x->val;
        ll l=max(x->child[0]->l,
            x->child[0]->sum+x->val);
        ll r=max(x->val, x->child[0]->r+x->val);
        ll mid=max(l, max(r, max(x->val,
            max(x->child[0]->mid,
            x->child[0]->r+x->val))));
        ll l1=max(l,
            x->child[0]->sum+x->val+x->child[1]->l);
        ll r1=max(x->child[1]->r,
            x->child[1]->sum+r);
        ll mid1=max(l1, max(r1,
            max(x->child[1]->mid, max(mid,
            r+x->child[1]->l))));
        x->l=l1, x->r=r1, x->mid=mid1;
    }
    inline void updpro(Node &x, ll val){
        x->add+=val;
        x->val+=val;
        x->sum+=x->ta*val;
        x->l+=x->ta*val;
        x->r+=x->ta*val;
        x->mid+=x->ta*val;
    }
    inline void push(Node &x){
        if(x==nil) return;
    }
};

```

```

if(x->add!=0){
    if(x->child[0]!=nil) updpro(x->child[0],
        x->add);
    if(x->child[1]!=nil) updpro(x->child[1],
        x->add);
    x->add=0;
}
if(x->t){
    swap(x->child[0], x->child[1]);
    x->child[0]->t=!x->child[0]->t;
    x->child[1]->t=!x->child[1]->t;
    x->t=false;
}
}
inline void set(Node *x, Node *y, ll d){
    x->child[d]=y;
    y->p=x;
}
inline ll get(Node *x, Node *y){
    return x->child[0]==y? 0 : 1;
}
inline ll rot(Node *&x, ll d){
    Node *y=x->child[d], *z=x->p;
    set(x, y->child[d^1], d);
    set(y, x, d^1);
    set(z, y, get(z, x));
    upd(x), upd(y);
}
inline void splay(Node *&x){
    push(x);
    while(x->p != nil){
        Node *y=x->p, *z=y->p;
        ll dy=get(y, x), dz=get(z, y);
        if(z==nil) rot(y, dy);
        else if(dy==dz) rot(z, dz), rot(y, dy);
        else rot(y, dy), rot(z, dz);
    }
    upd(x);
}
inline Node * getnode(Node *x, ll pos){
    while(push(x), x->child[0]->ta!=pos){
        pos<x->child[0]->ta? x=x->child[0]:
            (pos==x->child[0]->ta+1,
            x=x->child[1]);
    }
}

```

```

}
return splay(x), x;
}
inline void split(Node *x, ll l, Node * &t1,
    Node * &t2){
    if(l==0) t1=nil, t2=x;
    else{
        t1=getnode(x, l-1);
        t2=t1->child[1];
        t1->child[1]=t2->p=nil;
        upd(t1);
    }
}
inline Node * unir(Node *x, Node *y){
    if(x==nil) return y;
    x=getnode(x, x->ta-1);
    set(x, y, 1);
    upd(x);
    return x;
}
inline void AddRange(ll l, ll r, ll add){
    Node *t1, * t2, * t3;
    split(root, r, t1, t3);
    split(t1, l, t1, t2);
    t2->add+=add;
    t2->val+=add;
    t2->sum+=t2->ta * add;
    root=unir(t1, unir(t2, t3));
}
inline void modify(ll pos, ll val){
    root=getnode(root, pos);
    root->val=val;
    upd(root);
}
inline void AddVal(ll pos, ll val){
    Node *t1, *t2;
    Node *cur=new Node();
    cur->val=val, cur->p=nil, cur->t=false;
    cur->ta=1, cur->sum=val;
    cur->child[0]=cur->child[1]=nil;
    ll r=pos;
    if(r==root->ta || r==0){
        if(r==0) root=unir(cur, root);
        else root=unir(root, cur);
    }
}

```

```

return;
}
split(root, r, t1, t2);
root=unir(unir(t1, cur), t2);
}
inline void DelVal(ll pos){
    Node *t1, *t2, *t3;
    ll l=pos, r=pos+1;
    if(l==0 || r==root->ta){
        if(l==0){
            split(root, r, t1, t2);
            root=t2;
        }
        else{
            split(root, l, t1, t2);
            root=t1;
        }
        return;
    }
    split(root, r, t1, t3);
    split(t1, l, t1, t2);
    root=unir(t1, t3);
    delete t2;
    return;
}
inline void Reverse(ll l, ll r){
    Node * t1, * t2, * t3;
    split(root, r, t1, t3);
    split(t1, l, t1, t2);
    t2->t=!t2->t;
    root=unir(unir(t1, t2), t3);
}
inline void move(ll l, ll r, ll l1, ll r1){
    Node *t1, *t2, *t3, *t4, *t5;
    split(root, r1, t4, t5);
    split(t4, l1, t3, t4);
    split(t3, r, t2, t3);
    split(t2, l, t1, t2);
    root=unir(unir(t1,unir(t4, t3)), unir(t2,
        t5));
}
inline ll que(ll l, ll r){
    Node *t1, * t2, *t3;
    split(root, r, t1, t3);
}

```

```

    split(t1, l, t1, t2);
    ll ans=t2->mid;
    root=unir(unir(t1, t2), t3);
    return ans;
}
inline ll Permute(ll l, ll r){
    ll x=que(r-1, r);
    AddVal(l, x);
    DelVal(r);
}
inline Node * build1(ll l, ll r){
    if(l==r) return nil;
    ll mid=(l+r)>>1;
    Node *x=new Node();
    x->val=ar[mid];
    x->sum=ar[mid];
    x->mid=ar[mid];
    x->l=ar[mid];
    x->r=ar[mid];
    x->p=nil;
    x->t=false;
    x->add=0;
    set(x, build1(l, mid), 0);
    set(x, build1(mid+1, r), 1);
    upd(x);
    return x;
}
inline void build(){
    Node *root1=build1(0, n);
    root=root1;
}
}st;

int main(){
    ll n; scanf("%lld", &n);
    st.n=n;
    FER(i,0,n) scanf("%lld", &st.ar[i]);
    st.preprocess();
    st.build();
    ll q, l, r; scanf("%lld", &q);
    char s;
    FER(i,0,q){
        scanf(" %c", &s);

```

```

        if(s=='I'){
            scanf("%lld%lld", &l, &r); l--;
            st.AddVal(l, r);
        }
        else if(s=='D'){
            scanf("%lld", &l); l--;
            st.DelVal(l);
        }
        else if(s=='R'){
            scanf("%lld%lld", &l, &r); l--;
            st.modify(l, r);
        }
        else{
            scanf("%lld%lld", &l, &r); l--;
            ll froz=st.que(l, r);
            printf("%lld\n", froz);
        }
    }
    return 0;
}

```

1.16 ksetTree

```

#include <bits/stdc++.h>
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/detail/standard_policies.hpp>
using namespace __gnu_pbds;

tree<int, null_type, less<int>, rb_tree_tag,
    tree_order_statistics_node_update> X;
// El primer parametro int es el tipo de dato del
// set
// El segundo parametro es null_type para set, u
// otro tipo para map. Por ejemplo
// tree<string,int,...> es un map<string,int>
// El tercer parametro es la funcion de
// comparacin, por defecto usar less<T>
// El cuarto y el quinto se ponen como estn
// soportan las operaciones *x.find_by_order(k);
// x.order_of_key(key);
typedef tree<int, null_type, less<int>,
    rb_tree_tag,

```

```

    tree_order_statistics_node_update> kset;

/*Implementacion usada para el subregional de
Moscow 2008*/
template <class T> struct ord{
    bool operator() (const ii &x, const ii& y)
    const{
        if(x.fst != y.fst)
            return x.fst > y.fst;
        return x.snd < y.snd;
    }
};

typedef tree< ii,string, ord<ii>,rb_tree_tag,
    tree_order_statistics_node_update> kset;

int main(){

    kset x;

    return 0;
}

```

2 Flujos

2.1 Bipartite Matching

```

/* Bipartite matching BFS-DFS
Autor: elManco
*/

const int INF = 1e9;
int n, m, nil, match[N], dist[N];
vector<int> adj[N];

bool bfs(){
    queue<int> q;
    for(int i = 1; i <= n; ++i){
        if(match[i] == nil){

```

```

        dist[i] = 0;
        q.push(i);
    }
    else dist[i] = INF;
}
dist[nil] = INF;
while(!q.empty()){
    int u = q.front();
    q.pop();
    if(u == nil) continue;
    for(int v : adj[u]){
        if(dist[match[v]] == INF){
            dist[match[v]] = dist[u] + 1;
            q.push(match[v]);
        }
    }
}
return dist[nil] != INF;
}

bool dfs(int u){
    if(u != nil){
        for(int v : adj[u]){
            if(dist[match[v]] != dist[u] + 1)
                continue;
            if(dfs(match[v])){
                match[v] = u;
                match[u] = v;
                return true;
            }
        }
        dist[u] = INF;
        return false;
    }
    return true;
}

int bipMatching(){
    nil = n + m + 1;
    for(int i = 1; i <= n+m; ++i)
        match[i] = nil;
    int matching = 0;
    while(bfs()){
        for(int i = 1; i <= n; ++i)

```

```

        if(match[i] == nil and dfs(i))
            ++matching;
    }
    return matching;
}

```

2.2 Dinic Max Flow

```

/* Dinic Max Flow
Autor: Matias Hunicken
Instrucciones de uso:
    Min cut: nodes with dist>=0 vs nodes with
    dist<0
    Matching MVC: left nodes with dist<0 + right
    nodes with dist>0
*/

struct Dinic{
    int nodes,src,dst;
    vector<int> dist,q,work;
    struct edge {int to,rev;ll f,cap;};
    vector<vector<edge>> g;
    Dinic(int
        x):nodes(x),g(x),dist(x),q(x),work(x){}
    void add_edge(int s, int t, ll cap){
        g[s].pb((edge){t,SZ(g[t]),0,cap});
        g[t].pb((edge){s,SZ(g[s])-1,0,0});
    }
    bool dinic_bfs(){
        fill(ALL(dist),-1);dist[src]=0;
        int qt=0;q[qt++]=src;
        for(int qh=0;qh<qt;qh++){
            int u=q[qh];
            FER(i,0,SZ(g[u])){
                edge &e=g[u][i];int v=g[u][i].to;
                if(dist[v]<0&&e.f<e.cap)
                    dist[v]=dist[u]+1,q[qt++]=v;
            }
        }
        return dist[dst]>=0;
    }
    ll dinic_dfs(int u, ll f){

```

```

        if(u==dst)return f;
        for(int &i=work[u];i<SZ(g[u]);i++){
            edge &e=g[u][i];
            if(e.cap<=e.f)continue;
            int v=e.to;
            if(dist[v]==dist[u]+1){
                ll df=dinic_dfs(v,min(f,e.cap-e.f));
                if(df>0){e.f+=df;g[v][e.rev].f-=df;return
                    df;}
            }
        }
        return 0;
    }
    ll max_flow(int _src, int _dst){
        src=_src;dst=_dst;
        ll result=0;
        while(dinic_bfs()){
            fill(ALL(work),0);
            while(ll
                delta=dinic_dfs(src,INF))result+=delta;
        }
        return result;
    }
};

```

2.3 Edmonds Blossom

```

/* Edmonds Blossom - General matching para grafos
generales
Autor: Phibrain
*/

struct EdmondsBlossom{
    ll n;
    vi match, vis, tmp;
    set<ii> used;
    inline void join(ll a, ll b) {
        match[a]=b, match[b]=a;
    }
    inline bool dfs(ll k, vector<vi> &com, vi
        &blossom){
        vis[k] = 0;

```



```

FER(i, 0, n) if(com[k][i]){
    if(vis[i] == -1){
        vis[i] = 1;
        if(match[i] == -1 || dfs(match[i],
            com, blossom)) { join(k, i);
            return true;}
    }
    if(vis[i] == 0 || sz(blossom)){
        blossom.pb(i), blossom.pb(k);
        if(k == blossom[0]) { match[k] = -1;
            return true;}
        return false;
    }
}
return false;
}
inline bool augment(vector<vi> &com){
    FER(m, 0, n) if(match[m] == -1){
        vi blossom;
        vis = vi(n, -1);
        if(!dfs(m, com, blossom)) continue;
        if(sz(blossom) == 0) return true;

        ll base = blossom[0], s = sz(blossom);
        vector<vi> newcom = com;

        FER(i, 1, s-1) FER(j, 0, n)
            newcom[base][j] = newcom[j][base]
            |= com[blossom[i]][j];
        FER(i, 1, s-1) FER(j, 0, n)
            newcom[blossom[i]][j] =
            newcom[j][blossom[i]] = 0;
        newcom[base][base] = 0;

        if(!augment(newcom)) return false;

        ll k = match[base];
        if(k != -1) FER(i, 0, s)
            if(com[blossom[i]][k]){
                join(blossom[i], k);
                if(i&1) for(ll j = i + 1; j < s; j +=
                    2) join(blossom[j], blossom[j +
                        1]);
            }
    }
}

```

```

        else for(ll j = 0; j < i; j += 2)
            join(blossom[j], blossom[j + 1]);
        break;
    }
    return true;
}
return false;
}
inline ll edmonds(vector<vi> &com){
    ll res=0;
    match=vi(n, -1);
    while(augment(com)) res++;
    return res;
}
inline void preprocess(vector<vi> &com){
    tmp.clear();
    tmp.assign(n, 0);
    FER(i,0,n) com.pb(tmp);
    used.clear();
    match.clear();
    vis.clear();
}
inline void add(ll a, ll b, vector<vi> &com){
    if(a>b) swap(a, b);
    if(used.count({a, b})) return;
    used.insert({a, b});
    com[a][b]=com[b][a]=1;
}
};

```

2.4 Gomory Hu Tree

```

// Gomory Hu Tree 0 index base
// graph is stored in vector<pi> gph

#include <bits/stdc++.h>
using namespace std;
typedef long long lint;
typedef pair<int, int> pi;

const int MAXN = 205;
struct maxflow{

```

```

    struct edg{int pos, cap, rev;};
    vector<edg> gph[MAXN];

    void clear(){
        for(int i=0; i<MAXN; i++){
            gph[i].clear();
        }
    }

    void add_edge(int s, int e, int x){
        gph[s].push_back({e, x,
            (int)gph[e].size()});
        gph[e].push_back({s, 0,
            (int)gph[s].size()-1});
    }

    int dis[MAXN], pnt[MAXN];

    bool bfs(int src, int sink){
        memset(dis, 0, sizeof(dis));
        memset(pnt, 0, sizeof(pnt));
        queue<int> que;
        que.push(src);
        dis[src] = 1;
        while(!que.empty()){
            int x = que.front();
            que.pop();
            for(int i=0; i<gph[x].size(); i++){
                edg e = gph[x][i];
                if(e.cap > 0 && !dis[e.pos]){
                    dis[e.pos] = dis[x] + 1;
                    que.push(e.pos);
                }
            }
        }
        return dis[sink] > 0;
    }

    int dfs(int x, int sink, int f){
        if(x == sink) return f;
        for(; pnt[x] < gph[x].size(); pnt[x]++){
            edg e = gph[x][pnt[x]];
            if(e.cap > 0 && dis[e.pos] == dis[x] +
                1){

```

```

        int w = dfs(e.pos, sink, min(f,
            e.cap));
        if(w){
            gph[x][pnt[x]].cap -= w;
            gph[e.pos][e.rev].cap += w;
            return w;
        }
    }
    return 0;
}

lint match(int src, int sink){
    lint ret = 0;
    while(bfs(src, sink)){
        int r;
        while((r = dfs(src, sink, 2e9))) ret
            += r;
    }
    return ret;
}

};

struct gomory_hu{
    struct edg{ int s, e, x; };
    vector<edg> eds;
    maxflow mf;

    void clear(){
        eds.clear();
    }

    void add_edge(int s, int e, int x){
        eds.push_back({s, e, x});
    }

    bool vis[MAXN];

    void dfs(int x){
        if(vis[x]) return;
        vis[x] = 1;
        for(auto &i : mf.gph[x]){
            if(i.cap > 0) dfs(i.pos);
        }
    }
};

```

```

    }

    vector<pi> solve(int n){
        vector<pi> ret(n);
        for(int i=1; i<n; i++){
            for(auto &j : eds){
                mf.add_edge(j.s, j.e, j.x);
                mf.add_edge(j.e, j.s, j.x);
            }
            ret[i].first = mf.match(i,
                ret[i].second);
            memset(vis, 0, sizeof(vis));
            dfs(i);
            for(int j=i+1; j<n; j++){
                if(ret[j].second == ret[i].second
                    && vis[j]){
                    ret[j].second = i;
                }
            }
            mf.clear();
        }
        return ret;
    }
}gh;

vector<pi> gph[205];

int main(){
    int n, m;
    cin >> n >> m;
    for(int i=0; i<m; i++){
        int s, e, x;
        scanf("%d %d %d", &s, &e, &x);
        s--, e--;
        gh.add_edge(s, e, x);
    }
    auto w = gh.solve(n);
    for(int i=1; i<n; i++){
        gph[w[i].second].push_back(pi(i,
            w[i].first));
        gph[i].push_back(pi(w[i].second,
            w[i].first));
    }
    for(int i=0; i<n; i++){

```

```

        cout<<i<<endl;
        for(auto xd: gph[i]) cout<<xd.first<<"
            "<<xd.second<<endl;
    }
    return 0;
}

```

2.5 Lower Bound Upper Bound Flow

/* Max flow with upper and lower bounds

add(a, b, c, d): crea un nodo de "a" hacia "b" con lower value de c y upper value de "d". Todos los nodos son 1 base

Implementacion basada en verificar que el flujo es feasible y para encontrar el max flow usa binary search

La construccion del grafo esta dada por construct(sink, source, #de nodos) El grafo esta guardado en graph[] [] y adj1[]

Para reconstruir los caminos usar el weighted eulerian path

Usar Push Relabel flow!

```

*/
#define N1 101

ll graph[N1][N1];
vector<tri> adj[N1];
vii adj1[N1];

inline void add(ll a, ll b, ll d, ll c){
    adj[a].pb(tri{b, {d, c}});
}

inline ll solve(ll x, ll sn, ll sr, ll n,
    PushRelabel &pr){
    ll nod=0, ands=0, neg=0;

```

```

ll sale[N1], entra[N1], d[N1];
fill(sale, 0), fill(entra, 0);
FER(i,0,n+1) adj1[i].clear();
adj[nod].clear();
adj[nod].pb(tri{sn, {x, INF}});
FER(i,0,n+1){
    if(sz(adj[i])) for(auto xd: adj[i]){
        sale[i]+=xd.tm2;
        entra[xd.tm1]+=xd.tm2;
    }
}
ll t1=n+1, t2=n+2;
FER(i, 0, n+1) d[i]=sale[i]-entra[i];
FER(i, 0, n+1) (d[i]>0)? ands+=d[i] :
    neg+=-d[i];
FER(i, 0, n+1) {
    if(d[i]>0) pr.AddEdge(i, t2, d[i]);
    if(d[i]<0) pr.AddEdge(t1, i, -d[i]);
}
FER(i, 0, n+1){
    if(sz(adj[i])) for(auto xd: adj[i]){
        ll w=xd.tm3-xd.tm2;
        pr.AddEdge(i, xd.tm1, w);
    }
}
pr.AddEdge(sr, nod, INF);
ll froz=pr.GetMaxFlow(t1, t2);
if(froz==ands && neg==ands) return 1;
return 0;
}

inline void go(ll x, ll sn, ll sr, ll n){
    PushRelabel pr(n+3);
    ll pe=solve(x, sn, sr, n, pr);
    map<ii, ll> m;
    for(auto xd: pr.G){
        for(auto gg: xd){
            if(gg.flow>0) m[{gg.from,
                gg.to}]+=gg.flow;
        }
    }
    fill(graph, 0);
    FER(i, 1, n+1){
        if(sz(adj[i]))for(auto xd: adj[i]){

```

```

        ll w=xd.tm2+m[{i, xd.tm1}];
        if(w==0) continue;
        graph[i][xd.tm1]=w;
        adj1[i].pb(ii{xd.tm1, w});
    }
}

inline ll construct(ll sn, ll sr, ll n){
    ll ini=0, fin=INF-1;
    FER(i,0,100){
        if(ini+1==fin) break;
        ll mid=(ini+fin)>>1;
        PushRelabel pr(n+3);
        if(solve(mid, sn, sr, n, pr)) ini=mid;
        else fin=mid;
    }
    PushRelabel pr(n+3);
    ll pe=solve(ini, sn, sr, n, pr);
    if(pe && ini) {
        go(ini, sn, sr, n);
        return 1;
    }
    return 0;
}

```

2.6 Max Flow Extensions

```

// In Max Flow, to add a "demand" function d(u,v)
// which forces
// edge (u,v) to have a flow of at least d(u,v),
// let s, t be
// the source and sink respectively.

// This problem is reduced to a normal max flow
// in graph
// V' = V u {s', t'}
// with new capacities c', source s' and sink t':
// c'(s', v) = sum{ d(u,v), u in V }
// c'(v, t') = sum{ d(v,w), w in V }
// c'(u, v) = c(u,v) - d(u,v)
// c'(t, s) = INF

```

```

// There exists a solution in problem V if and
// only if the
// problem in graph V' has a saturating max flow
// (with |f| = sum{ d(u, v) / u,v in V }).
// If a max flow / min flow solution is needed,
// consider adding
// a node to control the total flow, controlling
// its
// capacity / demand in a binary search.

```

2.7 Min Cost Circulation

```

// Min Cost Circulation

```

```

struct Edge {
    int cap, flow;
    ll cost;
    Edge(int cap = 0, int flow = 0, ll cost = 0) :
        cap(cap), flow(flow), cost(cost) {}
};

struct MinCostCirculation{
    int s,t,n;
    ll maxFlow, minCost;
    vector<vector<Edge>> G; //nonexistent edge =
        Edge(0,0,0);
    vector<int> vis,parent;
    vector<ll> dist;
    MinCostCirculation (int _n = 0){
        n = _n;
        G.resize(n+1); dist.resize(n+1);
        vis.resize(n+1); parent.resize(n+1);
        //OK;
        FER(i,0,n+1) G[i].resize(n+1);
        minCost = 0;
    }
    void AddEdge(int from, int to, int cap, ll
        cost){
        G[from][to] = Edge(cap, 0, cost);
        G[to][from] = Edge(0, 0,-cost);
    }
}

```

```

void updateEdge(int from, int to, int ncap){
    G[from][to].cap = ncap;
}
void Augment(int u){
    FER(i,0,n+1) vis[i] = 0;
    while(!vis[u]){
        vis[u] = 1;
        u = parent[u];
    }
    int aux = u;
    ll addedCost = 0;
    ll delta = INF;
    do{
        int v = parent[u];
        addedCost += G[v][u].cost;
        delta = min(delta, (ll) G[v][u].cap -
                    G[v][u].flow);
        u = v;
    }while(u != aux);
    do{
        int v = parent[u];
        G[v][u].flow += delta;
        G[u][v].flow -= delta;
        u = v;
    }while(u != aux);
    minCost += addedCost*delta;
}

bool negCycle(){
    /*Vertex n is a dummy vertex => |V| = n+1*/
    FER(i,0,n) dist[i] = INF;
    dist[n] = 0;
    parent[n] = -1;
    FER(k,0,n) FER(i,0,n+1) FER(j,0,n+1){
        if(G[i][j].flow == G[i][j].cap) continue;
        if(dist[i] + G[i][j].cost < dist[j]){
            dist[j] = dist[i] + G[i][j].cost;
            parent[j] = i;
        }
    }
    FER(i,0,n+1) FER(j,0,n+1){
        if(G[i][j].flow == G[i][j].cap) continue;
        if(dist[i] + G[i][j].cost < dist[j]){
            parent[j] = i;

```

```

        Augment(j);
        return true;
    }
    return false;
}

ll GetMinCostCirculation(){
    while(negCycle());
    return minCost;
}
};



---



## 2.8 Min Cost Max Flow



---


const int MAXN = 5010;

const ll INF = 1e15;
struct edge { int dest; ll origcap, cap; ll cost;
              int rev; };

struct MinCostMaxFlow {

    vector<edge> adj[MAXN];
    ll dis[MAXN], cost;
    int source, target, iter;
    ll cap;
    edge* pre[MAXN];
    int queued[MAXN];
    MinCostMaxFlow (){}
    void AddEdge(int from, int to, ll cap, ll
                  cost) {
        adj[from].push_back(edge {to, cap, cap,
                                   cost, (int)adj[to].size()});
        adj[to].push_back(edge {from, 0, 0, -cost,
                                   (int)adj[from].size() - 1});
    }

    bool spfa() {
        FER(i,0,MAXN) queued[i] = 0;
        fill(dis, dis + MAXN, INF);
        queue<int> q;

```

```

        pre[source] = pre[target] = 0;
        dis[source] = 0;
        q.emplace(source);
        queued[source] = 1;
        while (!q.empty()) {
            int x = q.front();

            ll d = dis[x];
            q.pop();
            queued[x] = 0;
            for (auto& e : adj[x]) {
                int y = e.dest;
                ll w = d + e.cost;
                if (e.cap < 1 || dis[y] <= w) continue;
                dis[y] = w;
                pre[y] = &e;
                if(!queued[y]){
                    q.push(y);
                    queued[y] = 1;
                }
            }
        }
        edge* e = pre[target];

        if (!e) return 0;
        while (e) {
            edge& rev = adj[e->dest][e->rev];
            e->cap -= cap;
            rev.cap += cap;
            cost += cap * e->cost;
            e = pre[rev.dest];
        }
        return 1;
    }

    pair<ll,ll> GetMaxFlow(int S, int T) {
        cap = 1, source = S, target = T, cost = 0;
        while(spfa()) {}
        ll totflow = 0;
        for(auto e: adj[source]){
            totflow += (e.origcap - e.cap);
        }
        return make_pair(totflow, cost);
    }
}

```

```
};
```

2.9 Push Relabel Max Flow

```
/* Push Relabel Max Flow
   Autor: itu
   Instrucciones de uso:
   O(|V|^3), corre bien para n ~ 5000
   Todos los edges con flow > 0 tienen flujo
   corriendo
*/

struct Edge {
    int from, to, cap, flow, index;
    Edge(int from, int to, int cap, int flow, int
        index) :
        from(from), to(to), cap(cap), flow(flow),
        index(index) {}
};

struct PushRelabel {
    int N;
    vector<vector<Edge>> > G;
    vector<ll> excess;
    vector<int> dist, active, count;
    queue<int> Q;

    PushRelabel(int N) :
        N(N), G(N), excess(N), dist(N), active(N),
        count(2*N) {}

    void AddEdge(int from, int to, int cap) {
        G[from].push_back(Edge(from, to, cap, 0,
            G[to].size()));
        if (from == to) G[from].back().index++;
        G[to].push_back(Edge(to, from, 0, 0,
            G[from].size() - 1));
    }

    void Enqueue(int v) {
        if (!active[v] && excess[v] > 0) {
            active[v] = true; Q.push(v);
        }
    }
};
```

```
    }

    void Push(Edge &e) {
        int amt = int(min(excess[e.from], ll(e.cap -
            e.flow)));
        if (dist[e.from] <= dist[e.to] || amt == 0)
            return;
        e.flow += amt;
        G[e.to][e.index].flow -= amt;
        excess[e.to] += amt;
        excess[e.from] -= amt;
        Enqueue(e.to);
    }

    void Gap(int k) {
        for (int v = 0; v < N; v++) {
            if (dist[v] < k) continue;
            count[dist[v]]--;
            dist[v] = max(dist[v], N+1);
            count[dist[v]]++;
            Enqueue(v);
        }
    }

    void Relabel(int v) {
        count[dist[v]]--;
        dist[v] = 2*N;
        for (int i = 0; i < G[v].size(); i++)
            if (G[v][i].cap - G[v][i].flow > 0)
                dist[v] = min(dist[v], dist[G[v][i].to] + 1);
        count[dist[v]]++;
        Enqueue(v);
    }

    void Discharge(int v) {
        for (int i = 0; excess[v] > 0 && i <
            G[v].size(); i++)
            Push(G[v][i]);
        if (excess[v] > 0) {
            if (count[dist[v]] == 1) Gap(dist[v]);
            else Relabel(v);
        }
    }
};
```

```
ll GetMaxFlow(int s, int t) {
    count[0] = N-1;
    count[N] = 1;
    dist[s] = N;
    active[s] = active[t] = true;
    for (int i = 0; i < G[s].size(); i++) {
        excess[s] += G[s][i].cap;
        Push(G[s][i]);
    }

    while (!Q.empty()) {
        int v = Q.front();
        Q.pop();
        active[v] = false;
        Discharge(v);
    }

    ll totflow = 0;
    for (int i = 0; i < G[s].size(); i++) totflow
        += G[s][i].flow;
    return totflow;
}
};
```

3 Geometria

3.1 Circle Tangent

```
//Asumo que las circunferencias son diferentes
//Centro p y radio r1, centro q y radio r2
//Al menos un radio tiene que ser mayor que 0
//maneja el caso en que un circulo es punto
vector< Line > ComputeTgtInt(PT p, double r1, PT
    q, double r2){
    vector< Line > out;
    out.clear();
    if(equal(p, q)) return out;
    if(r1 > r2){
        swap(r1, r2);
        swap(p, q);
    }
}
```

```

}

double distcent = sqrt(dist2(p, q));
if(distcent + r1 - EPS < r2){
    return out;
}

vector< PT > ccinter = CircleCircleInter(p, q,
    r1, r2);
Line lin;
if(sz(ccinter) == 2) return out;
else{
    if(sz(ccinter) == 1){
        PT tmp = ccinter[0];
        lin.fst = tmp;
        lin.snd = tmp + RotateCCW90(p - tmp);
        out.pb(lin);
    }
    else{
        PT O1;
        O1 = (p * r2 + q * r1) / (r1 + r2);
        double distc = sqrt(dist2(q, O1));
        double Alfa = asin(r2 / distc);
        PT tmp1 = (p - O1) * cos(Alfa);
        PT tmp2 = (q - O1) * cos(Alfa);
        lin.fst = O1 + RotateCCW(tmp1, Alfa);
        lin.snd = O1 + RotateCCW(tmp2, Alfa);
        out.pb(lin);
        lin.fst = O1 + RotateCCW(tmp1, 2 * PI -
            Alfa);
        lin.snd = O1 + RotateCCW(tmp2, 2 * PI -
            Alfa);
        out.pb(lin);
    }
    return out;
}
}

//Asumo que las circunferencias son diferentes
//Centro p y radio r1, centro q y radio r2
//Siempre devuelve 0 o 2 tangentes
//En el caso de que solo haya 1, devuelve 2
//rectas que son
//las mismas salvo por EPS

```

```

vector< Line > ComputeTgtExt(PT p, double r1, PT
    q, double r2){
    vector< Line > out;
    out.clear();
    if(equal(p, q)) return out;
    if(r1 > r2){
        swap(r1, r2);
        swap(p, q);
    }

    double distcent = sqrt(dist2(p, q));
    if(distcent + r1 + EPS < r2){
        return out;
    }

    Line lin;
    PT tmp1, tmp2;
    if(abs(r1 - r2) < EPS){
        tmp1 = q - p;
        tmp1 = tmp1 * (r1 / distcent);
        tmp2 = p - q;
        tmp2 = tmp2 * (r2 / distcent);
        lin.fst = p + RotateCCW90(tmp1);
        lin.snd = q + RotateCW90(tmp2);
        out.pb(lin);
        lin.fst = p + RotateCW90(tmp1);
        lin.snd = q + RotateCCW90(tmp2);
        out.pb(lin);
    }
    else{
        double sinalfa = (r2 - r1) / distcent;
        sinalfa = max(sinalfa, 0.0);
        double Alfa = asin(sinalfa);
        PT O1;
        O1 = p + (p - q) * r1 / (r2 - r1);
        tmp1 = (p - O1) * cos(Alfa);
        tmp2 = (q - O1) * cos(Alfa);
        lin.fst = O1 + RotateCCW(tmp1, Alfa);
        lin.snd = O1 + RotateCCW(tmp2, Alfa);
        out.pb(lin);
        lin.fst = O1 + RotateCCW(tmp1, 2 * PI -
            Alfa);
        lin.snd = O1 + RotateCCW(tmp2, 2 * PI -
            Alfa);
    }
}

```

```

    out.pb(lin);
}
return out;
}

```

3.2 Convex Hull - Doubles

```

// INPUT: a vector of input points, unordered.
// OUTPUT: a vector of points in the convex
// hull,
// counterclockwise, starting with bottom
// left

```

```
#define REMOVE_REDUNDANT
```

```

typedef double T;
const T EPS = 1e-7;
struct PT {
    T x, y;
    PT() {}
    PT(T x, T y) : x(x), y(y) {}
    bool operator<(const PT &rhs) const {
        return mp(y,x) < mp(rhs.y,rhs.x);
    }
    bool operator==(const PT &rhs) const {
        return mp(y,x) == mp(rhs.y,rhs.x);
    }
};

T cross(PT p, PT q) { return p.x*q.y-p.y*q.x; }
T area2(PT a, PT b, PT c) {
    return cross(a,b) + cross(b,c) + cross(c,a);
}

#ifdef REMOVE_REDUNDANT
bool between(const PT &a, const PT &b, const PT
    &c) {
    return (fabs(area2(a,b,c)) < EPS &&
        (a.x-b.x)*(c.x-b.x) <= 0 &&
        (a.y-b.y)*(c.y-b.y) <= 0);
}
#endif

```

```

void ConvexHull(vector<PT> &pts) {
    sort(pts.begin(), pts.end());
    pts.erase(unique(pts.begin(), pts.end()),
               pts.end());
    vector<PT> up, dn;
    for (int i = 0; i < pts.size(); i++) {
        while (up.size() > 1 &&
               area2(up[up.size()-2], up.back(), pts[i])
               >= 0)
            up.pop_back();
        while (dn.size() > 1 &&
               area2(dn[dn.size()-2], dn.back(), pts[i])
               <= 0)
            dn.pop_back();
        up.push_back(pts[i]);
        dn.push_back(pts[i]);
    }
    pts = dn;
    for (int i = (int) up.size() - 2; i >= 1; i--)
        pts.push_back(up[i]);

#ifdef REMOVE_REDUNDANT
    if (pts.size() <= 2) return;
    dn.clear();
    dn.push_back(pts[0]);
    dn.push_back(pts[1]);
    for (int i = 2; i < pts.size(); i++) {
        if (between(dn[dn.size()-2],
                   dn[dn.size()-1], pts[i]))
            dn.pop_back();
        dn.push_back(pts[i]);
    }
    if (dn.size() >= 3 && between(dn.back(),
                                dn[0], dn[1])) {
        dn[0] = dn.back();
        dn.pop_back();
    }
    pts = dn;
#endif
}

```

3.3 Convex Hull - Enteros

```

ll dist(ii a, ii b){
    a.first -= b.first; a.second -= b.second;
    return ll(a.first)*a.first +
           ll(a.second)*a.second;
}

ll dot(ii o, ii a, ii b){
    return ll(a.first-o.first)*(b.first-o.first)
           +
           ll(a.second-o.second)*(b.second-o.second);
}

ll cross(ii o, ii a, ii b){
    return ll(a.first-o.first)*(b.second-o.second)
           - ll(a.second-o.second)*(b.first-o.first);
}

vector<ii> convexHull(vector<ii> &P){
    sort(P.begin(),P.end());
    P.erase(unique(P.begin(),P.end()),P.end());
    int n = sz(P);
    if(n == 1) return P;

    vector<ii> bot = {P[0]};
    FER(i,1,n){
        while(sz(bot) > 1 and
              cross(bot[sz(bot)-2],bot.back(),P[i])
              <= 0)
            bot.pop_back();
        bot.pb(P[i]);
    }
    bot.pop_back();

    vector<ii> up = {P[n-1]};
    for(int i = n-1; i >= 0; --i){
        while(sz(up) > 1 and
              cross(up[sz(up)-2],up.back(),P[i]) <= 0)
            up.pop_back();
        up.pb(P[i]);
    }
    up.pop_back();
}

```

```

    bot.insert(bot.end(),up.begin(),up.end());
    return bot;
}

```

3.4 Delaunay Triangulation

```

// Slow but simple Delaunay triangulation. Does
// not handle
// degenerate cases (from O'Rourke, Computational
// Geometry in C)
//
// Running time: O(n^4)
//
// INPUT:  x[] = x-coordinates
//         y[] = y-coordinates
//
// OUTPUT: triples = a vector containing m
//         triples of indices
//         corresponding to triangle vertices

```

```

#include<vector>
using namespace std;

typedef double T;

struct triple {
    int i, j, k;
    triple() {}
    triple(int i, int j, int k) : i(i), j(j), k(k)
    {}
};

vector<triple> delaunayTriangulation(vector<T>&
                                   x, vector<T>& y) {
    int n = x.size();
    vector<T> z(n);
    vector<triple> ret;

    for (int i = 0; i < n; i++)
        z[i] = x[i] * x[i] + y[i] * y[i];

    for (int i = 0; i < n-2; i++) {

```

```

for (int j = i+1; j < n; j++) {
for (int k = i+1; k < n; k++) {
    if (j == k) continue;
    double xn = (y[j]-y[i])*(z[k]-z[i]) -
                (y[k]-y[i])*(z[j]-z[i]);
    double yn = (x[k]-x[i])*(z[j]-z[i]) -
                (x[j]-x[i])*(z[k]-z[i]);
    double zn = (x[j]-x[i])*(y[k]-y[i]) -
                (x[k]-x[i])*(y[j]-y[i]);
    bool flag = zn < 0;
    for (int m = 0; flag && m < n; m++)
        flag = flag && ((x[m]-x[i])*xn +
                        (y[m]-y[i])*yn +
                        (z[m]-z[i])*zn <= 0);
    if (flag) ret.push_back(triple(i, j, k));
}
}
return ret;
}

int main(){
    T xs[]={0, 0, 1, 0.9};
    T ys[]={0, 1, 0, 0.9};
    vector<T> x(&xs[0], &xs[4]), y(&ys[0], &ys[4]);
    vector<triple> tri = delaunayTriangulation(x,
                                              y);

    //expected: 0 1 3
    //          0 3 2

    int i;
    for(i = 0; i < tri.size(); i++)
        printf("%d %d %d\n", tri[i].i, tri[i].j,
              tri[i].k);
    return 0;
}

```

3.5 Geometry Routines

// C++ routines for computational geometry

```

const ld INF = 1e100;
const ld EPS = 1e-12;
const ld PI = acos(-1);

struct PT {
    ld x, y;
    PT() {}
    PT(ld x, ld y) : x(x), y(y) {}
    PT(const PT &p) : x(p.x), y(p.y) {}
    PT operator + (const PT &p) const { return
        PT(x+p.x, y+p.y); }
    PT operator - (const PT &p) const { return
        PT(x-p.x, y-p.y); }
    PT operator * (ld c) const { return PT(x*c, y*c
        ); }
    PT operator / (ld c) const { return PT(x/c, y/c
        ); }
};

ld dot(PT p, PT q) { return p.x*q.x+p.y*q.y; }
ld norm(PT p) { return sqrt(dot(p,p)); }
ld dist2(PT p, PT q) { return dot(p-q,p-q); }
ld cross(PT p, PT q) { return p.x*q.y-p.y*q.x; }

ld angle(PT p){
    ld res = acos(p.x / norm(p));
    if (p.y > 0) return res;
    else return 2*PI - res;
}

// rotate a point CCW or CW around the origin
PT RotateCCW90(PT p) { return PT(-p.y,p.x); }
PT RotateCW90(PT p) { return PT(p.y,-p.x); }
PT RotateCCW(PT p, ld t) {
    return PT(p.x*cos(t)-p.y*sin(t),
              p.x*sin(t)+p.y*cos(t));
}

// project point c onto line through a and b
// assuming a != b
PT ProjectPointLine(PT a, PT b, PT c) {
    return a + (b-a)*dot(c-a, b-a)/dot(b-a, b-a);
}

```

```

// project point c onto line segment through a
// and b
PT ProjectPointSegment(PT a, PT b, PT c) {
    ld r = dot(b-a,b-a);
    if (fabs(r) < EPS) return a;
    r = dot(c-a, b-a)/r;
    if (r < 0) return a;
    if (r > 1) return b;
    return a + (b-a)*r;
}

// distance from c to segment between a and b
ld DistancePointSegment(PT a, PT b, PT c) {
    return sqrt(dist2(c, ProjectPointSegment(a, b,
        c)));
}

// distance between point (x,y,z) and plane
// ax+by+cz=d
ld DistancePointPlane(ld x, ld y, ld z,
                      ld a, ld b, ld c, ld d)
{
    return fabs(a*x+b*y+c*z-d)/sqrt(a*a+b*b+c*c);
}

// Whethes lines (a,b), (c,d) are
// parallel/collinear
bool LinesParallel(PT a, PT b, PT c, PT d) {
    return fabs(cross(b-a, c-d)) < EPS;
}

bool LinesCollinear(PT a, PT b, PT c, PT d) {
    return LinesParallel(a, b, c, d)
        && fabs(cross(a-b, a-c)) < EPS
        && fabs(cross(c-d, c-a)) < EPS;
}

// determine if line segment from a to b
// intersects with
// line segment from c to d
bool SegmentsIntersect(PT a, PT b, PT c, PT d) {
    if (LinesCollinear(a, b, c, d)) {
        if (dist2(a, c) < EPS || dist2(a, d) < EPS ||
            dist2(b, c) < EPS || dist2(b, d) < EPS)
            return true;
    }
}

```



```

    if (dot(c-a, c-b) > 0 && dot(d-a, d-b) > 0 &&
        dot(c-b, d-b) > 0) return false;
    return true;
}

if (cross(d-a, b-a) * cross(c-a, b-a) > 0)
    return false;
if (cross(a-c, d-c) * cross(b-c, d-c) > 0)
    return false;
return true;
}

// compute intersection of line passing through a
// and b
// with line passing through c and d, assuming
// that unique
// intersection exists; for segment intersection,
// check if
// segments intersect first
PT ComputeLineIntersection(PT a, PT b, PT c, PT
    d) {
    b=b-a; d=c-d; c=c-a;
    assert(dot(b, b) > EPS && dot(d, d) > EPS);
    return a + b*cross(c, d)/cross(b, d);
}

// compute center of circle given three points
PT ComputeCircleCenter(PT a, PT b, PT c) {
    b=(a+b)/2;
    c=(a+c)/2;
    return ComputeLineIntersection(b,
        b+RotateCW90(a-b), c, c+RotateCW90(a-c));
}

// determine if point is in a possibly non-convex
// polygon
// (by William Randolph Franklin); returns 1 for
// strictly
// interior points, 0 for strictly exterior
// points, and 0 or 1
// for the remaining points. Note that it is
// possible to
// convert this into an *exact* test using
// integer arithmetic

```

```

// by taking care of the division appropriately
// (making sure
// to deal with signs properly) and then by
// writing exact
// tests for checking point on polygon boundary
bool PointInPolygon(const vector<PT> &p, PT q) {
    bool c = 0;
    for (int i = 0; i < p.size(); i++){
        int j = (i+1)%p.size();
        if (((p[i].y <= q.y && q.y < p[j].y) ||
            (p[j].y <= q.y && q.y < p[i].y)) &&
            q.x < p[i].x + (p[j].x - p[i].x) * (q.y -
                p[i].y) /
                (p[j].y - p[i].y))
            c = !c;
        }
    return c;
}

// determine if point is on the boundary of a
// polygon
bool PointOnPolygon(const vector<PT> &p, PT q) {
    for (int i = 0; i < p.size(); i++){
        if (dist2(ProjectPointSegment(p[i],
            p[(i+1)%p.size()], q), q) < EPS)
            return true;
        }
    return false;
}

// compute intersection of line through points a
// and b with
// circle centered at c with radius r > 0
vector<PT> CircleLineIntersection(PT a, PT b, PT
    c, ld r) {
    vector<PT> ret;
    b = b-a;
    a = a-c;
    ld A = dot(b, b);
    ld B = dot(a, b);
    ld C = dot(a, a) - r*r;
    ld D = B*B - A*C;
    if (D < -EPS) return ret;
}

```

```

ret.push_back(c+a+b*(-B+sqrt(D+EPS))/A);
if (D > EPS)
    ret.push_back(c+a+b*(-B-sqrt(D))/A);
return ret;
}

// compute intersection of circle centered at a
// with radius r
// with circle centered at b with radius R
vector<PT> CircleCircleInter(PT a, PT b, ld r, ld
    R) {
    vector<PT> ret(0);
    ld d = sqrt(dist2(a, b));
    if (d > r+R + EPS || d+min(r, R) + EPS < max(r,
        R)) return ret;
    ld x = (d*d-R*R+r*r)/(2*d);
    ld y = sqrt(r*r-x*x);
    PT v = (b-a)/d;
    ret.push_back(a+v*x + RotateCCW90(v)*y);
    if (y > 0)
        ret.push_back(a+v*x - RotateCCW90(v)*y);
    return ret;
}

// Area or centroid of a (possibly nonconvex)
// polygon,
// assuming the coordinates are listed in a
// clockwise or
// counterclockwise order. Note that the centroid
// is often
// known as the "center of gravity" or "center of
// mass".
ld ComputeSignedArea(const vector<PT> &p) {
    ld area = 0;
    for(int i = 0; i < p.size(); i++) {
        int j = (i+1) % p.size();
        area += p[i].x*p[j].y - p[j].x*p[i].y;
    }
    return area / 2.0;
}

ld ComputeArea(const vector<PT> &p) {
    return fabs(ComputeSignedArea(p));
}

```

```

PT ComputeCentroid(const vector<PT> &p) {
    PT c(0,0);
    ld scale = 6.0 * ComputeSignedArea(p);
    for (int i = 0; i < p.size(); i++){
        int j = (i+1) % p.size();
        c = c + (p[i]+p[j])*(p[i].x*p[j].y -
            p[j].x*p[i].y);
    }
    return c / scale;
}

// Whether or not a given (CW or CCW) polygon is
// simple
bool IsSimple(const vector<PT> &p) {
    for (int i = 0; i < p.size(); i++) {
        for (int k = i+1; k < p.size(); k++) {
            int j = (i+1) % p.size();
            int l = (k+1) % p.size();
            if (i == l || j == k) continue;
            if (SegmentsIntersect(p[i], p[j], p[k], p[l]))
                return false;
        }
    }
    return true;
}

// Computes the circumcenter of a Triangle PQR
PT circumcenter(PT p, PT q, PT r) {
    PT a = p-r, b = q-r;
    PT c = PT(dot(a, (p + r)) / 2, dot(b, (q + r))
        / 2);
    return PT(dot(c, RotateCW90(PT(a.y, b.y))),
        dot(PT(a.x, b.x), RotateCW90(c))) / dot(a,
        RotateCW90(b));
}

```

3.6 Minkowski Sum

```

//Calcula suma de Minkowski en O(n + m)
//A y B deben estar en sentido antihorario
inline bool compare(PT a, PT b){

```

```

    // mas abajo, mas a la izquierda
    if(a.y < b.y) return 1;
    if(a.y == b.y) return a.x < b.x;
    return 0;
}

vector<PT> minkow_sum(const vector<PT>& a, const
    vector<PT>& b){
    vector< PT > out;
    out.clear();
    int lena = int(a.size());
    int lenb = int(b.size());
    int i = 0, j = 0;
    for(int q = 0; q < lena; ++q) if(compare(a[q],
        a[i])) i = q;
    for(int q = 0; q < lenb; ++q) if(compare(b[q],
        b[j])) j = q;
    ll pr;
    int nxti, nxtj;
    do{
        out.pb(a[i] + b[j]);
        nxti = (i + 1) % lena;
        nxtj = (j + 1) % lenb;
        pr = cross(a[nxti] - a[i], b[nxtj] - b[j]);
        if(pr > 0) i = nxti;
        else if(pr < 0) j = nxtj;
        else i = nxti, j = nxtj; // paralelas, subo
            en ambas
    }while((a[i] + b[j]) != out[0]);
    return out;
}

```

3.7 Polygon

```

int sgn(double x){return x<-EPS?-1:x>EPS;}
struct pol {
    int n; vector<PT> p;
    pol(){}
    pol(vector<PT> _p){p=_p;n=p.size();}
    double area(){
        double r=0.;
        FER(i,0,n) r+=p[i]%p[(i+1)%n];

```

```

        return abs(r)/2; // negative if CW, positive
            if CCW
    }
    PT centroid(){ // (barycenter)
        PT r(0,0);double t=0;
        FER(i,0,n){
            r=r+(p[i]+p[(i+1)%n])*(p[i]%p[(i+1)%n]);
            t+=p[i]%p[(i+1)%n];
        }
        return r/t/3;
    }
    bool has(PT q){ // O(n)
        FER(i,0,n)if(ln(p[i],p[(i+1)%n]).seghas(q))return
            true;
        int cnt=0;
        FER(i,0,n){
            int j=(i+1)%n;
            int k=sgn((q-p[j])%(p[i]-p[j]));
            int u=sgn(p[i].y-q.y),v=sgn(p[j].y-q.y);
            if(k>0&&u<0&&v>=0)cnt++;
            if(k<0&&v<0&&u>=0)cnt--;
        }
        return cnt!=0;
    }
    void normalize(){ // (call before haslog,
        remove collinear first)
        if(p[2].left(p[0],p[1]))
            reverse(p.begin(),p.end());
        int pi=min_element(p.begin(),
            p.end())-p.begin();
        vector<PT> s(n);
        FER(i,0,n)s[i]=p[(pi+i)%n];
        p.swap(s);
    }
    bool haslog(PT q){ // O(log(n)) only CONVEX.
        Call normalize first
        if(q.left(p[0],p[1])||q.left(p.back(),p[0]))return
            false;
        int a=1,b=p.size()-1; // returns true if
            point on boundary
        while(b-a>1){ // (change sign of EPS in
            left
            int c=(a+b)/2; // to return false in
                such case)

```

```

        if(!q.left(p[0],p[c]))a=c;
        else b=c;
    }
    return !q.left(p[a],p[a+1]);
}
PT farthest(PT v){ // O(log(n)) only CONVEX
    if(n<10){
        int k=0;
        FER(i,1,n)if(v*(p[i]-p[k])>EPS)k=i;
        return p[k];
    }
    if(n==SZ(p))p.pb(p[0]);
    PT a=p[1]-p[0];
    int s=0,e=n,ua=v*a>EPS;
    if(!ua&&v*(p[n-1]-p[0])<=EPS)return p[0];
    while(1){
        int m=(s+e)/2;PT c=p[m+1]-p[m];
        int uc=v*c>EPS;
        if(!uc&&v*(p[m-1]-p[m])<=EPS)return p[m];
        if(ua&&(!uc||v*(p[s]-p[m])>EPS))e=m;
        else if(ua||uc||v*(p[s]-p[m])>=-EPS)
            s=m,a=c,ua=uc;
        else e=m;
        assert(e>s+1);
    }
}
pol cut(ln l){ // cut CONVEX polygon by line l
    vector<PT> q; // returns part at left of l.pq
    FER(i,0,n){
        int d0=sgn(l.pq*(p[i]-l.p));
        int d1=sgn(l.pq*(p[(i+1)%n]-l.p));
        if(d0>=0)q.pb(p[i]);
        ln m(p[i],p[(i+1)%n]);
        if(d0*d1<0&&!(1/m))q.pb(l~m);
    }
    return pol(q);
}
double intercircle(circle c){ // area of
    intersection with circle
    double r=0.;
    FER(i,0,n){
        int j=(i+1)%n;double
        w=c.intertriangle(p[i],p[j]);
        if((p[j]-c.o)*(p[i]-c.o)>0)r+=w;
    }
}

```

```

        else r-=w;
    }
    return abs(r);
}
double callipers(){ // square distance of most
    distant points
    double r=0; // prereq: convex, ccw, NO
    COLLINEAR POINTS
    for(int i=0,j=n<2?0:1;i<j;++i){
        for(;;j=(j+1)%n){
            r=max(r,(p[i]-p[j]).norm2());
            if((p[(i+1)%n]-p[i])%
                (p[(j+1)%n]-p[j])<=EPS)break;
        }
    }
    return r;
}
};
// Dynamic convex hull trick
vector<pol> w;
void add(PT q){ // add(q), O(log^2(n))
    vector<PT> p={q};
    while(!w.empty()&&SZ(w.back().p)<2*SZ(p)){
        for(PT v:w.back().p)p.pb(v);
        w.pop_back();
    }
    w.pb(pol(chull(p)));
}
ll query(PT v){ // max(q*v:q in w), O(log^2(n))
    ll r=-INF;
    for(auto& p:w)r=max(r,p.farthest(v)*v);
    return r;
}

```

3.8 Rotating Callipers

/* Maximum Distance of a pair of points
in a Range Using Rotatin Calipers and Segment Tree
Note that to build the convex hull in a range
always there at least 2 points less for that
we can build the convexhull, for queries
we just construct what we need */

```

struct PT{
    ll x, y;
    PT(){}
    PT(ll x, ll y): x(x), y(y){}
};

PT points[1<<18];
ll tt[1<<18];

ll GetDist(PT &a, PT &b){
    return sqr(a.x-b.x)+sqr(a.y-b.y);
}

inline ll GetOri(ll id1, ll id2, ll id3){
    ll r=(points[id2].y-points[id1].y)*
        (points[id3].x-points[id1].x);
    r-=(points[id2].x-points[id1].x)*
        (points[id3].y-points[id1].y);
    return (r==0)? 0: ((r>0)? 1: -1);
}

inline bool compare(ll id1, ll id2){
    if(points[id1].x<points[id2].x) return true;
    if(points[id1].x==points[id2].x and
        points[id1].y<points[id2].y) return true;
    return false;
}

struct ConvexHull{
    vi uh, lh;
    ll count=0;
    inline void set(ll i){
        uh.resize(1, i);
        lh.resize(1, i);
        count=1;
    }
};
// Dinamic Segment Tree

struct ST{

```

```

vector<ConvexHull> t;
ll n;

inline ll Join(vi &v1, vi &v2, ll *tt){
    ll i=0, j=0, k=0, n1=sz(v1), n2=sz(v2);
    while(i<n1 && j<n2){
        if(compare(v1[i], v2[j])) tt[k++]=v1[i++];
        else tt[k++]=v2[j++];
    }
    while(i<n1) tt[k++]=v1[i++];
    while(j<n2) tt[k++]=v2[j++];
    return k;
}

inline ConvexHull Op(ConvexHull &t1,
    ConvexHull &t2){
    if(t1.count==0) return t2;
    if(t2.count==0) return t1;
    ConvexHull ty;
    ll count=Join(t1.uh, t2.uh, tt), l=0, r=0;
    FER(i,0,count){
        while(l>1 and GetOri(tt[l-2], tt[l-1],
            tt[i])<=0) --l;
        tt[l++]=tt[i];
    }
    ty.uh.insert(ty.uh.end(), tt, tt+l);
    count=Join(t1.lh, t2.lh, tt);
    FER(i,0,count){
        while(r>1 and GetOri(tt[r-2], tt[r-1],
            tt[i])>=0) --r;
        tt[r++]=tt[i];
    }
    ty.lh.insert(ty.lh.end(), tt, tt+r);
    ty.count=sz(ty.uh)+sz(ty.lh)-2;
    if(ty.count==0) ty.count=1;
    return ty;
}

ST(ll n): n(n), t(2*n){
    FER(i,0,n) t[i+n].set(i);
    IFR(i, n-1, 1) t[i]=Op(t[i<<1], t[i<<1|1]);
}

inline ConvexHull Query(ll l, ll r){

```

```

    ConvexHull ans;
    for(l+=n, r+=n; l<r; l>=1, r>=1){
        if(l&1) ans=Op(ans, t[l++]);
        if(r&1) ans=Op(t[--r], ans);
    }
    return ans;
}

inline ll que(ll l, ll r){
    if(r-l<=1) return GetDist(points[l],
        points[r]);
    ConvexHull cur=Query(l, r+1);
    ll lr=sz(cur.uh), rr=sz(cur.lh), i=0, j=rr-1;
    ll ans=0, curdist, val;
    while(i<lr-1 or j>0){
        PT &left=points[cur.uh[i]];
        PT &right=points[cur.lh[j]];
        curdist=GetDist(left, right);
        // first brute force
        if(curdist>ans) ans=curdist;
        if(i==lr-1) --j;
        else if(j==0) i++;
        else{
            // rotating calipers
            PT& t1=points[cur.uh[i+1]];
            PT& t2=points[cur.lh[j-1]];
            val=(t1.y-left.y)*(right.x-t2.x);
            val-=(t1.x-left.x)*(right.y-t2.y);
            if(val>0) i++;
            else --j;
        }
    }
    return ans;
}

int main(){
    fastio;
    ll n; cin>>n;
    FER(i,0,n) cin>>points[i].x>>points[i].y;
    ST st(n);
    ll q, l, r, froz; cin>>q;
    FER(i,0,q){

```

```

        cin>>l>>r; l--, r--;
        froz=st.que(l, r);
        cout<<froz<<endl;
    }
    return 0;
}

```

3.9 Voronoi Diagram

```

//Voronoi diagrams:  $O(n^2 \log n)$ 
//Convex hull:  $O(n \log n)$ 
// Requires PT, ComputeLineIntersection, dot,
    cross
#define MAXN 1024
double INF = 1e10;
double EPS = 1e-7;

typedef struct {
    int id;
    double x;
    double y;
    double ang;
} chp;

int n;
double x[MAXN], y[MAXN]; // Input points
chp inv[2*MAXN]; // Points after inversion for
    Hull
int vors;
int vor[MAXN]; // Set of points in convex hull;
    //starts at lefmost; last same as
    first!!
PT ans[MAXN][2];

int chpcmp(const void *aa, const void *bb) {
    double a = ((chp *)aa)->ang;
    double b = ((chp *)bb)->ang;
    if (a<b) return -1;
    else if (a>b) return 1;
    else return 0;
}

```

```

int orient(chp *a, chp *b, chp *c) {
    double s = a->x*(b->y-c->y) + b->x*(c->y-a->y) +
        c->x*(a->y-b->y);
    if (s>0) return 1;
    else if (s<0) return -1;
    else if (a->ang==b->ang && a->ang==c->ang)
        return -1;
    else return 0;
}

int convexHull(int n, chp *pt, int *ans) {
    int i, j, st, anses=0;

    qsort(pt, n, sizeof(chp), chpcmp);
    for (i=0; i<n; i++) pt[n+i] = pt[i];
    st = 0;
    for (i=1; i<n; i++) {
        if (pt[i].x<pt[st].x ||
            (pt[i].x==pt[st].x && pt[i].y<pt[st].y))
            st = i;
    }
    ans[anses++] = st;
    for (i=st+1; i<=st+n; i++) {
        for (j=anses-1; j; j--) {
            if (orient(pt+ans[j-1], pt+ans[j], pt+i)>=0)
                break;
            // Should change the above to strictly
            // greater,
            // if you don't want redundan points
            // If you want them, you might put an epsilon
            // in orient
        }
        ans[j+1] = i;
        anses = j+2;
    }
    for (i=0; i<anses; i++) ans[i] = pt[ans[i]].id;
    return anses;
}

int main(void) {
    int i, j, jj;
    double tmp;

    scanf("%d", &n);

```

```

    for (i=0; i<n; i++) scanf("%lf %lf", &x[i],
        &y[i]);
    for (i=0; i<n; i++) {
        x[n] = 2*(-INF)-x[i]; y[n] = y[i];
        x[n+1] = x[i]; y[n+1] = 2*INF-y[i];
        x[n+2] = 2*INF-x[i]; y[n+2] = y[i];
        x[n+3] = x[i]; y[n+3] = 2*(-INF)-y[i];
        for (j=0; j<n+4; j++) if (j!=i) {
            jj = j - (j>i);
            inv[jj].id = j;
            tmp = (x[j]-x[i])*(x[j]-x[i]) +
                (y[j]-y[i])*(y[j]-y[i]);
            inv[jj].x = (x[j]-x[i])/tmp;
            inv[jj].y = (y[j]-y[i])/tmp;
            inv[jj].ang = atan2(inv[jj].y, inv[jj].x);
        }
        vors = convexHull(n+3, inv, vor);
        // Build bisectors
        for (j=0; j<vors; j++) {
            ans[j][0].x = (x[i]+x[vor[j]])/2;
            ans[j][0].y = (y[i]+y[vor[j]])/2;
            ans[j][1].x = ans[j][0].x - (y[vor[j]]-y[i]);
            ans[j][1].y = ans[j][0].y + (x[vor[j]]-x[i]);
        }
        printf("Around (%lf, %lf)\n", x[i], y[i]);
        // List all intersections of the bisectors
        for (j=1; j<vors; j++) {
            PT vv;
            vv = ComputeLineIntersection(ans[j-1][0],
                ans[j-1][1],
                ans[j][0], ans[j][1]);
            printf("%lf, %lf\n", vv.x, vv.y);
        }
        printf("\n");
    }
    return 0;
}

```

4 Grafos

4.1 2-SAT

```

/* 2-Satisfiability
Autor: Phibrain
Instrucciones de uso:
    assig[i]: 0 pushear, 1: no pushear
    nodes para el nodo n >> true: 2*n, false:
        2*n + 1
*/

struct TWOSAT{
    vi adj[N], adjt[N];
    ll n;
    ll comp[N], vis[N], assig[N];
    vi eulerian;
    inline void dfs1(ll u){
        vis[u]=1;
        for(auto xd: adj[u]) if(vis[xd]==0) dfs1(xd);
        eulerian.pb(u);
    }
    inline void dfs2(ll u, ll rt){
        comp[u]=rt;
        for(auto xd: adjt[u]) if(comp[xd]==-1)
            dfs2(xd, rt);
    }
    inline ll solve(){
        fill(vis, 0), fill(comp, -1), fill(assig, 0);
        ll pos=0;
        FER(i,0,n) for(auto xd: adj[i])
            adjt[xd].pb(i);
        FER(i,0,n) if(vis[i]==0) dfs1(i);
        reverse(all(eulerian));
        for(auto xd: eulerian) if(comp[xd]==-1)
            dfs2(xd, pos++);
        FER(i,0,n/2) assig[i]=1;
        for(ll i=0; i<n; i+=2){
            if(comp[i]==comp[i+1]) return 0;
            assig[i/2]=(comp[i]>comp[i+1]);
        }
        return 1;
    }
};

```

4.2 BiconnectedComponents

```
vector<ii> st;
int tc,n,m,par[N];
int cnt,num[N],low[N];
vector<int> adj[N];
vector< vector<ii> > bcc;

void tarjan(int u, bool root = 0){
    num[u] = low[u] = cnt++;
    int child = 0;
    for(int v : adj[u]) if(v != par[u]){
        if(num[v] == -1){
            child++, par[v] = u;
            st.push_back(ii(u,v));
            tarjan(v);
            low[u] = min(low[u],low[v]);
            if((root && child > 1) || (!root &&
                num[u] <= low[v])){
                vector<ii> tmp;
                while(st.back() != ii(u,v))
                    tmp.pb(st.back()), st.pop_back();
                tmp.pb(st.back()), st.pop_back();
                bcc.pb(tmp);
            }
        }else if(num[v] < num[u]){
            low[u] = min(low[u],num[v]);
            st.pb(ii(u,v));
        }
    }
}

int main(){
    int TC,a,b;
    cin >> TC;
    for(tc = 1; tc <= TC; ++tc){
        cin >> n >> m;
        bcc.clear();
        for(int i = 0; i <= n; ++i)
            adj[i].clear();
```

```
        for(int i = 0; i < m; ++i){
            cin >> a >> b;
            adj[a].pb(b);
            adj[b].pb(a);
        }
        cnt = 0;
        for(int i = 0; i <= n; ++i)
            par[i] = num[i] = low[i] = -1;

        for(int i = 1; i <= n; ++i) if(num[i] == -1){
            tarjan(i,1);
            if(sz(st)) bcc.pb(st);
            st.clear();
        }
    }

    return 0;
}
```

4.3 Bridges and Articulation Points

```
/* Bridges and Articulation Points
Autor: Froz
Instrucciones de uso:
fill(id, -1) antes de usarlo
En bridge se guardaran los puentes
art[i] == True si el nodo i es articulacion
*/

const int N = 1e5 + 2;

int low[N], id[N], parent[N];
vector<int> adj[N];
int curr_id = 0;
int root, rootchild;

bool art[N];
vector<ii> bridges;

void dfs(int u) {
    low[u] = id[u] = curr_id++;
    for(int &v: adj[u]){
```

```
        if (id[v] == -1){
            parent[v] = u;
            if (u == root) rootchild++;
            dfs(v);
            if (low[v] >= id[u]) art[u] = true;
            if (low[v] > id[u]){
                bridges.pb({u, v});
            }
            low[u] = min(low[u], low[v]);
        }
        else if (v != parent[u]) low[u] =
            min(low[u], id[v]);
    }
}

//inside int main()
fill(id, -1);
FER(i, 0, n){
    if (id[i] == -1) {
        root = i; rootchild = 0; dfs(i);
        art[root] = (rootchild > 1);
    }
}
}
```

4.4 Chu Liu

// O(n * m) MST for directed graphs

```
typedef ll tw;const tw INF=1LL<<60;
struct edge {int src,dst;tw w;};
struct ChuLiu {
    int n,r;tw cost;bool found;
    vector<int> no,pr,mark;
    vector<vector<int> > comp,nx;
    vector<tw> mcost;
    vector<vector<edge> > h;
    ChuLiu(int n):n(n),h(n){}
    void add_edge(int x, int y, tw
        w){h[y].pb((edge){x,y,w});}
    void visit(int v, int s){
        if(mark[v]){
            vector<int> temp=no;found=true;
```

```

do {
    cost+=mcost[v];v=pr[v];
    if(v!=s)while(comp[v].size()>0){
        no[comp[v].back()]=s;
        comp[s].pb(comp[v].back());
        comp[v].pop_back();
    }
}while(v!=s);
for(int j:comp[s])if(j!=r)for(edge&
    e:h[j])
    if(no[e.src]!=s)e.w-=mcost[temp[j]];
}
mark[v]=true;
for(int
    i:nx[v])if(no[i]!=no[v]&&pr[no[i]]==v)
    if(!mark[no[i]]||i==s)
        visit(i,s);
}
tw doit(int _r){ // r: root (0(nm))
    r=_r;
    no.resize(n);comp.clear();comp.resize(n);
    FER(x,0,n)comp[x].pb(no[x]=x);
    for(cost=0;;){
        pr.clear();pr.resize(n,-1);
        mcost=vector<tw>(n,INF);
        FER(j,0,n)if(j!=r)for(edge e:h[j])
            if(no[e.src]!=no[j]&&e.w<mcost[no[j]])
                mcost[no[j]]=e.w,pr[no[j]]=no[e.src];
        nx.clear();nx.resize(n);
        FER(x,0,n)if(pr[x]>=0)nx[pr[x]].pb(x);
        bool stop=true;
        mark.clear();mark.resize(n);
        FER(x,0,n)if(x!=r&&!mark[x]&&
            !comp[x].empty()){
            found=false;visit(x,x);
            if(found)stop=false;
        }
        if(stop){
            FER(x,0,n)if(pr[x]>=0)cost+=mcost[x];
            return cost;
        }
    }
}
};

```

4.5 Erdos Gallai

```

/* Erdos Gallai Theorem
Autor: Froz
Instrucciones de uso:
    Dado un vector de N nodos con grado d[i],
    establecer si
    es posible tener un grafo con esa
    construccion.
*/

bool erdosgallai(vector<int> d) {
    sort(all(d), greater<int>());
    vector<ll> pd(sz(d));
    int n = sz(d), p = n-1;
    FER(i, 0, n) pd[i] = d[i] + (i > 0 ? pd[i-1] :
        0);
    FER(k, 1, n + 1){
        while(p >= 0 && d[p] < k) p--;
        ll sum;
        if (p >= k-1) sum = (p-k+1)*1ll*k + pd[n-1]
            - pd[p];
        else sum = pd[n-1] - pd[k-1];
        if (pd[k-1] > k*(k-1LL) + sum) return false;
    }
    return (pd[n-1] % 2 == 0);
}

```

4.6 Eulerian Path

```

// Finds Eulerian Path (visits every edge exactly
// once)
// CYCLE exists iff all edges even degree, all
// edges in
// same connected component.
// PATH exists iff cycle exists and once edge
// removed
// [ Hamiltonian (all vertices) is NP complete ]

```

```

struct Edge;
typedef list<Edge>::iterator iter;
struct Edge
{
    int next_vertex;
    iter reverse_edge;
    Edge(int next_vertex)
        :next_vertex(next_vertex) { }
};
const int max_vertices = ;
int num_vertices;
list<Edge> adj[max_vertices]; // adjacency list
vector<int> path;

void find_path(int v)
{
    while(adj[v].size() > 0)
    {
        int vn = adj[v].front().next_vertex;
        adj[vn].erase(adj[v].front().reverse_edge);
        adj[v].pop_front();
        find_path(vn);
    }
    path.push_back(v);
}

void add_edge(int a, int b)
{
    adj[a].push_front(Edge(b));
    iter ita = adj[a].begin();
    adj[b].push_front(Edge(a));
    iter itb = adj[b].begin();
    ita->reverse_edge = itb;
    itb->reverse_edge = ita;
}

```

4.7 Graph Utils

```

// Lexicographically smallest toposort

vector<int> g[MAXN];int n;
vector<int> tsort(){

```



```

vector<int> r;priority_queue<int> q;
vector<int> d(2*n,0);
FER(i,0,n)FER(j,0,g[i].size())d[g[i][j]]++;
FER(i,0,n)if(!d[i])q.push(-i);
while(!q.empty()){
    int x=-q.top();q.pop();r.pb(x);
    FER(i,0,g[x].size()){
        d[g[x][i]]--;
        if(!d[g[x][i]])q.push(-g[x][i]);
    }
}
return r; // if not DAG it will have less than
n elements
}

//Floyd - Warshall and Bellman Ford

// g[i][j]: weight of edge (i, j) or INF if
// there's no edge
// g[i][i]=0
ll g[MAXN][MAXN];int n;
void floyd(){ // O(n^3) . Replaces g with min
distances
    FER(k,0,n)FER(i,0,n)if(g[i][k]<INF)
        FER(j,0,n)if(g[k][j]<INF)
            g[i][j]=min(g[i][j],g[i][k]+g[k][j]);
}
bool inNegCycle(int v){return g[v][v]<0;}
bool hasNegCycle(int a, int b){ // true iff
there's neg cycle in between
    FER(i,0,n)if(g[a][i]<INF&&
        g[i][b]<INF&&g[i][i]<0)return true;
    return false;
}

```

4.8 Kruskal

```

/* Kruskal para Minimum Spanning Tree
Autor: Phibrain
*/

```

```

struct Kruskal{

```

```

ll id[N], n, ans;
vector<pair<ll,ii>> v;
vii adj[N];
inline void build(){
    FER(i,0,n) id[i]=i;
}
inline ll find(ll x){
    while(id[x]!=x) id[x]=id[id[x]], x=id[x];
    return x;
}
inline void unir(ll x, ll y){
    ll p=find(x), q=find(y);
    id[p]=id[q];
}
inline void clear(){
    fill(id,0);
    ans=0;
}
inline void go(ll l, ll r){
    FER(i,l,r){
        ll w=v[i].tm1, x=v[i].tm2, y=v[i].tm3;
        if(find(x)!=find(y)) ans+=w,
            unir(x,y),adj[x].pb({y,w}),
            adj[y].pb({x,w});
    }
}
};

```

4.9 Maximal Cliques

```

// Bron-Kerbosch algorithm for finding all the
// maximal cliques of a graph in O(3^(n/3))
// 3 ^ 13 = 1.6e6

// Call them using clique(0, (1LL << n) - 1, 0)
// n vertexs
ll adj[65];
// This algorithm finds all the maximal cliques
// containing an edge
// The cliques are found explicitly (the vertex
// of the cliques)
void clique(ll r, ll p, ll x) {

```

```

if (p == 0 && x == 0) {
    /* r is a maximal clique */
    /* Every 1 in r is a vertex of the clique
    Then, __builtin_popcountll(r) is the size of
    the clique*/
    return;
}
int pivot = -1;
int menor = INF;
for (int i = 0; i < n; i++) {
    if ( ((1LL << i) & p) || ((1LL << i) & x) ) {
        int x = __builtin_popcountll(p &
            (~adj[i]));
        if (x < menor) {
            pivot = i;
            menor = x;
        }
    }
}
for (int i = 0; i < n; i++) {
    if ((1LL << i) & p) {
        if (pivot != -1 && adj[pivot] & (1LL <<
            i)) continue;
        clique(r | (1LL << i), p & adj[i], x &
            adj[i]);
        p = p ^ (1LL << i);
        x = x | (1LL << i);
    }
}
}
}

```

```

// This one has the same idea, but is faster
// However, it only finds the size of the cliques
void clique2(int r, ll p, ll x){
    if(p == 0 && x == 0){
        // r is the size of the clique
    }
    if(p == 0) return;
    int u = __builtin_ctzll(p | x);
    ll c = p & ~ adj[u];
    while(c){
        int v = __builtin_ctzll(c); //Number of
        trailing zeros
        clique(r + 1, p & adj[v], x & adj[v]);
    }
}

```



```

    p ^= (1LL << v);
    x |= (1LL << v);
    c ^= (1LL << v);
}
}

```

4.10 Planar Dual Graph

```

vector<int> g[MAXN]; int n; // input graph (must
    be connected)
vector<int> gd[MAXN]; int nd; // output graph
vector<int> nodes[MAXN]; // nodes delimiting
    region (in CW order)
map<pair<int,int>,int> ps,es;
void get_dual(vector<PT> p){ // p: points
    corresponding to nodes
    ps.clear(); es.clear();
    FER(x,0,n){
        Cmp pc(p[x]); // (radial order of points)
        auto comp=[&](int a, int b){return
            pc(p[a],p[b]);};
        sort(g[x].begin(),g[x].end(),comp);
        FER(i,0,g[x].size())ps[mp(x,g[x][i])]=i;
    }
    nd=0;
    FER(xx,0,n)for(auto
        yy:g[xx])if(!es.count(mp(xx,yy))){
        int
            x=xx,y=yy;gd[nd].clear();nodes[nd].clear();
        while(!es.count(mp(x,y))){
            es[mp(x,y)]=nd;nodes[nd].pb(y);
            int z=g[y][(ps[mp(y,x)]+1)%g[y].size()];
            x=y;y=z;
        }
        nd++;
    }
    for(auto p:es){
        pair<int,int> q=mp(p.fst.snd,p.fst.fst);
        assert(es.count(q));
        if(es[q]!=p.snd)gd[p.snd].pb(es[q]);
    }
    FER(i,0,nd){

```

```

        sort(gd[i].begin(),gd[i].end());
        gd[i].erase(unique(gd[i].begin(),
            gd[i].end()),gd[i].end());
    }
}

```

4.11 Tarjan Strongly Connected Components

```

/* Tarjan's SCC
Autor: Halim
Instrucciones de uso:
    Inicializar d[N] en -1!
    No olvidarse de limpiar variables
    Es como un dfs, lanzar un for y correr si
        nodo actual
    no esta visitado.
*/
vector<int> g[N];
int d[N], low[N], scc[N];
bool stacked[N];
stack<int> s;
int ticks, current_scc;

void tarjan(int u){
    d[u] = low[u] = ticks++;
    s.push(u);
    stacked[u] = true;
    const vector<int> &out = g[u];
    for (int k=0, m=out.size(); k<m; ++k){
        const int &v = out[k];
        if (d[v] == -1){
            tarjan(v);
            low[u] = min(low[u], low[v]);
        }else if (stacked[v]){
            low[u] = min(low[u], low[v]);
        }
    }
    if (d[u] == low[u]){
        int v;

```

```

do{
    v = s.top();
    s.pop();
    stacked[v] = false;
    scc[v] = current_scc;
}while (u != v);
current_scc++;
}
}

```

4.12 TreeIsomorphism

```

vector< pair< vector<int>,int > >
Radixsort(vector< vector<int> > &vec){
    int lmax = 0, m = 0;
    for(vector<int> &v : vec){
        lmax = max(lmax,sz(v));
        for(int &x : v) m = max(m,x+1);
    }
    deque<int> dq;
    vector< queue<int> > q(m);
    vector< vector<int> > hold(lmax);
    for(int i = 0; i < sz(vec); ++i){
        hold[sz(vec[i])-1].pb(i);
    }
    for(int i = lmax-1; i >= 0; --i){
        for(int x : hold[i]){
            dq.push_front(x);
            while(sz(dq)){
                int x = dq.front();
                dq.pop_front();
                q[vec[x][i]].push(x);
            }
            for(int j = 0; j < m; ++j){
                while(sz(q[j])){
                    dq.push_back(q[j].front());
                    q[j].pop();
                }
            }
        }
    }
}

```

```

vector< pair< vector<int>,int > >
    ans(sz(vec));
for(int i = 0; i < sz(vec); ++i){
    int x = dq.front(); dq.pop_front();
    ans[i] = mp(vec[x],x);
}

return ans;
}

int vist[N];
int lev1,lev2,id1[N],id2[N],pai1[N],pai2[N];
vector<int> noleaf1[N],noleaf2[N];
vector<int> leaf1[N],leaf2[N],tup1[N],tup2[N];

void dfs1(Tree &tree, int u, int p, int dep = 0){
    pai1[u] = p;
    lev1 = max(lev1,dep);
    vist[u] = 1;
    int tam = 0;
    for(int v : tree[u]) if(!vist[v]){
        dfs1(tree,v,u,dep+1);
        tam++;
    }
    if(tam >= 1) noleaf1[dep].pb(u);
    else leaf1[dep].pb(u), id1[u] = 0;
}

void dfs2(Tree &tree, int u, int p, int dep = 0){
    pai2[u] = p;
    lev2 = max(lev2,dep);
    vist[u] = 1;
    int tam = 0;
    for(int v : tree[u]) if(!vist[v]){
        dfs2(tree,v,u,dep+1);
        tam++;
    }
    if(tam >= 1) noleaf2[dep].pb(u);
    else leaf2[dep].pb(u), id2[u] = 0;
}

bool isomorphic(Tree &t1, int r1, Tree &t2, int
    r2, int ban){
    for(int i = 0; i < N; ++i){

```

```

        vist[i] = 0;
        noleaf1[i].clear(),
            noleaf2[i].clear();
        leaf1[i].clear(), leaf2[i].clear();
        tup1[i].clear(), tup2[i].clear();
    }
    vist[ban] = 1;
    lev1 = lev2 = 0;
    dfs1(t1,r1,r1);
    FER(i,0,N) vist[i] = 0; vist[ban] = 1;
    dfs2(t2,r2,r2);
    if(lev1 != lev2) return false;
    vector<int> l1,l2;
    for(int x : leaf1[lev1]) l1.pb(x);
    for(int x : leaf2[lev2]) l2.pb(x);
    if(sz(l1) != sz(l2)) return false;

    for(int i = lev1-1; i >= 0; --i){
        vector< vector<int> > v1,v2;
        for(int u : l1){
            tup1[pai1[u]].pb(id1[u]);
        }
        for(int u : l2){
            tup2[pai2[u]].pb(id2[u]);
        }
        for(int u : noleaf1[i])
            v1.pb(tup1[u]);
        for(int u : noleaf2[i])
            v2.pb(tup2[u]);
        if(sz(v1) != sz(v2)) return false;

        vector< pair< vector<int>,int > >
            res1, res2;
        res1 = Radixsort(v1);
        res2 = Radixsort(v2);
        int cnt = 1;
        l1.clear(); l2.clear();

        for(int x : leaf1[i]) l1.pb(x);
        for(int x : leaf2[i]) l2.pb(x);
        for(int j = 0; j < sz(res1); ++j){
            if(res1[j].first !=
                res2[j].first) return
                false;

```

```

            if(j > 0 and res1[j].first
                != res1[j-1].first)
                cnt++;
            l1.pb(noleaf1[i][res1[j].second]);
            l2.pb(noleaf2[i][res2[j].second]);
            id1[noleaf1[i][res1[j].second]]
                = cnt;
            id2[noleaf2[i][res2[j].second]]
                = cnt;
        }
    }
    return true;
}

```

4.13 Weighted Eulerian Path

```

/*
    Dado un grafo dirigido con pesos y 2 nodos
    "a", "b"
    encuentra todos los caminos de "a" hacia "b"

    la idea clave es vacear todos los ciclos
    recursivamente
    mientras se hace el dfs de "a" hacia "b"
    ... El grafo esta guardado en graph[] [] y
    adj1[]

*/

ll graph[N1][N1];
vii adj1[N1];

vi ands, cycles[N1], path[N1], pru[N1], cam[N1];
vector<tri> tnt;
ll vis[N1], mark[N1], visC[N1], enc[N1];

inline void GoBack(ll u, ll v, ll &flag, vi &vec){
    if(flag) return;
    enc[u]++;
    vec.pb(u);
    for(auto xd: adj1[u]){

```

```

    if(flag) return;
    if(!graph[u][xd.ff]) continue;
    if(xd.ff==v){
        flag++;
        if(flag==1) vec.pb(v);
        return;
    }
    if(!enc[xd.ff]){
        GoBack(xd.ff, v, flag, vec);
        if(!flag) vec.pop_back();
    }
}

}

inline void fillout(ll u){
    vi cur;
    ll ans, flag;
    while(!visC[u]){
        flag=0;
        cur.clear();
        fill(enc, 0);
        GoBack(u, u, flag, cur);
        if(flag){
            ans=INF;
            FER(i, 1, sz(cur)) ans=min(ans,
                graph[cur[i-1]][cur[i]]);
            FER(i, 1, sz(cur))
                graph[cur[i-1]][cur[i]]-=ans;
            FER(j, 0, ans) FER(i, 0, sz(cur)-1)
                cycles[u].pb(cur[i]);
        }
        else{
            visC[u]++;
            break;
        }
    }
}

inline void dfs(ll u, ll v){
    if(mark[u]) return;
    mark[u]++;
    if(!visC[u]) fillout(u);

```

```

    for(auto xd: cycles[u]) {
        if(!mark[xd]) dfs(xd, v);
        path[v].pb(xd);
    }
}

inline void findallpaths(ll u, ll v, ll &flag){
    if(flag) return;
    vis[u]++;
    if(!mark[u]){
        dfs(u, u);
        path[u].pb(u);
        for(auto xd: path[u]) ands.pb(xd);
    }
    else ands.pb(u);
    for(auto xd: adj1[u]){
        if(flag) return;
        if(graph[u][xd.ff]==0) continue;
        if(xd.ff==v){
            flag++;
            if(flag==1) ands.pb(v), graph[u][xd.ff]--;
            return;
        }
        if(!vis[xd.ff]){
            graph[u][xd.ff]--;
            findallpaths(xd.ff, v, flag);
        }
    }
}

int main(){
    fastio;
    ll cnt=0;
    FER(i, 1, n+1) cnt+=graph[a][i];
    FER(i, 1, n+1) cnt-=graph[i][a];
    cout<<cnt<<endl;
    fill(mark, 0);
    FER(k, 0, cnt){
        FER(i, 1, n+1) vis[i]=0;
        ands.clear();
        t=0;
        findallpaths(a, b, t);
        for(auto xd: ands) cout<<xd<<" "; cout<<endl;
    }
}

```

```

    }
    return 0;
}

```

5 Math

5.1 Baby Step Giant Step

```

// Given a, b and m, find a value "x" that
// a^x=(b)mod(m)
// Complexity in sqrt(m)

inline ll GetExpo(ll a, ll b, ll c){
    ll n=sqrt(c)+1;
    ll ans=1, cur;
    map<ll, ll> m;
    FER(i, 1, n+1) ans=(ans*a)%c;
    cur=ans;
    FER(i, 1, n+1){
        if(!m[cur]) m[cur]=i;
        cur=(cur*ans)%c;
    }
    cur=b;
    FER(i, 0, n){
        if(m[cur]){
            return m[cur]*n-i;
        }
        cur=(cur*a)%c;
    }
}

int main(){
    fastio;
    ll t, ans; cin>>t;
    while(t--){
        ll a, b, m; cin>>a>>b>>m;
        if(a==b%m) {cout<<1<<endl; continue;}
        if(b==1 && m!=1) {cout<<0<<endl; continue;}
        ans=GetExpo(a, b, m);
        cout<<ans<<endl;
    }
}

```

```

    return 0;
}

```

5.2 Chinese Remainder Theorem

```

/* Chinese remainder theorem.
   Find z such that z % x[i] = a[i] for all i.
*/
ll crt(vector<ll> &a, vector<ll> &x) {
    ll z = 0;
    ll n = 1;
    for (int i = 0; i < x.size(); ++i)
        n *= x[i];

    for (int i = 0; i < a.size(); ++i) {
        ll tmp = (a[i] * (n / x[i])) % n;
        tmp = (tmp * mod_inv(n / x[i], x[i])) % n;
        z = (z + tmp) % n;
    }

    return (z + n) % n;
}

```

5.3 Cribas

```

// Criba en O(n)
// p[i] indica el valor del primo i-esimo
// A[i] indica que el menor factor primo de i
// es el primo A[i] - esimo
#define MAXN 100000

int A[MAXN + 1], p[MAXN + 1], pc = 0;

void sieve()
{
    for(int i=2; i<=MAXN; i++){
        if(!A[i]) p[A[i] = ++pc] = i;
        for(int j=1; j<=A[i] && i*p[j]<=MAXN; j++)
            A[i*p[j]] = j;
    }
}

```

```

}

//Criba para phi
int phi[MAX];

void CribaEuler(){
    FER(i,0, MAX) primo[i] = 1, phi[i] = 1;
    primo[0] = primo[1] = false;
    FER(i,2,MAX){
        if(primo[i]){
            phi[i] = i - 1;
            for(int j = i+i; j < MAX; j += i){
                primo[j] = false;
                int pot = 1, aux = j/i;
                while( aux % i == 0 ){
                    aux /= i, pot *= i;
                }
                phi[j] *= (i-1)*pot ;
            }
        }
    }
}

```

5.4 Euler Totient

```

//Euler Totient
//Finds all k <= n where gcd(k,n) == 1

int phi(int n){
    int res = n;
    for(int p = 2; p*p <= n; p++){
        if(n % p == 0){
            while(n % p == 0)
                n/=p;
            res -= res/p;
        }
    }
    if (n > 1) res -= res/n;
    return res;
}

```

5.5 FFT

```

//FFT

#define pi acos(-1)
struct cpx{
    double a, b;
    cpx() : a(0.0), b(0.0){}
    cpx(double a, double b) : a(a), b(b){}
    const cpx operator+(const cpx &c) const{return
        cpx(a+c.a, b+c.b);}
    const cpx operator-(const cpx &c) const{return
        cpx(a-c.a, b-c.b);}
    const cpx operator*(const cpx &c) const{return
        cpx(a*c.a-b*c.b, a*c.b+b*c.a);}
};

struct FFT{
    vector<cpx> data, roots;
    vi rev;
    ll s, n;
    inline void init(ll pw){
        s=pw, n=(1<<s);
        rev=vi(n), data=vector<cpx> (n),
            roots=vector<cpx> (n+1);
        FER(i,0,n) FER(j,0,s) if((i & (1<<j))!=0)
            rev[i]+=(1<<(s-j-1));
        roots[0]=cpx(1, 0);
        cpx mult=cpx(cos(2*pi/n), sin(2*pi/n));
        FER(i,1,n+1) roots[i]=roots[i-1]*mult;
    }
    inline void transform(bool inverse=false){
        vector<cpx> temp(n);
        FER(i,0,n) temp[i]=data[rev[i]];
        FER(i,0,n) data[i]=temp[i];
        FER(i,1,s+1){
            ll m=(1<<i), md2=m/2;
            ll start=0, incr=(1<<(s-i));
            cpx t;
            if(inverse) start=n, incr*=-1;
            for(ll k=0; k<n; k+=m){
                ll idx=start;
                FER(j,k,md2+k){
                    t=roots[idx]*data[j+md2];

```

```

        idx+=incr;
        data[j+md2]=data[j]-t;
        data[j]=data[j]+t;
    }
}
}
if(inverse) FER(i,0,n) data[i].a/=n,
data[i].b/=n;
}
};
inline void build(vi &a, FFT &f, ll y, ll n){
    f.init(y);
    FER(i, 0, sz(a)) f.data[i]=cpx(a[i], 0);
    FER(i,sz(a), n) f.data[i]=cpx(0, 0);
    f.transform();
}
inline vi GetConvolution(vi &a, vi &b){
    ll t1=sz(a), t2=sz(b), ta=t1+t2-1, x=0, n;
    while((1<<x)<ta) x++;
    n=1<<x;
    FFT f1, f2;
    build(a, f1, x, n), build(b, f2, x, n);
    FER(i,0,n) f1.data[i]=f1.data[i]*f2.data[i];
    f1.transform(true);
    vi froz(ta);
    FER(i,0,ta) froz[i]=(ll) (f1.data[i].a+0.5);
    return froz;
}
int main(){
    fastio;
    ll t; cin>>t;
    while(t--){
        ll n; cin>>n;
        vi v1, v2;
        FER(i,0,n+1){
            ll x; cin>>x;
            v1.pb(x);
        }
        FER(i,0,n+1) {
            ll x; cin>>x;
            v2.pb(x);
        }
        vi c=GetConvolution(v1, v2);
        for(auto xd: c) cout<<xd<<" "; cout<<endl;
    }
}

```

```

    }
    return 0;
}

```

5.6 FWHTAnd

```

// FWHT And
inline ll bp(ll a, ll n){
    if(n==0) return 1;
    if(n&1) return (a*bp(a, n-1))%mod;
    ll x=bp(a, n>>1);
    return (sqr(x)%mod+mod)%mod;
}

// Fast Walsh-Hadamard Transform for And
// multiplication
// take note about the matrix associated
// |-1 +1|
// |+1 +0|
// The inverse is not the same as the matrix, so
// it's necessary
// to have a bool parameter in the convolution

struct FWHTAnd{
    ll n;
    inline void GetInverse(vi &v, ll flag){
        n=1;
        while(n<sz(v)) n<<=1;
        v.resize(n);
        for(ll len=1; (len<<1)<=n; len<<=1){
            for(ll i=0; i<n; i+=(len<<1)){
                FER(j,0,len){
                    ll a=v[i+j], b=v[i+j+len];
                    if(flag){
                        v[i+j]=b;
                        v[i+j+len]=a+b;
                        v[i+len+j]%=mod;
                    }
                    else{
                        v[i+j]=mod-a+b;
                        v[i+j+len]=a;
                    }
                }
            }
        }
    }
}

```

```

        v[i+j]%=mod;
    }
}
}
}

}
}fwht;
int main(){
    fastio;
    ll n, top=0, ans=0; cin>>n;
    vi v1, v2;
    FER(i,0,n){
        ll x; cin>>x;
        top=max(top, x);
        v1.pb(x);
    }
    top++;
    v2.resize(top);
    v2[0]=1;
    for(auto xd: v1) v2[xd]=1;
    fwht.GetInverse(v2, 1);
    FER(i,0,fwht.n) v2[i]=bp(v2[i], n);
    fwht.GetInverse(v2, 0);
    FER(i,0,fwht.n) if(v2[i]) ans++;
    cout<<ans<<endl;
    return 0;
}

```

5.7 FWHTXor

```

// FWHT XOR
inline ll bp(ll a, ll n){
    if(n==0) return 1;
    if(n&1) return (a*bp(a, n-1))%mod;
    ll x=bp(a, n>>1);
    return (sqr(x)%mod+mod)%mod;
}

// Fast Walsh-Hadamard Transform for xor
// multiplication

```

```

// take note about the matrix associated
// |+1 +1|
// |+1 -1|
// The inverse is the same as the matrix, so it's
// not necessary
// to have a bool parameter in the convolution

struct FWHTXor{
    ll n;
    inline void GetInverse(vi &v){
        n=1;
        while(n<sz(v)) n<<=1;
        v.resize(n);
        for(ll len=1; (len<<1)<=n; len<<=1){
            for(ll i=0; i<n; i+=(len<<1)){
                FER(j,0,len){
                    ll a=v[i+j], b=v[i+j+len];
                    v[i+j]=(a+b)%mod;
                    v[i+j+len]=(mod+a-b)%mod;
                }
            }
        }
    }
}fwht;
int main(){
    fastio;
    ll n, k; cin>>n>>k;
    vi v;
    FER(i,0,k+1) v.pb(1);
    fwht.GetInverse(v);
    FER(i,0,fwht.n) v[i]=bp(v[i], n);
    fwht.GetInverse(v);
    ll inv=bp(fwht.n, mod-2), ans=0;
    FER(i,1,fwht.n){
        ans+=(v[i]*inv)%mod;
        ans%=mod;
    }
    cout<<ans<<endl;
    return 0;
}

```

5.8 Gauss Jordan

```

// O(n^3) Gauss-Jordan elimination with full
// pivoting.
// Takes a[][] nxn, b[][] nxm
// Returns b = a^{-1}b in b, a^{-1} in a,
// determinant
const double EPS = 1e-10;
typedef double T;
typedef vector<vector<double>> vvt;

T GaussJordan(vvt &a, vvt &b) {
    const int n = a.size();
    const int m = b[0].size();
    vi irow(n), icol(n), ipiv(n);
    T det = 1;

    for (int i = 0; i < n; i++) {
        int pj = -1, pk = -1;
        for (int j = 0; j < n; j++) if (!ipiv[j])
            for (int k = 0; k < n; k++) if (!ipiv[k])
                if (pj == -1 || fabs(a[j][k]) >
                    fabs(a[pj][pk])) {
                    pj = j; pk = k;
                }
        if (fabs(a[pj][pk]) < EPS) {
            cerr << "Matrix is singular." << endl;
            exit(0);
        }
        ipiv[pk]++;
        swap(a[pj], a[pk]);
        swap(b[pj], b[pk]);
        if (pj != pk) det *= -1;
        irow[i] = pj;
        icol[i] = pk;

        T c = 1.0 / a[pk][pk];
        det *= a[pk][pk];
        a[pk][pk] = 1.0;
        for (int p = 0; p < n; p++) a[pk][p] *= c;
        for (int p = 0; p < m; p++) b[pk][p] *= c;
        for (int p = 0; p < n; p++) if (p != pk) {
            c = a[p][pk];

```

```

            a[p][pk] = 0;
            for (int q = 0; q < n; q++) a[p][q] -=
                a[pk][q] * c;
            for (int q = 0; q < m; q++) b[p][q] -=
                b[pk][q] * c;
        }
    }

    for (int p = n-1; p >= 0; p--) if (irow[p] !=
        icol[p]) {
        for (int k = 0; k < n; k++)
            swap(a[k][irow[p]], a[k][icol[p]]);
    }
    return det;
}

```

5.9 Inverso Modular

```

/** Inverso Modular **/
#define MAX 100
#define MOD 1000000009

ll inverso[MAX];

void inv(){
    inverso[1] = 1;
    FER(i,2,MAX) inverso[i] = ( (MOD-MOD/i) *
        inverso[MOD%i] ) % MOD;
}

```

5.10 Matrix Determinant

```

/* Matrix Determinant */

ll matrix[1<<8][1<<8];

inline ll FindDeterminant(ll n, ll m){
    FER(i,0,n) FER(j,0,n) {
        matrix[i][j]%m;
        matrix[i][j]+=m;
    }
}

```

```

    if(matrix[i][j]>=m) matrix[i][j]-=m;
}
ll res=1;
FER(i,0,n){
    if(!matrix[i][i]){
        bool flag=false;
        FER(j,i+1,n){
            if(matrix[j][i]){
                flag=true;
                FER(k,i,n) swap(matrix[i][k],
                    matrix[j][k]);
                res=-res;
                break;
            }
        }
        if(!flag) return 0;
    }
    FER(j,i+1,n){
        while(matrix[j][i]){
            ll t=matrix[i][i]/matrix[j][i];
            FER(k,i,n){
                matrix[i][k]=(matrix[i][k]-
                    t*matrix[j][k])%m;
                swap(matrix[i][k], matrix[j][k]);
            }
            res=-res;
        }
    }
    res=(res*matrix[i][i])%m;
}
return (res+m)%m;
}

```

5.11 Matrix Exponentiation

```

void sum(ll &a, ll b){
    a += b;
    if(a >= MOD) a -= MOD;
}

void Init(ll a[N][N]){

```

```

    FER(i,0,N) FER(j,0,N) a[i][j] = (i == j);
}

void Cpy(ll a[N][N], ll b[N][N]){
    FER(i,0,N) FER(j,0,N) b[i][j] = a[i][j];
}

void Mult(ll a[N][N], ll b[N][N], ll c[N][N]){
    FER(i,0,N) FER(j,0,N) c[i][j] = 0;
    FER(i,0,N) FER(j,0,N) FER(k,0,N)
        sum(c[i][j], (a[i][k]*b[k][j])%MOD);
}

void BinPow(ll a[N][N], ll p, ll ans[N][N]){
    ll aux[N][N]; Init(ans);
    while(p){
        if(p&1LL) Mult(ans,a,aux), Cpy(aux,ans);
        p >>= 1; Mult(a,a,aux); Cpy(aux,a);
    }
}

```

5.12 Miller Rabin

```

//Miller-Rabin primality test

ll pow(ll a, ll b, ll c){
    ll ans = 1;
    while(b){
        if(b&1) ans = (1LL*ans*a)%c;
        a = (1LL*a*a)%c;
        b >>= 1;
    }
    return ans;
}

bool miller(ll p, ll it = 10){
    if(p<2) return 0;
    if(p!=2 && (p&1) == 0) return 0;
    ll s=p-1;
    while((s&1) == 0) s>>=1;
    while(it--){
        ll a = rand()%(p-1)+1, temp = s;

```

```

        ll mod = pow(a,temp,p);
        while(temp!= p-1 && mod!=1 && mod!=p-1){
            mod = (1LL*mod*mod)%p;
            temp<<=1;
        }
        if(mod!=p-1 && (temp&1) == 0) return 0;
    }
    return 1;
}

```

5.13 Mobius Function

```

// Mobius function

vi mo, lp, primes;
ll ar[1<<20], cnt[1<<20];

inline void ConstructMobius(ll n){
    lp.resize(n+1, -1), mo.resize(n+1, -1);
    mo[1]=1;
    FER(i, 2, n+1){
        if(lp[i]==-1) lp[i]=i, primes.pb(i);
        ll j=0;
        for(; j<sz(primes) && primes[j]*i<=n &&
            primes[j]< lp[i]; j++){
            lp[primes[j]*i]=primes[j],
            mo[primes[j]*i]=-mo[i];
        }
        if(primes[j]*i<=n) mo[primes[j]*i]=0;
    }
}

int main(){
    fastio;
    ConstructMobius(1e6);
    ll n; cin>>n;
    FER(i,0,n) cin>>ar[i], cnt[ar[i]]++;
    ll ans=0;
    FER(i, 1, 1e6+1){
        for(ll j=i<<1; j<=1e6; j+=i) cnt[i]+=cnt[j];
    }
}

```

```

FER(i, 1, 1e6+1)
    ans+=mo[i]*(sqr(cnt[i])-cnt[i])*(cnt[i]-2)/6;
cout<<ans<<endl;
return 0;
}

```

5.14 NTTDFT

```
//NTT
```

```

struct NTT{
    ll root_1, root, root_pw;
    inline ll bp(ll a, ll b){
        if(b==0) return 1;
        if(b&1) return (a*(bp(a, b-1)))%mod;
        ll x=bp(a, b>>1)%mod;
        x=(x+mod)%mod;
        return ((x*x)%mod+mod)%mod;
    }
    inline ll inverse(ll a){
        return bp(a, mod-2);
    }
    inline void build(){
        // Si: c*pow(2, k)+1=mod
        // Entonces root cumple:
        // pow(root, 1<<(pow(2, k))=1mod
        // Mejor complejidad=Primary root test.
        root_pw=1<<23;
        FER(i,2,1e5+1) if(bp(i, root_pw)==1){
            root=i; break;}
        root_1=inverse(root);
    }
    inline void fft(vi &a, bool inv){
        ll n=sz(a);
        for(ll i=1, j=0; i<n; i++){
            ll bt=n>>1;
            for(; j<bt; bt>>=1) j^=bt;
            j^=bt;
            if(i<j) swap(a[i], a[j]);
        }
        for(ll len=2; len<=n; len<<=1){
            ll wlen=inv? root_1: root;

```

```

            for(ll i=len; i<root_pw; i<<=1)
                wlen=(sqr(wlen)%mod+mod)%mod;
            for(ll i=0; i<n; i+=len){
                ll w=1;
                for(ll j=0; j<len/2; j++){
                    ll u=a[i+j], v=(a[i+j+len/2]*w)%mod;
                    a[i+j]=u+v<mod? u+v: u+v-mod;
                    a[i+j+len/2]=u-v>=0? u-v: u-v+mod;
                    w=((w*wlen)%mod+mod)%mod;
                }
            }
        }
        if(inv){
            ll n_1=inverse(sz(a));
            for(auto &xd: a)
                xd=((xd*n_1)%mod+mod)%mod;
        }
    }
    inline vi GetConvolution(vi a, vi b){
        ll n=1;
        while(n<sz(a)+sz(b)) n<<=1;
        a.resize(n), b.resize(n);
        fft(a, false), fft(b, false);
        FER(i,0,n) a[i]=((a[i]*b[i])%mod+mod)%mod;
        fft(a, true);
        return a;
    }
    inline vi BinPolyExp(vi &a, ll b){
        if(b==1) return a;
        ll n=1;
        while(n<sz(a)*b) n<<=1;
        a.resize(n);
        fft(a, false);
        for(auto &xd: a) xd=bp(xd, b);
        fft(a, true);
        return a;
    }
}
}ntt;

int main(){
    fastio;
    NTT ntt;
    ntt.build();

```

```

    ll n, k; cin>>n>>k;
    vi v(10);
    FER(i,0,k){
        ll x; cin>>x;
        v[x]=1;
    }
    ll ans=0;
    vi froz=ntt.BinPolyExp(v, n/2);
    for(auto xd: froz) ans=(ans+sqr(xd))%mod;
    ans=(ans+mod)%mod;
    cout<<ans<<endl;
    return 0;
}

```

5.15 Number Theory

```

// return a % b (positive value)
int mod(int a, int b) {
    return ((a%b)+b)%b;
}

// returns d = gcd(a,b); finds x,y such that d =
// ax + by
int extended_euclid(int a, int b, int &x, int &y)
{
    int xx = y = 0;
    int yy = x = 1;
    while (b) {
        int q = a/b;
        int t = b; b = a%b; a = t;
        t = xx; xx = x-q*xx; x = t;
        t = yy; yy = y-q*yy; y = t;
    }
    return a;
}

// finds all solutions to ax = b (mod n)
vi modular_linear_equation_solver(int a, int b,
    int n) {
    int x, y;
    vi solutions;
    int d = extended_euclid(a, n, x, y);

```



```

if (!(b%d)) {
    x = mod (x*(b/d), n);
    for (int i = 0; i < d; i++)
        solutions.push_back(mod(x + i*(n/d), n));
}
return solutions;
}

// computes b such that ab = 1 (mod n), returns
// -1 on failure
int mod_inverse(int a, int n) {
    int x, y;
    int d = extended_euclid(a, n, x, y);
    if (d > 1) return -1;
    return mod(x,n);
}

// Chinese remainder theorem: find z such that
// z % x[i] = a[i] for all i. Note that the
// solution is
// unique modulo M = lcm_i (x[i]). Return (z,M).
// On
// failure, M = -1. Note that we do not require
// the a[i]'s
// to be relatively prime.
// Note that it's NOT necessary that x[i] > 0
// But if they are positive, it makes MODULO > 0
// (convenient)
ii chinese_remainder(int x, int a, int y, int b) {
    int s, t;
    x = abs(x), y = abs(y); // x and y NEED to be
    // positive here
    int d = extended_euclid(x, y, s, t);
    if (a%d != b%d) return make_pair(0, -1);
    return make_pair(mod(s*b*x+t*a*y,x*y)/d, x*y/d);
}
ii chinese_remainder(const vi &x, const vi &a) {
    ii ret = make_pair(a[0], abs(x[0]));
    for (int i = 1; i < x.size(); i++) {
        ret = chinese_remainder(ret.second, ret.first,
            x[i], a[i]);
        if (ret.second == -1) break;
    }
    return ret;
}

```

```

}

// Criba en O(n)
// p[i] indica el valor del primo i-esimo
// A[i] indica que el menor factor primo de i
// es el primo A[i] -esimo
#define MAXN 100000
int A[MAXN + 1], p[MAXN + 1], pc = 0;
void sieve()
{
    for(int i=2; i<=MAXN; i++){
        if(!A[i]) p[A[i] = ++pc] = i;
        for(int j=1; j<=A[i] && i*p[j]<=MAXN; j++)
            A[i*p[j]] = j;
    }
}

// Lucas Teorem
// Cuando n y m son grandes y se pide
// comb(n,m)%MOD, donde
// MOD es un numero primo, se puede usar el
// Teorema de Lucas.
ll comb( ll n, ll k ){
    ll ans = 1;
    while( n > 0 ){
        ll ni = n%MOD, ki = k%MOD;
        n /= MOD; k /= MOD;
        if( ni - ki < 0 )return 0;
        ll temp = (FP[ki]*FP[ni-ki])%MOD;
        temp = (temp*F[ni])%MOD;
        ans = (ans*temp)%MOD;
    }
    return ans;
}

// Criba en O(n) real
ll vis[1000000005];
inline bool check(int prime[], int x){
    return prime[x/64] & (1<<((x>>1)&31));
}

inline void mark(int prime[], int x){
    prime[x/64] |= (1<<((x>>1)&31));
}

```

```

}

inline void bit(ll n){
    int prime[n/64];
    fill(prime, 0);
    for(ll i=3; sqr(i)<=n; i+=2){
        if(!check(prime, i)){
            for(ll j=sqr(i), k=i<<1; j<n; j+=k)
                mark(prime, j);
        }
    }
    printf("2\n");
    ll cnt=1;
    for(ll i=3; i<n; i+=2) if(!check(prime, i)) {
        cnt++;
        if(cnt%100==1) printf("%lld\n", i);
    }
}

int main(){
    bit(1e8);
    return 0;
}

```

5.16 PiFunction

```

/* Pi Function*/

ll vis[1000000005], cant[1000000005],
    phi[1000005][105];
vi primos;

inline ll FindPhi(ll x, ll p){
    if(!p) return x;
    if(primos[p-1]>=x) return 1;
    if(x<=1e5 && p<=1e2) return phi[x][p];
    return FindPhi(x, p-1)-FindPhi(x/primos[p-1],
        p-1);
}

inline ll PiFuncion(ll n){
    if(n<1e7) return cant[n];
    ll raizC2=sqrt(n), raizC3=pow(n, 1.0/3.0);
    ll m=PiFuncion(raizC3);
}

```

```

    ll ans=FindPhi(n, m)+m-1;
    for(ll i=m; primos[i]<=raizC2; i++){
        ans-=PiFuncion(n/primos[i]);
        ans+=PiFuncion(primos[i]);
        ans--;
    }
    return ans;
}
int main(){
    fastio;
    FER(i,2,1e7){
        cant[i]+=cant[i-1];
        if(vis[i]) continue;
        cant[i]++;
        primos.pb(i);
        for(ll j=i*i; j<1e7; j+=i) vis[j]++;
    }
    FER(i,0,101){
        FER(j,0,1e5+1) phi[j][i]=i?
            phi[j][i-1]-phi[j/primos[i-1]][i-1]: j;
    }
    ll n, ans=0; cin>>n;
    for(auto xd: primos){
        ll x=xd, y=n/xd;
        if(sqr(x)>n) break;
        ans+=PiFuncion(y)-PiFuncion(x);
    }
    for(auto xd: primos) {
        if(sqr(xd)*xd<=n) ans++;
        else break;
    }
    cout<<ans<<endl;
    return 0;
}

```

5.17 Pisano

// Find pisano of Fib[i]%x
 // i.e. Spoj pisano

```

inline ll bm(ll a, ll b, ll m){
    if(a==0) return 0LL;

```

```

    if(a&1) {
        ll x=bm(a-1, b, m);
        x+=b;
        if(x>=m) x-=m;
        return x;
    }
    ll x=bm(a>>1, b, m);
    x<<=1;
    if(x>=m) x-=m;
    return x;
}

struct matrix{
    ll ar[2][2];
    inline void clear(){
        fill(ar, 0);
    }
};

matrix ind;

inline matrix mul(matrix &a, matrix b, ll m){
    matrix ty;
    FER(i,0,2) FER(j,0,2){
        ll ans=0;
        FER(k,0,2){
            ans+=bm(a.ar[i][k],
                b.ar[k][j], m);
            if(ans>=m) ans-=m;
        }
        ty.ar[i][j]=ans;
    }
    return ty;
}

inline matrix bpmatrix(matrix &a, ll n, ll m){
    if(n==1) return a;
    if(n&1) return mul(a, bpmatrix(a, n-1, m),
        m);
    matrix x=bpmatrix(a, n>>1, m);
    return mul(x, x, m);
}

```

```

inline ll fib(ll n, ll m){
    matrix cur=bpmatrix(ind, n, m);
    return cur.ar[0][1];
}

// here we need the pollard rho algorithm
// asuming that we have that here

map<ll, ll> mapis;

inline bool testpisano(ll n, ll m){
    if(fib(1+n, m)==1 && fib(2+n, m)==1)
        return true;
    return false;
}

inline ll pisanoprime(ll n, ll m){
    ll resp=n, ans, cur;
    vii v;
    make(resp);
    construct(v);
    for(auto xd: v){
        ans=1LL, cur=-1;
        FER(i, 0, xd.ss+1){
            if(testpisano(resp/ans, m))
                cur=ans;
            ans*=xd.ff;
        }
        resp/=cur;
    }
    return resp;
}

inline ll pisano(ll n){
    if(mapis.count(n)) return mapis[n];
    ll p=n%5, m=n-1;
    if(p==1 || p==4) return
        mapis[n]=pisanoprime(m, n);
    m+=2, m<<=1;
    return mapis[n]=pisanoprime(m, n);
}

```

```

}

inline ll binpow(ll a, ll n){
    if(!n) return 1LL;
    if(n&1) return a*binpow(a, n-1);
    ll x=binpow(a, n>>1);
    return sqr(x);
}

inline ll findpisano(ll n){
    vii v;
    make(n);
    construct(v);
    ll ans, cur, resp=1LL;
    for(auto xd: v){
        ans=binpow(xd.ff, xd.ss-1);
        cur=pisano(xd.ff)*ans;
        ans=__gcd(resp, cur);
        resp/=ans;
        resp*=cur;
    }
    return resp;
}

int main(){
    fastio;
    build();
    // because of pollard rho template
    ind.ar[0][0]=1, ind.ar[0][1]=1;
    ind.ar[1][0]=1, ind.ar[1][1]=0;
    mapis[2]=3, mapis[3]=8, mapis[5]=20;
    ll t; cin>>t;
    while(t--){
        ll x; cin>>x;
        cout<<findpisano(x)<<endl;
    }
    return 0;
}

```

5.18 Pollard Rho + Divisors

```

/* Pollard Rho + Recursive Divisors
   Autor: Phibrain
*/

const int maxp = 1e6 + 1, maxv = 25, maxc =
    (int)1e4 + 1;
int ptot, pr[maxp], d[maxp], cnt;
ll p[maxc];
inline ll mod_add(ll x, ll y, ll p) {
    return (x += y) < p ? x : x - p;
}
inline ll mod_mul(ll x, ll y, ll p) {
    ll ret = x * y - (ll)((long double)x * y / p +
        0.5) * p;
    return ret < 0 ? ret + p : ret;
}
inline ll mod_pow(ll x, ll k, ll p) {
    ll ret = 1 % p;
    for( ; k > 0; k >>= 1, x = mod_mul(x, x, p))
        (k & 1) && (ret = mod_mul(ret, x, p));
    return ret;
}
bool miller_rabin(ll n) {
    if(n == 2) return 1;
    if(n < 2 || !(n & 1))
        return 0;
    ll s = 0, r = n - 1;
    for( ; !(r & 1); r >>= 1, ++s);
    for(int i = 0; pr[i] < n && pr[i] < maxv; ++i) {
        ll cur = mod_pow(pr[i], r, n), nxt;
        for(int j = 0; j < s; ++j) {
            nxt = mod_mul(cur, cur, n);
            if(nxt == 1 && cur != 1 && cur != n - 1)
                return 0;
            cur = nxt;
        }
        if(cur != 1) return 0;
    }
    return 1;
}
inline ll gcd(ll a, ll b) {

```

```

    int ret = 0;
    while(a) {
        for( ; !(a & 1) && !(b & 1); ++ret, a >>= 1, b
            >>= 1);
        for( ; !(a & 1); a >>= 1);
        for( ; !(b & 1); b >>= 1);
        if(a < b)
            swap(a, b);
        a -= b;
    }
    return b << ret;
}
inline ll pollard_rho(ll n) {
    static ll seq[maxp];
    while(1) {
        ll x = rand() % n, y = x, c = rand() % n;
        ll *px = seq, *py = seq, tim = 0, prd = 1;
        while(1) {
            *py++ = y = mod_add(mod_mul(y, y, n), c, n);
            *py++ = y = mod_add(mod_mul(y, y, n), c, n);
            if((x = *px++) == y) break;
            ll tmp = prd;
            prd = mod_mul(prd, abs(y - x), n);
            if(!prd) return gcd(tmp, n);
            if(++tim == maxv) {
                if((prd = gcd(prd, n)) > 1 && prd < n)
                    return prd;
                tim = 0;
            }
        }
        if(tim && (prd = gcd(prd, n)) > 1 && prd < n)
            return prd;
    }
}
inline void decompose(ll n) {
    for(int i = 0; i < cnt; ++i)
        if(n % p[i] == 0) {
            p[cnt++] = p[i];
            n /= p[i];
        }
    if(n < maxp) {
        for( ; n > 1; p[cnt++] = d[n], n /= d[n]);
    } else if(miller_rabin(n)) {
        p[cnt++] = n;
    }
}

```

```

    } else {
        ll fact = pollard_rho(n);
        decompose(fact), decompose(n / fact);
    }
}

inline void build(){
    FER(i, 2, maxp) {
        if(!d[i]) pr[ptot++] = d[i] = i;
        for(int j = 0, k; (k = i * pr[j]) < maxp; ++j)
            {
                d[k] = pr[j];
                if(d[i] == pr[j]) break;
            }
    }
}

inline void construct(vii &v){
    sort(p, p+cnt);
    for(int i = 0; i < cnt; ) {
        ll ctr=0;
        for(ll pp = p[i]; i < cnt && p[i] == pp; ++i,
            ++ctr);
        v.pb({p[i-1], ctr});
    }
    cnt=0;
}

inline void make(ll n){
    decompose(n);
}

vi froz[N], v[N];
ll ja;
inline void go(ll n, ll t, vi &tt){
    if(n>=t){
        for(auto xd: tt) froz[ja].pb(xd);
        ja++;
        tt.pop_back();
        return;
    }
    FER(i,0,sz(v[n])){
        tt.pb(v[n][i]);
        go(n+1, t, tt);
    }
    if(sz(tt)) tt.pop_back();
}

```

```

inline void solve(ll t){
    vi tt;
    go(0, t, tt);
}

inline void kotaro(ll n, vi &v){
    FER(i, 0, n){
        ll t = 1;
        for(auto xd: froz[i]) t *= xd;
        v.pb(t);
    }
}

int main(){
    build();
    ll n; cin>>n;
    make(n);
    vii v1;
    construct(v1);
    cout<<"La factorizacion es: "<<endl;
    for(auto xd: v1){
        cout<<xd.ff<<" "<<xd.ss<<endl;
    }
    cout<<"Los divisores son: "<<endl;
    ja=0;
    FER(i,0,sz(v1)){
        ll t=1;
        ll ta=v1[i].ss+1, num=v1[i].ff;
        FER(j,0,ta) {
            v[i].pb(t);
            t*=v1[i].ff;
        }
    }
    solve(sz(v1));
    vi tnt;
    kotaro(ja, tnt);
    sort(all(tnt));
    for(auto xd: tnt) cout<<xd<<" "; cout<<endl;
    return 0;
}

```

5.19 Primality Test (pq prime)

```

// Primality test for [0,3*10^18] (Lucas97)
const ll MOD = 1e9 + 7;
const int MN = 1e6 + 2;
bool primos[MN * 5]; // 1 if is prime, 0 otherwise
int cnt = 0; // cnt denotes the number of primes
int prime[MN * 5];

ll mulmod(ll a, ll b, ll c) {
    ll x = 0, y = a % c;
    while (b) {
        if (b & 1) x = (x + y) % c;
        y = (y << 1) % c;
        b >>= 1;
    }
    return x % c;
}

ll fastPow(ll x, ll n, ll mod) {
    ll ret = 1;
    while (n) {
        if (n & 1) ret = mulmod(ret, x, mod);
        x = mulmod(x, x, mod);
        n >>= 1;
    }
    return ret;
}

bool isPrime(ll n) {
    // Verificar primero los casos pequenos (hasta 1e6)
    if(n <= 5 * MN - 10){
        if(!primos[n]) return 1;
        else return 0;
    }
    // Fin de verificacion
    ll d = n - 1;
    int s = 0;
    while (d % 2 == 0) {
        s++;
        d >>= 1;
    }
}

```

```
// These values will work for any number
// smaller than 3*10^18
//(Only 9 are necessary)
int a[11] = { 2, 3, 5, 7, 11, 13, 17, 19, 23,
             29, 31 };
for(int i = 0; i < 11; i++) {
    bool comp = fastPow(a[i], d, n) != 1;
    if(comp) for(int j = 0; j < s; j++) {
        ll fp = fastPow(a[i], (1LL << (ll)j)*d, n);
        if (fp == n - 1) {
            comp = false;
            break;
        }
    }
    if(comp) return false;
}
return true;
}
```

5.20 Primitive Root

// If a primitive root exists, there are
phi(phi(n)) of them */

```
//Primitive root modulo p(prime p>=2)
//Complexity: O(plogp)
int generator(int p){
    int phi = p - 1, np = phi ;
    vector<int>fact;
    for(int i = 2; i*i<=np ; ++i){
        if(np%i == 0){
            fact.pb(i);
            while(np%i == 0) np/=i;
        }
    }
    if(np > 1) fact.pb(np);
    for(int ret = 1; ret < p ; ++ret){
        bool ok = 1;
        for(int i = 0; i<fact.size(); ++i){
            if( binpow(ret,phi/fact[i],p) == 1LL){
                ok = 0; break;
            }
        }
    }
}
```

```
    }
    if(ok) return ret;
}
return -1;
}

//Probabilistic method , Complexity: O(loglogp *
p)
ll prob_generator(ll p){
    ll g , order;
    while(1){
        g = rand() % (p - 1) + 1;
        ll G = 1;
        for(order = 1 ;; ++order){
            G = (G * g) % p;
            if(G == 1) break;
        }
        if(order == p-1) break;
    }
    return g;
}
```

5.21 Row Reduced Form

```
// O(n^3) RREF via Gauss-Jordan with partial
pivoting.
// Takes a[][] nxm matrix, converts to rref,
returns rank
const double EPS = 1e-10;
typedef double T;
typedef vector<T> VT;
typedef vector<VT> VVT;
int rref(VVT &a) {
    int n = a.size();
    int m = a[0].size();
    int r = 0;
    for (int c = 0; c < m && r < n; c++) {
        int j = r;
        for (int i = r+1; i < n; i++)
            if (fabs(a[i][c]) > fabs(a[j][c])) j = i;
        if (fabs(a[j][c]) < EPS) continue;
        swap(a[j], a[r]);
    }
}
```

```
T s = 1.0 / a[r][c];
for (int j = 0; j < m; j++) a[r][j] *= s;
for (int i = 0; i < n; i++) if (i != r) {
    T t = a[i][c];
    for (int j = 0; j < m; j++) a[i][j] -= t *
        a[r][j];
}
r++;
}
return r;
}
```

5.22 Simplex Method

```
/* Simplex Method
Autor: el Vasito
*/
```

```
vector<int> X,Y;
vector<vector<double>> > A;
vector<double> b,c;
double z;
int n,m;
void pivot(int x,int y){
    swap(X[Y],Y[X]);
    b[X]/=A[X][Y];
    FER(i,0,m)if(i!=y)A[X][i]/=A[X][Y];
    A[X][Y]=1/A[X][Y];
    FER(i,0,n)if(i!=x&&abs(A[i][Y])>EPS){
        b[i]-=A[i][Y]*b[X];
        FER(j,0,m)if(j!=y)A[i][j]-=A[i][Y]*A[X][j];
        A[i][Y]-=A[i][Y]*A[X][Y];
    }
    z+=c[Y]*b[X];
    FER(i,0,m)if(i!=y)c[i]-=c[Y]*A[X][i];
    c[Y]=-c[Y]*A[X][Y];
}
pair<double,vector<double>> simplex( // maximize
c^T x s.t. Ax<=b, x>=0
vector<vector<double>> > _A, vector<double>
_b, vector<double> _c){
```

```
// returns pair (maximum value, solution
// vector)
A=_A;b=_b;c=_c;
n=b.size();m=c.size();z=0.;
X=vector<int>(m);Y=vector<int>(n);
FER(i,0,m)X[i]=i;
FER(i,0,n)Y[i]=i+m;
while(1){
    int x=-1,y=-1;
    double mn=-EPS;
    FER(i,0,n)if(b[i]<mn)mn=b[i],x=i;
    if(x<0)break;
    FER(i,0,m)if(A[x][i]<=-EPS){y=i;break;}
    assert(y>=0); // no solution to Ax<=b
    pivot(x,y);
}
while(1){
    double mx=EPS;
    int x=-1,y=-1;
    FER(i,0,m)if(c[i]>mx)mx=c[i],y=i;
    if(y<0)break;
    double mn=1e200;
    FER(i,0,n)if(A[i][y]>EPS&&b[i]/A[i][y]<mn)
        mn=b[i]/A[i][y],x=i;
    assert(x>=0); // c^T x is unbounded
    pivot(x,y);
}
vector<double> r(m);
FER(i,0,n)if(Y[i]<m)r[Y[i]]=b[i];
return mp(z,r);
}
```

5.23 Simpson Integral

```
// Numerical Integration with Simpson's Rule
// Error is f^4(x) * (b-a)^5 / 2880

int n = 500; // n tiene que ser par
ld f( ld x ){ return sqrt( 1 + 1.0/x ) / x; }
int main(){
    int a , b ;
    while( cin >> a >> b ){
```

```
ld dt = ( b - a )/(ld)n;
ld I = 0 , pos = a;
for( int i = 0 ; i < n ; i +=2 )
    I += dt * ( f( pos ) + 4*f( pos + dt ) +
        f( pos = pos + dt + dt ) );
printf( "%.2f\n" , double( I/3.0 ) );
}
}
```

5.24 Stirling Numbers Second Kind

```
// ..... Stirling Numbers Second Kind
// .....

// Building second kind of stirling numbers
// based on stir(n, k)=k*stir(n-1, k)+stir(n-1,
// k-1)
// stir(i, 0)=stir(0, i)=0, stir(0, 0)=1
// pow(x, p)= sum[k={1,...,p}]{stir(p,
// k)*k!*comb(x, k)
// sum[x={1, ..., n}]{pow(x, p)=
// sum[k={1,...,p}]{stir(p, k)*k!*comb(n+1, k+1)}

ll stir[1<<10][1<<10], dp[1<<10], dpi[1<<10];

inline ll bp(ll a, ll n, ll m){
    if(!n) return 1LL%m;
    if(n&1) return (a*bp(a, n-1, m))%m;
    ll x=bp(a, n>>1, m);
    return (x*x)%m;
}

inline void Build(ll n, ll k, ll m){
    FER(i,0,n) stir[i][0]=0;
    FER(i,0,n) stir[0][i]=0;
    stir[0][0]=1;
    FER(i,1,n+1){
        FER(j,1,k+1){
            stir[i][j]=(j*stir[i-1][j]+stir[i-1][j-1])%m;
        }
    }
    dp[0]=dp[1]=1;
```

```
dpi[1]=1;
FER(i,2,n) {
    dp[i]=(dp[i-1]*i)%m;
    dpi[i]=(dpi[i-1]*bp(i, m-2, m))%m;
}

inline ll FindSum(ll n, ll k, ll m){
    ll ans=0, cur=1, coe=(n), comb=n+1;
    if(!k) return n;
    FER(i,1,k+1){
        cur=(dp[i]*stir[k][i])%m;
        comb=(comb*coe)%m;
        cur=(comb*cur)%m;
        cur=(cur*dpi[i+1])%m;
        ans+=cur;
        if(ans>=m) ans-=m;
        coe--;
    }
    return ans;
}

int main(){
    Build(350, 350, mod);
    ll t, n, k, froz, ands, tnt; scanf("%lld",
    &t);
    while(t--){
        scanf("%lld%lld", &n, &k);
        froz=FindSum(n, k, mod);
        ands=FindSum(n, k+1, mod);
        tnt=(froz*(n+1)-ands+mod)%mod;
        if(tnt>=mod) tnt-=mod;
        if(tnt<0) tnt+=mod;
        printf("%lld\n", tnt);
    }
    return 0;
}
```

5.25 Teorema de Lucas

```
ll comb[105][105];
```

```
//Devuelve la comb(n,k) % m para n,k grandes y m
pequeno

ll lucas( ll n , ll k , ll m ){
    //Se puede precalcular la combinatoria afuera
    FER(i,0,52) FER(j,0,52){
        if( j == 0 ) comb[i][0] = 1;
        else if( j > i ) comb[i][j] = 0;
        else comb[i][j] = ( comb[i-1][j] +
            comb[i-1][j-1] ) % m;
    }

    ll ans = 1, x, y;

    while( n ){
        x = n % m, y = k % m;
        ans = ( ans * comb[x][y] ) % m;
        n /= m, k /= m;
    }
    return ans;
}
```

6 Misc

6.1 Centroid Decomposition Phibrain

//Centroid Decomposition Phibrain

```
struct CD{
    ll gid, n;
    vi graph[N], adj[N];
    ll tsz[N], p[N], id[N], rt[N];
    ll tsz1[N], p1[N], d1[N];
    inline void add(ll a, ll b){
        graph[a].pb(b);
        graph[b].pb(a);
    }
    inline ll dfs(ll u, ll pp){
        tsz[u]=1;
        for(auto xd: graph[u]) if(xd!=pp &&
            p[xd]==-1) tsz[u]+=dfs(xd, u);
```

```
        return tsz[u];
    }
    inline void descompose(ll u, ll pp, ll sb, ll
        prev){
        for(auto xd: graph[u]) if(xd!=pp &&
            p[xd]==-1 && (tsz[xd]*2) > sb)
            {descompose(xd, u, sb, prev); return;}
        p[u]=prev; ll pe;
        for(auto xd: graph[u]) if(p[xd]==-1)
            pe=dfs(xd, -1), descompose(xd, u,
            tsz[xd], u);
    }
    inline ll dfs1(ll u, ll pp, ll depth){
        p1[u]=pp, d1[u]=depth, tsz1[u]=1;
        for(auto xd: adj[u]) if(xd!=pp)
            tsz1[u]+=dfs1(xd, u, depth+1);
        return tsz1[u];
    }
    inline void make(ll u){
        ll pe=dfs1(u, -1, 0);
    }
    inline void go(ll u, ll root){
        id[u]=gid++, rt[u]=root;
        ll w=-1, bc=-1;
        for(auto xd: adj[u]) if(xd!=p1[u] &&
            tsz1[xd]>w) w=tsz1[xd], bc=xd;
        if(bc!=-1) go(bc, root);
        for(auto xd: adj[u]) if(xd!=p1[u] && xd!=bc)
            go(xd, xd);
    }
    inline ll lca(ll a, ll b){
        while(rt[a]!=rt[b]) d1[rt[a]]>d1[rt[b]]?
            a=p1[rt[a]]: b=p1[rt[b]];
        return d1[a]>d1[b]? b: a;
    }
    inline void build(ll u){
        FER(i,0,N) p[i]=-1;
        ll ja=dfs(u, -1);
        descompose(u, -1, tsz[u], -2);
        ll nod;
        FER(i,0,n) if(p[i]==-2) {nod=i; break;}
        FER(i,0,n) if(i!=nod) adj[i].pb(p[i]),
            adj[p[i]].pb(i);
        gid=0;
    }
```

```
        make(nod);
        go(nod, nod);
    }
}cd;

//cd.add(a, b); cd.build(0); rooted in 0
```

6.2 Centroid Decomposition itu

//Centroid Decomposition itu

```
const int MAXCD = 1e5 + 5;
struct CentroidD{
    //MAXCD: tamaño maximo del grafo
    vector< int > graph[MAXCD];
    int sub[MAXCD]; //size de subtree en CD tree
    int cpar[MAXCD]; //padre en el centroid tree
    //la raiz tiene padre -2

    void add_edge(int a, int b){
        graph[a].pb(b);
        graph[b].pb(a);
    }

    void dfs(int cur, int parent){
        sub[cur] = 1;
        for(int i = 0; i < sz(graph[cur]); ++i){
            int to = graph[cur][i];
            if(to != parent && cpar[to] == -1){
                dfs(to, cur);
                sub[cur] += sub[to];
            }
        }
    }

    void decompose(int cur, int parent, int sb,
        int prevc){
        for(int i = 0; i < sz(graph[cur]); ++i){
            int to = graph[cur][i];
            if(to != parent && cpar[to] == -1 && (2 *
                sub[to] > sb)){
                decompose(to, cur, sb, prevc);
```

```

        return;
    }
}
cpar[cur] = prevc;
for(int i = 0; i < sz(graph[cur]); ++i){
    int to = graph[cur][i];
    if(cpar[to] == -1){
        dfs(to, -1);
        decompose(to, cur, sub[to], cur);
    }
}
}

void init(int start){
    for(int i = 0; i < MAXCD; ++i) cpar[i] = -1;
    dfs(start, -1);
    decompose(start, -1, sub[start], -2);
}
};

```

6.3 Closest Pair

//Closest Pair Algorithm with Sweep
//Complexity: $O(n \log n)$

```

#define MAX_N 100000
#define px second
#define py first
typedef pair<long long, long long> point;

```

```

int N;
point P[MAX_N];
set<point> box;

```

```

bool compare_x(point a, point b){ return
    a.px < b.px; }

```

```

inline double dist(point a, point b){
    return

```

```

    sqrt((a.px-b.px)*(a.px-b.px)+(a.py-b.py)*(a.py-b.py));
}

```

```

double closest_pair(){

```

```

    if(N<=1) return -1;
    sort(P,P+N,compare_x);
    double ret = dist(P[0],P[1]);
    box.insert(P[0]);
    set<point> :: iterator it;
    for(int i = 1,left = 0;i<N;++i){
        while(left<i &&
            P[i].px-P[left].px>ret)
            box.erase(P[left++]);
        for(it =
            box.lower_bound(make_pair(P[i].py-ret,P[i].py+ret));
            it!=box.end() &&
            P[i].py+ret>=*it.py;++it)
            ret = min(ret, dist(P[i],*it));
        box.insert(P[i]);
    }
    return ret;
}

```

6.4 Convex Hull Trick

```

// Simple Hull
struct HullSimple { // Upper envelope for Maximum.
    // Special case: strictly increasing slope in
    // insertions,
    // increasing value in queries.
    deque<pair<ll, ll> > dq;
    ld cross(pair<ll, ll> l1, pair<ll, ll> l2){
        return (ld)(l2.snd - l1.snd) / (ld)(l1.fst -
            l2.fst);
    }
    void insert_line(ll m, ll b){
        pair<ll,ll> line = mp(m,b);
        while (sz(dq) > 1 && cross(line,
            dq[sz(dq)-1]) <=
            cross(dq[sz(dq)-1], dq[sz(dq)-2]))
            dq.pop_back();
        dq.pb(mp(m,b));
    }
    ll eval(pair<ll, ll> line, ll x){
        return line.fst * x + line.snd;
    }
};

```

```

}

ll eval(ll x){
    while (sz(dq) > 1 && eval(dq[0], x) <
        eval(dq[1],x))
        dq.pop_front();
    return eval(dq[0],x);
}

};

// Compile with g++ -std=c++11 file.cpp -o file
typedef long double ld;
const ll is_query = -(1LL<<62);
struct Line {
    ll m, b;
    mutable function<const Line*> succ;
    bool operator<(const Line& rhs) const {
        if (rhs.b != is_query) return m < rhs.m;
        const Line* s = succ();
        if (!s) return 0;
        ll x = rhs.m;
        return b - s->b < (s->m - m) * x;
    }
};

// Upper envelope for Maximum
struct HullDynamic : public multiset<Line> {
    bool bad(iterator y) {
        auto z = next(y);
        if (y == begin()) {
            if (z == end()) return 0;
            return y->m == z->m && y->b <= z->b;
        }
        auto x = prev(y);
        if (z == end()) return y->m == x->m && y->b
            <= x->b;
        return (x->b - y->b)*(z->m - y->m) >=
            (y->b - z->b)*(y->m - x->m);
    }
    void insert_line(ll m, ll b) {
        auto y = insert({ m, b });
        y->succ = [=] { return next(y) == end() ? 0:
            &*next(y); };
        if (bad(y)) { erase(y); return; }
    }
};

```



```

while (next(y) != end() && bad(next(y)))
    erase(next(y));
while (y != begin() && bad(prev(y)))
    erase(prev(y));
}
ll eval(ll x) {
    auto l = *lower_bound((Line) { x, is_query
        });
    return l.m * x + l.b;
}
};

```

6.5 Dates

```

// Months: [1,12], Days: [1,31], Years: [0, 9999]
string dayOfWeek[7] = {"Mon", "Tue", "Wed",
    "Thu", "Fri", "Sat", "Sun"};
// During a leap year, february is 29 and NOT 28
int daysPerMonth[12] = {31, 28, 31, 30, 31, 30,
    31, 31, 30, 31, 30, 31}
// Decides whether a year is leap in Gregorian
calendar
int isLeap(int y){
    if (y%4) return 0;
    if (y%100) return 1;
    if (y%400) return 0;
    return 1;
}
// converts Gregorian date to integer (Julian day
number)
int dateToInt (int m, int d, int y){
    return
        1461 * (y + 4800 + (m - 14) / 12) / 4 +
        367 * (m - 2 - (m - 14) / 12 * 12) / 12 -
        3 * ((y + 4900 + (m - 14) / 12) / 100) / 4 +
        d - 32075;
}
// converts integer (Julian day number) to
Gregorian date: month/day/year
void intToDate (int jd, int &m, int &d, int &y){
    int x, n, i, j;
    x = jd + 68569;

```

```

n = 4 * x / 146097;
x -= (146097 * n + 3) / 4;
i = (4000 * (x + 1)) / 1461001;
x -= 1461 * i / 4 - 31;
j = 80 * x / 2447;
d = x - 2447 * j / 80;
x = j / 11;
m = j + 2 - 12 * x;
y = 100 * (n - 49) + i + x;
}

```

6.6 Divide and Conquer Trick

```

//Divide and Conquer Trick
//Solves dp[i][j] = min k < j {dp[i - 1][k] +
    C[k][j]}
//Think about the base cases and set them on main
//The most important part is to set the method
for getval
//Hint: Should be some dp or segment tree
void go(ll i, ll l, ll r, ll optl, ll optr){
    if(l >= r) return;
    ll m = (l + r) >> 1;
    ll opt = -1;
    dp[i][m] = -1;
    FER(k, optl, min(optr, m) + 1){
        ll c = dp[i-1][k] + getval(k + 1, m + 1);
        if(c > dp[i][m]){
            dp[i][m] = c; opt = k;
        }
    }
    go(i, l, m, optl, opt);
    go(i, m + 1, r, opt, optr);
}

int main(){
    FER(i, 0, n) dp[0][i] = //base cases!;
    FER(i, 1, m) go(i, 0, n, 0, n);
    cout << dp[m - 1][n - 1] << endl;
    return 0;
}

```

6.7 Fractions

```

struct Frac{
    ll a,b;
    Frac(){}
    Frac(ll _a, ll _b) : a(_a), b(_b){
        if(b < 0) a = -a, b = -b;
    }
    void toZero(){ a = 0; }
    long double getReal(){ return (long
        double)a / b; }
    Frac operator * (ll x){ return
        Frac(a*x,b); }
    Frac operator + (Frac other){
        return Frac(a*other.b + b *
            other.a,b * other.b);
    }
    bool positive(){ return a > 0;}
    bool negative(){ return a < 0;}
    bool operator == (Frac &other){
        return a * other.b == b * other.a;
    }
    bool operator < (const Frac &other) const{
        return a * other.b < b * other.a;
    }
    bool operator > (const Frac &other) const{
        return a * other.b > b * other.a;
    }
    bool operator <= (const Frac &other) const{
        return a * other.b <= b * other.a;
    }
    bool operator >= (const Frac &other) const{
        return a * other.b >= b * other.a;
    }
};

```

6.8 IO formatting

```
// Fast number input
#define getcx getchar_unlocked
inline void read(int &number) {
    number = 0;
    int ch = getcx();
    while (ch < '0' || ch > '9')
        ch = getcx();
    while(ch >= '0' && ch <= '9'){
        number = (number << 3) + (number << 1) + ch
            - '0';
        ch = getcx();
    }
}

int main(){
    cout << setprecision(5) << fixed; // Amount of
        decimal
    cout.setf(ios::showpoint); // Show decimal
        point
    cout.setf(ios::showpos); // Show + in positiv
    cout << hex << dec << endl; // Hexadecimal /
        Normal
}

```

6.9 Longest Increasing Subsequence

```
// Simple O( nlogn ) Longest Increasing
    Subsequence
// Answer is stored in array b[N]

```

```
int LIS( vi &a ){
    int b[N];
    int sz = 0;
    FER(i,0,a.size()){
        int j = lower_bound( b , b + sz , a[ i ] ) -
            b;
        // (lower) a < b < c
        // (upper) a <= b <= c
        b[ j ] = a[ i ];
        if( j == sz ) sz++;
    }
}

```

```
return sz;
}

```

6.10 MinRotation

```
// finds the point to rotate to make s has the
    smallests values first
int min_rotation(vl s) {
    int a=0, N=sz(s);
    s.insert(s.end(),all(s));
    FOR(b,N) FOR(i,N) {
        if (a+i == b || s[a+i] < s[b+i]) {b
            += max(0, i-1); break;}
        if (s[a+i] > s[b+i]) { a = b;
            break; }
    }
    return a;
}

// rotates v
vl rot_min(vl v) {
    int x = min_rotation(v);
    rotate(v.begin(),v.begin()+x,v.end());
    return v;
}

```

6.11 Mo algorithm

```
map<ll , ll> m;
ll z;
inline ll OnlineCom(ll x){
    return m.count(x)? m[x]: m[x]=++z;
}

struct Que{
    ll l, r, t, id, lq, rq;
    Que(){}
    Que(ll l, ll r, ll t, ll id, ll lq, ll rq) :
        l(l), r(r), t(t), id(id), lq(lq), rq(rq){}
}Q[1<<18];

```

```
struct Upd{
    ll p, last, nxt;
    Upd(){}
    Upd(ll p, ll last, ll nxt) : p(p), last(last),
        nxt(nxt){}
}U[1<<18];

```

```
struct MO{
    ll n, curR, curL, q, t;
    ll ans[1<<18], ar[1<<18], cnt[1<<18],
        fre[1<<18];
    MO(){}
    inline void Add(ll val){
        fre[cnt[val]]--;
        fre[++cnt[val]]++;
    }
    inline void Rem(ll val){
        fre[cnt[val]]--;
        fre[--cnt[val]]++;
    }
    inline void Retime(ll i, ll cur){
        if(i>=curL && i<curR){
            Add(cur);
            Rem(ar[i]);
        }
        ar[i]=cur;
    }
    inline void Query(ll l, ll r, ll timer, ll id){
        while(t<timer) Retime(U[t].p, U[t].nxt), t++;
        while(t>timer) t--, Retime(U[t].p,
            U[t].last);
        while(curL<l) Rem(ar[curL++]);
        while(curL>l) Add(ar[--curL]);
        while(curR>r) Rem(ar[--curR]);
        while(curR<r) Add(ar[curR++]);
        ll mex=1;
        while(fre[mex++]>0);
        ans[id]=mex-1;
    }
    inline void build(){
        sort(Q, Q+q, [&](Que &a, Que &b){
            if(a.lq!=b.lq) return a.lq<b.lq;
            if(a.rq!=b.rq) return a.rq<b.rq;

```

```

        return a.t<b.t;
    }
};
curL=0, curR=0;
FER(i,0,q) Query(Q[i].l, Q[i].r, Q[i].t,
    Q[i].id);
}
}mo;

int main(){
    fastio;
    ll n, q, add; cin>>n>>q;
    add=(ll) pow(n, 2.0/3.0);
    mo.n=n;
    FER(i,0,n) {
        cin>>mo.ar[i];
        mo.ar[i]=OnlineCom(mo.ar[i]);
    }
    ll type, l, r, t1=0, t2=0;
    FER(i,0,q){
        cin>>type>>l>>r; l--;
        if(type==1){
            Q[t1]={l, r, t2, t1, l/add, (r-1)/add};
            t1++;
        }
        else{
            U[t2].p=l, U[t2].nxt=OnlineCom(r);
            t2++;
        }
    }
    mo.q=t1, mo.t=t2;
    FER(i,0,t2){
        U[i].last=mo.ar[U[i].p];
        mo.ar[U[i].p]=U[i].nxt;
    }
    mo.build();
    FER(i,0,t1) cout<<mo.ans[i]<<endl;
    return 0;
}

```

6.12 Parallel Binary Search

```

// Paralel Binary Search
// Key idea is to maintain a node with the range
// and depth
// then use a queue mantaining a ni current nivel
// and nt current index
// in a grid, given heights of a the mountain,
// find the smallest height that
// are in a path from 2 given coordinates.
// i.e. Mountainers cf gym 102021 M

```

```

ll val[1<<19];
ll nn, mm;

struct node{
    ll l, r, d;
    vi v;
    node(){
        node(ll l, ll r, ll d): l(l), r(r), d(d){}
        inline void clear(){
            l=r=d=0;
            v.clear();
        }
    };

struct Kruskal{
    ll n, id[1<<19];
    vector<tri> v;
    inline void build(){
        FER(i,0,n) id[i]=i;
    }
    inline ll find(ll x){
        while(id[x]!=x) id[x]=id[id[x]],
            x=id[x];
        return x;
    }
    inline void unir(ll a, ll b){
        ll p=find(a), q=find(b);
        id[p]=id[q];
    }
    inline ll go(ll lim, ll idx){
        FER(i,idx,sz(v)){
            ll w=v[i].tm1, a=v[i].tm2,
                b=v[i].tm3;
            if(w>=lim){

```

```

                return i;
            }
            ll node=a*mm+b;
            FER(t1, -1, 2) FER(t2, -1,
                2){
                if(abs(t1)+abs(t2)!=1)
                    continue;
                ll x=a+t1, y=b+t2;
                if(x>=0 && x<nn &&
                    y>=0 && y<mm){
                    ll cur=x*mm+y;
                    if(val[cur]<lim){
                        unir(cur,
                            node);
                    }
                }
            }
            return sz(v);
        }
    }kr;

    ii queries[1<<19];
    ll ans[1<<19];

    int main(){
        fastio;
        ll n, m, k; cin>>n>>m>>k;
        nn=n, mm=m;
        kr.n=n*m+1;
        FER(i,0,n){
            FER(j,0,m) {
                ll w; cin>>w;
                ll node=i*m+j;
                val[node]=w;
                kr.v.pb({w, {i, j}});
            }
        }
        kr.build();
        sort(all(kr.v));
        ll ini=0;
        FER(i,0,k){
            ll x1, y1; cin>>x1>>y1; x1--, y1--;
            ll node1=x1*m+y1;

```

```

    ll x2, y2; cin>>x2>>y2; x2--, y2--;
    ll node2=x2*m+y2;
    if(node1==node2){
        ans[i]=val[node1];
        continue;
    }
    queries[i]={node1, node2};
}
node cur, cur1, cur2;
cur.l=0, cur.r=1e9, cur.d=0;
FER(i,0,k) if(!ans[i]) cur.v.pb(i);
queue<node> q;
q.push(cur); cur.clear();
ll ni=-1, nt=0;
while(sz(q)){
    cur=q.front(); q.pop();
    if(cur.d!=ni) {
        kr.build();
        ni=cur.d, nt=0;
    }
    ll mid=(cur.l+cur.r)>>1;
    nt=kr.go(mid, nt);
    if(cur.l+1==cur.r){
        nt=kr.go(cur.r, nt);
        for(auto xd: cur.v){
            ll a=queries[xd].ff,
                b=queries[xd].ss;
            ll p1=kr.find(a),
                p2=kr.find(b);
            (p1==p2)?
                ans[xd]=mid:
                ans[xd]=-1;
        }
        continue;
    }
    for(auto xd: cur.v){
        ll a=queries[xd].ff,
            b=queries[xd].ss;
        ll p1=kr.find(a),
            p2=kr.find(b);
        (p1==p2)? cur1.v.pb(xd):
            cur2.v.pb(xd);
    }
}

```

```

    cur1.l=cur.l, cur1.r=mid,
    cur1.d=cur.d+1;
    cur2.l=mid, cur2.r=cur.r,
    cur2.d=cur.d+1;
    if(sz(cur1.v)) q.push(cur1);
    if(sz(cur2.v)) q.push(cur2);
    nt=kr.go(cur.r, nt);
    cur.clear(), cur1.clear(),
    cur2.clear();
}
FER(i,0,k){
    assert(ans[i]>=0);
    cout<<ans[i]<<endl;
}
return 0;
}

```

6.13 Sack Guni

```

//Sack Guni DS
//Solves Lomsat Gelral:
//Sum of all dominating colors in the subtree of
//vertex v
//Color c is dominating in the subtree of vertex v
//if there are no other colors that appear in the
//subtree
//of vertex v more times than color c

ll tsz[1<<17], d[1<<17], big[1<<17], col[1<<17];
ll sum[1<<17], cnt[1<<17], fre[1<<17], val,
    ans[1<<17];
vi adj[1<<17];

struct ST{
    ll n, t[1<<19];
    inline ll Op(ll &val1, ll &val2){
        return val1+val2;
    }
    inline void Upd(ll p, ll val, ll id, ll l,
        ll r){
        if(l>p || r<=p) return;
        if(l+1==r && l==p){

```

```

            t[id]+=val;
            return;
        }
        ll mid=(l+r)>>1;
        Upd(p, val, id<<1, l, mid);
        Upd(p, val, id<<1|1, mid, r);
        t[id]=Op(t[id<<1], t[id<<1|1]);
    }

    inline ll find(ll id, ll l, ll r){
        if(l+1==r && t[id]!=0) return t[id];
        ll valor=t[id<<1|1], mid=(l+r)>>1;
        if(!valor) return find(id<<1, l,
            mid);
        return find(id<<1|1, mid, r);
    }

    inline void build() { fill(t, 0);}
    inline void upd(ll p, ll val) { Upd(p,
        val, 1, 0, n);}
    inline ll que() { return find(1, 0, n);}
}st;

inline void add(ll v, ll p, ll x){
    ll valor=fre[col[v]];
    st.upd(valor, -col[v]);
    fre[col[v]]+=x;
    valor=fre[col[v]];
    st.upd(valor, col[v]);
    for(auto xd: adj[v]) if(xd!=p && !big[xd])
        add(xd,v,x);
}

inline void dfs(ll u, ll pp, ll depth){
    d[u]=depth, tsz[u]=1;
    for(auto xd: adj[u]) if(xd!=pp) dfs(xd, u,
        depth+1), tsz[u]+=tsz[xd];
}

inline void go(ll u, ll p, ll x){
    ll w=-1, bc=-1;
    for(auto xd: adj[u]) if(xd!=p &&
        tsz[xd]>w) w=tsz[xd], bc=xd;

```

```

for(auto xd: adj[u]) if(xd!=bc && xd!=p)
    go(xd, u, 0);
if(bc!=-1) go(bc, u, 1), big[bc]=1;
add(u, p, 1);
ans[u]=st.que();
if(bc!=-1) big[bc]=0;
if(x==0) add(u, p, -1);
}

int main(){
    fastio;
    ll n; cin>>n;
    st.n=n;
    FER(i, 1, n+1) cin>>col[i];
    FER(i, 0, n-1){
        ll a, b; cin>>a>>b; //a--, b--;
        adj[a].pb(b);
        adj[b].pb(a);
    }
    st.build();
    dfs(1, -1, 0);
    go(1, -1, 1);
    FER(i, 1, n+1) cout<<ans[i]<<" ";
    cout<<endl;
    return 0;
}

```

6.14 Stable Marriage

```

// Stable Marriage Problem
#define N 505
int n;
int pref_men[N][N], pref_women[N][N], inv[N][N];
int cont[N], husband[N], wife[N];
void stable_marriage(){
    FER(i,0,n) FER(j,0,n) inv[i][pref_women[i][j]]
        = j;
    clr(cont, 0);
    clr(husband, -1);
    int m, w, dumped;
    FER(i,0,n){

```

```

        m = i;
        while(m >= 0){
            while(1){
                w = pref_men[m][cont[m]];
                ++cont[m];
                if(husband[w] < 0 || inv[w][m] <
                    inv[w][husband[w]])
                    break;
            }
            dumped = husband[w];
            husband[w] = m;
            wife[m] = w;
            m = dumped;
        }
    }
}

```

6.15 Unordered Map

```

unordered_map<int,int> mp;
mp.reserve(1024); // power of 2 is better
mp.max_load_factor(0.25); // 0.75 used in java

```

7 Strings

7.1 Aho Corasick - Iterative

```

//Iterative Aho - Corasick
#define N 1050
#define MOD 1000003

inline char f(char ch){
    if(ch >= 'a' and ch <= 'z') return ch - 'a';
    if(ch >= 'A' and ch <= 'Z') return ch - 'A' +
        26;
    return ch - '0' + 52;
}

```

```

struct AhoCorasick{
    int cnt;
    char pch[N];
    int t[N][27], gt[N][27], link[N], slink[N], p[N];
    int leaf[N];

    AhoCorasick(){
        cnt = 1;
        p[0] = 0;
        fill(t, -1);
        fill(gt, -1);
        fill(link, -1);
        fill(slink, -1);
        fill(leaf, 0);
    }

    void add_string(string &s, int id){
        int u = 0;
        for(char ch : s){
            int c = f(ch);
            if(t[u][c] == -1){
                t[u][c] = cnt++;
                p[cnt-1] = u;
            }
            u = t[u][c];
            pch[u] = ch;
        }
        leaf[u] = 1;
    }

    int get_link(int v){
        if(link[v] == -1){
            if(!v or !p[v]) link[v] = 0;
            else link[v] = go(get_link(p[v]), pch[v]);
        }
        return link[v];
    }

    int go(int v, char ch){
        int c = f(ch);
        if(gt[v][c] == -1){
            if(t[v][c] != -1) gt[v][c] = t[v][c];
            else gt[v][c] = (v == 0) ? 0 :
                go(get_link(v), ch);
        }
    }
}

```

```

    }
    return gt[v][c];
}

int get_superlink(int u){
    if(slink[u] == -1){
        int lk = get_link(u);
        if(lk == 0) slink[u] = 0;
        else if(leaf[lk]) slink[u] = lk;
        else slink[u] = get_superlink(lk);
    }
    return slink[u];
}
};

```

7.2 Aho Corasick - Phibrain

```

struct T{
    ll nxt[1<<6], fail, cnt;
    inline void clear(){
        fill(nxt, 0);
        fail=cnt=0;
    }
};

inline ll Find(char &s){
    if(s>='a' and s<='z') return s-'a';
    if(s>='A' and s<='Z') return s-'A'+26;
    if(s>='0' and s<='9') return s-'0'+52;
    return 62;
}

struct AhoCorasick{
    vector<T> t;
    T nil;
    ll id, nod;
    vi End, Q, ans, cnt;
    vector<string> set;
    inline void init(){
        nod=0;
        t.clear();
        End.clear();
    }
};

```

```

Q.clear();
ans.clear();
cnt.clear();
nil.clear();
set.clear();
t.pb(nil);
End.pb(-1);
Q.pb(-1);
cnt.pb(0);
}

inline void Add(string &st){
    ll cur=0, idx;
    for(auto xd: st){
        idx=Find(xd);
        if(!t[cur].nxt[idx])
            t[cur].nxt[idx]=++nod, t.pb(nil),
            cnt.pb(0);
        cur=t[cur].nxt[idx];
    }
    End.pb(cur);
    cnt[cur]++;
    set.pb(st);
}

inline void Build(){
    id=1;
    FER(i,0,1<<6) if(t[0].nxt[i])
        Q.pb(t[0].nxt[i]);
    while(id<=sz(Q)-1){
        ll u=Q[id++];
        cnt[u]+=cnt[t[u].fail];
        FER(i,0,1<<6){
            ll v=t[u].nxt[i];
            if(!v){
                t[u].nxt[i]=t[t[u].fail].nxt[i];
                continue;
            }
            Q.pb(v);
            t[v].fail=t[t[u].fail].nxt[i];
        }
    }
}

inline void AddText(string &st){

```

```

    ll cur=0, idx;
    for(auto xd: st){
        idx=Find(xd);
        cur=t[cur].nxt[idx];
        t[cur].cnt++;
    }
}

inline void PushAll(){
    ans.resize(sz(End));
    IFR(i, sz(Q)-1, 1)
        t[t[Q[i]].fail].cnt+=t[Q[i]].cnt;
    FER(i, 1, sz(ans)) ans[i]=t[End[i]].cnt;
}

inline ll Get(string &st){
    ll cur=0, idx, ans=0, n;
    for(auto xd: st){
        idx=Find(xd);
        cur=t[cur].nxt[idx];
        ans+=cnt[cur];
    }
    return ans;
}
}ac[20][2];

inline void Upd(ll n, ll idx, string &s){
    ll id=-1;
    FER(i, 0, 20){
        ll val=(n>>i)&1;
        if(!(val)) {id=i; break;}
    }
    ac[id][idx].init();
    FER(i, 0, id){
        for(auto xd: ac[i][idx].set)
            ac[id][idx].Add(xd);
        ac[i][idx].init();
    }
    ac[id][idx].Add(s);
    ac[id][idx].Build();
}

```

```

inline void Query(string &s){
    ll ans=0;
    FER(i, 0, 20){
        if(sz(ac[i][0].set)) ans+=ac[i][0].Get(s);
        if(sz(ac[i][1].set)) ans-=ac[i][1].Get(s);
    }
    cout<<ans<<endl;
    cout.flush();
}
int main(){
    fastio;
    ll q, t, tot1=0, tot2=0; cin>>q;
    string s;
    while(q--){
        cin>>t>>s;
        if(t==1){
            Upd(tot1, 0, s);
            tot1++;
        }
        else if(t==2){
            Upd(tot2, 1, s);
            tot2++;
        }
        else{
            Query(s);
        }
    }
    return 0;
}

```

7.3 KMP

// KMP + KMP Automata

```

void kmp(string &cad){
    p[0] = 0;
    for(int i = 1; i < m; ++i){
        p[i] = p[i-1];
        while(p[i] > 0 and cad[p[i]] != cad[i])
            p[i] = p[p[i]-1];
        if(cad[p[i]] == cad[i]) p[i]++;
    }
}

```

```

}

int go[N][300];

vector<int> kmp(string &s){
    int n = sz(s);
    vector<int> p(n);
    p[0] = 0;
    for(int i = 1; i < n; ++i){
        p[i] = p[i-1];
        while(p[i] and s[p[i]] != s[i]) p[i] =
            p[p[i]-1];
        if(s[p[i]] == s[i]) p[i]++;
    }
    return p;
}

void process(string &s){
    int n = sz(s);
    vector<int> vec = kmp(s);
    fill(go, 0);
    for(int i = 0; i < n; ++i) go[i][s[i]] = i+1;
    for(int i = 1; i <= n; ++i)
        for(char j = 'a'; j <= 'z'; ++j){
            if(go[i][j]) continue;
            go[i][j] = go[vec[i-1]][j];
        }
}

void search(string &pat, string &text){
    int cnt = 0;
    vector<int> vec = kmp(pat);
    for(char c : text){
        while(cnt and c != pat[cnt]) cnt =
            vec[cnt-1];
        if(c == pat[cnt]) cnt++;
        if(cnt == sz(pat)){
            // pattern found
            cnt = vec[cnt-1];
        }
    }
}

```

7.4 Manacher

```

// O(n) Manacher's algorithm for finding all
// palindromes
int n;
char s[200200];
char aux[100100];
int p[200200];

int main(){
    scanf("%s", aux, &n);
    s[0] = '^';
    s[1] = '#';
    FER(i, 0, n){
        s[2*i+2] = aux[i];
        s[2*i+3] = '#';
    }
    s[2*n+2] = '\0';
    int c = 0, r = 0;
    FER(i, 0, 2*n+2){
        if(i > r) p[i] = 0;
        else p[i] = min(r-i, p[2*c-i]);
        while(s[i+p[i]+1] == s[i-p[i]-1]) p[i]++;
        if(i + p[i] > r){
            c = i;
            r = i + p[i];
        }
    }

    printf("%s\n", s);
    FER(i, 0, 2*n+2) {
        printf("%d", p[i]);
    }
    printf("\n");
    return 0;
}

```

7.5 Palindromic Tree

// adamant's palindromic tree online
O(n*log(|E|)) construction

```
// Tutorial: http://adilet.org/blog/25-09-14/
// Add/Delete operation can be supported in
// O(logn) by doing
// check(link[v]), v = slink[v] in get_link
// (periodicity -> same initial char)
const int maxn = 5e5, sigma = 26, INF = 1e9;
int s[maxn], len[maxn], link[maxn],
    to[maxn][sigma];
int n, last, sz;
// All these optional (palindromic factoring)
int d[maxn], slink[maxn], dpe[maxn], dpo[maxn];
int anse[maxn], anso[maxn], prve[maxn],
    prvo[maxn];

void init(){ // Call with n=0
    s[n++] = -1;
    link[0] = 1;
    len[1] = -1;
    sz = 2;
    anse[0] = 0;
    anso[0] = INF;
}

int get_link(int v){
    while(s[n - len[v] - 2] != s[n - 1]) v =
        link[v];
    return v;
}

ii getmin(int v, int* ans, int* dp, int* prv){
    dp[v] = ans[n - (len[slink[v]] + d[v]) - 1];
    int best = n - (len[slink[v]] + d[v]) - 1;
    if (d[v] == d[link[v]]){
        if (dp[v] > dp[link[v]]){
            dp[v] = dp[link[v]];
            best = prv[n-1-d[v]];
        }
    }
    return mp(dp[v] + 1, best);
}

void add_letter(int c){
    s[n++] = c;
    last = get_link(last);
```

```
if(!to[last][c]) {
    len[sz] = len[last] + 2;
    link[sz] = to[get_link(link[last])][c];
    d[sz] = len[sz] - len[link[sz]];
    if (d[sz] == d[link[sz]]) slink[sz] =
        slink[link[sz]];
    else slink[sz] = link[sz];
    to[last][c] = sz++;
}
last = to[last][c];

anse[n-1] = INF;
for (int v = last; len[v] > 0; v = slink[v]){
    ii acte = getmin(v, anso, dpe, prve);
    if (act.fst < anse[n-1]){
        anse[n-1] = act.fst;
        prve[n-1] = act.snd;
    }
}

anso[n-1] = INF;
for (int v = last; len[v] > 0; v = slink[v]){
    ii act = getmin(v, anse, dpo, prvo);
    if (act.fst <= anso[n-1]){
        anso[n-1] = act.fst;
        prvo[n-1] = act.snd;
    }
}
}
```

7.6 Suffix Array

```
//Suffix array, fast enough for 1e5
// p[i]: order of the suffix starting at i
// r: Suffix array itself (indexes of ordered
//     starts of suffixes)
// h: kasai array

struct SA{
    ll n,t;
    ll p[N], r[N], h[N];
    string s;
```

```
ll rmq[M][N], flog2[N];
inline void fix_index(ll b, ll e){
    ll lastpk, pk, d;
    lastpk=p[r[b]+t];
    d=b;
    FER(i,b,e){
        if(((pk=p[r[i]+t]) !=lastpk) && (b>lastpk ||
            pk>=e)){
            lastpk = pk;
            d=i;
        }
        p[r[i]]=d;
    }
}

inline void suf_arr(){
    s[n++]='$';
    ll bc[256];
    FER(i,0,256) bc[i]=0;
    FER(i,0,n) bc[(ll)s[i]]++;
    FER(i,1,256) bc[i]+=bc[i-1];
    IFR(i,n-1,0) r[--bc[(ll)s[i]]]=i;
    IFR(i,n-1,0) p[i]=bc[(ll)s[i]];
    for(t=1;t<n;t<=1){
        for(ll i=0,j=1;i<n;i=j++){
            while(j<n && p[r[j]]==p[r[i]]) ++j;
            if(j-i>1){
                sort(r+i,r+j, [&](const ll &i, const ll
                    &j){return p[i+t]<p[j+t];});
                fix_index(i,j);
            }
        }
    }
}

inline void initlcp(){
    ll tam=0,j;
    FER(i,0,n-1){
        j=r[p[i]-1];
        while(s[i+tam]==s[j+tam]) ++tam;
        h[p[i]-1]=tam;
        if(tam>0) --tam;
    }
}

inline void makelcp(){
    initlcp();
```



```

FER(i,0,n-1) rmq[0][i]=h[i];
ll lg=0,pw=1;
do{
    FER(i,pw,2*pw) flog2[i]=lg;
    lg++;pw*=2;
    FER(i,0,n-1){
        if(i+pw/2 < n-1)
            rmq[lg][i]=min(rmq[lg-1][i],
                rmq[lg-1][i+pw/2]);
        else rmq[lg][i]=rmq[lg-1][i];
    }
} while(pw<n);
}
inline ll lcp(ll i, ll j){
    if(i==j) return n-r[i]-1;
    ll lg=flog2[j-i], pw=(1<<lg);
    return min(rmq[lg][i], rmq[lg][j-pw]);
}
inline void build(){
    fill(p,0);
    fill(r,0);
    fill(h,0);
    fill(rmq,0);
    fill(flog2,0);
    n=sz(s);
    suf_arr();
    makelcp();
}
}sa;

```

7.7 Suffix Automaton

```

// O(n) Online suffix automaton construction
// len[u]: Max length of a string accepted by u
// link[u]: Suffix link of u
// Link edges give the suffix tree of reverse(s)
// Terminal nodes can be obtained by
// traversing last's links

```

```

const int MAX = 1000000;
int len[MAX*2];
int link[MAX*2];

```

```

map<char,int> adj[MAX*2];
int sz, last;

// To reuse, clear adj[]
void sa_init() {
    sz = last = 0;
    len[0] = 0;
    link[0] = -1;
    sz++;
}

void sa_extend (char c) {
    int cur = sz++;
    len[cur] = len[last] + 1;
    int p;
    for (p=last; p!=-1 && !adj[p].count(c); p =
        link[p])
        adj[p][c] = cur;
    if (p == -1)
        link[cur] = 0;
    else {
        int q = adj[p][c];
        if (len[p] + 1 == len[q])
            link[cur] = q;
        else {
            int clone = sz++;
            len[clone] = len[p] + 1;
            adj[clone] = adj[q];
            link[clone] = link[q];
            for (; p != -1 && adj[p][c] == q; p =
                link[p])
                adj[p][c] = clone;
            link[q] = link[cur] = clone;
        }
    }
    last = cur;
}

```

7.8 Z-Algorithm

```

//Zfun(i) devuelve la longitud del maximo prefijo
que empieza en i

```

```

vi Zfun(string s){
    vi Z(s.sz,0);
    int l = 0, r = 0;
    FER(i,1,sz(s)){
        if ( i<=r ) Z[i] = min(Z[i-1], r-i+1);
        while ( i+Z[i]<s.sz and s[i+Z[i]]==s[Z[i]] )
            Z[i]++;
        if ( i+Z[i]-1>r ) l = i, r = i+Z[i]-1;
    }
    return Z;
}

```

8 Templates

8.1 Makefile

```

CXX = g++
CXXFLAGS = -std=c++14 -Wall -Wextra
            -Wno-sign-compare -O2 -g

all: %
%: %.cpp
    $(CXX) $(CXXFLAGS) -o $@ $@.cpp

```

8.2 Random Generator

```

//Mersenne Twister Random Generator
//Note: If you want 64-bit random numbers,
//      just declare it as mt19937_64.

mt19937 rng(chrono::steady_clock::now().
    time_since_epoch().count());

```

8.3 Script

```

# Gracias men de la uni

```

```
echo "Compiling gen.cpp"
g++ gen.cpp -std=c++14 -o gen
echo "Compiling ac.cpp"
g++ ac.cpp -std=c++14 -o ac
echo "Compiling main.cpp"
g++ main.cpp -std=c++14 -o main
```

```
((count = 0))
((testCases = 1000))
for((i = 1; ; i++)); do
    echo "Running on test ${i}"
    ./gen $i > test
    ./ac < test > out1
    ./main < test > out2
    diff -w out1 out2 || break
    ((count++))
done
```

```
if [ $count -eq $testCases ]
then
    echo "Accepted"
else
    ((count++))
    echo "Wrong Answer on test ${count}"
    more test
    echo "Answer : "
    more out1
    echo "Output : "
    more out2
fi
```

8.4 Stack Size

```
#include <sys/resource.h>

int main (int argc, char **argv){
    const rlim_t kStackSize = 64L * 1024L * 1024L;
    // min stack size = 64 Mb
    struct rlimit rl;
    int result;
```

```
    result = getrlimit(RLIMIT_STACK, &rl);
    if (result == 0)
    {
        if (rl.rlim_cur < kStackSize)
        {
            rl.rlim_cur = kStackSize;
            result = setrlimit(RLIMIT_STACK, &rl);
            if (result != 0)
            {
                fprintf(stderr, "setrlimit returned
                    result = %d\n", result);
            }
        }
    }

    // ...

    return 0;
}
```

8.5 Template

```
#pragma GCC optimize ("Ofast,unroll-loops")
#pragma GCC target ("sse,sse2,sse3,ssse3,sse4")
#pragma GCC target
    ("popcnt,abm,mmx,avx,tune=native")

#include <bits/stdc++.h>
using namespace std;
#define sz(x) int(x.size())
#define pb push_back
#define FER(i,a,b) for(ll i = ll(a); i < ll(b); ++i)
#define IFR(i,a,b) for(ll i = ll(a); i >= ll(b); --i)
#define all(v) (v).begin(),(v).end()
#define mp make_pair
#define ff first
#define ss second
#define tm1 first
#define tm2 second.first
```

```
#define tm3 second.second
#define fill(x,v) memset(x,v,sizeof(x))
#define fastio
    ios_base::sync_with_stdio(0);cin.tie(0);cout.tie(0)
#define sqr(x) (x)*(x)
#define bas 987625403
#define N 100010
typedef long double ld;
typedef long long ll;
typedef pair<ll,ll> ii;
typedef pair<ll,ii> tri;
typedef vector<ll> vi;
typedef vector<ii> vii;
#define trace(...) fff(#__VA_ARGS__,__VA_ARGS__)

template<typename t> void fff(const char *x, t&&
    val1){
    cout << x << " : " << val1 << endl;
}

template<typename t1, typename... t2> void
    fff(const char *x, t1&& val1, t2&&... val2){
    const char *xd = strchr(x+1,',');
    cout.write(x,xd-x)<< " : " << val1 << " | ";
    fff(xd+1,val2...);
}

int main(){
    fastio;
    return 0;
}
```

8.6 Vim Configuration (vimrc)

```
set number
set autoindent
set ts=4
set mouse=a
```