The Calculation of the Temperature Rise and Load Capability of Cable Systems

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IN 1932 D. M. Simmons[1] published a series of articles entitled, "Calculation of the Electrical Problems of Underground Cables." Over the intervening 25 years this work has achieved the status of a handbook on the subject. During this period, however, there have been numerous developments in the cable art, and much theoretical and experimental work has been done with a view to obtaining more accurate methods of evaluating the parameters involved. The advent of the pipe-type cable system has emphasized the desirability of a more rational method of calculating the performance of cables in duct in order that a realistic comparison may be made between the two systems.

In this paper the authors have endeavored to extend the work of Simmons by presenting under one cover the basic principles involved, together with more recently developed procedures for handling such problems as the effect of the loading cycle and the temperature rise of cables in various types of duct structures. Included as well are expressions required in the evaluation of the basic parameters for certain specialized allied procedures. It is thought that a work of this type will be useful not only as a guide to engineers entering the field and as a reference to the more experienced, but particularly as a basis for setting up computation methods for the preparation of industry load capability and a-c/d-c ratio compilations.

The calculation of the temperature rise of cable systems under essentially steady-state conditions, which includes the effect of operation under a repetitive load cycle, as opposed to transient temperature rises due to the sudden application of large amounts of load, is a relatively simple procedure and involves only the application of the thermal equivalents of Ohm's and Kirchoff's Laws to a relatively simple thermal circuit. Because this circuit usually has a number of parallel paths with heat flows entering at several points, however, care must be exercised in the method used of expressing the heat flows and thermal resistances involved, and differing methods are used by various engineers. The method employed in this paper has been selected after careful consideration as being the most consistent and most readily handled over the full scope of the problem.

All losses will be developed on the basis of watts per conductor foot. The heat flows and temperature rises due to dielectric loss and to current-produced losses will be treated separately, and, in the latter case, all heat flows will be expressed in terms of the current produced loss originating in one foot of conductor by means of multiplying factors which take into account the added losses in the sheath and conduit.

In general, all thermal resistances will be developed on the basis of the per conductor heat flow through them. In the case of underground cable systems, it is convenient to utilize an effective thermal resistance for the earth portion of the thermal circuit which includes the effect of the loading cycle and the mutual heating effect of the other cable of the system. All cables in the system will be considered to carry the same load currents and to be operating under the same load cycle.

The system of nomenclature employed is in accordance with that adopted by the Insulated Conductor Committee as standard, and differs appreciably from that used in many of the references. This system represents an attempt to utilize in so far as possible the various symbols appearing in the American Standards Association Standards for Electrical Quantities, Mechanics, Heat and Thermo-Dynamics, and Hydraulics, when these symbols can be used without ambiguity. Certain symbols which have long been used by cable engineers have been retained, even though they are in direct conflict with the above-mentioned standards.

Nomenclature

(AF) = attainment factor, per unit (pu)
A_s = cross-section area of a shielding tape or skid wire, square inches
α = thermal diffusivity, square inches per hour
CI = conductor area, circular inches
d = distance, inches
d_{12} etc. = from center of cable no. 1 to center of cable no. 2 etc.
d_{12}' etc. = from center of cable no. 1 to image of cable no. 2 etc.
d_{1i} etc. = from center of cable no. 1 to a point of interference
d_{1i}' etc. = from image of cable no. 1 to a point of interference
D = diameter, inches
D_0 = inside of annular conductor
D_c = outside of conductor
D_i = outside of insulation
D_s = outside of sheath
D_{sm} = mean diameter of sheath
D_j = outside of jacket
D_s' = effective (circumscribing circle) of several cables in contact
D_p = inside of duct wall, pipe or conduit
D_e = diameter at start of the earth portion of the thermal circuit
D_x = fictitious diameter at which the effect of loss factor commences
E = line to neutral voltage, kilovolts (kv)
ϵ = coefficient of surface emissivity
ϵ_r = specific inductive capacitance of insulation
f = frequency, cycles per second
F, F_{int} = products of ratios of distances
$F(x)$ = derived Bessel function of x (Table III and Fig. 1)
G = geometric factor
G_1 = applying to insulation resistance (Fig. 2 of reference 1)
G_2 = applying to dielectric loss (Fig. 2 of reference 1)
G_b = applying to a duct bank (Fig. 2)
I = conductor current, kiloamperes
k_s = skin effect correction factor for annular and segmental conductors
k_p = relative transverse conductivity factor for calculating conductor proximity effect
l = lay of a shielding tape or skid wire, inches
L = depth of reference cable below earth's surface, inches
L_b = depth to center of a duct bank (or backfill), inches
(lf) = load factor, per unit
(LF) = loss factor, per unit
n = number of conductors per cable
n' = number of conductors within a stated diameter
N = number of cables or cable groups in a system
P = perimeter of a duct bank or backfill, inches
$\cos\phi$ = power factor of the insulation
q_s = ratio of the sum of the losses in the conductors and sheaths to the losses in the conductors
q_e = ratio of the sum of the losses in the conductors, sheath and conduit to the losses in the conductors
R = electrical resistance, ohms
R_{dc} = d-c resistance of conductor
R_{ac} = total a-c resistance per conductor
R_s = d-c resistance of sheath or of the parallel paths in a shield-skid wire assembly
$\bar{R}$ = thermal resistance (per conductor losses) thermal ohm-feet
$\bar{R}_i$ = of insulation
$\bar{R}_j$ = of jacket
$\bar{R}_{sd}$ = between cable surface and surrounding enclosure

Paper 57-660, recommended by the AIEE Insulated Conductors Committee and approved by the AIEE Technical Operations Department for presentation at the AIEE Summer General Meeting, Montreal, Que., Canada, June 24–28, 1957. Manuscript submitted March 20, 1957; made available for printing April 18, 1957.

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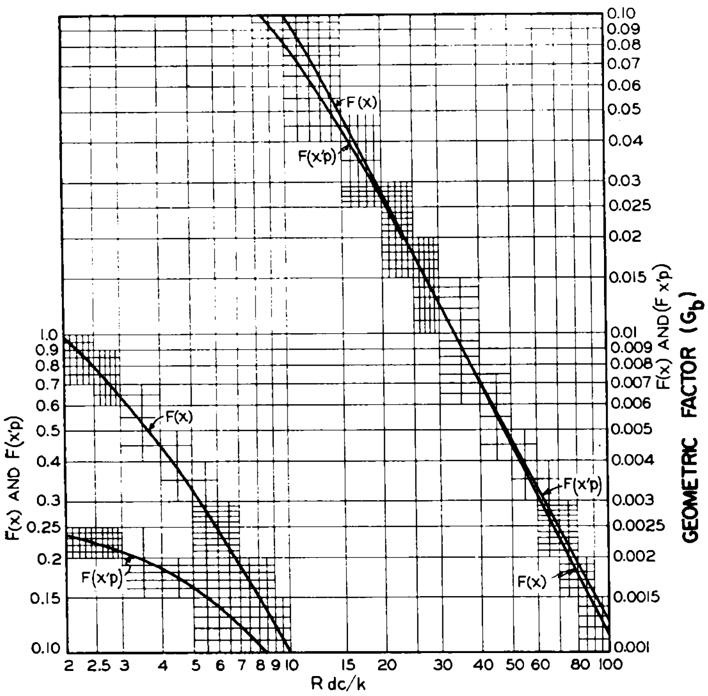
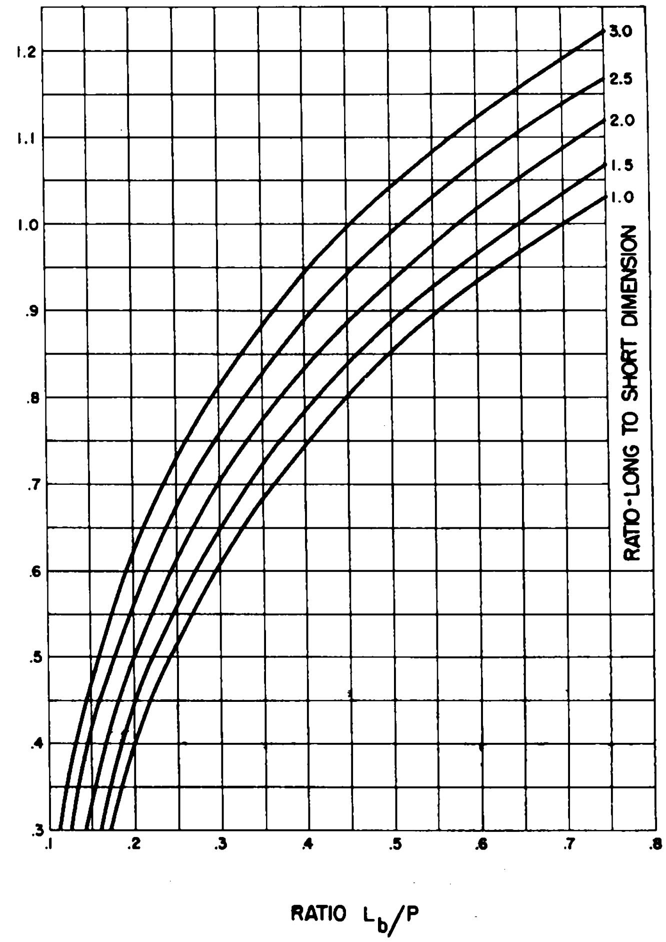


Fig. 1 (above). F(x) and F(x_p') as functions of R_{dc}/k

Fig. 2 (right). G_b for a duct bank



$\bar{R}_d$ = of duct wall or asphalt mastic covering
$\bar{R}_{se}$ = total between sheath and diameter D_e including $\bar{R}_j$, $\bar{R}_{sd}$ and $\bar{R}_d$
$\bar{R}_e$ = between conduit and ambient
$\bar{R}_e'$ = effective between diameter D_e and ambient earth including the effects of loss factor and mutual heating by other cables
$\bar{R}_{ca}'$ = effective between conductor and ambient for conductor loss
R_{ct}' = effective transient thermal resistance of cable system
$\bar{R}_{da}'$ = effective between conductor and ambient for dielectric loss
$\bar{R}_{int}$ = of the interference effect
$\bar{R}_{ya}$ = between a steam pipe and ambient earth
ρ = electrical resistivity, circular mil ohms per foot
$\bar{\rho}$ = thermal resistivity, degrees centigrade centimeters per watt
s = distance in a 3-conductor cable between the effective current center of the conductor and the axis of the cable, inches
S = axial spacing between adjacent cables, inches
t, T = thickness (as indicated), inches
T = temperature, degrees centigrade
T_a = of ambient air or earth
T_c = of conductor
T_m = mean temperature of medium
ΔT = temperature rise, degrees centigrade
ΔT_c = of conductor due to current produced losses
ΔT_d = of conductor due to dielectric loss
ΔT_{int} = of a cable due to extraneous heat source
τ = inferred temperature of zero resistance, degrees centigrade (C) (used in correcting R_{dc} and R_s to temperatures other than 20 C)
V_w = wind velocity, miles per hour
W = losses developed in a cable, watts per conductor foot
W_c = portion developed in the conductor
W_s = portion developed in the sheath or shield
W_p = portion developed in the pipe or conduit
W_d = portion developed in the dielectric
X_m = mutual reactance, conductor to sheath or shield, microhms per foot
Y = the increment of a-c/d-c ratio, pu
Y_c = due to losses originating in the conductor, having components Y_{cs} due to skin effect and Y_{cp} due to proximity effect
Y_s = due to losses originating in the sheath or shield, having components Y_{sc} due to circulating current effect and Y_{se} due to eddy current effect
Y_p = due to losses originating in the pipe or conduit
Y_a = due to losses originating in the armor

General Considerations of the Thermal Circuit

The Calculation of Temperature Rise

The temperature rise of the conductor of a cable above ambient temperature may be considered as being composed of a temperature rise due to its own losses, which may be divided into a rise due to current produced (I^2R) losses (hereinafter referred to merely as losses) in the conductor, sheath and conduit ΔT_c and the rise produced by its dielectric loss ΔT_d. Thus

$$T_c - T_a = \Delta T_c + \Delta T_d \text{ degrees centigrade} \tag{1}$$

Each of these component temperature rises may be considered as the result of a rate of heat flow expressed in watts per foot through a thermal resistance expressed in thermal ohm feet (degrees centigrade feet per watt); in other words, the radial rise in degrees centigrade for a heat flow of one watt uniformly distributed over a conductor length of one foot.

Since the losses occur at several positions in the cable system, the heat flow in the thermal circuit will increase in steps. It is convenient to express all heat flows in terms of the loss per foot of conductor, and thus

$$\Delta T_c = W_c(\bar{R}_i + q_s\bar{R}_{se} + q_e\bar{R}_e) \text{ degrees centigrade} \tag{2}$$

in which W_c represents the losses in one conductor and $\bar{R}_i$ is the thermal resistance of the insulation, q_s is the ratio of the sum of the losses in the conductors and sheath to the losses in the conductors, $\bar{R}_{se}$ is the total thermal resistance between sheath and conduit, q_e is the ratio of the sum of the losses in conductors, sheath and conduit, to the conductor losses, and $\bar{R}_e$

is the thermal resistance between the conduit and ambient.

In practice, the load carried by a cable is rarely constant and varies according to a daily load cycle having a load factor (lf). Hence, the losses in the cable will vary according to the corresponding daily loss cycle having a loss factor (LF). From an examination of a large number of load cycles and their corresponding load and loss factors, the following general relationship between load factor and loss factor has been found to exist.[2]

$$(LF)=0.3\,(lf)+0.7\,(lf)^2 \text{ per unit} \quad (3)$$

In order to determine the maximum temperature rise attained by a buried cable system under a repeated daily load cycle, the losses and resultant heat flows are calculated on the basis of the maximum load (usually taken as the average current for that hour of the daily load cycle during which the average current is the highest, i.e. the daily maximum one-hour average load) on which the loss factor is based and the heat flow in the last part of the earth portion of the thermal circuit is reduced by the factor (LF). If this reduction is considered to start at a point in the earth corresponding to the diameter D_x,[3] equation 2 becomes

$$\Delta T_c=W_c[\bar{R}_i+q_s\bar{R}_{se}+q_e(\bar{R}_{ex}+(LF)\bar{R}_{xa})] \text{ degrees centigrade} \quad (4)$$

In effect this means that the temperature rise from conductor to D_x is made to depend on the heat loss corresponding to the maximum load whereas the temperature rise from diameter D_x to ambient is made to depend on the average loss over a 24-hour period. Studies indicate that the procedure of assuming a fictitious critical diameter D_x at which an abrupt change occurs in loss factor from 100% to actual will give results which very closely approximate those obtained by rigorous transient analysis. For cables or duct in air where the thermal storage capacity of the system is relatively small, the maximum temperature rise is based upon the heat flow corresponding to maximum load without reduction of any part of the thermal circuit.

When a number of cables are installed close together in the earth or in a duct bank, each cable will have a heating effect upon all of the others. In calculating the temperature rise of any one cable, it is convenient to handle the heating effects of the other cables of the system by suitably modifying the last term of equation 4. This is permissible since it is assumed that all the cables are carrying equal currents, and are operating on the same load cycle. Thus for an N-cable system

$$\Delta T_c=W_c(\bar{R}_i+q_s\bar{R}_{se}+q_e[\bar{R}_{ex}+(LF)\times(\bar{R}_{xa})+(N-1)\bar{R}_{pa}]) \quad (5)$$

$$=W_c(\bar{R}_i+q_e\bar{R}_{se}+q_eR_e') \text{ degrees centigrade} \quad (5A)$$

where the term in parentheses is indicated by the effective thermal resistance $\bar{R}_e'$.

The temperature rise due to dielectric loss is a relatively small part of the total temperature rise of cable systems operating at the lower voltages, but at higher voltages it constitutes an appreciable part and must be considered. Although the dielectric losses are distributed throughout the insulation, it may be shown that for single conductor cable and multiconductor shielded cable with round conductors the correct temperature rise is obtained by considering for transient and steady state that all of the dielectric loss W_d occurs at the middle of the thermal resistance between conductor and sheath or alternately for steady-state conditions alone that the temperature rise between conductor and sheath for a given loss in the dielectric is half as much as if that loss were in the conductor. In the case of multiconductor belted cables, however, the conductors are taken as the source of the dielectric loss.[1]

The resulting temperature rise due to dielectric loss ΔT_d may be expressed

$$\Delta T_d=W_d\bar{R}_{da}' \text{ degrees centigrade} \quad (6)$$

in which the effective thermal resistance $\bar{R}_{da}'$ is based upon $\bar{R}_i$, $\bar{R}_{se}$, and $\bar{R}_e'$(at unity loss factor) according to the particular case. The temperature rise at points in the cable system other than at the conductor may be determined readily from the foregoing relationships.

The Calculation of Load Capability

In many cases the permissible maximum temperature of the conductor is fixed and the magnitude of the conductor current (load capability) required to produce this temperature is desired. Equation 5(A) may be written in the form

$$\Delta T_c=I^2R_{dc}(1+Y_c)\bar{R}_{ca}' \text{ degrees centigrade} \quad (7)$$

in which the quantity R_{dc} $(1+Y_c)$ which will be evaluated later represents the effective electrical resistance of the conductor in microhms per foot, and which when multiplied by I^2 (I in kiloamperes) will equal the loss W_c in watts per conductor foot actually generated in the conductor; and $\bar{R}_{ca}'$ is the effective thermal resistance of the thermal circuit.

$$\bar{R}_{ca}'=\bar{R}_i+q_s\bar{R}_{se}+q_e\bar{R}_e' \text{ thermal ohm-feet} \quad (8)$$

From equation 1 it follows that

$$I=\sqrt{\frac{T_c-(T_a+\Delta T_d)}{R_{dc}(1+Y_c)\bar{R}_{ca}'}} \text{ kiloamperes} \quad (9)$$

Table I. Electrical Resistivity of Various Materials

Material	ρ Circular Mil Ohms per Foot at 20 C	τ, C
Copper (100% IACS*)	10.371	234.5
Aluminum (61% IACS)	17.002	228.1
Commercial Bronze (43.6% IACS) (90 Cu–10 Zn)	23.8	564
Brass (27.3% IACS) (70 Cu–30 Zn)	38.0	912
Lead (7.84% IACS)	132.3	236

* International Annealed Copper Standard.

Calculation of Losses and Associated Parameters

Calculation of D-C Resistances

The resistance of the conductor may be determined from the following expressions which include a lay factor of 2%; see Table I.

$$R_{dc}=\frac{1.02\rho_c}{CI} \text{ microhms per foot at 20 C} \quad (10)$$

$$=\frac{12.9}{CI} \text{ for 100\% IACS copper conductor at 75 C} \quad (10A)$$

$$=\frac{21.2}{CI} \text{ for 61\% IACS aluminum at 75 C} \quad (10B)$$

where CI represents the conductor size in circular inches and where ρ_c represents the electrical resistivity in circular mil ohms per foot. To determine the value of resistance at temperature T multiply the resistance at 20 C by $(\tau+T)/(\tau+20)$ where τ is the inferred temperature of zero resistance.

The resistance of the sheath is given by the expressions

$$R_s=\frac{\rho_s}{4D_{sm}t} \text{ microhms per foot at 20 C} \quad (11)$$

$$R_s=\frac{37.9}{D_{sm}t} \text{ for lead at 50 C} \quad (11A)$$

$$=\frac{4.75}{D_{sm}t} \text{ for 61\% aluminum at 50 C} \quad (11B)$$

where D_{sm} is the mean diameter of the sheath and t is its thickness, both in inches

$$D_{sm}=D_s-t \text{ inches} \quad (12)$$

The resistance of intercalated shields or skid wires may be determined from the expression

$$R_s \text{ (per path)}=\frac{\pi\rho_s}{4A_s}\sqrt{1+\left(\frac{\pi D_{sm}}{l}\right)^2} \text{ microhms per foot at 20 C} \quad (13)$$

where A_s is the cross-section area of the tape or skid wire and l is its lay. The over-all resistance of the shield and skid wire assembly, particularly for nonintercalated shields, should be determined by electrical measurement when possible.

Calculation of Losses

It is convenient to develop expressions for the losses in the conductor, sheath and pipe or conduit in terms of the components of the a-c/d-c ratio of the cable system which may be expressed as follows[4]

$$R_{ac}/R_{dc}=1+Y_c+Y_s+Y_p \qquad (14)$$

The a-c/d-c ratio at conductor is $1+Y_c$

and at sheath or shield is $1+Y_c+Y_s$

and at pipe or conduit is $1+Y_c+Y_s+Y_p$

The corresponding losses physically generated in the conductor, sheath, and pipe are

$$W_c=I^2R_{dc}(1+Y_c) \text{ watts per conductor foot} \qquad (15)$$

$$W_s=I^2R_{dc}Y_s \text{ watts per conductor foot} \qquad (16)$$

$$W_p=I^2R_{dc}Y_p \text{ watts per conductor foot} \qquad (17)$$

This permits a ready determination of the losses if the segregated a-c/d-c ratios are known, and conversely, the a-c/d-c ratio is readily obtained after the values of Y_c, Y_s and Y_p have been calculated.

It follows from the definitions of q_s and q_e that

$$q_s=\frac{W_c+W_s}{W_c}=1+\frac{Y_s}{1+Y_c} \qquad (18)$$

$$q_e=\frac{W_c+W_s+W_p}{W_c}=1+\frac{Y_s+Y_p}{1+Y_c} \qquad (19)$$

The factor Y_c is the sum of two components, Y_{cs} due to skin effect and Y_{cp} due proximity effect.

$$W_c=I^2R_{dc}(1+Y_{cs}+Y_{cp}) \text{ watts per conductor foot} \qquad (20)$$

The skin effect may be determined from the skin effect function $F(x)$

$$Y_{cs}=F(x_s) \qquad (21)$$

$$x_s=0.875\sqrt{\frac{fk_s}{R_{dc}}}=\frac{6.80}{\sqrt{R_{dc}/k_s}} \text{ at 60 cycles} \qquad (22)$$

in which the factor k_s depends upon the conductor construction. For solid or conventional conductors appropriate values of k_s will be found in Table II. The function $F(x)$ may be obtained from Table III or from the curves of Fig. 1 in terms of the ratio R_{dc}/k at 60 cycles.

For annual conductors

$$k_s=\frac{D_c-D_0}{D_c+D_0}\left(\frac{D_c+2D_0}{D_c+D_0}\right)^2 \qquad (23)$$

in which D_c and D_0 represent the outer and inner diameters of the annular conductor. In comparison with the rigorous Bessel function solution for the skin effect in an isolated tubular conductor, it has been found that the 60-cycle skin effect of annular conductor when computed by equation 23 will not be in error by more than 0.01 in absolute magnitude for copper or aluminum IPCEA (Insulated Power Cable Engineers Association) filled

Table II. Recommended Values of k_s and k_p

Conductor Construction	Coating on Strands	Treatment	k_s	k_p
Concentric round	None	None	1.0	1.0
Concentric round	Tin or alloy	None	1.0	1.0
Concentric round	None	Yes	1.0	0.80
Compact round	None	Yes	1.0	0.6
Compact segmental	None	None	0.435	0.6
Compact segmental	Tin or alloy	None	0.5	0.7
Compact segmental	None	Yes	0.435	0.37
Compact sector	None	Yes	1.0	(see note)

Notes:

1. The term "treated" denotes a completed conductor which has been subjected to a drying and impregnating process similar to that employed on paper power cable.

2. Proximity effect on compact sector conductors may be taken as one-half of that for compact round having the same cross-sectional area and insulation thickness.

3. Proximity effect on annular conductors may be approximated by using the value for a concentric round conductor of the same cross-sectional area and spacing. The increased diameter of the annular type and the removal of metal from the center decreases the skin effect but, for a given axial spacing, tends to result in an increase in proximity.

4. The values listed above for compact segmental refer to four segment constructions. The "uncoated-treated" values may also be taken as applicable to four segment compact segmental with hollow core (approximately 0.75 inch clear). For "uncoated-treated" six segment hollow core compact segmental limited test data indicates k_s and k_p values of 0.39 and 0.33 respectively.

Table III. Skin Effect in % in Solid Round Conductor and in Conventional Round Concentric Strand Conductors

100 F(x), Skin Effect %

x	0	1	2	3	4	5	6	7	8	9
0.3	0.00	0.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
0.4	0.01	0.01	0.02	0.02	0.02	0.02	0.02	0.03	0.03	0.03
0.5	0.03	0.04	0.04	0.04	0.05	0.05	0.05	0.06	0.06	0.06
0.6	0.07	0.07	0.08	0.08	0.09	0.10	0.10	0.11	0.11	0.12
0.7	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19	0.19	0.20
0.8	0.21	0.22	0.24	0.25	0.26	0.28	0.29	0.30	0.31	0.33
0.9	0.34	0.36	0.38	0.39	0.41	0.43	0.45	0.47	0.48	0.50
1.0	0.52	0.54	0.56	0.58	0.61	0.63	0.65	0.68	0.70	0.73
1.1	0.76	0.79	0.81	0.84	0.87	0.90	0.94	0.97	1.00	1.03
1.2	1 07	1.11	1.14	1.18	1.22	1.26	1.30	1.34	1.38	1.42
1.3	1.47	1.52	1.56	1.61	1.66	1.71	1.76	1.81	1.86	1.92
1.4	1.97	2.02	2.08	2.14	2.20	2.26	2.32	2.39	2.45	2.52
1.5	2.58	2.65	2.72	2.79	2.86	2.93	3.01	3.08	3.16	3.24
1.6	3.32	3.40	3.49	3.57	3.66	3.75	3.83	3.92	4.02	4.11
1.7	4.21	4.30	4.40	4.50	4.60	4.70	4.81	4.91	5.02	5.13
1.8	5.24	5.35	5.47	5.58	5.70	5.82	5.94	6.06	6.19	6.31
1.9	6.44	6.57	6.70	6.83	6.97	7.11	7.24	7.38	7.53	7.67
2.0	7.82	7.96	8.11	8.26	8.42	8.57	8.73	8.89	9.05	9.21
2.1	9.38	9.54	9.71	9.88	10.05	10.22	10.40	10.58	10.76	10.94
2.2	11.13	11.31	11.50	11.69	11.88	12.07	12.27	12.47	12.67	12.87
2.3	13.07	13.27	13.48	13.68	13.90	14.11	14.33	14.54	14.76	14.98
2.4	15.21	15.43	15.66	15.89	16.12	16.35	16.58	16.82	17.16	17.30
2.5	17.54	17.78	18.03	18.27	18.52	18.78	19.03	19.28	19.54	19.80
2.6	20.06	20.32	20.58	20.85	21.12	21.38	21.65	21.93	22.20	22.48
2.7	22.75	23.03	23.31	23.60	23.88	24.17	24.45	24.74	25.03	25.33
2.8	25.62	25.92	26.21	26.51	26.81	27.11	27.42	27.72	28.03	28.34
2.9	28.65	28.96	29.27	29.58	29.90	30.21	30.53	30.85	31.17	31.49
3.0	31.81	32.13	32.45	32.78	33.11	33.44	33.77	34.10	34.43	34.77
3.1	35.10	35.44	35.78	36.11	36.45	36.79	37.13	37.47	37.82	38.16
3.2	38.50	38.85	39.20	39.55	39.89	40.24	40.59	40.94	41.29	41.65
3.3	42.00	42.35	42.71	43.06	43.42	43.78	44.14	44.49	44.85	45.21
3.4	45.57	45.93	46.29	46.66	47.02	47.38	47.74	48.11	48.47	48.84
3.5	49.20	49.57	49.94	50.30	50.67	51.04	51.40	51.77	52.14	52.51
3.6	52.88	53.25	53.62	53.99	54.36	54.73	55.10	55.48	55.85	56.22
3.7	56.59	56.96	57.33	57.71	58.08	58.45	58.82	59.20	59.57	59.94
3.8	60.31	60.69	61.06	61.44	61.81	62.18	62.56	62.93	63.30	63.68
3.9	64.05	64.42	64.80	65.17	65.55	65.92	66.29	66.67	67.04	67.41
4.0	67.79	68.16	68.53	68.91	69.28	69.65	70.03	70.40	70.77	71.14
4.1	71.52	71.89	72.26	72.63	73.00	73.38	73.75	74.12	74.49	74.86
4.2	75.23	75.60	75.97	76.34	76.71	77.08	77.45	77.82	78.19	78.56
4.3	78.93	79.30	79.67	80.04	80.41	80.78	81.14	81.51	81.88	82.25
4.4	82.61	82.98	83.35	83.61	84.08	84.45	84.81	85.18	85.55	85.91
4.5	86.28	86.64	87.01	87.37	87.73	88.10	88.46	88.82	89.19	89.55
4.6	89.91	90.28	90.64	91.00	91.37	91.73	92.09	92.45	92.81	93.17
4.7	93.53	93.89	94.25	94.61	94.97	95.33	95.69	96.05	96.41	96.77
4.8	97.13	97.49	97.85	98.21	98.57	98.92	99.28	99.64	100.00	100.35
4.9	100.71	101.07	101.42	101.78	102.14	102.49	102.85	103.21	103.56	103.92

Table IV. Mutual Reactance at 60 Cycles, Conductor to Sheath (or Shield)

$D_{sm}/2S$	0	1	2	3	4	5	6	7	8	9
0.4	21.1	20.5	19.9	19.4	18.9	18.3	17.8	17.4	16.9	16.4
0.3	27.7	26.9	26.2	25.5	24.8	24.1	23.5	22.9	22.2	21.6
0.2	37.0	35.9	34.8	33.8	32.8	31.9	31.0	30.1	29.3	28.4
0.1	52.9	50.7	48.7	46.9	45.2	43.6	42.1	40.7	39.4	38.2

core conductors up through 5.0 *CI* and for hollow core concentrically stranded copper or aluminum oil-filled cable conductors up through 4.0 *CI*.

For values of x_p below 3.5, a range which appear to cover most cases of practical interest at power frequencies, the conductor proximity effect for cables in equilateral triangular formation in the same or in separate ducts may be calculated from the following equation based on an approximate expression given by Arnold[6] (equation 7) for a system of three homogeneous, straight, parallel, solid conductors of circular cross section arranged in equilateral formation and carrying balanced 3-phase current remote from all other conductors or conducting material. The empirical transverse conductance factor k_p is introduced to make the expression applicable to stranded conductors. Experimental results suggest the values of k_p shown in Table II.

$$Y_{cp}=F(x_p)\left(\frac{D_c}{S}\right)^2\times\left[\frac{1.18}{F(x_p)+0.27}+0.312\left(\frac{D_c}{S}\right)^2\right] \quad (24)$$

$$x_p=\frac{6.80}{\sqrt{R_{dc}/k_p}} \text{ at 60 cycles} \quad (25)$$

When the second term in the brackets is small with respect to the first term as it usually is, equation 24 may be written

$$Y_{cp}=4F(x_p)\left[\frac{0.295(D_c/S)^2}{F(x_p)+0.27}\right]=4\left(\frac{D_c}{S}\right)^2 F(x_p') \quad (24A)$$

where the function $F(x_p')$ is shown in Fig. 1.

The average proximity effect for conductors in cradle configuration in the same duct or in separate ducts in a formation approximating a regular polygon may also be estimated from equation 24 and 24(A). In such cases, S should be taken as the axial spacing between adjacent conductors.

The factor Y_s is the sum of two factors, Y_{sc} due to circulating current effect and Y_s due to eddy current effects.

$$W_s=I^2R_{dc}(Y_{sc}+Y_{se}) \text{ watts per conductor foot} \quad (26)$$

Because of the large sheath losses which result from short-circuited sheath operation with appreciable separation between metallic sheathed single conductor cables, this mode of operation is usually restricted to triplex cable or three single-conductor cables contained in the same duct. The circulating current effect in three metallic sheathed single-conductor cables arranged in equilateral configuration is given by

$$Y_{sc}=\frac{R_s/R_{dc}}{1+(R_s/X_m)^2} \quad (27)$$

When $(R_s/X_m)^2$ is large with respect to unity as usually is the case of shielded nonleaded cables, equation 27 reduces to

$$Y_{sc}=\frac{X_m}{R_sR_{dc}} \text{ approximately} \quad (27A)$$

$$X_m=0.882f\log 2S/D_{sm} \text{ microhms per foot} \quad (28)$$

$$=52.9\log 2S/D_{sm} \text{ microhms per foot at 60 cycles} \quad (28A)$$

where S is the axial spacing of adjacent cables. For a cradled configuration X_m may be approximated from

$$X_m=52.9\log\frac{2.52S}{D_{sm}}\sqrt[6]{1-\left(\frac{S}{D_p-S}\right)^2} \text{ microhms per foot at 60 cycles} \quad (29)$$

$$=52.9\log 2.3\ S/D_{sm} \text{ approximately} \quad (29A)$$

Table IV provides a convenient means for determining X_m for cables in equilateral configuration.

The eddy-current effect for single-conductor cables in equilateral configuration with open-circuited sheaths is

$$Y_{se}=\frac{3R_s/R_{dc}}{\left(\frac{5.2R_s}{f}\right)^2+\frac{1}{5}\left(\frac{2S}{D_{sm}}\right)}\times\left(\frac{D_{sm}}{2S}\right)^2\left[1+\frac{5}{12}\left(\frac{D_{sm}}{2S}\right)^2\right] \quad (30)$$

when $(5.2\ R_s/f)^2$ is large in respect to 1/5 $(2\ S/D_{sm})$ as in the case of lead sheaths.

$$Y_{se}=\frac{396}{R_sR_{ds}}\left(\frac{D_{sm}}{2S}\right)^2\left[1+\frac{5}{12}\left(\frac{D_{sm}}{2S}\right)^2\right] \text{ approximately at 60 cycles} \quad (30A)$$

When the sheaths are short-circuited, the sheath eddy loss will be reduced and may be approximated by multiplying equations 30 or 30(A) by the ratio

$$R_s^2/(R_s^2+X_m^2)$$

In computing average eddy current for cradled configuration, S should be taken equal to the axial spacing and not to the geometric-mean spacing. Equations 30 and 30(A) may be used to compute the eddy-current effect for single-conductor cables installed in separate ducts. Strictly speaking, these equations apply only to three cables in equilateral configuration but can be used to estimate losses in large cable groups when latter are so oriented as to approximate a regular polygon.

The eddy-current effect for a 3-conductor cable is given by Arnold.[6]

$$Y_{se}=\frac{3R_s}{R_{dc}}\left[\frac{(2s/D_{sm})^2}{\left(\frac{5.2R_s}{f}\right)^2+1}+\frac{(2s/D_{sm})^4}{4\left(\frac{5.2R_s}{f}\right)^2+1}+\frac{(2s/D_{sm})^8}{16\left(\frac{5.2R_s}{f}\right)^2+1}\cdots\right] \quad (31)$$

When $(5.2R_s/f)^2$ is large with respect to unity,

$$Y_{se}=\frac{396}{R_sR_{dc}}\left(\frac{2s}{D_{sm}}\right)^2 \text{ approximately at 60 cycles} \quad (31A)$$

$$s=1.155T+0.60\times\text{the } V \text{ gauge depth for compact sectors}$$

$$=1.155T+0.58\ D_c \text{ for round conductors} \quad (32)$$

and T is the insulation thickness, including thickness of shielding tapes, if any. While equation 31(A) will suffice for lead sheath cables, equation 31 should be used for aluminum sheaths.

On 3-conductor shielded paper lead cable it is customary to employ a 3- or 5-mil copper tape or bronze tape intercalated with a paper tape for shielding and binder purposes. The lineal d-c resistance of a copper tape 5 mils by 0.75 inch is about 2,200 microhms per foot of tape at 20 C. The d-c resistance per foot of cable will be equal to the lineal resistance of the tape multiplied by the lay correction factor as given by the expression under the square-root sign in equation 13. In practice the lay correction factor may vary from 4 to 12 or more resulting in shielding and binder assembly resist-

Table V. Specific Inductive Capacitance of Insulations

Material	ϵ_r	
Polyethylene	2.3	
Paper insulation (solid type)	3.7	(IPCEA value)
Paper insulation (other types)	3.3–4.2	
Rubber and rubber-like compounds	5	(IPCEA value)
Varnished cambric	5	(IPCEA value)

ances of approximately 10,000 or more microhms per foot of cable. Even on the assumption that the assembly resistance is halved because of contact with adjacent conductors and the lead sheath computations made using equations 27 and 30 show that the resulting circulating and eddy current losses are a fraction of 1% on sizes of practical interest. For this reason it is customary to assume that the losses in the shielding and binder tapes of 3-conductor shielded paper lead cable are negligible. In cases of nonleaded rubber power cables where lapped metallic tapes are frequently employed, tube effects may be present and may materially lower the resistance of the shielding assembly and hence increase the losses to a point where they are of practical significance.

An exact determination of the pipe loss effect Y_p in the case of single-conductor cables installed in nonmagnetic conduit or pipe is a rather involved procedure as indicated in reference 7. Equation 31 may be used to obtain a rough estimate of Y_p for cables in cradled formation on the bottom of a nonmagnetic pipe, however by taking the average of the results obtained for wide triangular spacing with $s=(D_p-D_s)/2$ and for close triangle spacing at the center of the pipe with $s=0.578\,D_s$. The mean diameter of the pipe and its resistance per foot should be substituted for D_{sm} and R_s respectively.

For magnetic pipes or conduit the following empirical relationships[8] may be employed

$$Y_p=\frac{1.54s-0.115D_p}{R_{dc}} \text{ (3-conductor cable)} \quad (33)$$

$$Y_p=\frac{0.89S-0.115D_p}{R_{dc}} \text{ (single-conductor, close triangular)} \quad (34)$$

$$Y_p=\frac{0.34S+0.175D_p}{R_{dc}} \text{ (single-conductor, cradled)} \quad (35)$$

These expressions apply to steel pipe[8] and should be multiplied by 0.8 for iron conduit.[9]

The expressions given for Y_c and Y_s above should be multiplied by 1.7 to find the corresponding in-pipe effects for magnetic pipe or conduit for both triangular and cradled configurations.

Calculation of Dielectric Loss

The dielectric loss W_d for 3-conductor shielded and single-conductor cable is given by the expression

$$W_d=\frac{0.00276E^2\epsilon_r\cos\phi}{\log(2T+D_c)/D_c} \text{ watts per conductor foot at 60 cycles} \quad (36)$$

and for 3-conductor belted cable by[1]

$$W_d=\frac{0.019E^2\epsilon_r\cos\phi}{G_2} \text{ watts per conductor foot at 60 cycles} \quad (37)$$

where E is the phase to neutral voltage in kilovolts, ϵ_r is the specific inductive capacitance of the insulation (Table V) T is its thickness and $\cos\phi$ is its power factor. The geometric factor G_2 may be found from Fig. 2 of reference 1.

For compact sector conductors the dielectric loss may be taken equal to that for a concentric round conductor having the same cross-sectional area and insulation thickness.

Calculation of Thermal Resistance

Thermal Resistance of the Insulation

For a single conductor cable,

$$\bar{R}_i=0.012\bar{\rho}_i\log D_i/D_c \quad \text{thermal ohm-feet} \quad (38)$$

where $\bar{\rho}_i$ is the thermal resistivity of the insulation (Table VI) and D_i is its diameter. In multiconductor cables there is a multipath heat flow between the conductor and sheath. The following expression[1] represents an equivalent value which, when multiplied by the heat flow from one conductor, will produce the actual temperature elevation of the conductor above the sheath.

$$\bar{R}_i=0.00522\bar{\rho}_iG_1 \quad \text{thermal ohm-feet} \quad (39)$$

Values of the geometric factor G_1 for 3-conductor belted and shielded cables are given in Fig. 2 and Table VIII respectively of reference 1. On large size sector conductors with relatively thin insulation walls (i.e. ratios of insulation thickness to conductor diameter of the order of 0.2 or less); values of G_1 for 3-conductor shielded cable as determined by back calculation, on the basis of an assumed insulation resistivity, from laboratory heat-run temperature-rise data, have not always confirmed theoretical values, and, in some cases, have yielded G_1 values which approach those for a nonshielded, nonbelted construction.

Table VI. Thermal Resistivity of Various Materials

Material	$\bar{\rho}$, C Cm/Watt
Paper insulation (solid type)	700 (IPCEA value)
Varnished cambric	600 (IPCEA value)
Paper insulation (other types)	500–550
Rubber and rubber-like	500 (IPCEA value)
Jute and textile protective covering	500
Fiber duct	480
Polyethylene	450
Transite duct	200
Somastic	100
Concrete	85

Thermal Resistance of Jackets, Duct Walls, and Somastic Coatings

The equivalent thermal resistance of relatively thin cylindrical sections such as jackets and fiber duct walls may be determined from the expression

$$\bar{R}=0.0104\bar{\rho}n'\left(\frac{t}{D-t}\right) \quad \text{thermal ohm-feet} \quad (40)$$

with appropriate subscripts applied to $\bar{R}$, $\bar{\rho}$, and D in which D represents the outside diameter of the section and t its thickness. n' is the number of conductors contained with the section contributing to the heat flow through it.

Thermal Resistance between Cable Surface and Surrounding Pipe, Conduit, or Duct Wall

Theoretical expressions for the thermal resistance between a cable surface and a surrounding enclosure are given in reference 10. As indicated in Appendix I, these have been simplified to the general form

$$\bar{R}_{sd}=\frac{n'A}{1+(B+CT_m)D_s'} \quad \text{thermal ohm-feet} \quad (41)$$

in which A, B, and C are constants, D_s' represents the equivalent diameter of the cable or group of cables and n' the number of conductors contained within D_s'. T_m is the mean temperature of the intervening medium. The constants A, B, and C

Table VII. Constants for Use in Equations 41 and 41(A)

Condition	A	B	C	A'	B'
In metallic conduit	17	3.6	0.029	3.2	0.19
In fiber duct in air	17	2.1	0.016	5.6	0.33
In fiber duct in concrete	17	2.3	0.024	4.6	0.27
In transite duct in air	17	3.0	0.014	4.4	0.26
In transite duct in concrete	17	2.9	0.029	3.7	0.22
Gas-filled pipe cable at 200 psi	3.1	1.16	0.0053	2.1	0.68
Oil-filled pipe cable	0.84	0	0.0065	2.1	2.45

D_s' = 1.00 × diameter of cable for one cable
1.65 × diameter of cable for two cables
2.15 × diameter of cable for three cables
2.50 × diameter of cable for four cables

given in Table VII have been determined from the experimental data given in references 10 and 11.

If representative values of $T_m=60$ C are assumed, equation 41 reduces to

$$\bar{R}_{sd}=\frac{n'A'}{D_s'+B'} \text{ thermal ohm-feet} \quad (41A)$$

It should be noted that in the case of ducts, $\bar{R}_{sd}$ is calculated to the inside of the duct wall and the thermal resistance of the duct wall should be added to obtain $\bar{R}_{sd}$.

Thermal Resistance from Cables, Conduits, or Ducts Suspended in Air

The thermal resistance $\bar{R}_e$ between cables, conduits, or ducts suspended in still air may be determined from the following expression which is developed in Appendix I.

$$\bar{R}_e=\frac{15.6n'}{D_s'[(\Delta T/D_s')^{1/4}+1.6\epsilon(1+0.0167T_m)]} \text{ thermal ohm-feet} \quad (42)$$

In this equation ΔT represents the difference between the cable surface temperature T_s and ambient air temperature T_a in degrees centigrade, T_m the average of these temperatures and ϵ the coefficient of emissivity of the cable surface. Assuming representative values of $T_s=60$ and $T_a=30$ C, and a range in D_s' of from 2 to 10 inches, equation 42 may be simplified to

$$\bar{R}_e=\frac{9.5n'}{1+1.7D_s'(\epsilon+0.41)} \text{ thermal ohm-feet} \quad (42A)$$

The value of ϵ may be taken as equal to 0.95 for pipes, conduits or ducts, and painted or braided surfaces, and from 0.2 to 0.5 for lead and aluminum sheaths, depending upon whether the surface is bright or corroded. It is interesting to note that equation 42(A) checks the IPCEA method of determining R_e very closely with $\epsilon=0.41$ for diameters up to 3.5 inches. In the IPCEA method $\bar{R}_e=0.00411\ n'B/D_s'$ where $B=650+314\ D_s'$ for

$D_s'=0-1.75$ inches and $B=1{,}200$ for larger values of D_s'

Effective Thermal Resistance Between Cables, Ducts, or Pipes, and Ambient Earth

As previously indicated, an effective thermal resistance $\bar{R}_e'$ may be employed to represent the earth portion of the thermal circuit in the case of buried cable systems. This effective thermal resistance includes the effect of loss factor and, in the case of a multicable installation, also the mutual heating effects of the other cables of the system. In the case of cables in a concrete duct bank, it is desirable to further recognize a difference between the thermal resistivity of the concrete $\bar{\rho}_c$ and the thermal resistivity of the surrounding earth $\bar{\rho}_e$.

The thermal resistance between any point in the earth surrounding a buried cable and ambient earth is given by the expression[12]

$$\bar{R}_{pa}=0.012\bar{\rho}_e \log d'/d \quad \text{thermal ohm-feet} \quad (43)$$

in which $\bar{\rho}_e$ is the thermal resistivity of the earth, d' is the distance from the image of the cable to the point P, and d is the distance from the cable center to P. From this equation and the principles discussed in references 3, 12, and 13, the following expressions may be developed, applicable to directly buried cables and to pipe-type cables.

$$\bar{R}_e'=0.012\bar{\rho}_e n'\times\left[\log\frac{D_x}{D_e}+(LF)\log\left[\left(\frac{4L}{D_x}\right)F\right]\right] \text{ thermal ohm-feet} \quad (44)$$

in which D_e is the diameter at which the earth portion of the thermal circuit commences and n' is the number of conductors contained within D_e. The fictitious diameter D_x at which the effect of loss factor commences is a function of the diffusivity of the medium α and the length of the loss cycle.[3]

$$D_x=1.02\sqrt{\alpha(\text{length of cycle in hours})} \text{ inches} \quad (45)$$

The empirical development of this equation is discussed in Appendix III. For a daily loss cycle and a representative value of $\alpha=2.75$ square inches per hour for earth, D_x is equal to 8.3 inches. It should be noted that the value of D_x obtained from equation 45 is applicable for pipe diameters exceeding D_x, in which case the first term of equation 44 is negative.

The factor F accounts for the mutual heating effect of the other cables of the cable system, and consists of the product of the ratios of the distance from the reference cable to the image of each of the other cables to the distance to that cable. Thus,

$$F=\left(\frac{d_{12}'}{d_{12}}\right)\left(\frac{d_{13}'}{d_{13}}\right)\cdots\left(\frac{d_{1N}'}{d_{1N}}\right)(N-1 \text{ terms}) \quad (46)$$

It will be noted that the value of F will vary depending upon which cable is selected as the reference, and the maximum conductor temperature will occur in the cable for which $4LF/D_x$ is a maximum. N refers to the number of cables or pipes, and F is equal to unity when $N=1$.

When the cable system is contained within a concrete envelope such as a duct bank, the effect of the differing thermal resistivity of the concrete envelope is conveniently handled by first assuming that the thermal resistivity of the medium is that of concrete $\bar{\rho}_c$ throughout and then correcting that portion lying beyond the concrete envelope to the thermal resistivity of the earth $\bar{\rho}_e$. Thus

$$\bar{R}_e'=0.012\bar{\rho}_c n'\times\left[\log\frac{D_x}{D_e}+(LF)\log\left[\left(\frac{4L}{D_x}\right)F\right]\right]+0.012(\bar{\rho}_e-\bar{\rho}_c)n'N(LF)G_b \text{ thermal ohm-feet} \quad (44A)$$

The geometric factor G_b, as developed in Appendix II is a function of the depth to the center of the concrete enclosure L_b and its perimeter P, and may be found conveniently from Fig. 2 in terms of the ratio L_b/P and the ratio of the longest to short dimension of the enclosure.

For buried cable systems T_a should be taken as the ambient temperature at the depth of the hottest cable. As indicated in reference 12, the expressions used throughout this paper for the thermal resistance and temperature rise of buried cable systems are based on the hypothesis suggested by Kennelly applied in accordance with the principle of superposition. According to this hypothesis, the isothermal-heat flow field and temperature rise at any point in the soil surrounding a buried cable can be represented by the steady-state solution for the heat flow between two parallel cylinders (constituting a heat source and sink) located in a vertical plane in an infinite medium of uniform temperature and thermal resistivity with an axial separation between cylinders of twice the actual depth of burial and with source and sink respectively generating and absorbing heat at identical rates, thereby resulting in the temperature of the horizontal midplane between cylinders (i.e., corresponding to the surface of the earth) remaining, by symmetry, undisturbed.

The principle of superposition, as applied to the case at hand, can be stated in thermal terms as follows: If the thermal network has more than one source of temperature rise, the heat that flows at any point, or the temperature drop between any two points, is the sum of the heat flows and temperature drops at these points which would exist if each source of temperature rise were considered separately. In the case at hand, the sources of heat flow and temperature rise to be superimposed are, namely, the heat from the cable, the outward flow of heat from the core of the earth, and the inward heat flow solar radiation, and, when present, the heat flow from interfering sources. By employing as the ambient temperature in the calculations the temperature at the depth of burial of the hottest cable, the combined heat flow from earth core and solar radiation sources is superimposed upon that produced at the surface of the hottest cable by the heat flow from that cable and interfering sources which are calculated separately with all other heat flows absent. The combined heat flow from earth core and solar sources results in an earth temperature which decreases with depth in summer; increases with depth in winter; remains about constant at any given depth on the average over a year; approximates constancy at all depths at midseason, and in turn results in flow of heat from cable sources to earth's surface, directly to surface in midseason and winter and indirectly to surface in summer.

Factors which tend to invalidate the combined Kennelly-superposition principle method are departure of the temperature of the surface of earth from a true isothermal (as evidenced by melting of snow in winter directly over a buried steam main) and nonuniformity of thermal resistivity (due to such phenomena as radial and vertical migration of moisture). The extent to which the Kennelly-superposition principle method is invalidated, however, is not of practical importance provided that an over-all or effective thermal resistivity is employed in the Kennelly equation.

Special Conditions

Although the majority of cable temperature calculations may be made by the foregoing procedure, conditions frequently arise which require somewhat specialized treatment. Some of these are covered herein.

Emergency Ratings

Under emergency conditions it is frequently necessary to exceed the stated normal temperature limit of the conductor T_c and to set an emergency temperature limit T_c'. If the duration of the emergency is long enough for steady-state conditions to obtain, then the emergency rating I' may be found by equation 9 substituting T_c' for T_c and correcting R_{dc} for the increased conductor temperature.

If the duration of the emergency is less than that required for steady-state conditions to obtain, the emergency rating of the line may be determined from

$$I'=\sqrt{\frac{T_c'-I^2R_{dc}(1+Y_c)(\bar{R}_{ca}'-\bar{R}_{ct}')-(T_a+\Delta T_d)}{R_{dc}'(1+Y_c)\bar{R}_{ct}'}}\quad \text{kiloamperes} \quad (47)$$

in which $\bar{R}_{ct}'$ is the effective transient thermal resistance of the cable system for the stated period of time. Procedures for calculating $\bar{R}_{ct}'$ for times up to several hours are given in reference 14, and for longer times in references 15-17.

The Effect of Extraneous Heat Sources

In the case of multicable installations the assumption has been made that all cables are of the same size and are similarly loaded. When this is not the case the temperature rise or load capability of one particular equal cable group may be determined by treating the heating effect of other cable groups separately, introducing an interference temperature rise ΔT_{int} in equations 1 and 9. Thus

$$T_c-T_a=\Delta T_c+\Delta T_d+\Delta T_{int}\quad \text{degrees centigrade} \quad (1A)$$

$$I=\sqrt{\frac{T_c-(T_a+\Delta T_d+\Delta T_{int})}{R_{dc}(1+Y_c)\bar{R}_{ca}'}}\quad \text{kiloamperes} \quad (9A)$$

in which ΔT_{int} represents the sum of a number of interference effects, for each of which

$$\Delta T_{int}=[W_cq_e(LF)+W_d]\bar{R}_{int}\quad \text{degrees centigrade} \quad (48)$$

$$\bar{R}_{int}=0.012\bar{\rho}_e n' \log F_{int}\quad \text{thermal ohm-feet} \quad (49)$$

$$F_{int}=\frac{(d_{1t}')(d_{2t}')(d_{3t}')\ldots d_{Nt}'}{(d_{1t})(d_{2t})(d_{3t})\ldots d_{Nt}}\ (N\text{ terms}) \quad (50)$$

where the parameters apply to each system which may be considered as a unit. For cables in duct

$$\bar{R}_{int}=0.012n'[\bar{\rho}_c \log F_{int}+N(\bar{\rho}_e-\bar{\rho}_c)G_b]\quad \text{thermal ohm-feet} \quad (49A)$$

Because of the mutual heating between cable groups, the temperature rise of the interfering groups should be rechecked. If all the cable groups are to be given mutually compatible ratings, it is necessary to evaluate W_c for each group by successive approximations, or by setting up a system of simultaneous equations, substituting for W_c its value by equation 15 and solving for I.

In case ΔT_{int} or a component of it is produced by an adjacent steam main, the temperature of the steam T_y rather than the heat flow from it is usually given. Thus

$$\Delta T_{int}=\left[\frac{T_y-T_a}{\bar{R}_{ya}}\right]\bar{R}_{int}\quad \text{degrees centigrade} \quad (51)$$

where $\bar{R}_{ya}$ is the thermal resistance between the steam pipe and ambient earth.

Aerial Cables

In the case of aerial cables it may be desirable to consider both the effects of solar radiation which increases the temperature rise and the effect of the wind which decreases it.[24] Under maximum sunlight conditions, a lead-sheathed cable will absorb about 4.3 watts per foot per inch of profile[18] which must be returned to the atmosphere through the thermal resistance $\bar{R}_e/n'$. This effect is conveniently treated as an interference temperature rise according to the relationship

$$\Delta T_{int}=4.3D_s'\bar{R}_e/n'\quad \text{degrees centigrade} \quad (47A)$$

For black surfaces this value should be increased about 75%.

As indicated in Appendix II, the following expression for $\bar{R}_e$ may be used where V_w is the velocity of the wind in miles per hour

$$\bar{R}_e=\frac{3.5n'}{D_s'(\sqrt{V_w/D_s'}+0.62\epsilon)}\quad \text{thermal ohm-feet} \quad (42B)$$

Use of Low-Resistivity Backfill

In cases where the thermal resistivity of the earth is excessively high, the value of $\bar{R}_e'$ may be reduced by backfilling the trench with soil or sand having a lower value of thermal resistivity. Equation 44(A) may be used for this case if $\bar{\rho}_f$, the thermal resistivity of the backfill is substituted for $\bar{\rho}_c$, and G_b applies to the zone having the backfill in place of the zone occupied by the concrete.

Single-Conductor Cables in Duct with Solidly Bonded Sheaths

The relatively large and unequal sheath losses in the three phases which may result from this type of operation may be determined from Table VI of reference 1. It will be noted that

$$Y_{sc1}=\left(\frac{R_s}{R_{dc}}\right)\left(\frac{I_{s1}^2}{I^2}\right);\quad Y_{sc2}=\left(\frac{R_s}{R_{dc}}\right)\left(\frac{I_{s2}^2}{I^2}\right);\quad Y_{sc3}=\left(\frac{R_s}{R_{dc}}\right)\left(\frac{I_{s3}^2}{I^2}\right) \quad (52)$$

where expressions for I_{s1}^2/I^2 etc., appear in the table. The resulting unequal values of Y_c in the three phases will yield unequal values of q_s, and equation 5 becomes for phase no. 1, the instance given as equation 5(A) on the following page.

$$\Delta T_{c1} = W_c[\bar{R}_t + q_{s1}\{\bar{R}_{se} + \bar{R}_{ex} + (LF)\bar{R}_{xp}\} + Nq_{sa}(LF)\bar{R}_{pa}] \quad \text{thermal ohm-feet} \quad (5A)$$

where q_{sa} is the average of q_{s1}, q_{s2}, and q_{s3}.

Armored Cables

In multiconductor armored cables a loss occurs in the armor which may be considered as an alternate to the conduit or pipe loss. If the armor is nonmagnetic, the component of armor loss Y_a to be used instead of Y_p in equations 14 and 19 may be calculated by the equations for sheath loss substituting the resistance and mean diameter of the armor for those of the sheath. In calculating the armor resistance, account should be taken of the spiralling effect for which equation 13 suitably modified may be used. If the armor is magnetic, one would expect an increase in the factors Y_c and Y_s in equation 14 since this occurs in the case of magnetic conduit. Unfortunately, no simple procedure is available for calculating these effects. A rough estimate of the inductive effects may be made by using the procedure given above for magnetic conduit.

A simple method of approximating the losses in single conductor cables with steel-wire armor at spacings ordinarily employed in submarine installations is to assume that the combined sheath and armor current is equal to the conductor current.[1] The effective a-c resistance of the armor may be taken as 30 to 60% greater than its d-c resistance corrected for lay as indicated above. If more accurate calculations are desired references 19 and 20 will be found useful.

Effect of Forced Cooling

The temperature rise of cables in pipes or tunnels may be reduced by forcing air axially along the system. Similarly, in the case of oil-filled pipe cable, oil may be circulated through the pipe. Under these conditions, the temperature rise is not uniform along the cable and increases in the direction of flow of the cooling medium. The solution of this problem is discussed in reference 21.

Appendix I

Development of Equations 41, 42, and Table VII

Theoretical and semiempirical expressions for the thermal resistance between cables and an enclosing pipe or duct wall are given in reference 10. Further data on the thermal resistance between cables and fiber and transite ducts are given in reference 11. For purposes of cable rating, it is desirable to develop standardized expressions for these thermal resistances based upon all of the data available and including the effect of the temperature of the intervening medium.

Table VIII. Constants for Use in Equation 53

Condition	a	b	c	Average ΔT
Cable in metallic conduit	0.07	0.121	0.0017	20
Cable in fiber duct in air	0.07	0.036	0.0009	20
Cable in fiber duct in concrete	0.07	0.043	0.0014	20
Cable in transite duct in air	0.07	0.086	0.0008	20
Cable in transite duct in concrete	0.07	0.079	0.0016	20
Gas-filled pipe-type cable at 200 psi	0.07	0.121	0.0017	10

The theoretical expression for the case where the intervening medium is air or gas as presented in reference 10 may be generalized in the following form:

$$\bar{R}_{sd} = \frac{n'}{D_s'\left[a\left(\frac{\Delta T P^2}{D_s'}\right)^{1/4} + b + cT_m\right]} \quad (53)$$

where

$\bar{R}_{sd}$ = the effective thermal resistance between cable and enclosure in thermal ohm-feet
D_s' = the cable diameter or equivalent diameter of three cables in inches
ΔT = the temperature differential in degrees centigrade
P = the pressure in atmospheres
T_m = mean temperature of the medium in degrees centigrade
n' = number of conductors involved

The constants a, b, and c in this equation have been established empirically as follows: Considering $b+cT_m$ as a constant for the moment, the analysis given in reference 10 results in a value of $a=0.07$. With a thus established, the data given in reference 10 for cable in pipe, and in reference 11 for cable in fiber and transite ducts were analyzed in similar manner to give the values of b and c which are shown in Table VIII.

In order to avoid a reiterative calculation procedure, it is desirable to assume a value for ΔT since its actual value will depend upon $\bar{R}_{sd}$ and the heat flow. Fortunately, as ΔT occurs to the 1/4 power in equation 53, the use of an average value as indicated in Table VIII will not introduce a serious error.

By further restricting the range of D_s' to 1–4 inches for cable in duct or conduit and to 3–5 inches for pipe-type cables, equation 53 is reduced to equation 41.

$$\bar{R}_{sd} = \frac{n'A}{1+(B+CT_m)D_s'} \quad \text{thermal ohm-feet} \quad (41)$$

in which the values of the constants A, B, and C appear in Table VII.

In the case of oil-filled pipe cable, the analysis given in reference 10 gives the following expression

$$\bar{R}_{sd} = \frac{n'}{0.60+0.025(D_s'^3 T_m^3 \Delta T)^{1/4}} \quad \text{thermal ohm-feet} \quad (54)$$

Assuming an average value of $\Delta T = 7$ C and a range of 150–350 for $D_s'T_m$, equation 54 reduces to equation 41 with the values of A, B, and C given in Table VII.

In the case of cables or pipes suspended in still air, the heat loss by radiation may be determined by the Stefan-Bolzmann formula

$$n'W\,(\text{radiation}) = 0.139 D_s'\epsilon[(T_s+273)^4-(T_a+273)^4]10^{-8} \quad \text{watts per foot} \quad (55)$$

where ϵ is the coefficient of emissivity of the cable or pipe surface. Over the limited temperature range in which we are interested, equation 55 may be simplified to[10]

$$n'W\,(\text{radiation}) = 0.102 D_s'\Delta T\epsilon \times (1+0.0167T_m) \quad \text{watts per foot} \quad (55A)$$

Over the same temperature range the heat loss by convection from horizontal cables or pipes is given with sufficient accuracy by the expression

$$n'W(\text{convection}) = 0.064\, D_s'\Delta T(\Delta T/D_s')^{1/4} \quad \text{watts per foot} \quad (56)$$

in which the numerical constant 0.064 has been selected for the best fit with the carefully determined test results reported by Heilman[22] on 1.3, 3.5 and 10.8-inch diameter black pipes ($\epsilon=0.95$). Incidentally, this value also represents the best fit with the test data on 1.9–4.5 inch diameter black pipes reported by Rosch.[23] For vertical cables or pipes the value of this numerical constant may be increased by 22%.[22]

Combining equations 55(A) and 56 we obtain the relationship

$$\bar{R}_e = \frac{\Delta T}{n'W\,(\text{total})} = \frac{15.6n'}{D_s'[(\Delta T/D_s')^{1/4}+1.6\epsilon(1+0.0167T_m)]} \quad \text{thermal ohm-feet} \quad (42)$$

If the cable is subjected to wind having a velocity of V_w miles per hour, the following expression derived from the work of Schurig and Frick[24] should be substituted for the convection component.

$$n'W\,(\text{convection}) = 0.286 D_s'\Delta T\sqrt{V_w/D_s'} \quad \text{watts per foot} \quad (56A)$$

Combining equations 55(A) and 56(A) with $T_m = 45$ C

$$\bar{R}_e = \frac{\Delta T}{n'W\,(\text{total})} = \frac{3.5n'}{D_s'(\sqrt{V_w/D_s'}+0.62\epsilon)} \quad \text{thermal ohm-feet} \quad (42B)$$

Appendix II

Determination of the Geometric Factor G_b for Duct Bank

Considering the surface of the duct bank to act as an isothermal circle of radius r_b, the thermal resistance between the duct bank and the earth's surface will be a logarithmic function of r_b and L_b the distance of the center of the bank below the surface. Using the long form of the Kennelly Formula[13] we may define the geometric factor G_b as

$$G_b=\log\frac{L_b+\sqrt{L_b^2-r_b^2}}{r_b}$$
$$=\log\left[L_b/r_b+\sqrt{(L_b/r_b)^2-1}\right] \quad (57)$$

In order to evaluate r_b in terms of the dimensions of a rectangular duct bank, let the smaller dimension of the bank be x and the larger dimension by y. The radius of a circle inscribed within the duct bank touching the sides is

$$r_1=x/2 \quad (58)$$

and the radius of a larger circle embracing the four corners is

$$r_2=\frac{\sqrt{x^2+y^2}}{2} \quad (59)$$

Let us assume that the circle of radius r_b lies between these circles and the magnitude of r_b is such that it divides the thermal resistance between r_1 and r_2 in direct relation to the portions of the heat field between r_1 and r_2 occupied and unoccupied by the duct bank. Thus

$$\log\frac{r_b}{r_1}=\frac{xy-\pi r_1^2}{\pi(r_2^2-r_1^2)}\left(\log\frac{r_2}{r_1}\right) \quad \text{or}$$
$$\log\frac{r_2}{r_b}=\frac{\pi r_2^2-xy}{\pi(r_2^2-r_1^2)}\left(\log\frac{r_2}{r_1}\right)$$

from which

$$\log\ r_b=\frac{1}{2}\frac{x}{y}\left(\frac{4}{\pi}-\frac{x}{y}\right)\log\left(1+\frac{y^2}{x^2}\right)+\log\frac{x}{2} \quad (60)$$

It is desirable to derive r_b in terms of the perimeter P of the duct bank. Thus

$$P=2(x+y)=4\frac{x}{2}(1+y/x)$$

and therefore

$$\log\frac{x}{2}=\log\frac{P}{4(1+y/x)} \quad (61)$$

The curves of Fig. 2 have been developed from equations 57, 60, and 61 for several values of the ratio y/x. It should be noted in passing that the value of $r_b=0.112P$ used in reference 13 applies to a y/x ratio of about 2/1 only.

Appendix III

Empirical Evaluation of D_x

In order to evaluate the effect of a cyclic load upon the maximum temperature rise of a cable system simply, it is customary to assume that the heat flow in the final portion of the thermal circuit is reduced by a factor equal to the loss factor of the cyclic load. The point at which this reduction commences may be conveniently expressed in terms of a fictitious diameter D_x. Thus

$$\bar{R}_{ca}'=\bar{R}_{cx}+(LF)\bar{R}_{xa} \text{ thermal ohm-feet} \quad (62)$$

For greater accuracy, it is desirable to establish the value of D_x empirically rather than to assume that D_x is equal to the diameter D_e at which the earth portion of the thermal circuit commences.

Equation 62 may be written in the form

$$\bar{R}_{ca}'=\bar{R}_{ce}+\bar{R}_{ex}+(LF)(\bar{R}_{ea}-\bar{R}_{ex})$$
$$\text{thermal ohm-feet} \quad (62\text{A})$$

In terms of the attainment factor (AF), one may write

$$\bar{R}_{ca}'=(AF)\bar{R}_{ca}=(AF)(\bar{R}_{ce}+\bar{R}_{ea})$$
$$\text{thermal ohm-feet} \quad (63)$$

Equating equations 62(A) and 63 obtains the relationship

$$\bar{R}_{ex}=(1-x)\bar{R}_{ea}-x\bar{R}_{ce} \text{ thermal ohm-feet} \quad (64)$$

where

$$x=\frac{1-(AF)}{1-(LF)} \quad (65)$$

Since

$$\bar{R}_{ex}=0.012n'\bar{\rho}\log D_x/D_e$$
$$\text{thermal ohm-feet} \quad (66)$$

$$\log D_x/D_e=\frac{83}{n'\bar{\rho}}[(1-x)\bar{R}_{ea}-x\bar{R}_{ce}] \quad (67)$$

The first paper of reference 3 presents the results of a study in which a number of typical daily loss cycles and also sinusoidal loss cycles of the same loss factor were applied to a number of typical buried cable systems. The results indicated that in all cases the sinusoidal loss cycle of the same loss factor adequately expressed the maximum temperature rise which was obtained with any of the actual loss cycles considered.

An analysis by equations 65 and 67 of the calculated values of attainment factors for sinusoidal loss cycles given in Table II and the corresponding cable system parameters given in Table I of the first paper of reference 3 yields a most probable value of $D_x=8.3$ inches. As indicated in the third paper of reference 3, however, theoretically D_x should vary as the square root of the product of the diffusivity and the time length of the loading cycle. Hence as the diffusivity was taken as 2.75 square inches per hour in the above,

$$D_x=1.02\times\sqrt{\alpha_e\times\text{length of cycle in hours}} \text{ inches} \quad (45)$$

Table IX. Comparison of Values of % (AF) for Sinusoidal Loss Cycles at 30% Loss Factor

System	Description, Inches	% (AF) Neher	% (AF) Shanklin	% (AF) Wiseman
I	4.5 pipe	63/63	61/62	63/65
II	6.6 pipe	56/56	60/57	53/60
III	8.6 pipe	56/56	59/58	54/63
IV	10.6 pipe	58/58	61/59	55/53
V	0.5 cable		80/80	
VI	1.5 cable	77/76	77/76	77/77
VII	1.9 cable	71/71		
VIII	2.0 cable		63/62	
IX	2.0 cable		75/74	
X	3.4 cable		77/78	
Xa*	3.4 cable	83/80	83/81	
XI	3.7 cable	76/74	74/73	
XII	4.2 cable	70/66	70/67	
XIII	4.5 cable	69/64	65/64	61/63

* Diffusivity = 4.7 square inches per hour.

Table IX presents a comparison of the values of per cent attainment factor for sinusoidal loss cycles at 30% loss factor as calculated by equations 45, 66, 62(A), and 63 and as they appear in Table II of the first paper of reference 3.

Appendix IV. Calculations for Representative Cable Systems

15-Kv 350-MCM—3-Conductor Shielded Compact Sector Paper and Lead Cable Suspended in Air

$D_c=0.616$ (equivalent round); $V=$ gauge depth $=0.539$ inch

$D_s=2.129$; $T=0.175$ inch; $t=0.120$ inch

$$T_c=81\text{ C};\ R_{dc}=\frac{12.9}{0.350}\left(\frac{234.5+81}{234.5+75}\right)$$
$$=37.6 \text{ microhms per foot} \quad (\text{Eq. 10A})$$

$$D_{sm}=2.129-0.120=2.009 \text{ inches} \quad (\text{Eq. 12})$$

$$R_s=\frac{37.9}{2.009(0.120)}=157 \text{ microhms per foot at 50 C} \quad (\text{Eq. 11A})$$

$k_s=1.0$; $k_p=0.6$ (equivalent round) (Table II)

$R_{dc}/k_s=37.6$; $Y_{cs}=0.008$ (Eq. 21 and Fig. 1)

$$S=0.616+2(0.175+0.008)=0.982 \text{ inches}$$

$R_{dc}/k_p=62.6$; $F(x_p')=0.003$ (Fig. 1)

$$Y_{cp}=\frac{1}{2}\left[4\left(\frac{0.616}{0.982}\right)^2\right]0.003=0.002$$
(Eq. 24A, and note to Table II)

$$1+Y_c=1+0.008+0.002=1.010$$

$$s=1.155(0.175+0.008)+0.60(0.539)$$
$$=0.534 \text{ inch} \quad (\text{Eq. 32})$$

$$Y_s=Y_{se}=\frac{396}{157(37.6)}\left\{\frac{2(0.534)}{2.009}\right\}^2=0.019$$
(Eq. 31A)

$$R_{ac}/R_{dc}=1.010+0.019=1.029 \quad (\text{Eq. 14})$$

$$q_s=q_e=1+\frac{0.019}{1.010}=1.019 \quad (\text{Eqs. 18–19})$$

$\epsilon_r=3.7$ (Table V); $E=15/\sqrt{3}=8.7$; $\cos\phi=0.022$

$$W_d=\frac{0.00276\,(8.7)^2[3.7(0.022)]}{\log\dfrac{2(0.175)+0.681}{0.681}}$$
$$=0.094 \text{ watt per conductor foot}$$
(Eq. 36 and text)

(Note: In computing dielectric loss on

sector conductors, the equivalent diameter of the conductor is taken equal to that of a concentric round conductor, i.e., 0.681 inch for 350 MCM.)

$\bar{\rho}_i=700$ (Table VI); $G_1=0.45$ (Table VIII of reference 1)

$\bar{R}_i=0.00522\{700(0.45)\}=1.64$ thermal ohm-feet (Eq. 39)

$n'=3$; $\epsilon=0.41$ (assumed)

$$\bar{R}_e=\frac{9.5(3)}{1+1.7[2.129(0.41+0.41)]}$$

$=7.18$ thermal ohm-feet (Eq. 42A)

$\bar{R}_{ca}=1.64+1.019(7.18)=8.96$ thermal ohm-feet (Eq. 8)

$\Delta T_d=0.094(0.82+7.18)=0.75$ C (Eq. 6)

$T_a=40$ C (assumed)

$$I=\sqrt{\frac{81-(40+0.8)}{37.6[1.010(8.96)]}}$$

$=0.344$ kiloampere (Eq. 9)

If the cable is outdoors in sunlight and subjected to an 0.84 mile per hour wind

$$\bar{R}_e=\frac{3.5(3)}{2.129[\sqrt{0.84/2.129}+0.62(0.41)]}$$

$=5.59$ thermal ohm-feet (Eq. 42B)

$\bar{R}_{ca}'=1.64+1.019(5.59)=7.34$ thermal ohm-feet (Eq. 8)

$$\Delta T_{int}=(4.3)(2.129)\left(\frac{5.59}{3}\right)=17.1 \text{ C}$$

(Eq. 47A)

$T_a=30$ C (assumed)

$$I=\sqrt{\frac{81-(30+0.6+17.1)}{(37.6)(1.010)(7.34)}}$$

$=0.346$ kiloampere (Eq. 9)

In this particular case the net effect of solar radiation and an 0.84 mile per hour wind is to effectively raise the ambient temperature by 10 degrees, which is a rough estimating value commonly used. It should be noted, however, that this will not always be true, and the procedure outlined above is preferable.

69-Kv 1,500-MCM—Single-Conductor Oil-Filled Cable in Duct

Two identical cable circuits will be considered in a 2 by 3 fiber and concrete duct structure having the dimensions shown in Fig. 3.

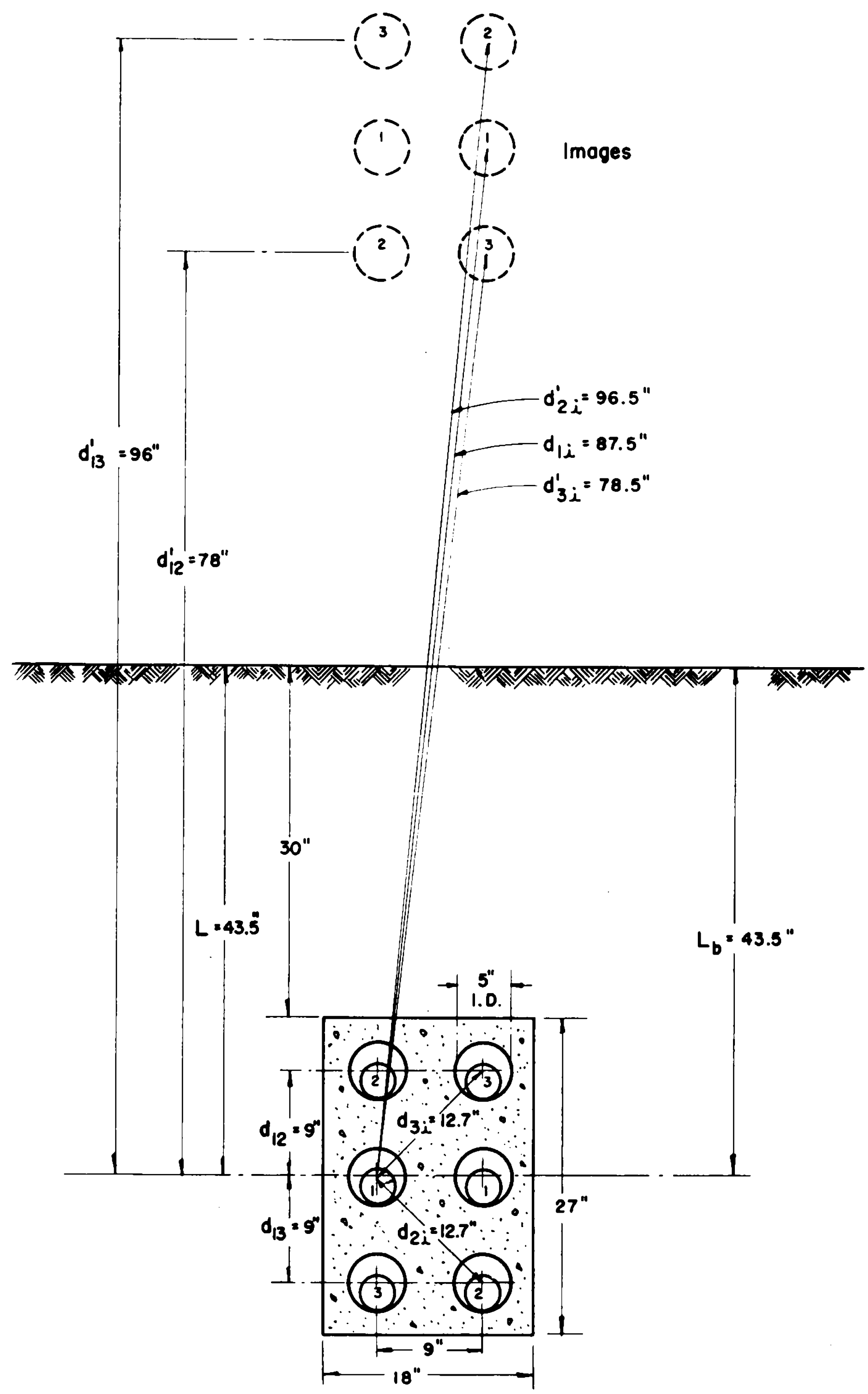


Fig. 3. Assumed duct bank configuration for typical calculations on 69-kv 1,500-MCM oil-filled cable (Appendix IV)

$D_o=0.600$; $D_c=1.543$; $D_i=2.113$; $T=0.285$; $D_s=2.373$; $t=0.130$ inches

$$T_c=75 \text{ C};\ R_{dc}=\frac{12.9}{1.50}=8.60$$

microhms per foot (Eq. 10A)

$D_{sm}=2.373-0.130=2.243$ inches (Eq. 12)

$$R_s=\frac{37.9}{(2.243)(0.130)}=130 \text{ microhms}$$

per foot at 50 C (Eq. 11A)

$$k_s=\frac{1.543-0.600}{1.543+0.600}\left(\frac{1.543+1.200}{1.543+0.600}\right)^2$$

$=0.72$; $k_p=0.8$ (Eq. 23 and Table II)

$R_{dc}/k_s=11.9$; $Y_{cs}=0.075$ (Eq. 21 and Fig. 1)

$S=9.0$ (Fig. 3); $R_{dc}/k_p=10.75$; $F(x_p')=0.075$ (Fig. 1)

$$Y_{cp}=4\left(\frac{1.412}{9.0}\right)^2 0.075=0.007 \quad \text{(Eq. 24A)}$$

$1+Y_c=1+0.075+0.007=1.082$

Assuming the sheaths to be open-circuited, $Y_{sc}=0$

$$Y_s=Y_{se}=\frac{396}{130(8.60)}\left(\frac{2.243}{2(9.0)}\right)^2\times$$

$$\left[1+\frac{5}{12}\left(\frac{2.243}{2(9.0)}\right)^2\right]=0.006 \quad \text{(Eq. 30A)}$$

$R_{ac}/R_{dc}=1.082+0.006=1.088$ (Eq. 14)

$$q_s=q_e=1+\frac{0.006}{1.082}=1.006 \quad \text{(Eqs. 18–19)}$$

$\epsilon_r=$ (Table V); $E=69/\sqrt{3}=40$; $\cos\phi=0.005$

$$W_d=\frac{0.00276(40)^2(3.5)(0.005)}{\log\dfrac{2.113}{1.543}}$$

$=0.57$ watt per conductor foot (Eq. 36)

$\bar{\rho}_i = 550$ (Table VI)

$$\bar{R}_i = 0.012\left(550 \log \frac{2.113}{1.543}\right) = 0.90 \text{ thermal ohm-foot} \quad \text{(Eq. 38)}$$

$$n' = 1;\ \bar{R}_{sd} = \frac{1(4.6)}{2.37+0.27} = 1.74 \text{ thermal ohm-feet} \quad \text{(Eq. 41A)}$$

$\bar{\rho}_d = 480$ (Table VI); $t = 0.25$; $D_e = 5.0+0.5 = 5.50$ for fiber duct

$$\bar{R}_d = \frac{0.0104(480)(0.25)}{5.50-0.25} = 0.24 \text{ thermal ohm-foot} \quad \text{(Eq. 40)}$$

$\bar{\rho}_e = 120$ (asumed); $\bar{\rho}_c = 85$ (Table VI); $L = L_b = 43.5$ inches (Fig. 3)

$N = 6$; $(LF) = 0.80$ (assumed);

$$F = \left(\frac{96}{9}\right)\left(\frac{78}{9}\right)\left(\frac{96.5}{12.7}\right)\left(\frac{87.5}{9}\right)\left(\frac{78.5}{12.7}\right) = 42{,}200 \quad \text{(Fig. 3 and Eq. 46)}$$

$$L_b/P = \frac{43.5}{2(18+27)} = 0.483;\ \frac{27}{18} = 1.5;\ G_b = 0.87 \quad \text{(Fig. 2)}$$

$$\bar{R}_e' \text{ (at 80\% loss factor)} = (0.012)(85)(1)\times \left(\log\frac{8.3}{5.5} + 0.80 \log\left[\frac{4(43.5)}{8.3}(42{,}200)\right]\right) + 0.012(120-85)(1)(6)(0.80)(0.87) = 6.79 \text{ thermal ohm-feet} \quad \text{(Eq. 44A)}$$

$\bar{R}_e'$ (at unity loss factor) $= 8.44$ thermal ohm-feet (Eq. 44A)

$$\bar{R}_{ca}' = 0.90+1.006\,(1.74+0.24+6.79) = 9.72 \text{ thermal ohm-feet} \quad \text{(Eq. 8)}$$

$$\Delta T_d = 0.57\left(\frac{0.90}{2}+1.74+0.24+8.44\right) = 6.2 \text{ C} \quad \text{(Eq. 6)}$$

$T_a = 25$ C (assumed);

$$I = \sqrt{\frac{75-(25+6.2)}{8.60(1.082)(9.72)}} = 0.696 \text{ kiloampere} \quad \text{(Eq. 9)}$$

To illustrate the case where the cable circuits are not identical, consider the second circuit to have 750-MCM conductors. For the first circuit,

$N = 3$; $(LF) = 0.80$ (assumed);

$$F = \left(\frac{96}{9}\right)\left(\frac{78}{9}\right) = 92.4 \quad \text{(Eq. 46)}$$

$$\bar{R}_e' = 0.012(85)(1)\times \left[\log\frac{8.3}{5.5} + 0.80 \log\left(\frac{4(43.5)}{8.3}\,92.4\right)\right] + 0.012(120-85)(1)(3)(0.80)(0.87) = 3.74 \text{ thermal ohm-feet} \quad \text{(Eq. 44A)}$$

$$F_{int} = \left(\frac{96.4}{12.7}\right)\left(\frac{87.5}{9}\right)\left(\frac{78.5}{12.7}\right) = 456 \quad \text{(Eq. 50)}$$

$$\bar{R}_{int} = 0.012(1)\times [85 \log 456 + 3(120-85)(0.87)] = 3.81 \text{ thermal ohm-feet} \quad \text{(Eq. 49)}$$

$$\bar{R}_{ca}' = 0.90+1.006(1.74+0.24+3.74) = 6.65 \text{ thermal ohm-feet} \quad \text{(Eq. 8)}$$

$$\Delta T_d = 0.57\,(0.45+1.75+0.24+4.63) = 4.0 \text{ C} \quad \text{(Eq. 6)}$$

$$W_{c1} = (I_1^2)(8.60)(1.082) = 9.31\, I_1^2 \text{ watts per conductor foot} \quad \text{(Eq. 15)}$$

$$\Delta T_{int} = (9.31 I_1^2[(1.006)(0.80)+0.57])3.81 = 2.17+28.5 I_1^2 \text{ degrees centigrade in circuit no. 2} \quad \text{(Eq. 48)}$$

Similar calculations for the second circuit yield the following values.

$\bar{R}_{ca}' = 7.18$; $\Delta T_d = 3.4$; $W_{c2} = 17.44 I_2^2$; $\Delta T_{int} = 1.71+53.2 I_2^2$ in circuit no. 1

$$I_1^2 = \frac{75-(25+4.0+1.71+53.2 I_2^2)}{(9.31)(6.65)}\times = 0.715-0.859 I_2^2 \quad \text{(Eq. 9A)}$$

$$I_2^2 = \frac{75-(25+3.4+2.17+28.5 I_1^2)}{(17.44)(7.18)}\times = 0.355-0.228 I_1^2 \quad \text{(Eq. 9A)}$$

Solving simultaneously $I_1 = 0.714$; $I_2 = 0.487$ kiloampere.

138-Kv 2,000-MCM High-Pressure Oil-Filled Pipe-Type Cable 8.625-Inch-Outside-Diameter Pipe

The cable shielding will consist of an intercalated 7/8(0.003)-inch bronze tape—1-inch lay, and a single 0.1(0.2)-inch *D*-shaped brass skid wire—1.5-inch lay. The cables will lie in cradled configuration.

$D_c = 1.632$; $D_i = 2.642$; $T = 0.505$; $D_s = 2.661$; $D_p = 8.125$

$$T_c = 70 \text{ C};\ R_{dc} = \left(\frac{12.9}{2.00}\right)\left(\frac{234.5+70}{234.5+75}\right) = 6.35 \text{ microhms per foot} \quad \text{(Eq. 10A)}$$

For shielding tape $A_s = 7/8(0.003) = 0.00263$; $l = 1.0$; $\rho = 23.8$; $\tau = 564$ (Table I)

$$R_s = \frac{23.8\pi}{4(0.00263)}\sqrt{1+\left(\frac{2.66\pi}{1}\right)^2}\times \left(\frac{564+50}{564+20}\right) = 62{,}900 \text{ microhms per foot at 50 C} \quad \text{(Eq. 13)}$$

For skid wire $A_s = \frac{1}{2}\pi(0.1)^2 = 0.0157$; $l = 1.5$; $\rho = 38$; $\tau = 912$ (Table I)

$$R_s = \frac{38\pi}{4(0.0157)}\sqrt{1+\left(\frac{2.66\pi}{1.5}\right)^2}\times \left(\frac{912+50}{912+20}\right) = 11{,}100 \text{ microhms per foot at 50 C} \quad \text{(Eq. 13)}$$

$$R_s\,(\text{net}) = \left[\frac{(62.9)(11.1)}{(62.9)(11.1)}\right]1{,}000 = 9{,}435 \text{ microhms per foot at 50 C}$$

$k_s = 0.435$; $k_p = 0.37$ (Table II)

$R_{dc}/k_s = 14.6$; $Y_{cs} = 0.052(1.7) = 0.088$ (Eq. 21, Fig. 1, and text)

$S = 2.66+0.10 = 2.76$; $R_{dc}/k_p = 17.2$; $F(x_p') = 0.035$ (Fig. 1)

$$Y_{cp} = 4\left(\frac{1.632}{2.76}\right)^2(0.035)(1.7) = 0.083 \quad \text{(Eq. 24A and text)}$$

$$1+Y_c = 1+0.088+0.083 = 1.171$$

$$X_m = 52.9 \log\frac{(2.3)(2.76)}{2.66} = 20.0 \text{ microhms per foot} \quad \text{(Eq. 29A)}$$

$$Y_s = Y_{sc} = \frac{(20.0)^2(1.7)}{(9.435)(6.35)} = 0.011 \quad \text{(Eq. 27A and text)}$$

$$Y_p = \frac{(0.34)(2.76)+(0.175)(8.13)}{6.35} = 0.372 \quad \text{(Eq. 35)}$$

$$R_{ac}/R_{dc} = 1.171+0.011+0.372 = 1.554 \quad \text{(Eq. 14)}$$

$$q_s = 1+\frac{0.011}{1.171} = 1.009;\ q = 1+\frac{0.011+0.372}{1.171} = 1.327 \quad \text{(Eqs. 18–19)}$$

$\epsilon_r = 3.5$ (Table V); $E = 138/\sqrt{3} = 80$; $\cos\phi = 0.005$

$$W_d = \frac{0.00276(80)^2(3.5)(0.005)}{\log\frac{2.642}{1.632}} = 1.48 \text{ watts per conductor foot} \quad \text{(Eq. 36)}$$

$\bar{\rho}_i = 550$ (Table VI); $\bar{R}_i = 0.012\times$

$$\left(550 \log\frac{2.642}{1.632}\right) = 1.38 \text{ thermal ohm-feet} \quad \text{(Eq. 38)}$$

$n' = 3$; $D_s' = 2.15(2.66) = 5.72$;

$$\bar{R}_{sd} = \frac{3(2.1)}{5.72+2.45} = 0.77 \text{ thermal ohm-foot} \quad \text{(Eq. 41A)}$$

$\rho_d = 100$ (Table VI); $t = 0.50$; $D_e = 8.63+1.0 = 9.63$ for 1/2-inch wall of asphalt mastic

$$\bar{R}_d = \frac{0.0104(100)(3)(0.50)}{9.63-0.50} = 0.17 \text{ thermal ohm-foot} \quad \text{(Eq. 40)}$$

Assume $\bar{\rho}_e = 80$, $L = 36$ inches, $(LF) = 0.85$; $N = 1$, $F = 1$

$$\bar{R}_e' \text{ (at 85\% loss factor)} = 0.012(80)(3)\times \left[\log\frac{8.3}{9.63} + 0.85 \log\left(\frac{4(36)}{8.3}(1)\right)\right] = 2.85 \text{ thermal ohm-feet} \quad \text{(Eq. 44)}$$

$\bar{R}_e'$ (at unity loss factor) $= 3.38$ thermal ohm-feet (Eq. 44)

$$\bar{R}_{ca}' = 1.38+1.009(0.77)+1.327(0.17+2.85) = 6.17 \text{ thermal ohm-feet} \quad \text{(Eq. 8)}$$

$$\Delta T_d = 1.48(0.69+0.77+0.17+3.38) = 7.4 \text{ C} \quad \text{(Eq. 6)}$$

$T_a = 25$ C (assumed);

$$I = \sqrt{\frac{70-(25+7.4)}{(6.35)(1.171)(6.17)}} = 0.905 \text{ kiloampere} \quad \text{(Eq. 9)}$$

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Discussion

C. C. Barnes (Central Electricity Authority, London, England): This paper is an excellent and up-to-date study of a most important subject. For 25 years D. M. Simmons' articles have been used for fundamental study on current rating problems, but the numerous cable developments and changes in installation techniques introduced in recent years have made a modern assessment of this subject very necessary. The essential duty of a power cable is that it should transmit the maximum current (or power) for specified installation conditions. There are three main factors which determine the safe continuous current that a cable will carry.

1. The maximum permissible temperature at which its components may be operated with a reasonable factor of safety.

2. The heat-dissipating properties of the cable.

3. The installation conditions and ambient conditions obtaining.

In Great Britain the basic reference document is ERA (The British Electrical and Allied Industries Research Association) report *F/T131*[1] published in 1939, and in 1955 revised current rating tables for solid-type cables up to and including 33 kv were published in ERA report *F/T183*. A more detailed report summarizing the method of computing current ratings for solid-type, oil-filled, and gas-pressure cables is now being finalized and will be published as ERA report *F/T187* some time in 1958.

Until recent years current ratings in Great Britain have usually been considered on a continuous basis, but the importance of taking into consideration cyclic ratings has now been carefully studied, since continued high metal prices have forced cable users to review carefully the effects of cyclic loadings. A report has recently been issued in which a simple method is presented for the rapid calculation of cyclic ratings.[2]

Table V gives specific inductive capacitance values for paper as: paper insulation (solid type), 3.7 (IPCEA value); paper insulation (other type), 3.3-4.2. Is it possible to list the other types and their appropriate specific inductive capacitance values or alternatively simply use an average specific inductive capacitance value of 3.7, for example, for all types of paper insulation?

Reference is made to the adoption of the hypothesis suggested by Kennelly as the basis of the paper—this is a logical approach but it appears to differ from the basis of computing ratings hitherto adopted in the United States. An amplification of the authors' viewpoint on this important issue will be welcomed.

With reference to the use of low-resistivity backfill, recent studies in Great Britain have shown that the method of backfilling cable trenches deserves careful consideration as attention to this point can result in increases up to 20% in load currents.

Equation 43 gives the thermal resistance between any point in the earth surrounding a buried cable and ambient earth. It is

Table X. Temperature Limits* for Belted-, Screened- and HSL†-Type Cables

System Voltage and Type of Cable	Laid Direct or in Air: Lead Sheathed, Armored	Laid Direct or in Air: Lead Sheathed, Unarmored	Laid Direct or in Air: Aluminum Sheathed, Armoured or Unarmored	In Ducts: Lead Sheathed, Armored	In Ducts: Lead Sheathed, Unarmored	In Ducts: Aluminum Sheathed, Armored or Unarmored
1.1 kv						
Single-core		80	80		60	80
Twin and multicore belted	80	80	80	80	60	80
3.3 kv and 6.6 kv						
Single-core		80	80		60	80
Three-core belted-type	80	80	80	80	60	80
11 kv						
Single-core		70	70		50	70
Three-core belted-type	65	65	65	65	50	65
Three-core screened-type	70	70	70	70	50	70
22 kv						
Single-core		65	65		50	65
Three-core belted-type	55	55		55	50	
Three-core screened-type	65	65	65	65	50	65
Three-core (SL‡ or SA§)	65		65	65		65
33 kv (screened)						
Single-core		65			50	
Three-core	65	65		65	50	
Three-core HSL	65			65		

* Measured in degrees centigrade.
† Hochstater separate lead.
‡ Separate lead sheathed.
§ Separate aluminum sheathed.