Application of Data Science and Information Technologies in Neurosciences
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Integrate and Fire Neuron Model f-I curve

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1 Exercise 1

The Leaky integrate-and-fire (LIF) model is one of the simplest mathematical spiking model of a neuron. The basic idea of LIF neuron was proposed in 1907 by Louis Édouard Lapicque, and it was long before the electrophysiology of a neuron was understood.

A neural membrane can be modeled as a simple resistor-capacitor (RC) circuit, where the model neuron spikes whenever the membrane potential surpasses a threshold. In reality, neurons spike because of voltage-dependent ion channels, however our model ignores the detailed biophysical mechanisms behind spiking. Besides these simplifications and apparent limitations the leaky integrate and fire is a useful model to understand how neurons respond to a current input. Our intuition tells us that the rate at which spikes are produced is positively correlated with the strength of the input current delivered to the cell.

The membrane potential dynamics of a LIF neuron is described by the following differential equation:

$$C_m \frac{dV_m}{dt} = (E_L - V_m)R_m + I_{app} \tag{1}$$

where C_m is the membrane capacitance, V_m is the membrane potential at every time, E_L is the leak potential or resting potential of the membrane, R_m is the membrane resistance and I_{app} is the applied current at each time.

For the purpose of this exercise we were given some fixed parameters to build our model that refer to the properties of the modeled membrane and are presented in the Table 1

Parameter	Symbol	Value
Leak Potential	E_L	$-70mV$
Resistance	R_m	$5M\Omega$
Conductance	C_m	$2nF$
Potential Threshold	E_L	$-50mV$
Potential Reset	E_L	$-65mV$

Table 1: The given fixed parameters to build our model.

Using the aforementioned parameters we also calculated some other parameters used by our model:

- **Leak conductance:** $G_L = \frac{1}{R_m}$
- **Membrane Time Constant:** $\tau_m = \frac{C_m}{G_L}$

To that end, we build a model by applying the **Euler's Forward Method** to Eq. (1) that calculates the membrane potential at a time point (eq. (2)), given that a specific current (I_{app}) is applied. If the membrane potential at a point is greater than the given threshold (V_{th}), then the neuron gives a spike and the membrane potential returns to a given reset value (V_{reset}).

$$V(i) = V(i-1) + dt \times \left(I(i) + \frac{E_L - V(i-1)}{Rm}\right) \times \frac{1}{Cm} \tag{2}$$

Using `matlab`, we simulate a LIF neuron model. More specifically, we set a loop for integrating through time and within it, we used the Forward Euler method to update the membrane potential. Within the same loop, we implemented the LIF spike recording and membrane potential re-setting mechanism. Our model simulates for $2s$ with a time step of $dt = 0.1ms$. The application of the current begins at $0.5s$ and ends at $1.5s$. We applied a current, $I_{app} = 4.001nA$, whose value was greater than the minimum applied current needed for the neuron to produce spikes and we present the results in the Figure 1:

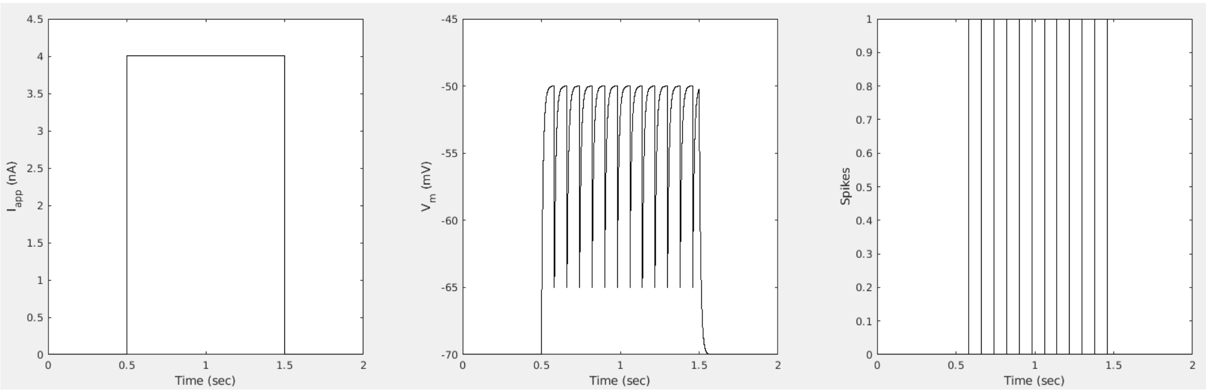

Figure 1: Current (I_{app}), membrane potential (V_m), and spike versus time after the application of $I_{app} = 4.001nA$.

2 Exercise 2

The minimum applied current needed for the neuron to produce spikes is also known as the threshold current. We calculate the threshold current (I_{th}) using the equation for the steady-state of the membrane potential (V_m^{ss}):

$$V_m^{ss} = E_L + \frac{I_{app}}{G_L} \iff G_L(V_m^{ss} - E_L) = I_{app}$$

$$\text{Using that } V_m^{ss} = V_{th} \text{ and } I_{app} = I_{th} \iff I_{th} = G_L(V_{th} - E_L)$$

where G_L is the leak conductance (the inverse of membrane resistance $G_L = \frac{1}{R_m}$), V_{th} is the given membrane potential threshold (artificial parameter of our LIF model) and E_L is the Leak potential (or membrane resting potential).

To that end we calculate the threshold current as follows:

$$I_{th} = G_L(V_{th} - E_L) = \frac{1}{R_m}(V_{th} - E_L) =$$

$$= \frac{1}{5M\Omega}(-50mV + 70mV) = 4 \times 10^{-3-6}A = 4nA$$

Furthermore, confirmed our calculation using our constructed LIF neuron model, by applying three different currents to it. One slightly lower, one slightly higher and the actual threshold current. We present the results in the Figure 2.

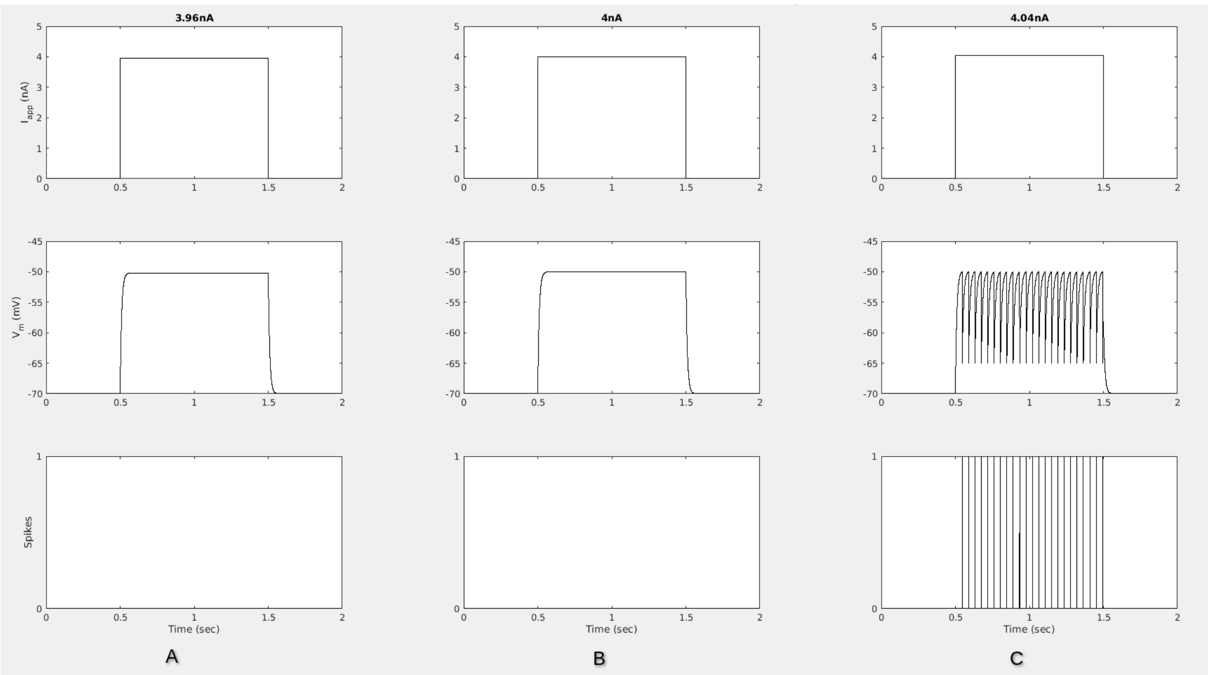

Figure 2: Current (I_{app}), membrane potential (V_m), and spike versus time after the application of **A** $I_{app} < I_{th}$, **B** $I_{app} = I_{th}$ and **C** $I_{app} > I_{th}$.

3 Exercise 3

For the purpose of this particular exercise, we want to apply ten different currents to our LIF model, so that the average firing rate is between 0 and 100 Hz.

The selection of the applied current values, is made using the ISI and I_{app} equations (3,4):

$$ISI = T = -\tau_m \ln\left(\frac{V_m^{ss} - V_{th}}{V_m^{ss} - V_{reset}}\right) = \tau_m \ln\left(\frac{V_m^{ss} - V_{reset}}{V_m^{ss} - V_{th}}\right) \tag{3}$$

$$f(I_{app}) = \frac{1}{ISI} = \frac{1}{\tau_m \ln\left(\frac{V_m^{ss} - V_{reset}}{V_m^{ss} - V_{th}}\right)} = \frac{1}{\tau_m \ln\left(\frac{E_L + \frac{I_{app}}{G_L} - V_{reset}}{E_L + \frac{I_{app}}{G_L} - V_{th}}\right)} \tag{4}$$

In order to have a firing rate of $0Hz$ ($f(I)$=$0Hz$), we should obviously apply a current of $I_{app} \leq 4nA$. In order to find the current that should be applied to have a firing rate of $100Hz$ ($f(I)$=$100Hz$), we use eq. 3 and 4 as follows:

$$f(I_{app}) = \frac{1}{ISI} = 100Hz \iff ISI = 0.01sec \iff$$

$$ISI = \tau_m \ln\left(\frac{V_m^{ss} - V_{reset}}{V_m^{ss} - V_{th}}\right) = 0.01 \iff e^{0.01} = e^{\tau_m} \times \frac{V_m^{ss} - V_{reset}}{V_m^{ss} - V_{th}} \iff$$

$$e^{0.01-\tau_m} = \frac{V_m^{ss} - V_{reset}}{V_m^{ss} - V_{th}} \iff (e^{0.01-\tau_m})(V_m^{ss} - V_{th}) = V_m^{ss} - V_{reset} \iff$$

$$V_m^{ss}(1 - e^{0.01-\tau_m}) = V_{reset} - e^{0.01-\tau_m} \times V_{th} \iff V_m^{ss} = \frac{V_{reset} - e^{0.01-\tau_m}}{1 - e^{0.01-\tau_m}} = \mathbf{-0.0413mV}$$

Using the $V_m^{ss} = -0.0413mV$, we calculate the maximum current needed so that our model has a firing rate of $100Hz$, as follows:

$$I_{app}^{max} = (Vmss_max - E_L)/Rm = \mathbf{5.74nA}$$

To that end, we apply ten different currents , in range I_{th} and I_{app}^{max}, with those values included and we plot the f-I curve, where the firing rate is calculated both **experimentally**, $f(I\_app)$=$\frac{\#\text{ of spikes}}{\Delta t}$, where $\Delta t = 1s$, refer to the time we apply the current in each trial, as well as **using the eq. 4**. We present the results in figure 3.

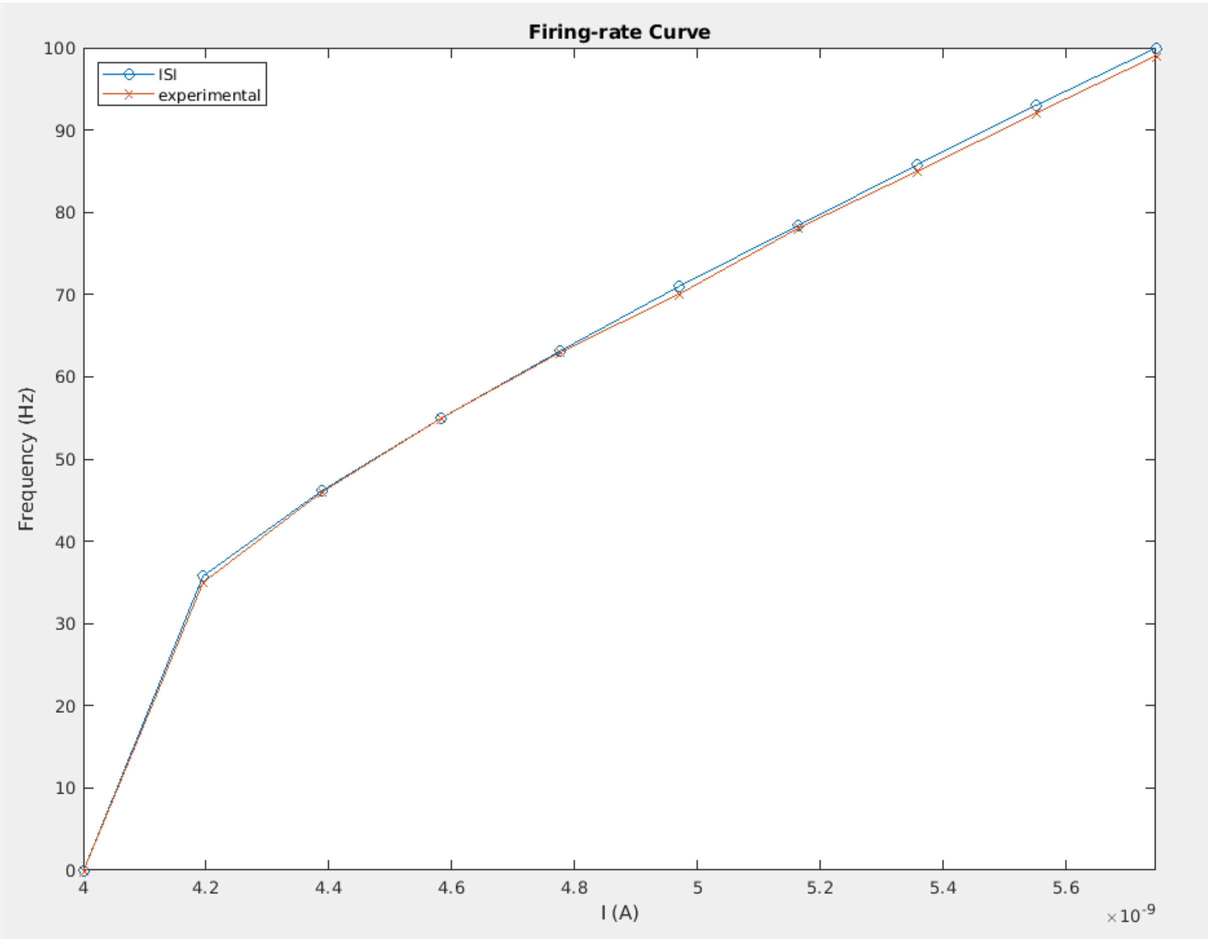

Figure 3: Firing-rate versus current $(f - I)$ curve of ten different currents, so that the firing rate is in range 0 to $100Hz$, calculated experimentally and by using ISI.

4 Exercise 4

For the purpose of exercise 4, we add noise in the membrane potential at each time step, and observe the impact it has on the $f - I$ curve.

We used 10 randomly selected currents that are in the range $[I_{min}, I_{max}]$ and I_th included, where $I_{min} = I_{th} - I_{th}/100$ and $I_{max} = I_{th} + I_{th}/100$. Furthermore, for the `sigma_I`, we selected the following values, `sigma_I` $= [0, 10^{-12}, 10^{-5}, 10^{-4}, 10^{-3}, 10^{-2}]$, from 0 to 0.01.

As presented in Figure 4, our model (`sigma_I = 0`) does not provide spikes for $I_{app} \leq I_{th} = 4nA$. However, when we add noise in the membrane potential at each time step, our model seems to produce spikes for $I_{app} \leq I_{th}$. Moreover, we note that for larger values of noise, `sigma_I` $\geq 10^{-12}$, the firing rate of our model increases as well. This is expected, since the more noise we add in the calculated membrane potential, the easier the condition $V_m > V_{th}$ is satisfied and thus our model produces spikes.

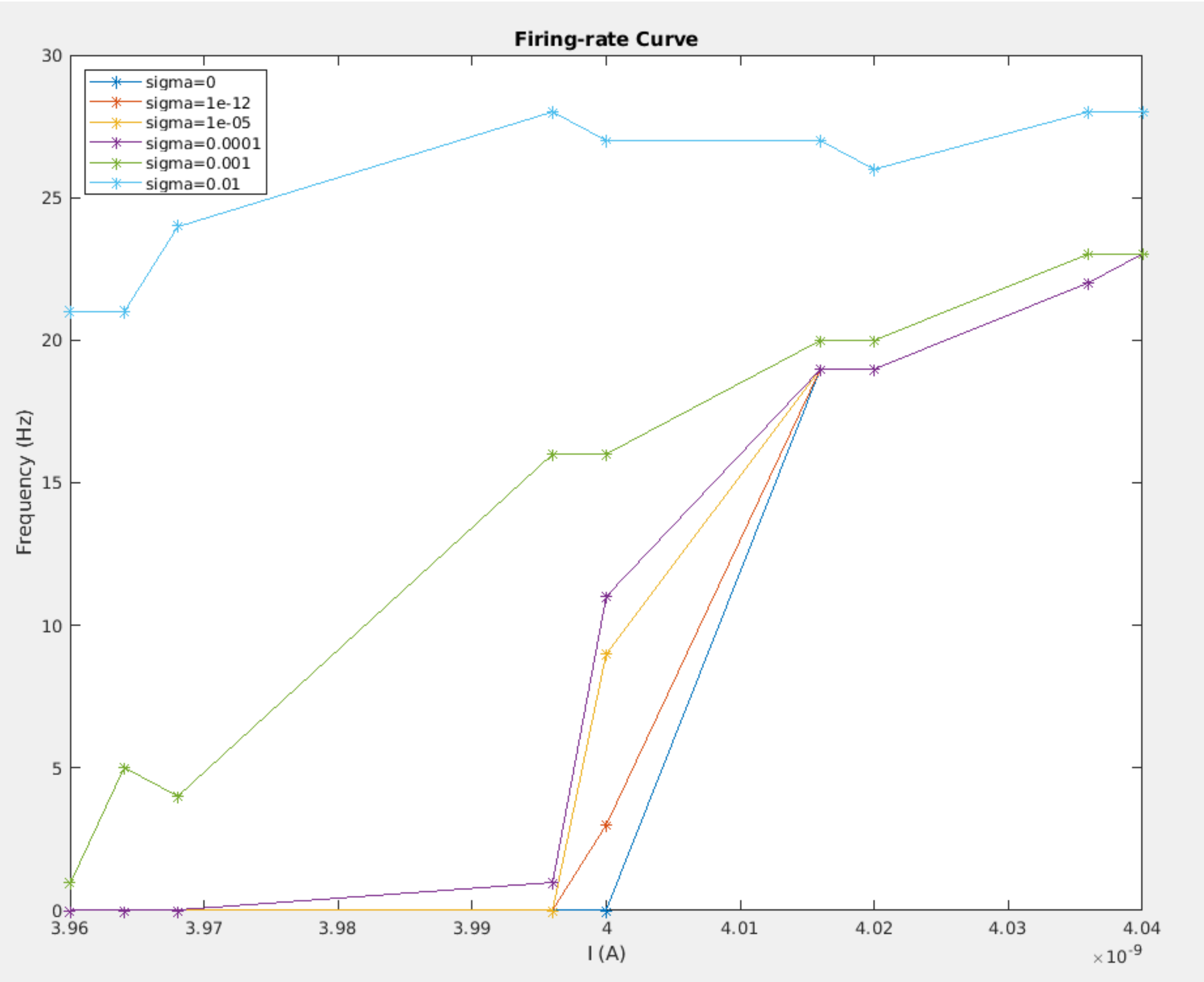

Figure 4: Firing rate $(f - I)$ curve, for ten different currents after adding different noise values to potential at each time step.

By decreasing the time step dt (a factor of ten), we obtain the results presented in Figure 5. As it is shown, now our model needs larger values of `sigma_I` in order to produce spikes for $I_{app} \leq I_{th}$. Moreover, we observe that the firing rate does not increases as much as in the previous experiment, without the alteration of the time step dt.

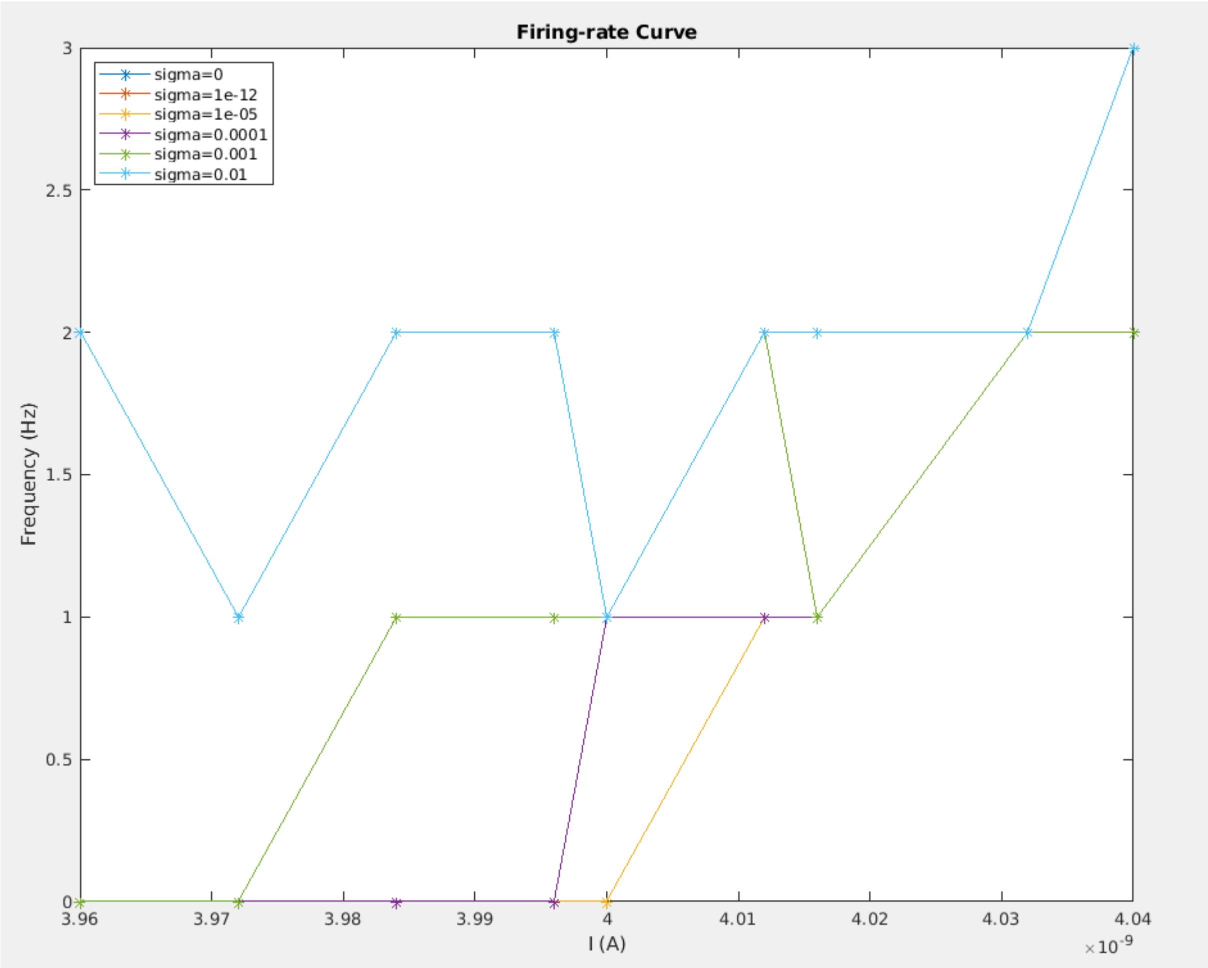

Figure 5: Firing rate $(f - I)$ curve, for ten different currents after adding different noise values to potential at each time step and decreasing the time step dt (a factor of ten).