

## Problem jankovAxiom

Input formula:  $\neg a \vee \neg\neg a$

Logic: GL

### Proved

Clauses in  $R_0$  (7) are defined at the end of the document

Implication clauses in  $X_0$  (2):

$$\lambda_0 = (\tilde{p}_0 \rightarrow \perp) \rightarrow \tilde{p}_1$$

$$\lambda_1 = (a \rightarrow \perp) \rightarrow \tilde{p}_0$$

Substitution

$$\tilde{p}_0 \mapsto \neg a$$

$$\tilde{p}_1 \mapsto \neg\neg a$$

$$\tilde{p}_2 \mapsto \neg a \vee \neg\neg a$$

$$\tilde{g} \mapsto \text{input formula}$$

### Start

(1)  $R_0 \vdash_c \tilde{g} ?$

No( $\emptyset$ )

New world:  $w_0$

$W$		$\lambda$ s.t. $w \not\vdash_W \lambda$
$w_0$	$\emptyset$	$\lambda_0, \lambda_1$

Selected:  $\langle w_0, \lambda_0 = (\tilde{p}_0 \rightarrow \perp) \rightarrow \tilde{p}_1 \rangle$

(2)  $R_0, w_0, \tilde{p}_0 \vdash_c \perp ?$

No( $\{\tilde{g}, \tilde{p}_0, \tilde{p}_2\}$ )

New world:  $w_1$

$W$		$\lambda$ s.t. $w \not\vdash_W \lambda$
$w_1$	$\tilde{g}, \tilde{p}_0, \tilde{p}_2$	$\emptyset$
$w_0$	$\emptyset$	$\lambda_1$

Selected:  $\langle w_0, \lambda_1 = (a \rightarrow \perp) \rightarrow \tilde{p}_0 \rangle$

(3)  $R_0, w_0, a \vdash_c \perp ?$

No( $\{a, \tilde{g}, \tilde{p}_1, \tilde{p}_2\}$ )

New world:  $w_2$

$W$		$\lambda$ s.t. $w \not\models_W \lambda$
$w_2$	$a, \tilde{g}, \tilde{p}_1, \tilde{p}_2$	$\emptyset$
$w_1$	$\tilde{g}, \tilde{p}_0, \tilde{p}_2$	$\emptyset$
$w_0$	$\emptyset$	$\emptyset$

Check the obtained model mod0 (see file mod0.png)

### Semantic failure

Learned axiom:

$$(a \rightarrow \neg a) \vee (\neg a \rightarrow a)$$

New clauses after clausification (6):

$$\tilde{p}_3 \rightarrow \tilde{p}_4$$

$$a \rightarrow \tilde{p}_5$$

$$\tilde{p}_3 \wedge \tilde{p}_5 \rightarrow a$$

$$a \wedge \tilde{p}_4 \rightarrow \tilde{p}_3$$

$$a \wedge \tilde{p}_3 \rightarrow \perp$$

$$\tilde{p}_4 \vee \tilde{p}_5$$

New implication clauses after clausifications (3):

$$\lambda_4 = (\tilde{p}_3 \rightarrow a) \rightarrow \tilde{p}_5$$

$$\lambda_3 = (a \rightarrow \perp) \rightarrow \tilde{p}_3$$

$$\lambda_2 = (a \rightarrow \tilde{p}_3) \rightarrow \tilde{p}_4$$

$$R_1 = R_0 + \text{new clauses}$$

Substitution

$$\tilde{p}_0 \mapsto \neg a$$

$$\tilde{p}_1 \mapsto \neg \neg a$$

$$\tilde{p}_2 \mapsto \neg a \vee \neg \neg a$$

$$\tilde{p}_3 \mapsto \neg a$$

$$\tilde{p}_4 \mapsto a \rightarrow \neg a$$

$$\tilde{p}_5 \mapsto \neg a \rightarrow a$$

$$\tilde{g} \mapsto \text{input formula}$$

Learned axiom with the substitution applied

$$(a \rightarrow \neg a) \vee (\neg a \rightarrow a)$$

### Restart 1 (semantic)

$$(4) R_1 \vdash_c \tilde{g} ?$$

No( $\{ \tilde{p}_4 \}$ )

New world:  $w_3$

$W$		$\lambda$ s.t. $w \not\models_W \lambda$
$w_3$	$\tilde{p}_4$	$\lambda_0, \lambda_1, \lambda_3, \lambda_4$

Selected:  $\langle w_3, \lambda_0 = (\tilde{p}_0 \rightarrow \perp) \rightarrow \tilde{p}_1 \rangle$

(5)  $R_1, w_3, \tilde{p}_0 \vdash_c \perp ?$

No( $\{ \tilde{g}, \tilde{p}_0, \tilde{p}_2, \tilde{p}_4 \}$ )

New world:  $w_4$

$W$		$\lambda$ s.t. $w \not\models_W \lambda$
$w_4$	$\tilde{g}, \tilde{p}_0, \tilde{p}_2, \tilde{p}_4$	$\lambda_3, \lambda_4$
$w_3$	$\tilde{p}_4$	$\lambda_1, \lambda_3, \lambda_4$

Selected:  $\langle w_4, \lambda_4 = (\tilde{p}_3 \rightarrow a) \rightarrow \tilde{p}_5 \rangle$

(6)  $R_1, w_4, \tilde{p}_3 \vdash_c a ?$

No( $\{ \tilde{g}, \tilde{p}_0, \tilde{p}_2, \tilde{p}_3, \tilde{p}_4 \}$ )

New world:  $w_5$

$W$		$\lambda$ s.t. $w \not\models_W \lambda$
$w_5$	$\tilde{g}, \tilde{p}_0, \tilde{p}_2, \tilde{p}_3, \tilde{p}_4$	$\emptyset$
$w_4$	$\tilde{g}, \tilde{p}_0, \tilde{p}_2, \tilde{p}_4$	$\lambda_3$
$w_3$	$\tilde{p}_4$	$\lambda_1, \lambda_3$

Selected:  $\langle w_4, \lambda_3 = (a \rightarrow \perp) \rightarrow \tilde{p}_3 \rangle$

(7)  $R_1, w_4, a \vdash_c \perp ?$

Yes( $\{ a, \tilde{p}_0 \}$ )

$R_1, a, \tilde{p}_0 \vdash_c \perp$

Learned basic clause:  $\tilde{p}_0 \rightarrow \tilde{p}_3$

$R_2 = R_1 + \text{learned basic clause}$

**Restart 2 (basic)**

- (8)  $R_2 \vdash_c \tilde{g} ?$   
 No( $\{a, \tilde{p}_5\}$ )  
 New world:  $w_6$

$W$		$\lambda$ s.t. $w \not\models_W \lambda$
$w_6$	$a, \tilde{p}_5$	$\lambda_0$

Selected:  $\langle w_6, \lambda_0 = (\tilde{p}_0 \rightarrow \perp) \rightarrow \tilde{p}_1 \rangle$

- (9)  $R_2, w_6, \tilde{p}_0 \vdash_c \perp ?$   
 Yes( $\{a, \tilde{p}_0\}$ )  
 $R_2, a, \tilde{p}_0 \vdash_c \perp$   
 Learned basic clause:  $a \rightarrow \tilde{p}_1$   
 $R_3 = R_2 + \text{learned basic clause}$

**Restart 3 (basic)**

- (10)  $R_3 \vdash_c \tilde{g} ?$   
 No( $\{\tilde{p}_3, \tilde{p}_4\}$ )  
 New world:  $w_7$

$W$		$\lambda$ s.t. $w \not\models_W \lambda$
$w_7$	$\tilde{p}_3, \tilde{p}_4$	$\lambda_0, \lambda_1$

Selected:  $\langle w_7, \lambda_0 = (\tilde{p}_0 \rightarrow \perp) \rightarrow \tilde{p}_1 \rangle$

- (11)  $R_3, w_7, \tilde{p}_0 \vdash_c \perp ?$   
 No( $\{\tilde{g}, \tilde{p}_0, \tilde{p}_2, \tilde{p}_3, \tilde{p}_4\}$ )  
 New world:  $w_8$

$W$		$\lambda$ s.t. $w \not\models_W \lambda$
$w_8$	$\tilde{g}, \tilde{p}_0, \tilde{p}_2, \tilde{p}_3, \tilde{p}_4$	$\emptyset$
$w_7$	$\tilde{p}_3, \tilde{p}_4$	$\lambda_1$

Selected:  $\langle w_7, \lambda_1 = (a \rightarrow \perp) \rightarrow \tilde{p}_0 \rangle$

- (12)  $R_3, w_7, a \vdash_c \perp ?$   
 Yes( $\{a, \tilde{p}_3\}$ )  
 $R_3, a, \tilde{p}_3 \vdash_c \perp$   
 Learned basic clause:  $\tilde{p}_3 \rightarrow \tilde{p}_0$   
 $R_4 = R_3 + \text{learned basic clause}$

**Restart 4 (basic)**

- (13)  $R_4 \vdash_c \tilde{g} ?$   
 No( $\{ \tilde{p}_5 \}$ )  
 New world:  $w_9$

$W$		$\lambda$ s.t. $w \not\models_W \lambda$
$w_9$	$\tilde{p}_5$	$\lambda_0, \lambda_1, \lambda_2, \lambda_3$

Selected:  $\langle w_9, \lambda_0 = (\tilde{p}_0 \rightarrow \perp) \rightarrow \tilde{p}_1 \rangle$

- (14)  $R_4, w_9, \tilde{p}_0 \vdash_c \perp ?$   
 Yes( $\{ \tilde{p}_0, \tilde{p}_5 \}$ )  
 $R_4, \tilde{p}_0, \tilde{p}_5 \vdash_c \perp$   
 Learned basic clause:  $\tilde{p}_5 \rightarrow \tilde{p}_1$   
 $R_5 = R_4 + \text{learned basic clause}$

**Restart 5 (basic)**

- (15)  $R_5 \vdash_c \tilde{g} ?$   
 No( $\{ \tilde{p}_4 \}$ )  
 New world:  $w_{10}$

$W$		$\lambda$ s.t. $w \not\models_W \lambda$
$w_{10}$	$\tilde{p}_4$	$\lambda_0, \lambda_1, \lambda_3, \lambda_4$

Selected:  $\langle w_{10}, \lambda_0 = (\tilde{p}_0 \rightarrow \perp) \rightarrow \tilde{p}_1 \rangle$

- (16)  $R_5, w_{10}, \tilde{p}_0 \vdash_c \perp ?$   
 No( $\{ \tilde{g}, \tilde{p}_0, \tilde{p}_2, \tilde{p}_3, \tilde{p}_4 \}$ )  
 New world:  $w_{11}$

$W$		$\lambda$ s.t. $w \not\models_W \lambda$
$w_{11}$	$\tilde{g}, \tilde{p}_0, \tilde{p}_2, \tilde{p}_3, \tilde{p}_4$	$\emptyset$
$w_{10}$	$\tilde{p}_4$	$\lambda_1, \lambda_3$

Selected:  $\langle w_{10}, \lambda_1 = (a \rightarrow \perp) \rightarrow \tilde{p}_0 \rangle$

- (17)  $R_5, w_{10}, a \vdash_c \perp ?$   
 Yes( $\{ a, \tilde{p}_4 \}$ )  
 $R_5, a, \tilde{p}_4 \vdash_c \perp$   
 Learned basic clause:  $\tilde{p}_4 \rightarrow \tilde{p}_0$   
 $R_6 = R_5 + \text{learned basic clause}$

**Restart 6 (basic)**

(18)  $R_6 \vdash_c \tilde{g} ?$

Yes( $\emptyset$ )

$R_6 \vdash_c \tilde{g}$

**Goal proved****Problem description**

Restarts: 6 (5 basic, 1 semantic)

Learned axioms (1):

$(a \rightarrow \neg a) \vee (\neg a \rightarrow a)$

Flat clauses  $R_0$  (7):

$\tilde{g} \rightarrow \tilde{p}_2$

$\tilde{p}_0 \rightarrow \tilde{p}_2$

$a \wedge \tilde{p}_0 \rightarrow \perp$

$\tilde{p}_1 \rightarrow \tilde{p}_2$

$\tilde{p}_0 \wedge \tilde{p}_1 \rightarrow \perp$

$\tilde{p}_2 \rightarrow \tilde{g}$

$\tilde{p}_2 \rightarrow \tilde{p}_0 \vee \tilde{p}_1$

Implication clauses  $X_0$  (2):

$\lambda_0 = (\tilde{p}_0 \rightarrow \perp) \rightarrow \tilde{p}_1$

$\lambda_1 = (a \rightarrow \perp) \rightarrow \tilde{p}_0$

Clauses added in basic restarts (5):

$\tilde{p}_0 \rightarrow \tilde{p}_3$

$a \rightarrow \tilde{p}_1$

$\tilde{p}_3 \rightarrow \tilde{p}_0$

$\tilde{p}_5 \rightarrow \tilde{p}_1$

$\tilde{p}_4 \rightarrow \tilde{p}_0$

Clauses added in semantic restarts (6):

$\tilde{p}_3 \rightarrow \tilde{p}_4$

$a \rightarrow \tilde{p}_5$

$\tilde{p}_3 \wedge \tilde{p}_5 \rightarrow a$

$a \wedge \tilde{p}_4 \rightarrow \tilde{p}_3$

$a \wedge \tilde{p}_3 \rightarrow \perp$

$\tilde{p}_4 \vee \tilde{p}_5$

Implication clauses learned in semantic restarts (3):

$$\lambda_2 = (a \rightarrow \tilde{p}_3) \rightarrow \tilde{p}_4$$

$$\lambda_3 = (a \rightarrow \perp) \rightarrow \tilde{p}_3$$

$$\lambda_4 = (\tilde{p}_3 \rightarrow a) \rightarrow \tilde{p}_5$$

Substitution

$$\tilde{p}_0 \mapsto \neg a$$

$$\tilde{p}_1 \mapsto \neg\neg a$$

$$\tilde{p}_2 \mapsto \neg a \vee \neg\neg a$$

$$\tilde{p}_3 \mapsto \neg a$$

$$\tilde{p}_4 \mapsto a \rightarrow \neg a$$

$$\tilde{p}_5 \mapsto \neg a \rightarrow a$$

$$\tilde{g} \mapsto \text{input formula}$$