heap

- *complete binary tree (review)*
- *heap and priority queues (Chapter 9)*
- *binary heap and min-heap*
- *max-heap demo*
- *max-heap coding*
- *heap sort (Chapter 7)*

heap coding: heap.h

Heap ADT: A **one - based** and **one dimensional array** is used to simplify parent and child calculations.

```
struct Heap {
  int *nodes;          // an array of nodes
  int capacity;        // array size of node or key, item
  int N;               // the number of nodes in the heap
  bool (*comp)(Heap*, int, int);
  Heap(int capa = 2) {
    capacity = capa;
    nodes = new int[capacity];
    N = 0;
    comp = nullptr;
  };
  ~Heap() {};
};
using heap = Heap*;
```

heap coding: heap.h

```
void clear(heap hp);                    // deallocate heap
int size(heap hp);                      // return nodes in heap currently
int level(int n);                       // return level based on num of nodes
int capacity(heap hp);                   // return its capacity (array size)
int reserve(heap hp, int capa);         // reserve the array size (= capacity)
int full(heap hp);                      // return true/false
int empty(heap hp);                     // return true/false
void grow(heap hp, int key);            // add a new key
void trim(heap hp);                     // delete a queue
int heapify(heap hp);                   // convert a complete BT into a heap

// helper functions to support grow/trim functions
int less(heap hp, int i, int j);        // used in max heap
int more(heap hp, int i, int j);        // used in min heap
void swim(heap hp, int k);              // bubble up
void sink(heap hp, int k);              // tickle down
// helper functions to check heap invariant
int heapOrdered(heap hp);               // is heap[1..N] a heap?
```

Promotion in a heap: swim

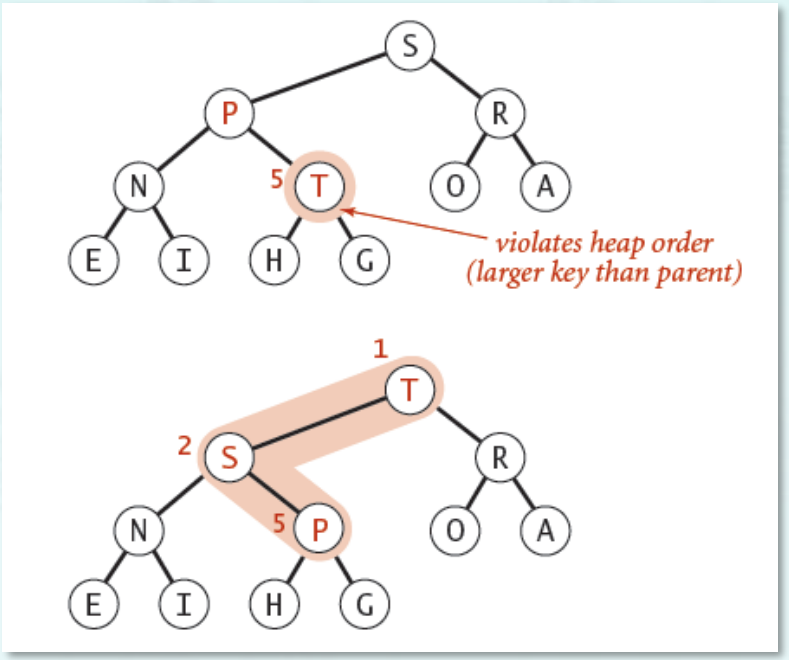
- To eliminate the violation:
  - Swap key in child with key in parent.
  - Repeat until **heap order** restored.

swim up

or

sink down

This is a maxheap example.



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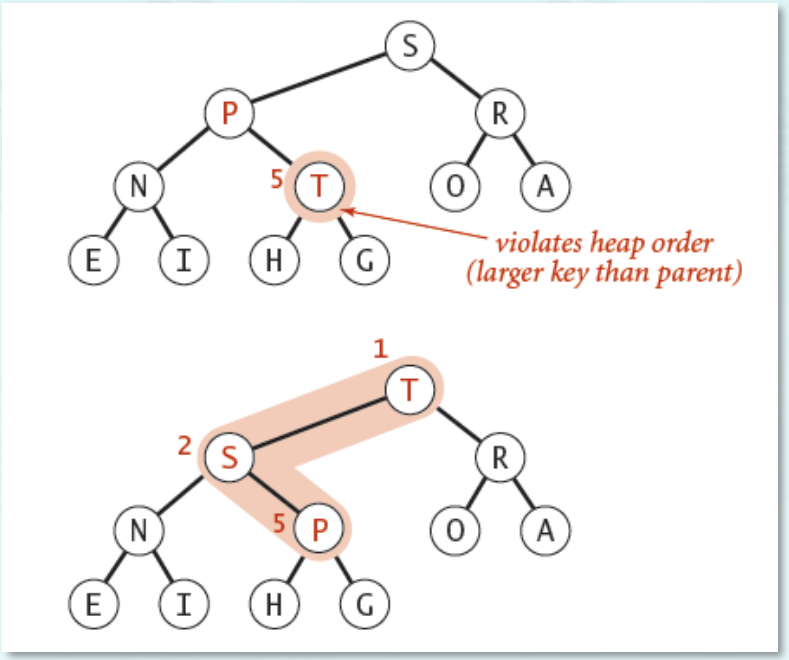
guess a name?

```
bool      (heap h, int p, int c) {
    return h->nodes[p] < h->nodes[c];
}
```



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bool ____(heap h, int p, int c) {
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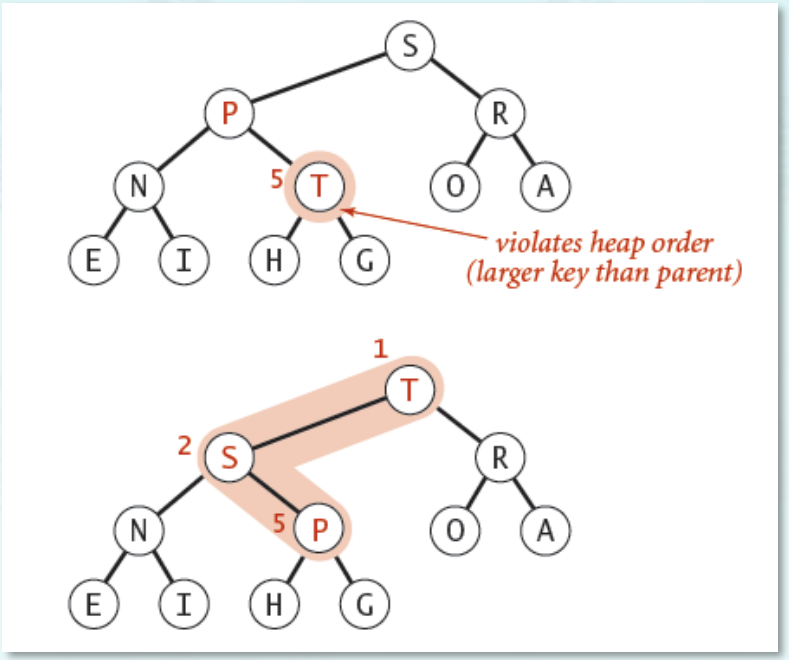
}
```

```
void ____(heap h, int p, int c) {
    int item = h->nodes[p];
    h->nodes[p] = h->nodes[c];
    h->nodes[c] = item;
}
```



Promotion in a heap: swim

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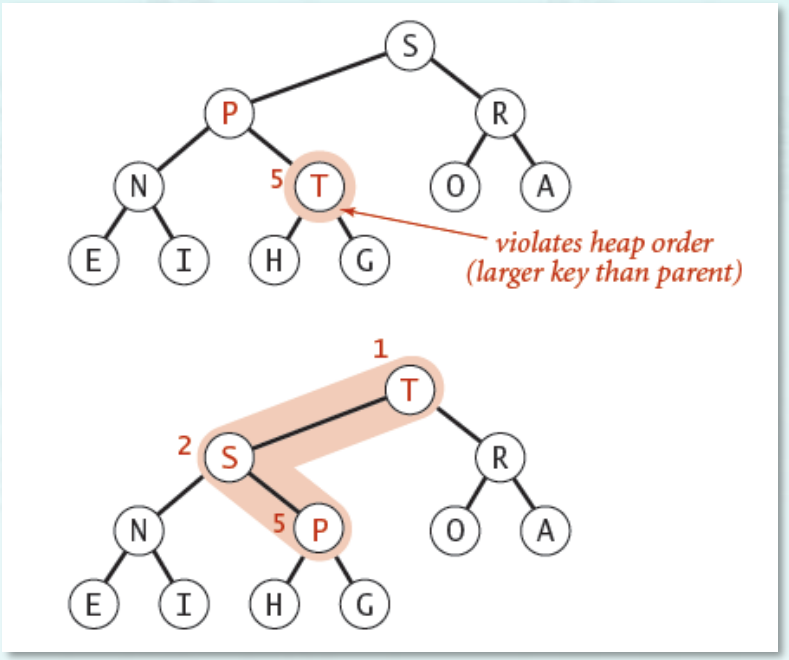
  - Repeat until heap order restored.

```
bool less(heap h, int p, int c) {
    return h->nodes[p] < h->nodes[c];
}
```

```
void swap(heap h, int p, int c) {
    int item = h->nodes[p];
    h->nodes[p] = h->nodes[c];
    h->nodes[c] = item;
}
```



Promotion in a heap: swim

- To eliminate the violation:
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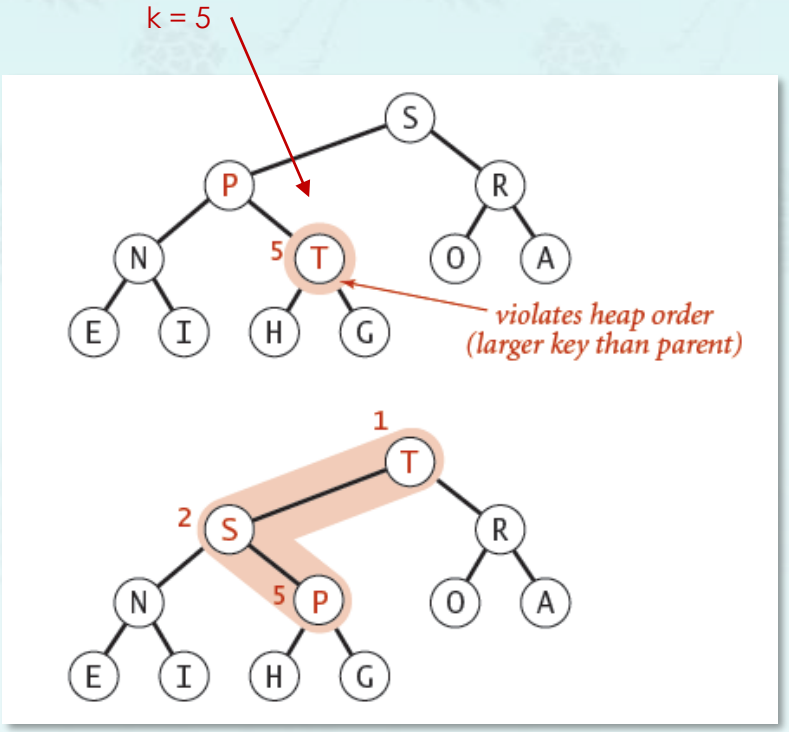
guess a name?

```
void      (heap h, int k)
{
  while (not reached at root &&
          k's parent key < k's key)
  {
                                    
  }
}
```



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  {
          [                    ]

  }
}
```

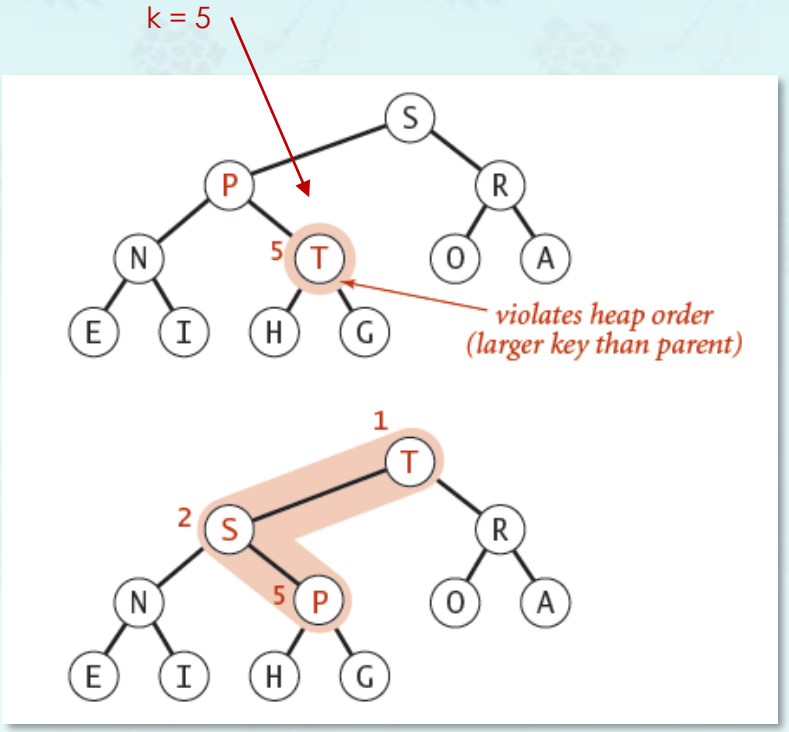

Promotion in a heap: swim

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  - Swap key in child with key in parent.
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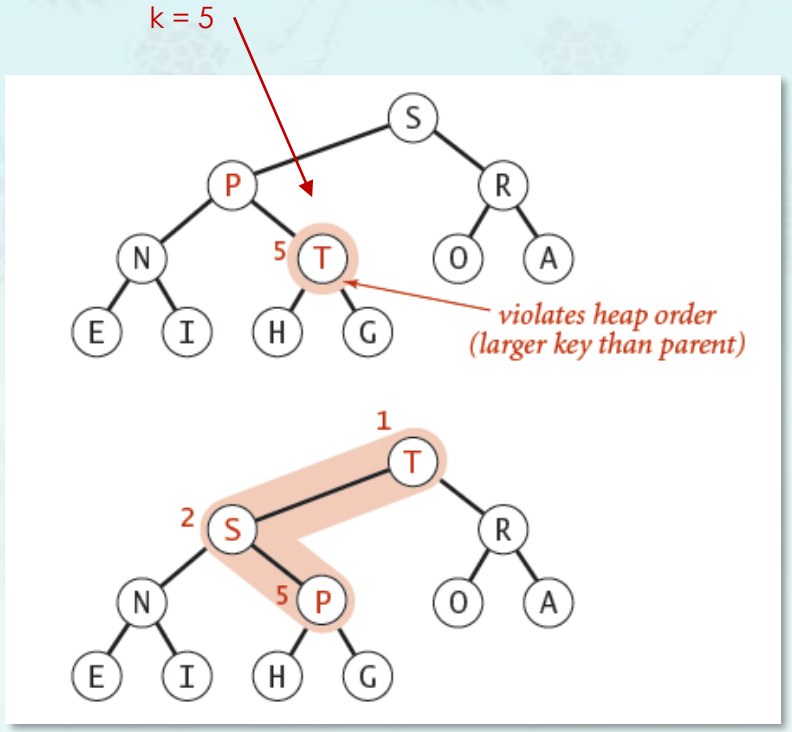
```
void swim(heap h, int k)
{
  while (not reached at root &&
          k's parent key < k's key)
  {
        swap k and its parent
        go for the next
  }
}
```



Promotion in a heap: swim

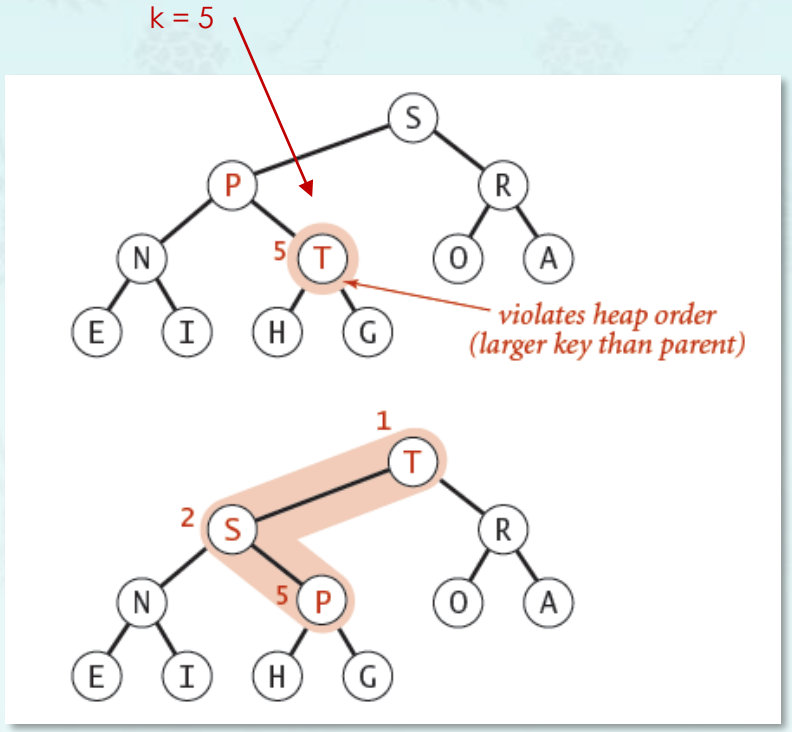
- To eliminate the violation:
  - Swap key in child with key in parent.
  - Repeat until heap order restored.

not reached at root

parent of k

```
void swim(h ap h, int k)
{
  while (
  {


  }
}
```



Promotion in a heap: swim

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  - Swap key in child with key in parent.
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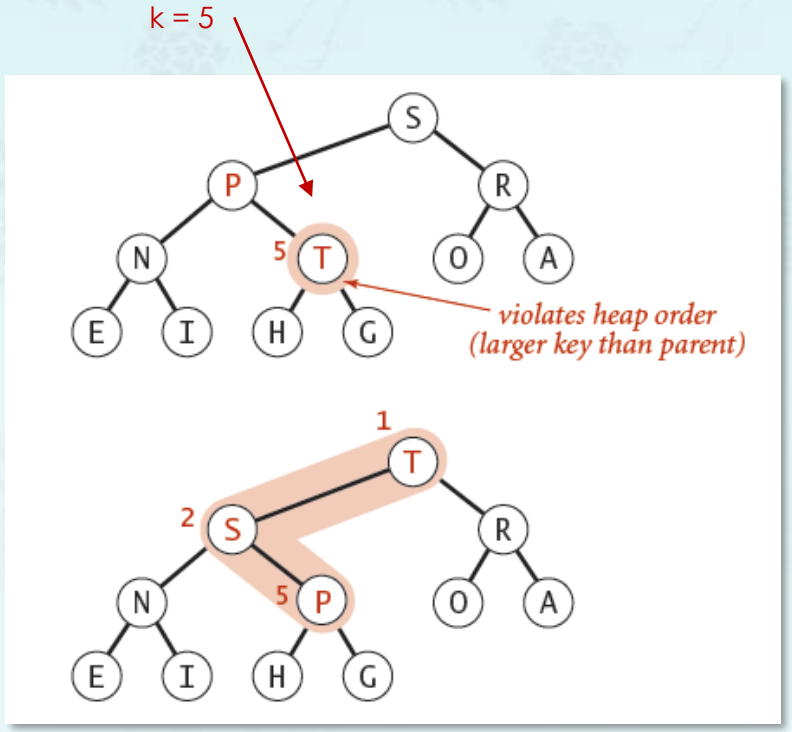
not reached at root

parent(k/2) is less its child(k)

```
void swim(h ap h, int k)
{
  while (k > 1 &&
  {


  }
}
```



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  - Swap key in child with key in parent.
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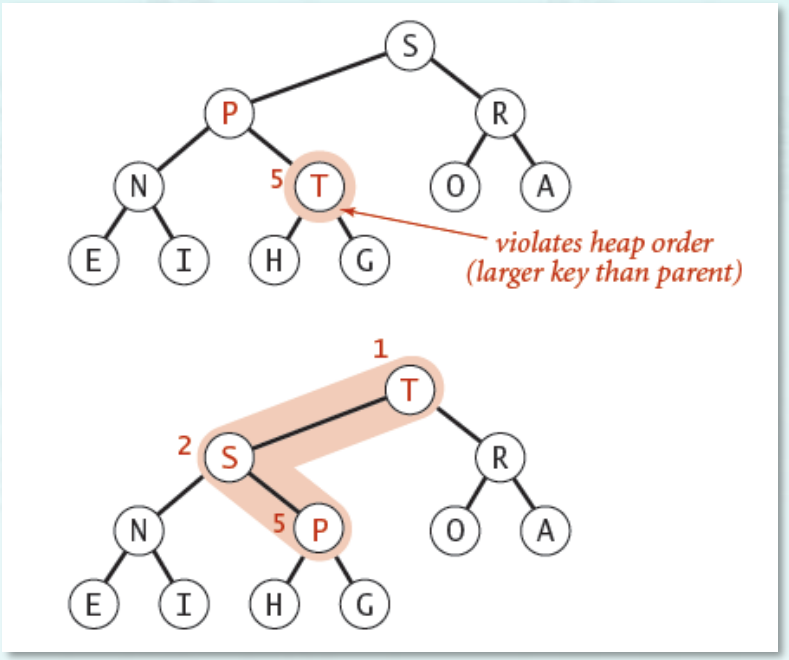
not reached at root

parent(k/2) is less its child(k)

```
void swim(h ap h, int k)
{
  while (k > 1 && less(h, k / 2, k))
  {
    
  }
}
```

swap parent(k/2) and its child(k)



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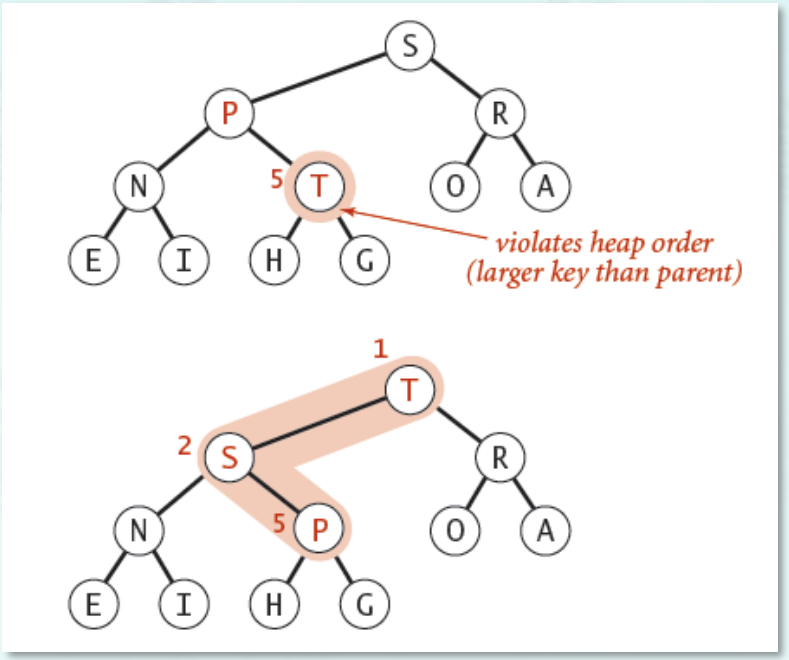
swap parent(k/2) and its child(k)

move up one level



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void swim(h ap h, int k)
{
  while (k > 1 && less(h, k / 2, k))
  {
    swap(h, k / 2, k);
    k = k / 2;
  }
}
```

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swap parent(k/2) and its child(k)

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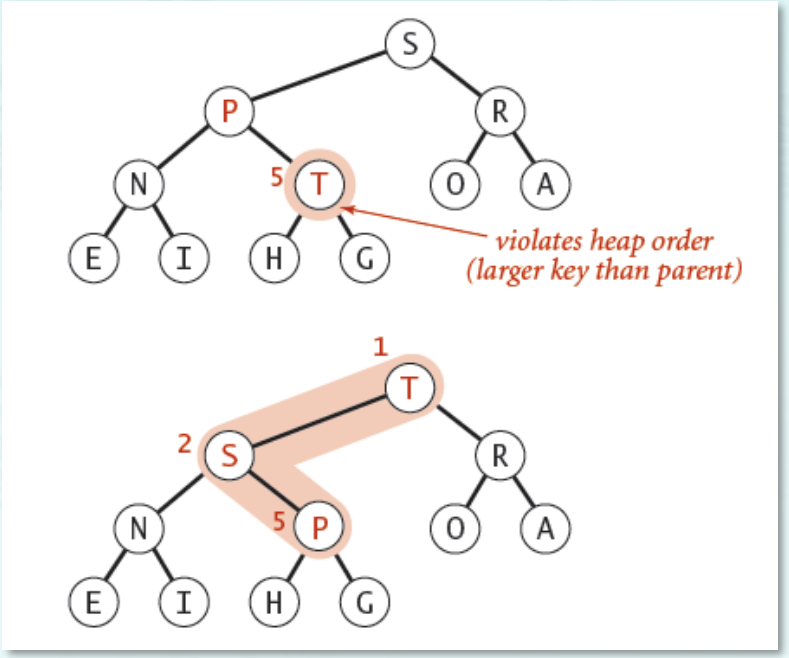
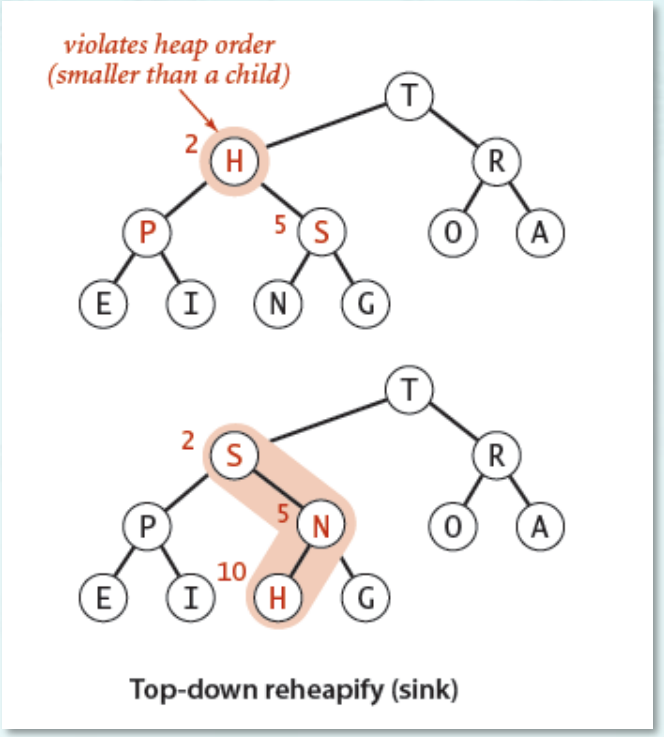


swim up

or

sink down



Top-down reheapify (sink)

Demotion in a heap: sink

- Parent's key becomes **smaller** than one (or both) of its children's.
- To eliminate the violation:
  - Swap key in parent with key in **larger** child (of two)
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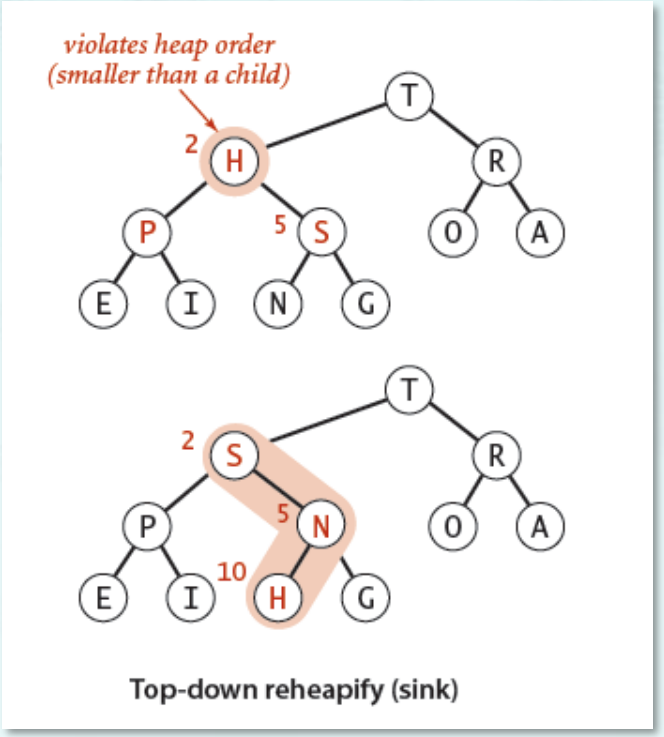
Why not smaller child?



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void sink(heap h, int k)
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   while (k's child not reached the last)
   {
      find the larger child of k, let it be j. (j = 5)

      if k's key is not less than j's key, break;
      swap k and j since k's key > j's key
      set k to the next node wh
   }
}
```
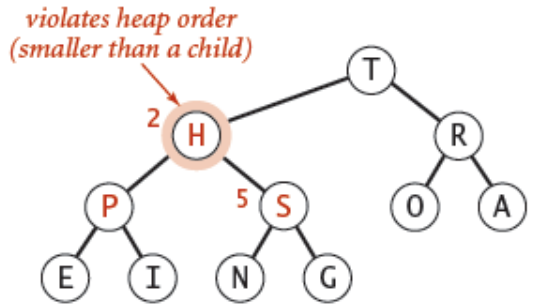

k = 2



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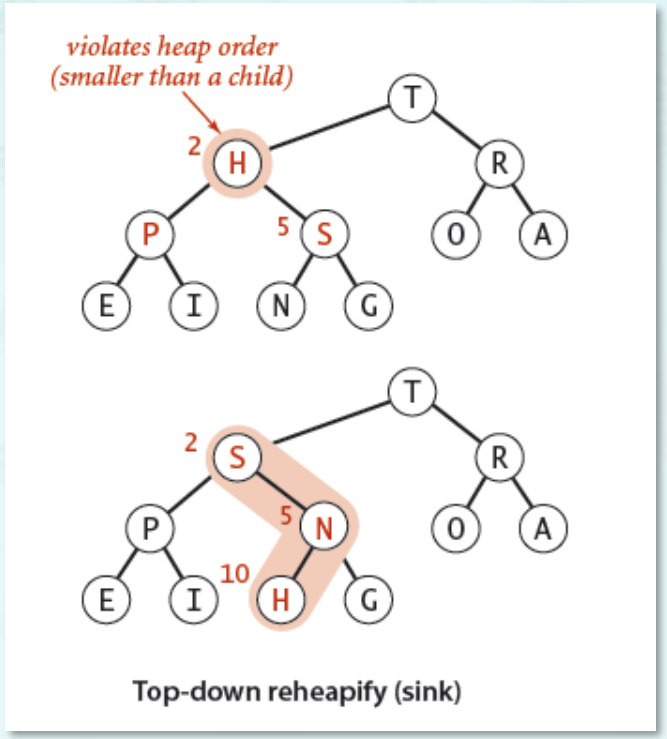

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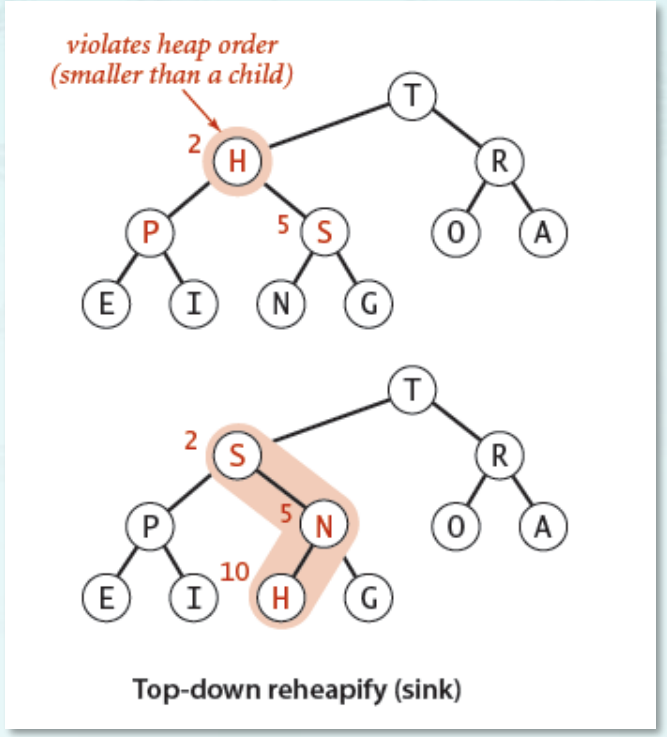

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void sink(heap h, int k)
{
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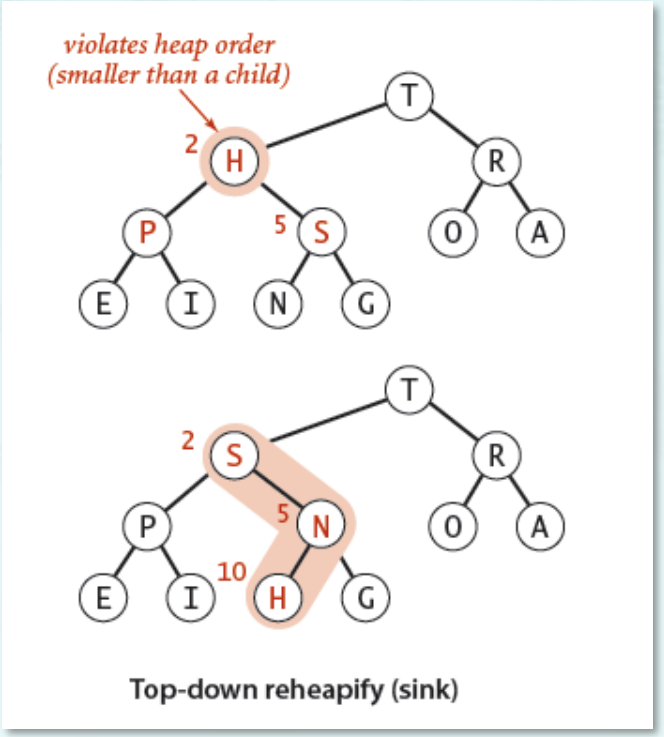
}
```

children of node **k** are **2k** and **2k+1**



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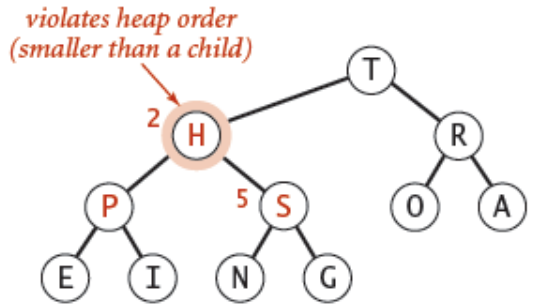

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{
   while (2 * k <= h->N)
   {
      int j = 2 * k;

      if (j < h->N && less(h, j, j + 1)) j++;

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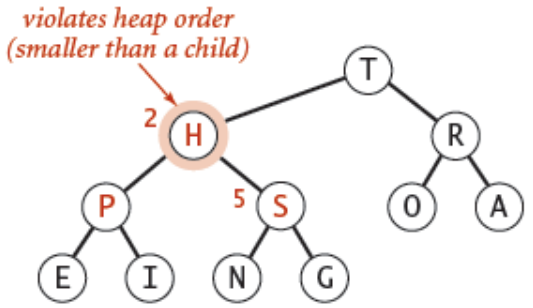

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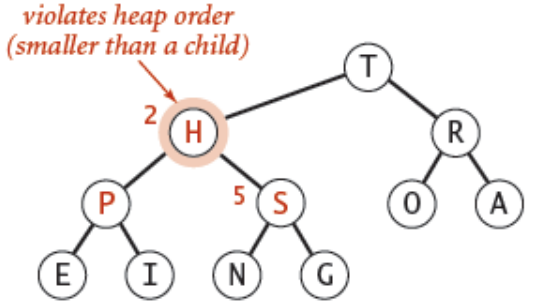

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{
   while (2 * k <= h->N)
   {
      int j = 2 * k;

      if (j < h->N && less(h, j, j + 1)) j++;
      if (!less(h, k, j)) break;
```
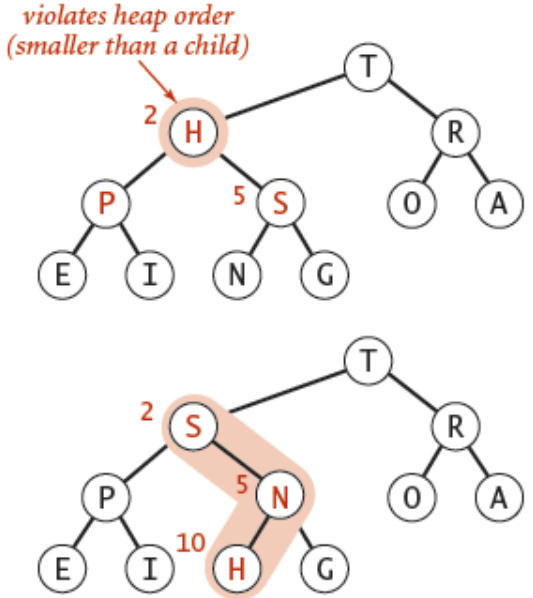

children of node **k** are **2k** and **2k+1**

swap k and j since k's key > j's key
set k to the next node (which is j.)

```
}
```



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      swap(h, k, j);
      set k to the next node (which is j.)
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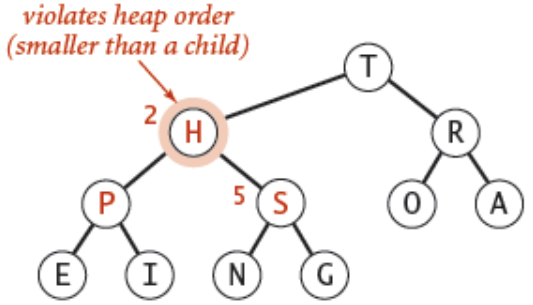

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      swap(h, k, j);
      k = j;
   }
}
```

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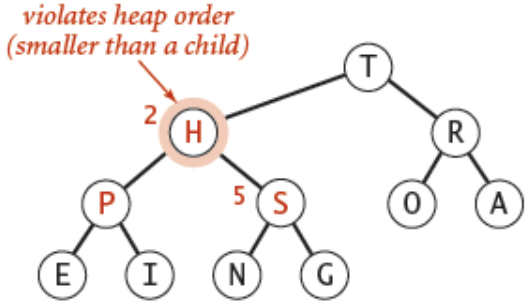


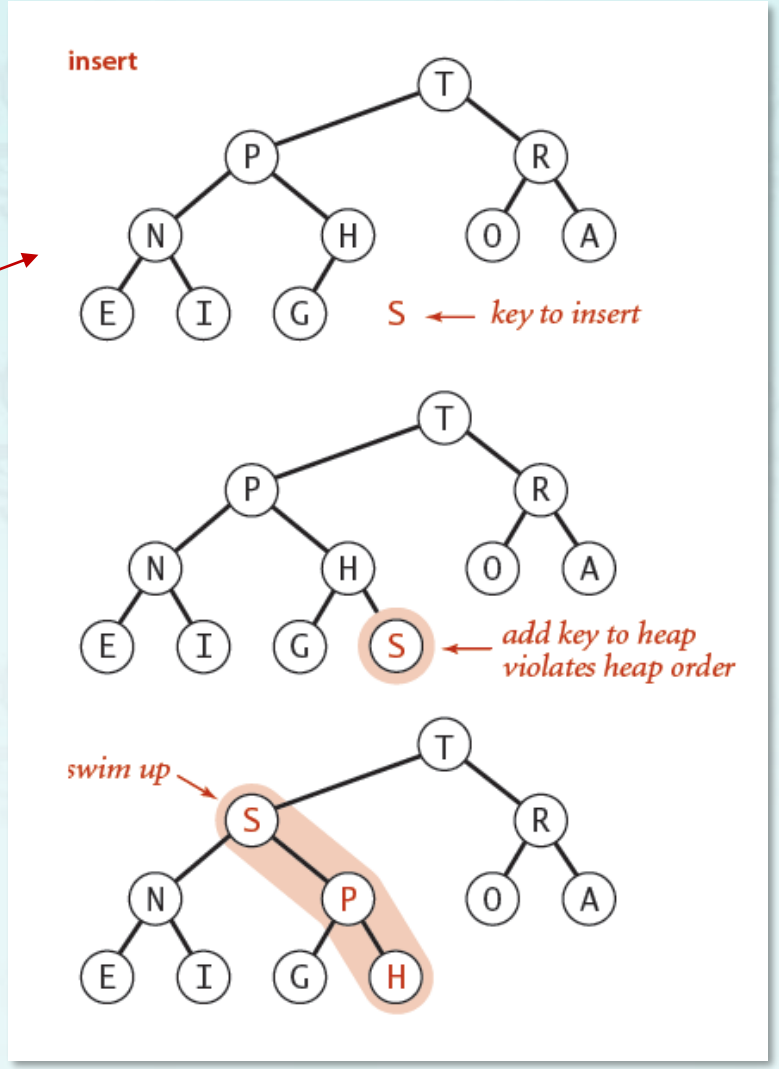
Top-down reheapify (sink)

heap coding

- Insert: Add node at end, then **swim** it up.
  - Cost: At most 1 + log N compares.

What is N now?

Insert



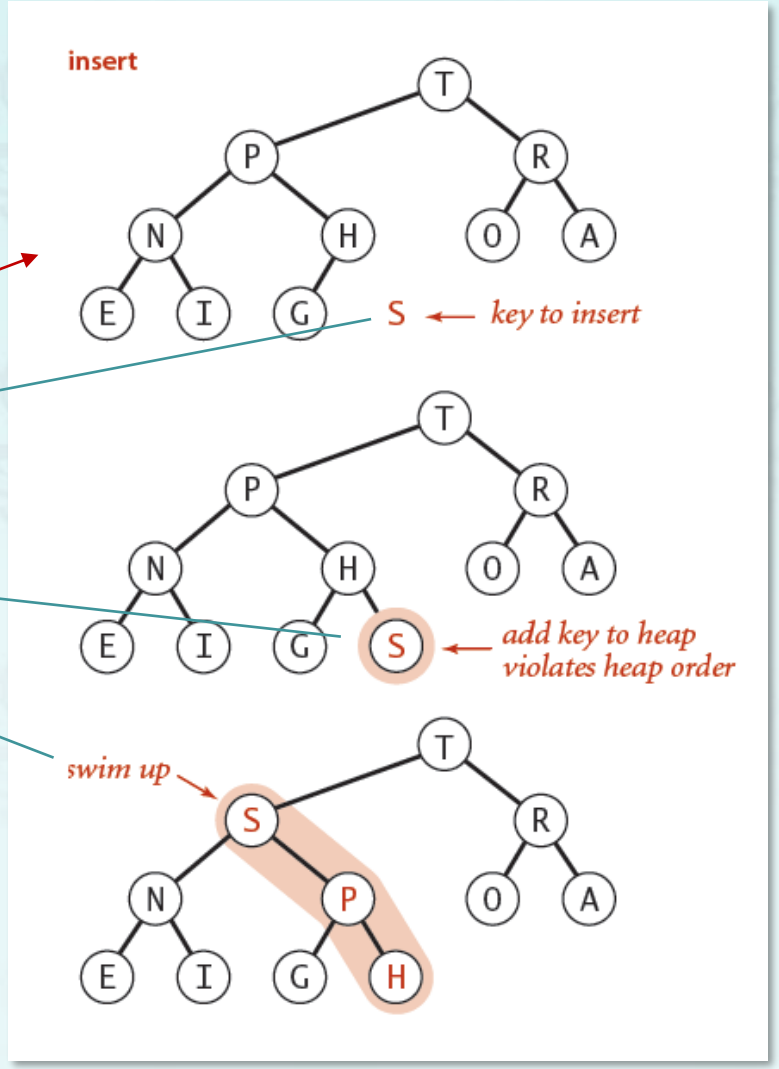
heap coding

Insertion in a heap:

- Insert: Add node at end, then **swim** it up.
  - Cost: At most 1 + log N compares.

What is N now?

```
void insert(heap h, int key)
{
   
}
```




```
struct Heap {
  int *nodes;        // an array of nodes
  int capacity;      // array size of node or key, item
  int N;             // the number of nodes in the heap
  //
};
using heap = *Heap;
```

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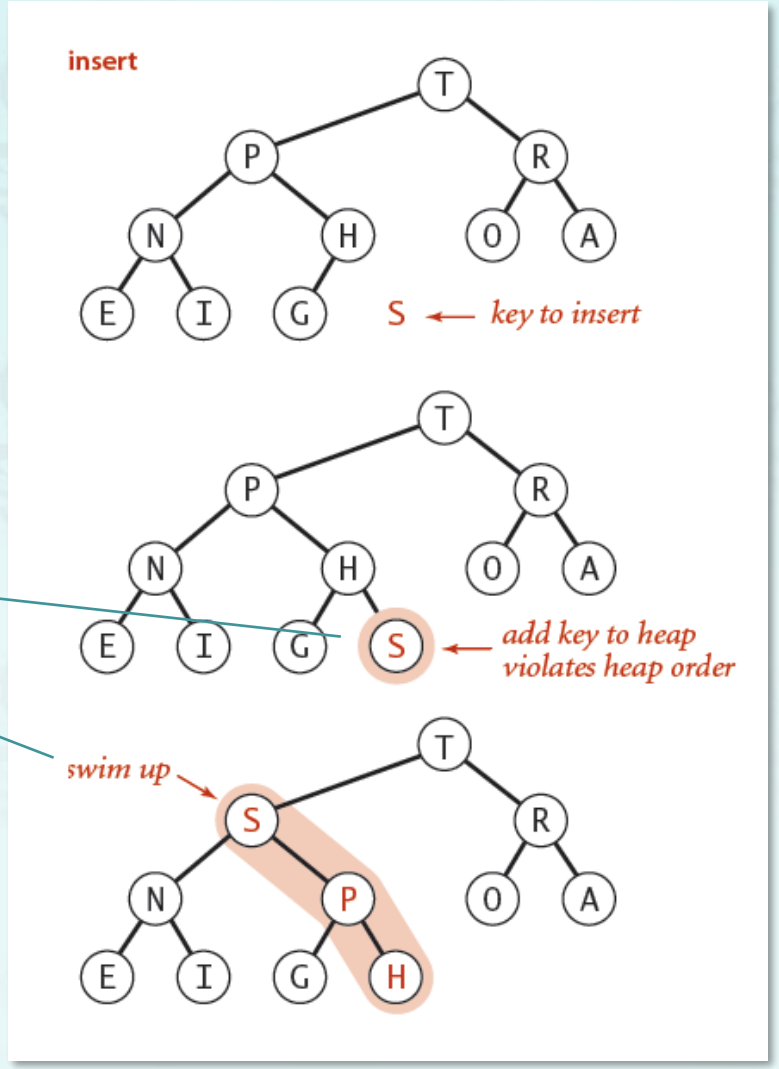


```
void insert(heap h, int key)
{
   h->nodes[++h->N] = key;
   
}
```

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  int capacity;      // array size of node or key, item
  int N;             // the number of nodes in the heap
  //
};
using heap = *Heap;
```

```
void swim(heap h, int k)
void sink(heap h, int k)
```

heap coding

Insertion in a heap:

- Insert: Add node at end, then **swim** it up.
  - Cost: At most 1 + log N compares.

```
void insert(heap h, int key)
{
   h->nodes[++h->N] = key;
   swim(h, h->N);
}
```

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struct Heap {
  int *nodes;        // an array of nodes
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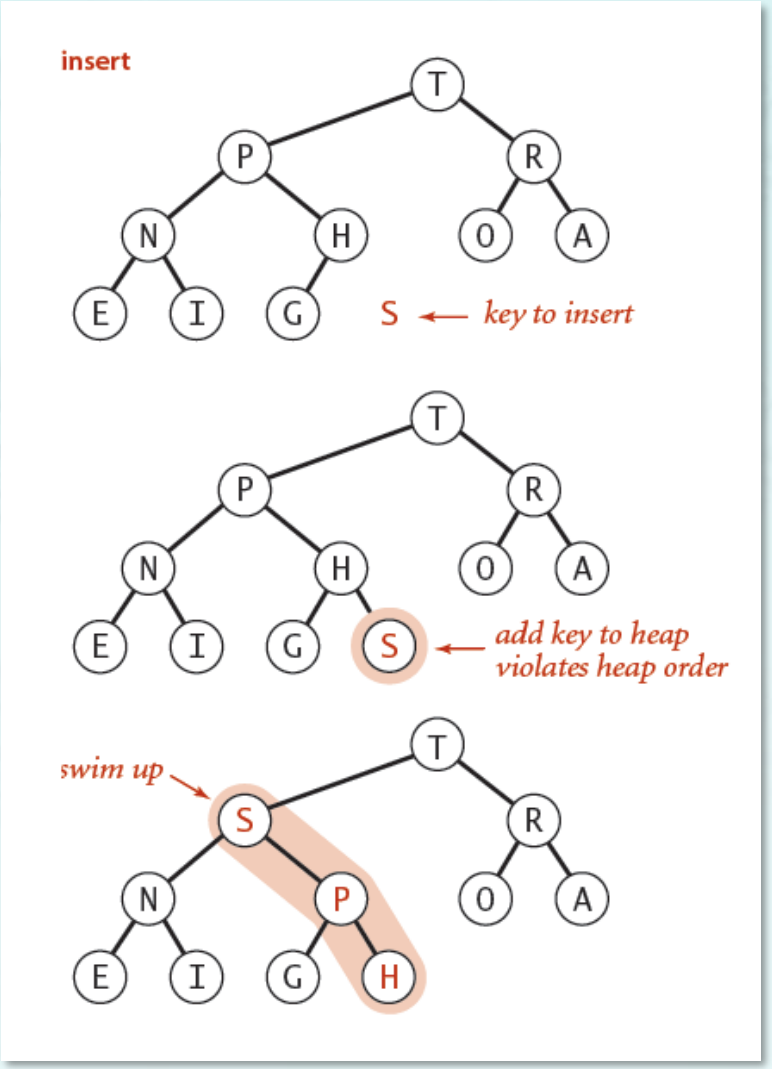
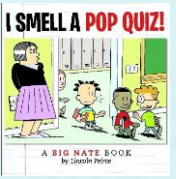
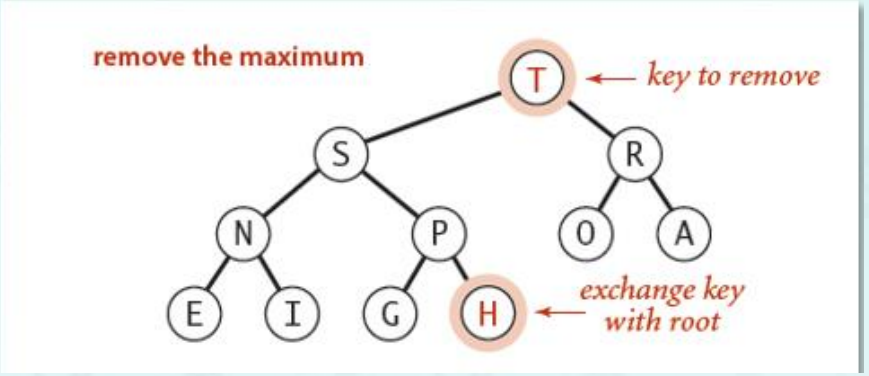




heap coding

(1) Delete the root (max or min) in a heap:

(2) How many times do comparisons occur for n nodes ? (select one):
n, 2n, n^2, 2 log n, n log n, log n




```
void delete(heap h) {


}
```

← **2 or 3 lines of code**

```
void swim(heap h, int k)
void sink(heap h, int k)
```

```
bool less(heap h, int p, int c)
```

```
void swap(heap h, int p, int c)
```

heap coding

Delete the root (max or min) in a heap:

- **Delete root:** Swap root with node at end, then sink it down.
- **Cost:** At most 2 log N compares.

```
void delete(heap h) {



}
```

```
    swap(h, ..., ... );
    sink(h, ...);
}
```

```
void swim(heap h, int k)
void sink(heap h, int k)
```

```
bool less(heap h, int p, int c)
```

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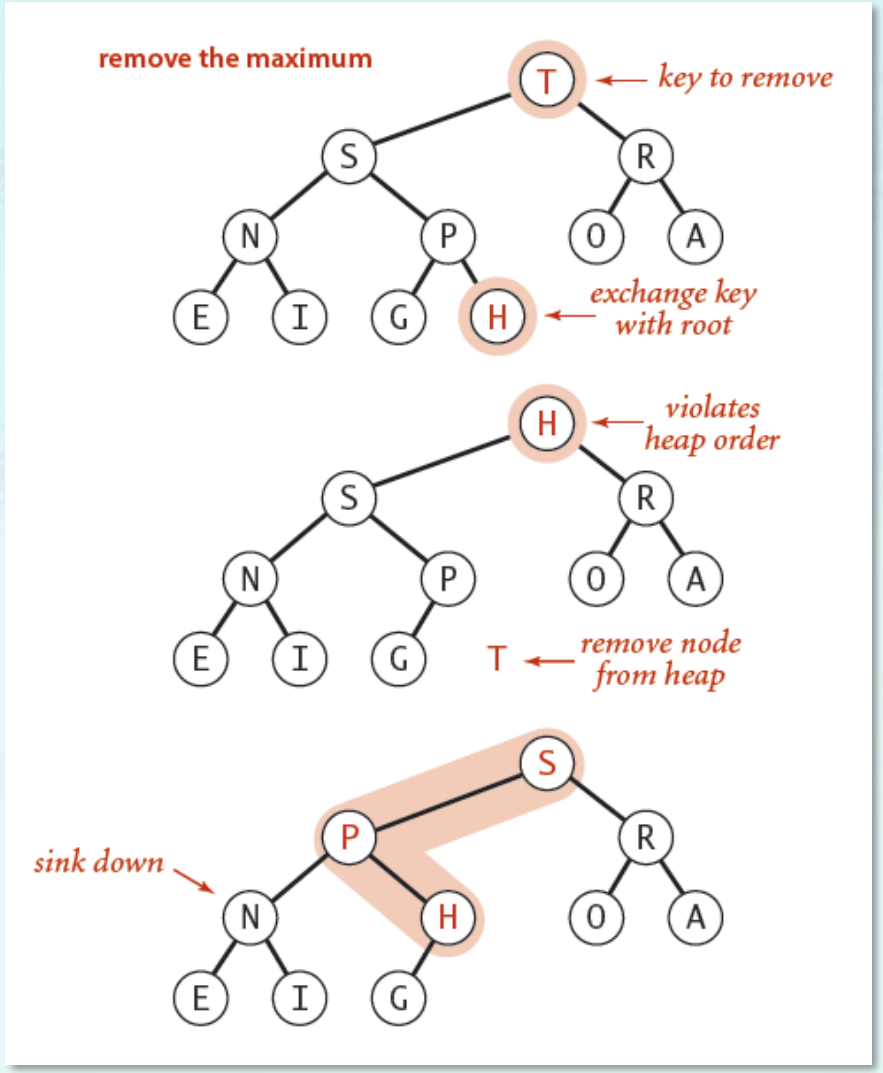

heap coding

Delete the root (max or min) in a heap:

- **Delete root:** Swap root with node at end, then sink it down.
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```
void delete(heap h) {

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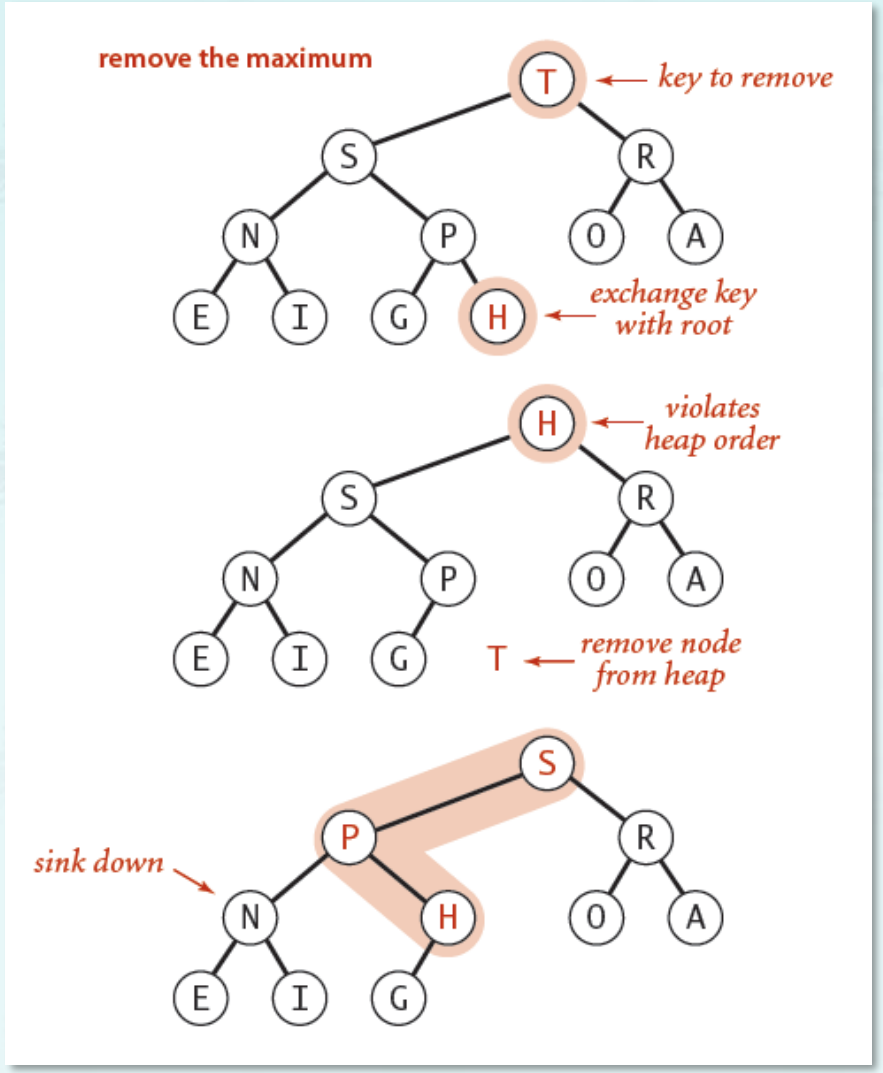

heap coding

Delete the root (max or min) in a heap:

- **Delete root:** Swap root with node at end, then sink it down.
- **Cost:** At most 2 log N compares.

```
void delete(heap h) {
    swap(h, 1, h->N--);

}
```

```
void swim(heap h, int k)
void sink(heap h, int k)
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```
bool less(heap h, int p, int c)
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void swap(heap h, int p, int c)
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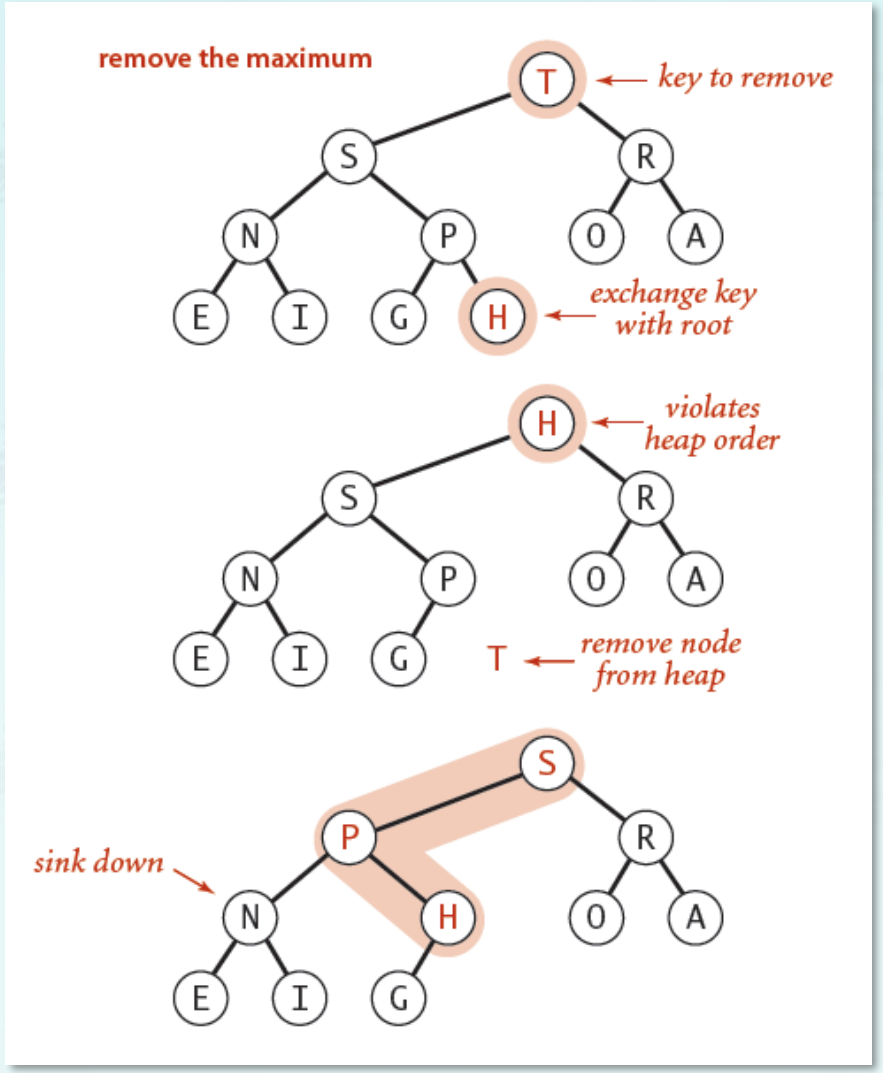

heap coding

Delete the root (max or min) in a heap:

- **Delete root:** Swap root with node at end, then sink it down.
- **Cost:** At most 2 log N compares.

```
void delete(heap h) {
    swap(h, 1, h->N--);
    sink(h, 1);
}
```

```
void swim(heap h, int k)
void sink(heap h, int k)
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```
bool less(heap h, int p, int c)
```

```
void swap(heap h, int p, int c)
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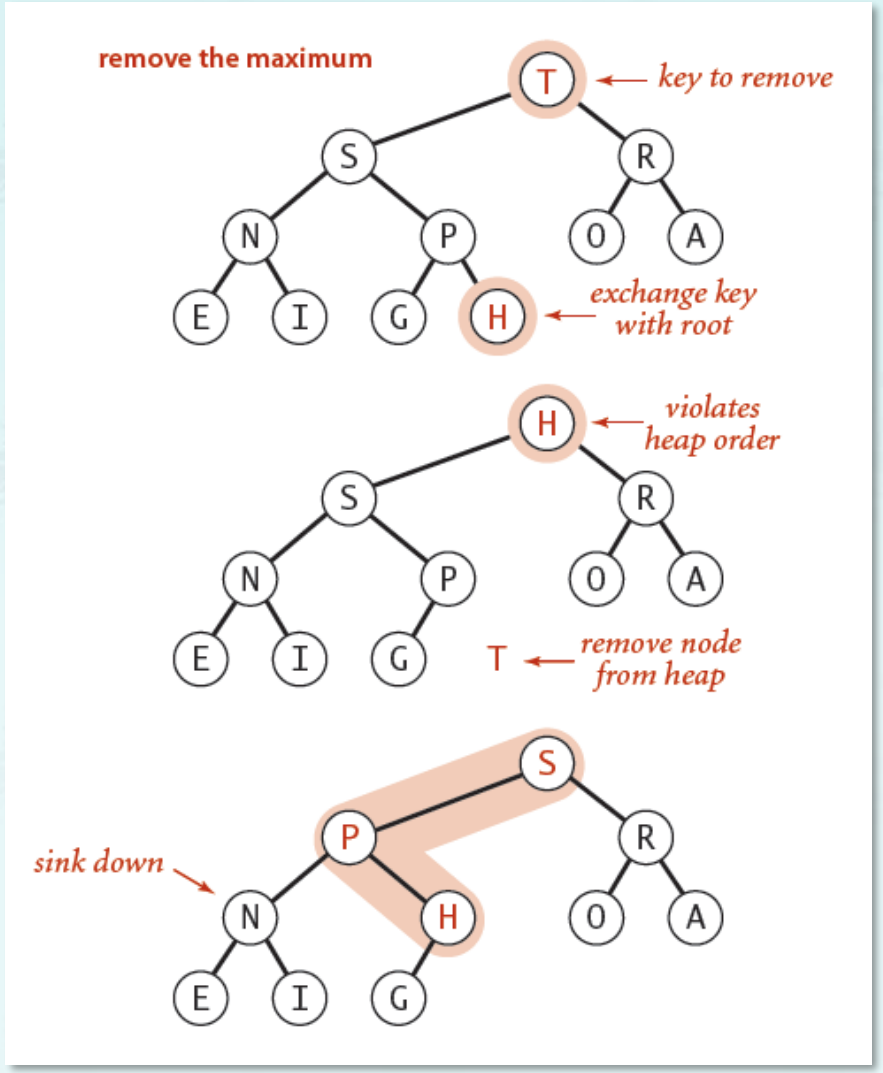


Heap Coding

```
void clear(heap hp);                    // deallocate heap
int size(heap hp);                      // return nodes in heap currently
int level(int n);                       // return level based on num of nodes
int capacity(heap hp);                   // return its capacity (array size)
int reserve(heap hp, int capa);         // reserve the array size (= capacity)
int full(heap hp);                      // return true/false
int empty(heap hp);                     // return true/false
void grow(heap hp, int key);            // add a new key
void trim(heap hp);                     // delete a queue
int heapify(heap hp);                   // convert a complete BT into a heap

// helper functions to support grow/trim functions
int less(heap hp, int i, int j);        // used in max heap
int more(heap hp, int i, int j);        // used in min heap
void swim(heap hp, int k);              // bubble up
void sink(heap hp, int k);              // tickle down
// helper functions to check heap invariant
int heapOrdered(heap hp);               // is heap[1..N] a heap?
```

Heap Coding

```
// return the number of items in heap
int size(heap hp) {
    return heap->N;
}
```

```
// Is this heap empty?
int empty(heap hp) {
    return (heap->N == 0) ? true : false;
}
```

```
// Is this heap full?
int full(heap hp) {
    return (heap->N == heap->capacity - 1) ? true : false;
}
```

Heap Coding

```
int less(heap hp, int i, int j) {
    return heap->nodes[i] < heap->nodes[j];
}
```

```
void swap(heap hp, int i, int j) {
    int t = heap->nodes[i];
    heap->nodes[i] = heap->nodes[j];
    heap->nodes[j] = t;
}
```

```
void swim(heap hp, int k) {

}
```

```
void sink(heap hp, int k) {

}
```

Heap Coding

```
void grow(heap hp, int key) {

    cout << "YOUR CODE HERE\n";

    // add key @ ++heap->N

    // swim up @ heap->N

}
```

```
void trim(heap hp) {
    if (empty(heap)) return;

    cout << "YOUR CODE HERE\n";

}
```

Heap Coding

newCBT()	with a given array, instantiate a new complete binary tree its result is neither maxheap nor minheap.
heapify()	make a complete binary tree into a max/minheap
heapsort()	use max/min-heap to sort elements in heap
heapprint()	convert heap to tree for display purpose only

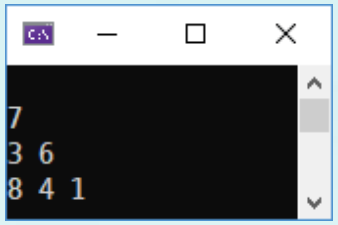
newCBT() – convert an int array to CBT

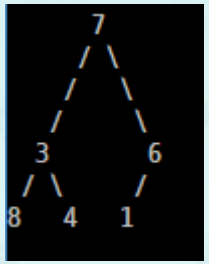
```
// instantiates a CBT with given data and its size.
heap newCBT(int *a, int n) {
    int capa = ?

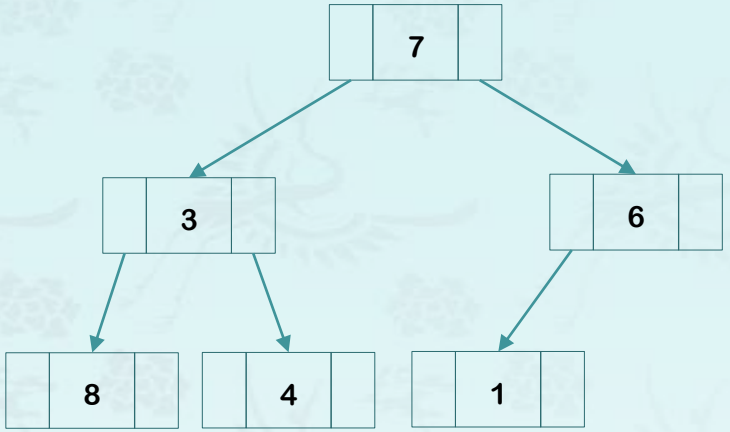
    heap p = new Heap{ capa };

    p->N = n;
    for (int i = 0; i < n; i++)
        p->nodes[i + 1] = a[i];
    return p;
}
```

heapprint() – convert heap to tree using queue



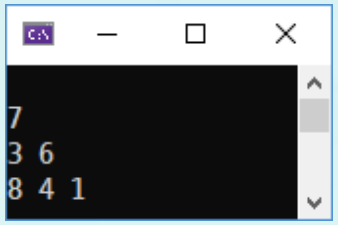


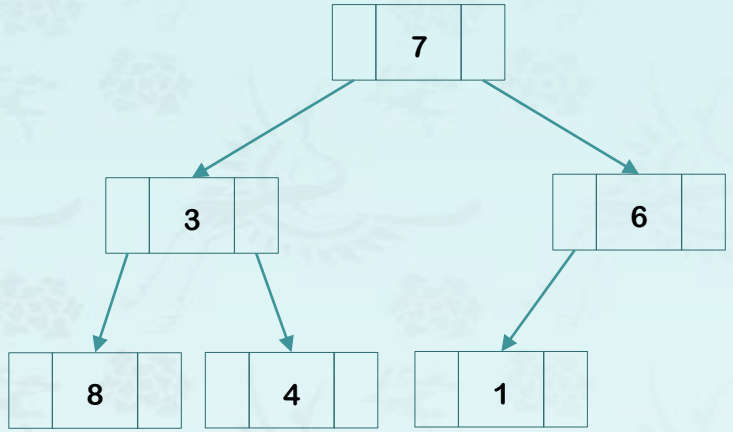



```
struct Heap {
  int *nodes;
  int capacity;
  int N;
  bool (*comp)(Heap*, int, int);
  Heap(int capa = 2) {
    capacity = capa;
    nodes = new int[capacity];
    N = 0;
    comp = nullptr;
  };
  ~Heap() {};
};
using heap = Heap*;
```

```
struct TreeNode {
  int key;
  TreeNode  *left;
  TreeNode  *right;
  TreeNode(const int k = 0,
    TreeNode* l = nullptr,
    TreeNode* r = nullptr) {
    key = k; left = l;right =r;
  }
  ~TreeNode() {}
};
using tree = TreeNode*;
```

heapprint() – convert heap to tree using queue





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2. Enqueue the root node.
3. Loop through from **the CBT nodes[2] to nodes[N]**
   A. Make a **new node from nodes[].**
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   C. If the left of the tree node doesn't exist,
      set the new node to the left of the tree node.
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   D. If this tree node is full, pop (or dequeu) it.
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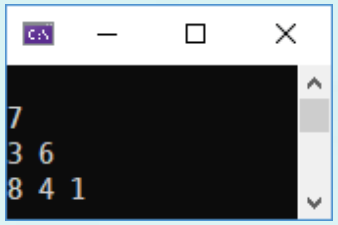
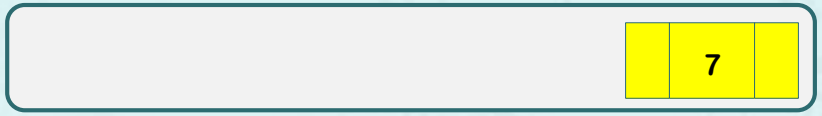
4. treeprint(root)

heapprint() – convert heap to tree using queue



Queue



0	1	2	3	4	5	6	7
	7	3	6	8	4	1	

`hp->nodes[]` `hp->N`

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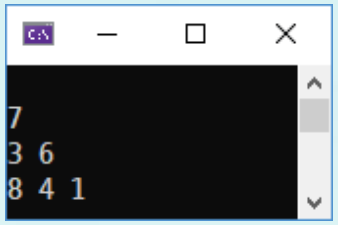

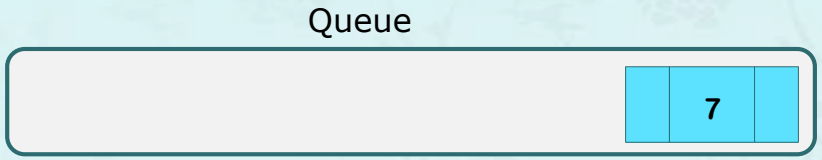

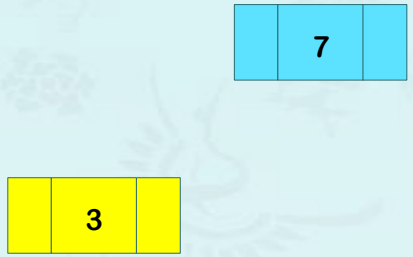

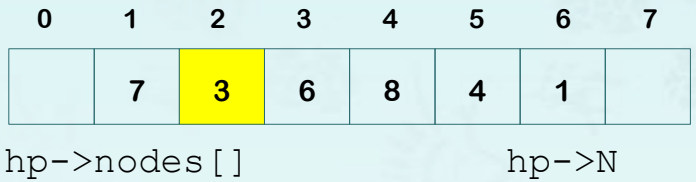

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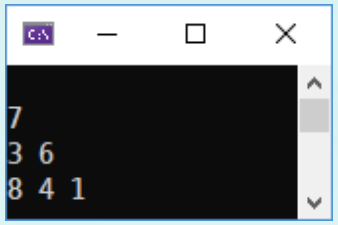

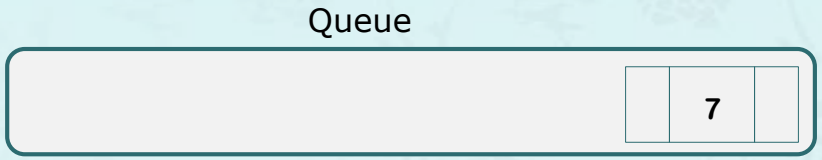

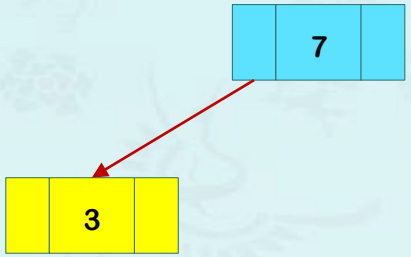

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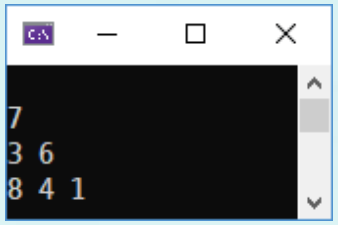

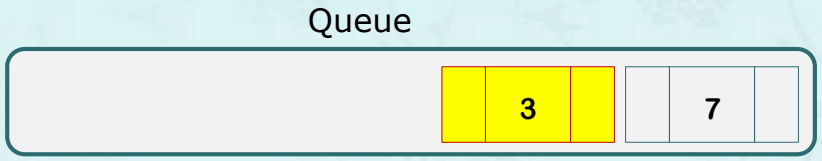

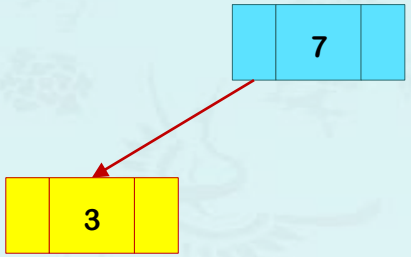

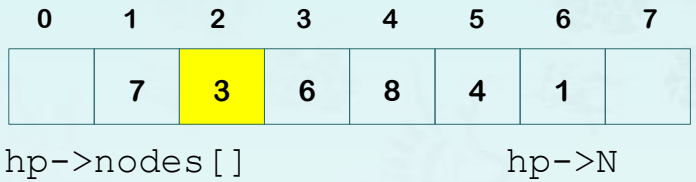

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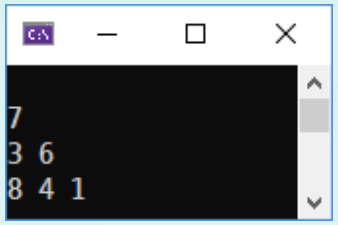

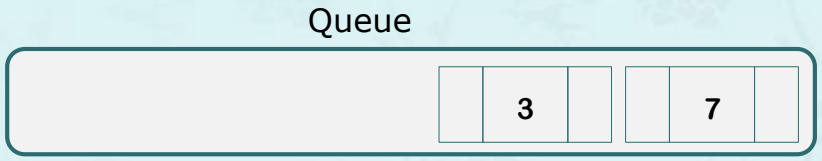

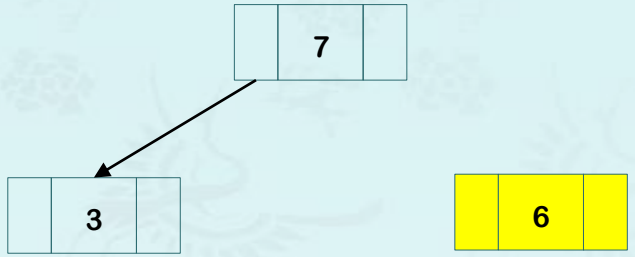

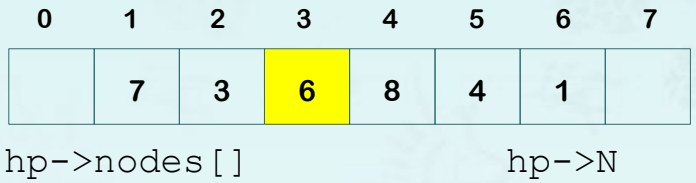

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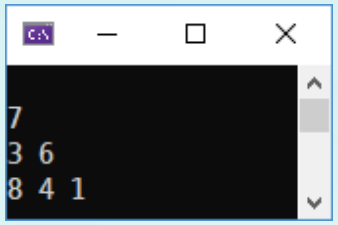

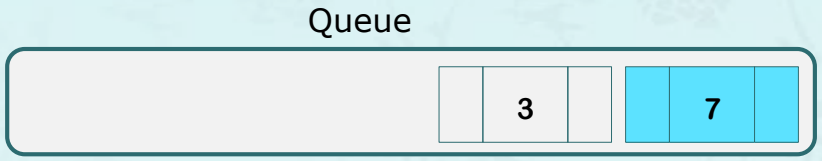

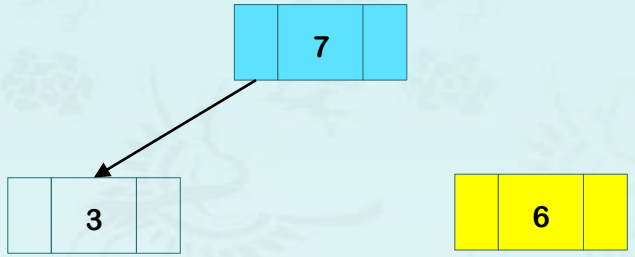

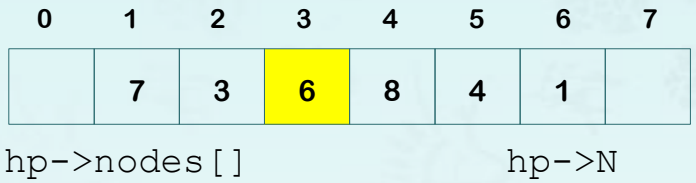

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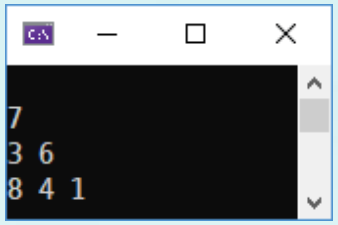

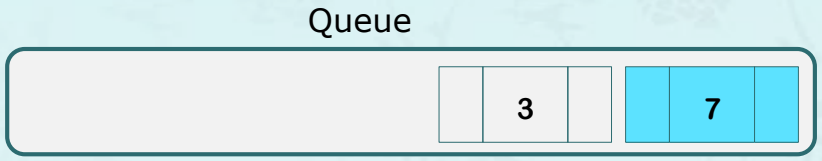

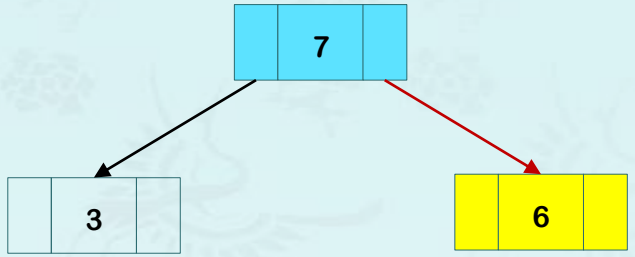

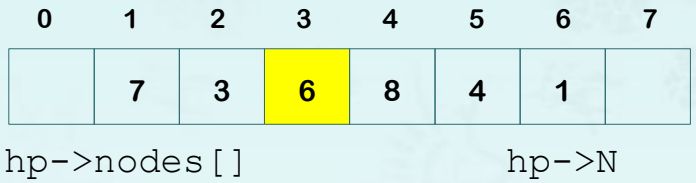

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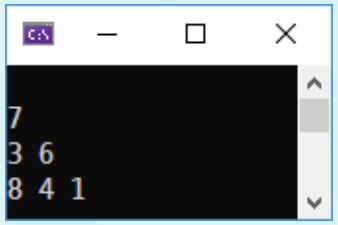

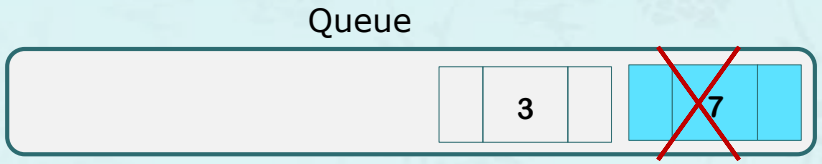

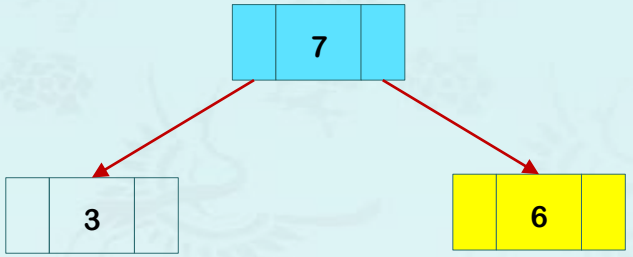

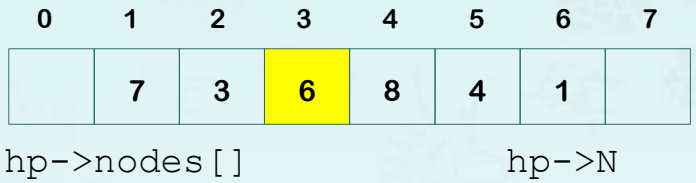

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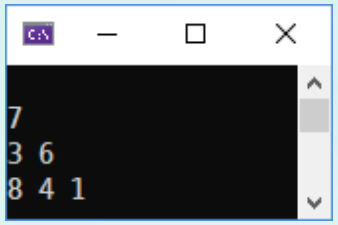

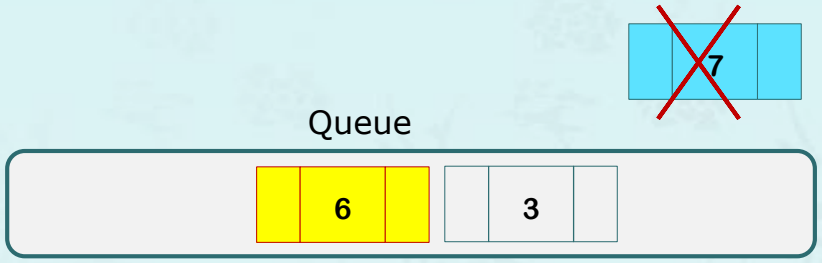

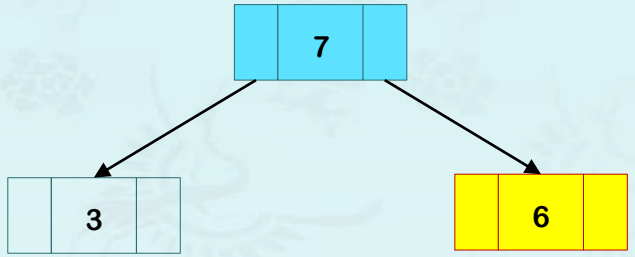

0	1	2	3	4	5	6	7
	7	3	6	8	4	1	

`hp->nodes[]` `hp->N`

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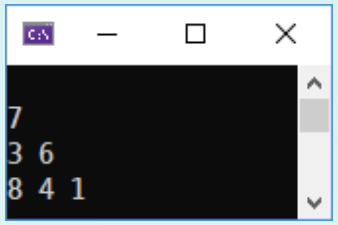

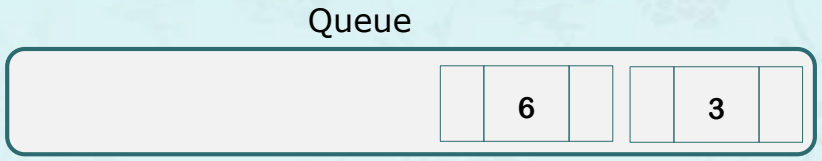

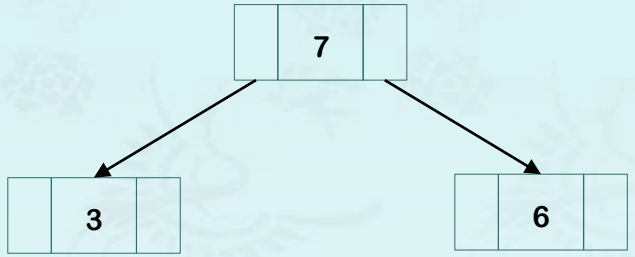

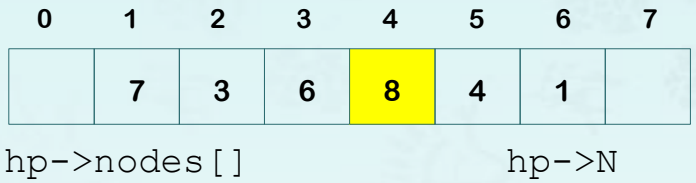

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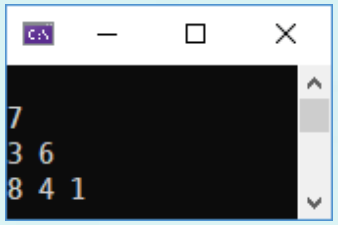

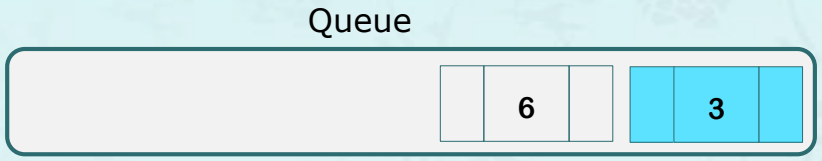

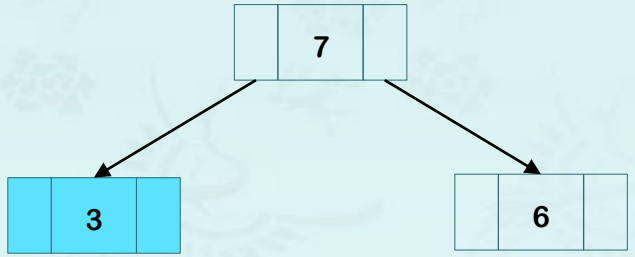

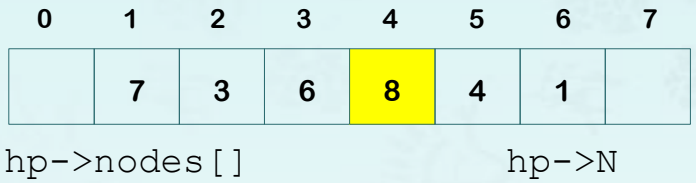

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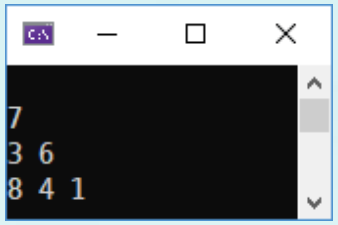

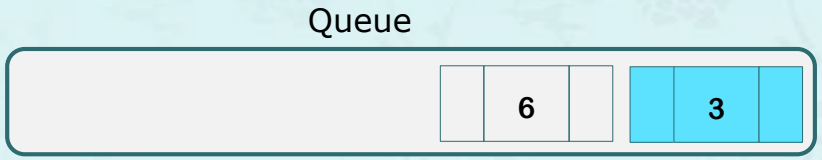

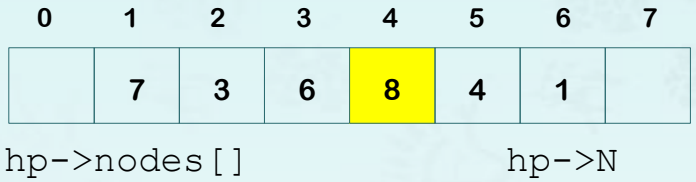

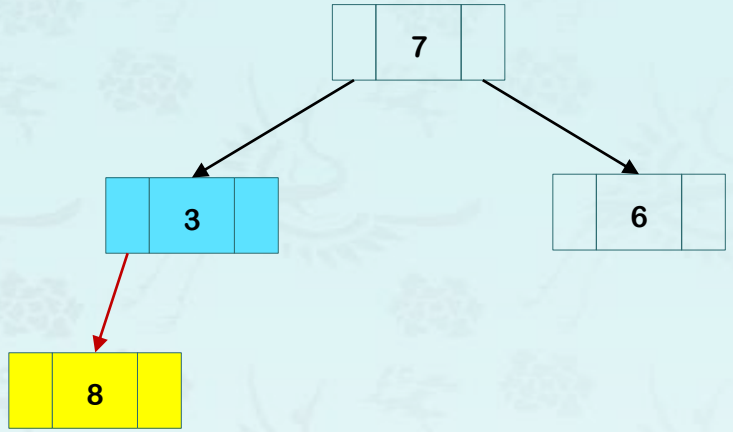

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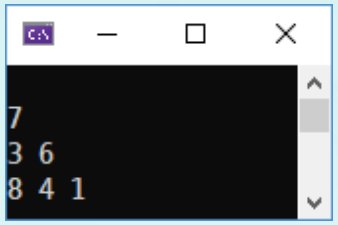

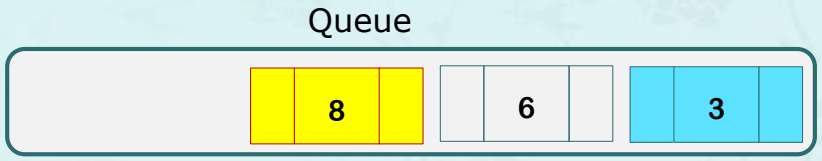

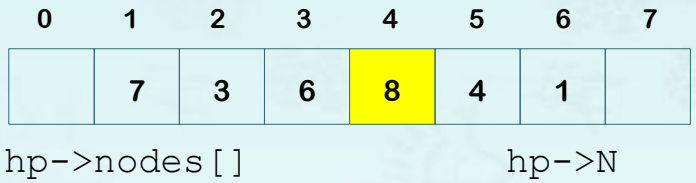

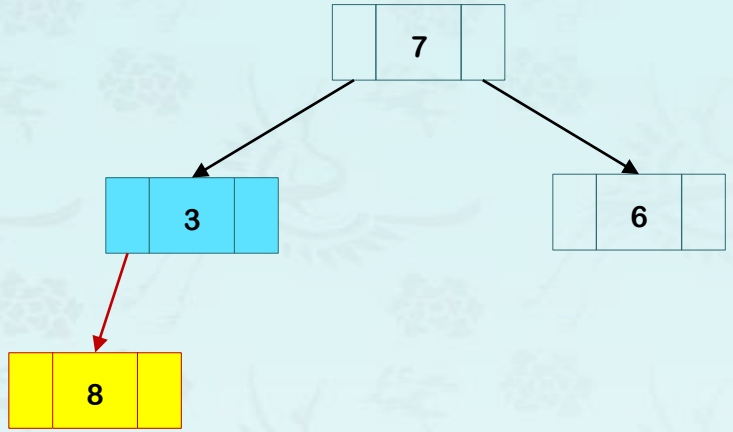

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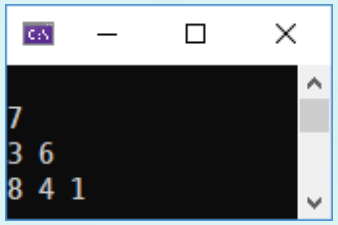

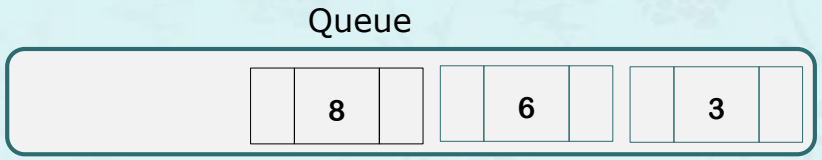

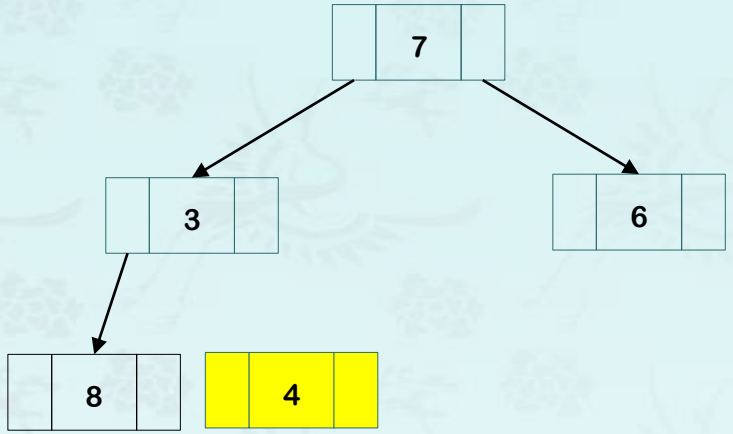

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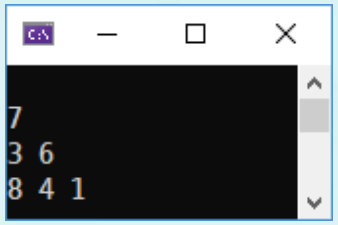

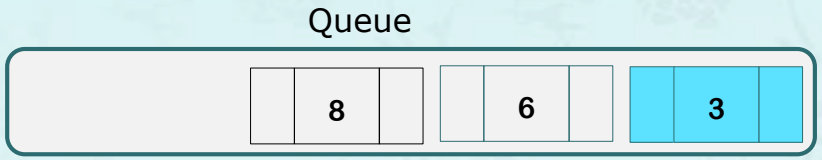

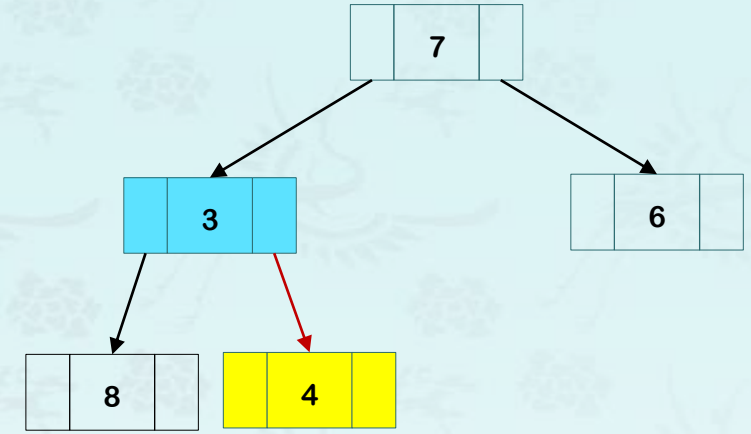

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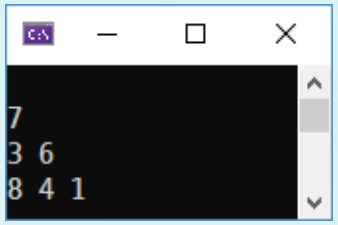

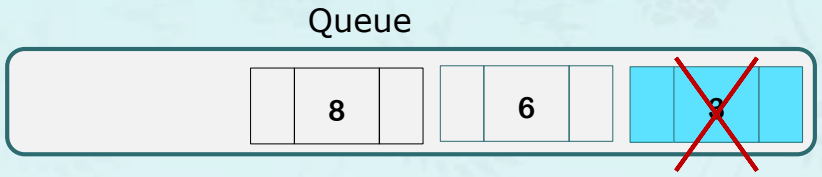

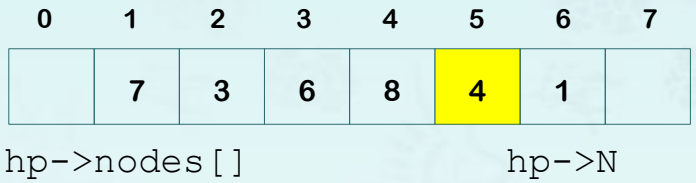

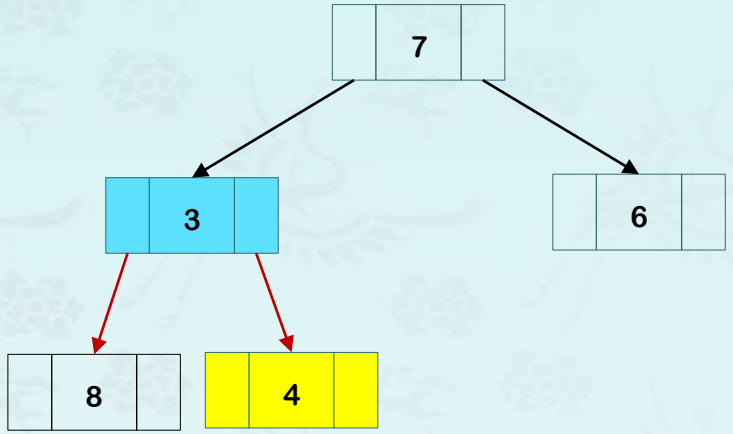

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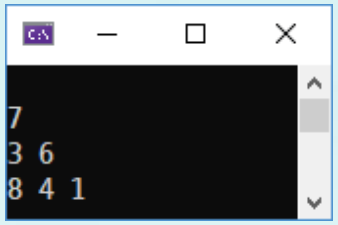

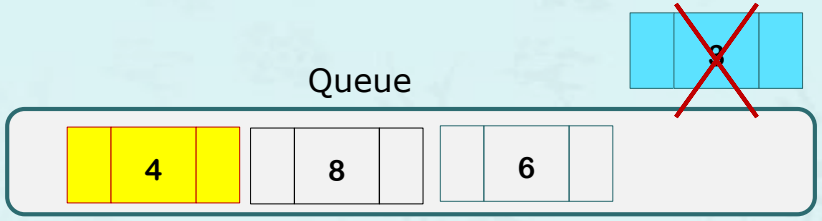

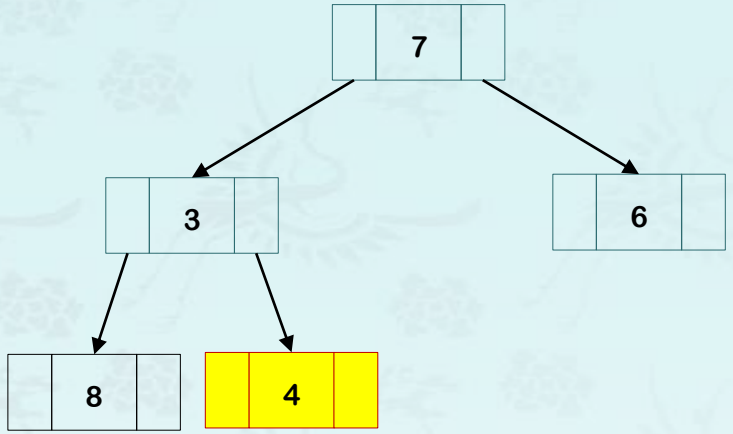

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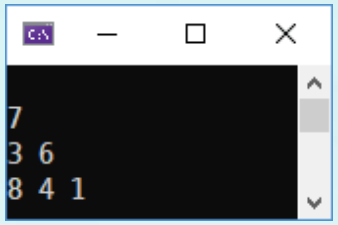

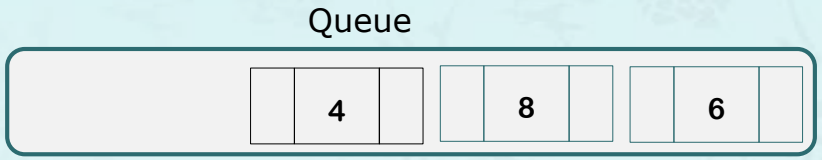

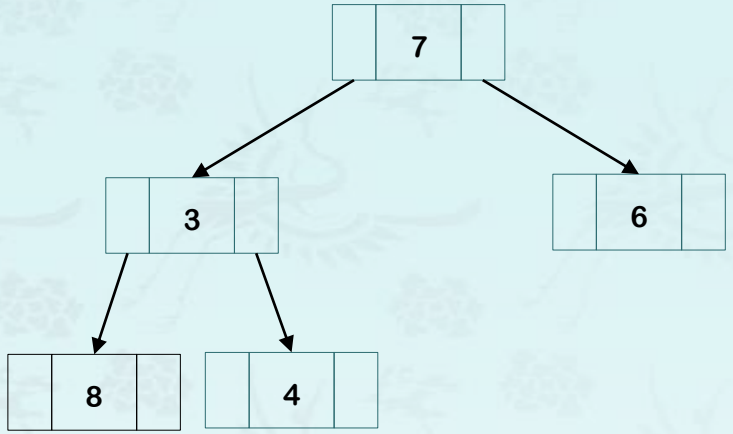

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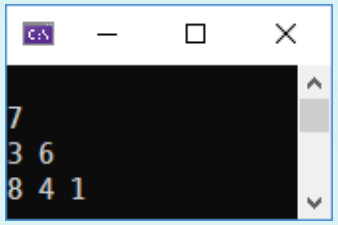
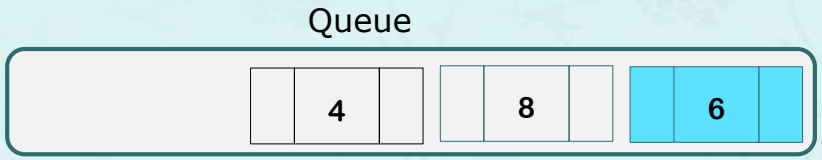
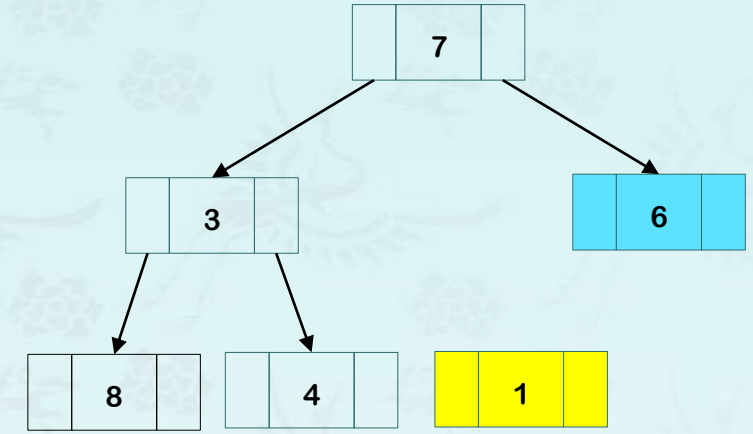
4. treeprint(root)

heapprint() – convert heap to tree using queue







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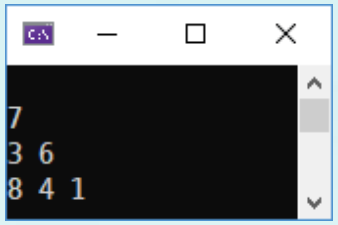

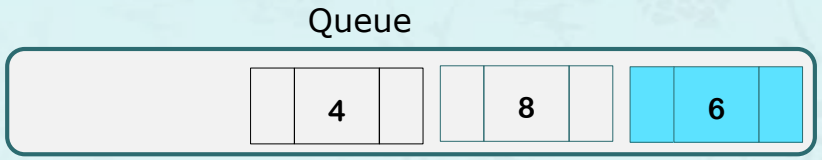

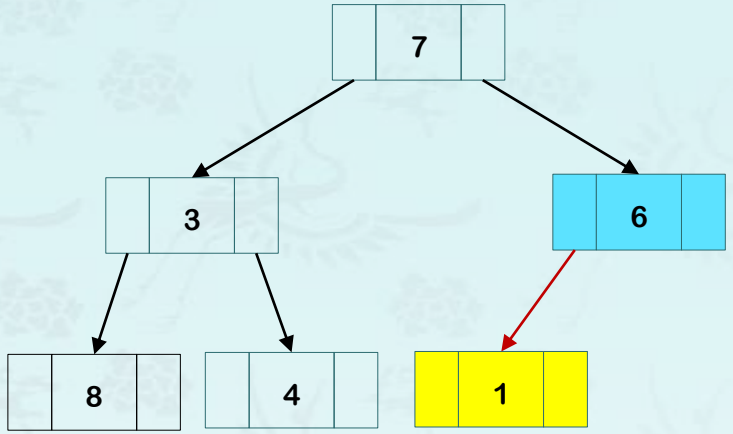

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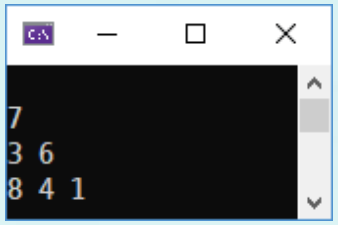

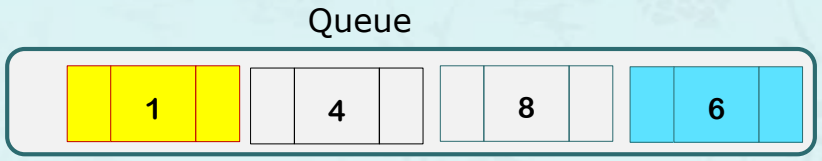

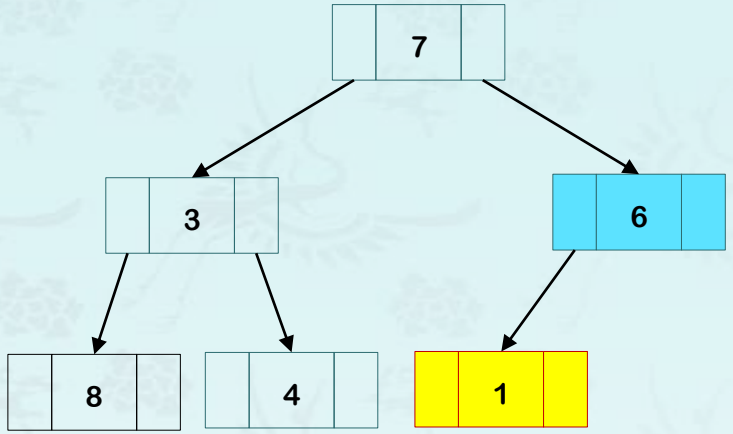

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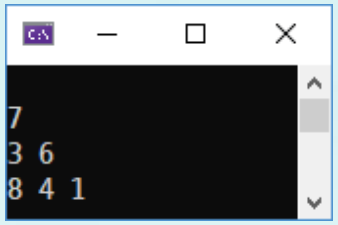

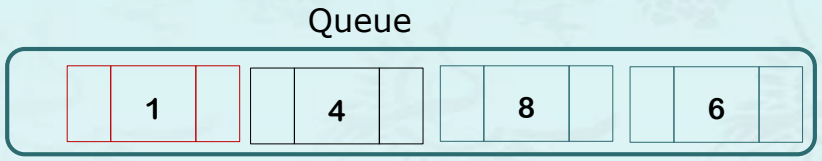

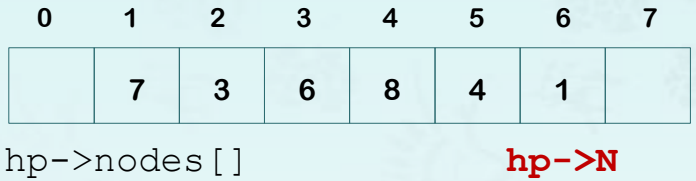

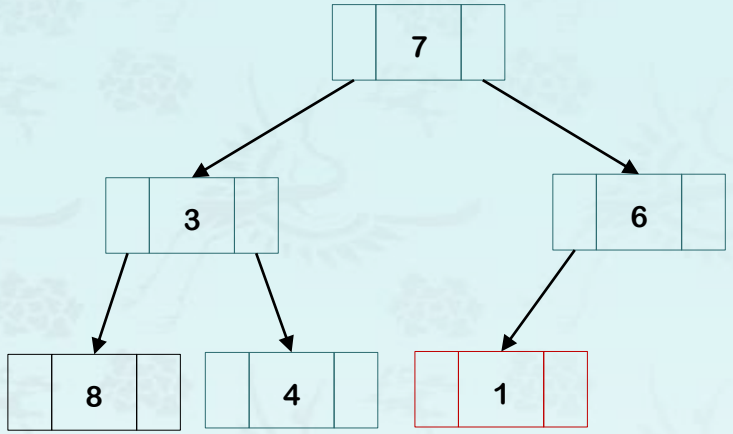

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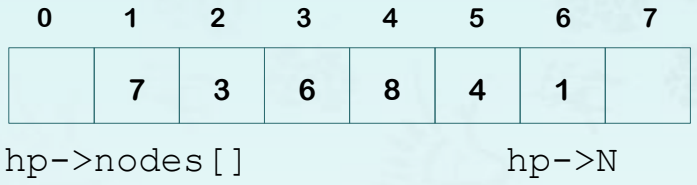
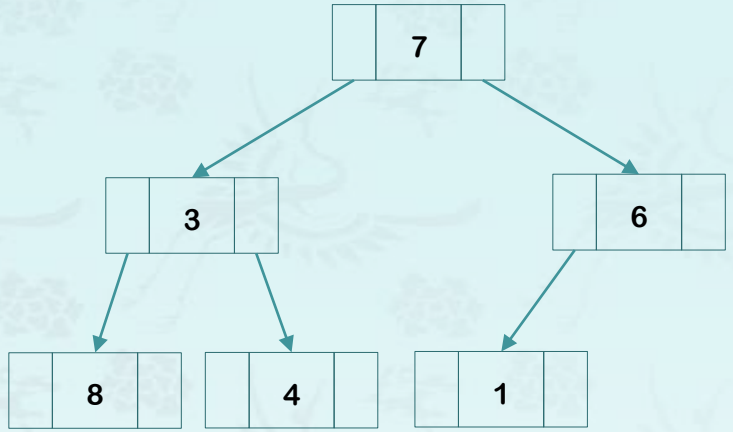
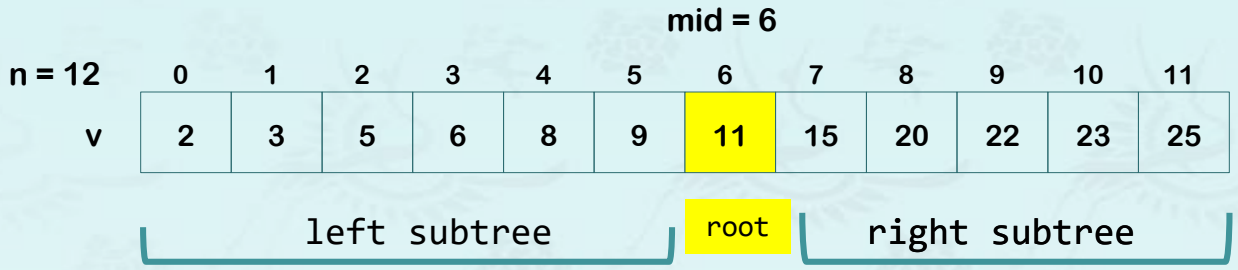
heapprint() – convert heap to tree using recursion

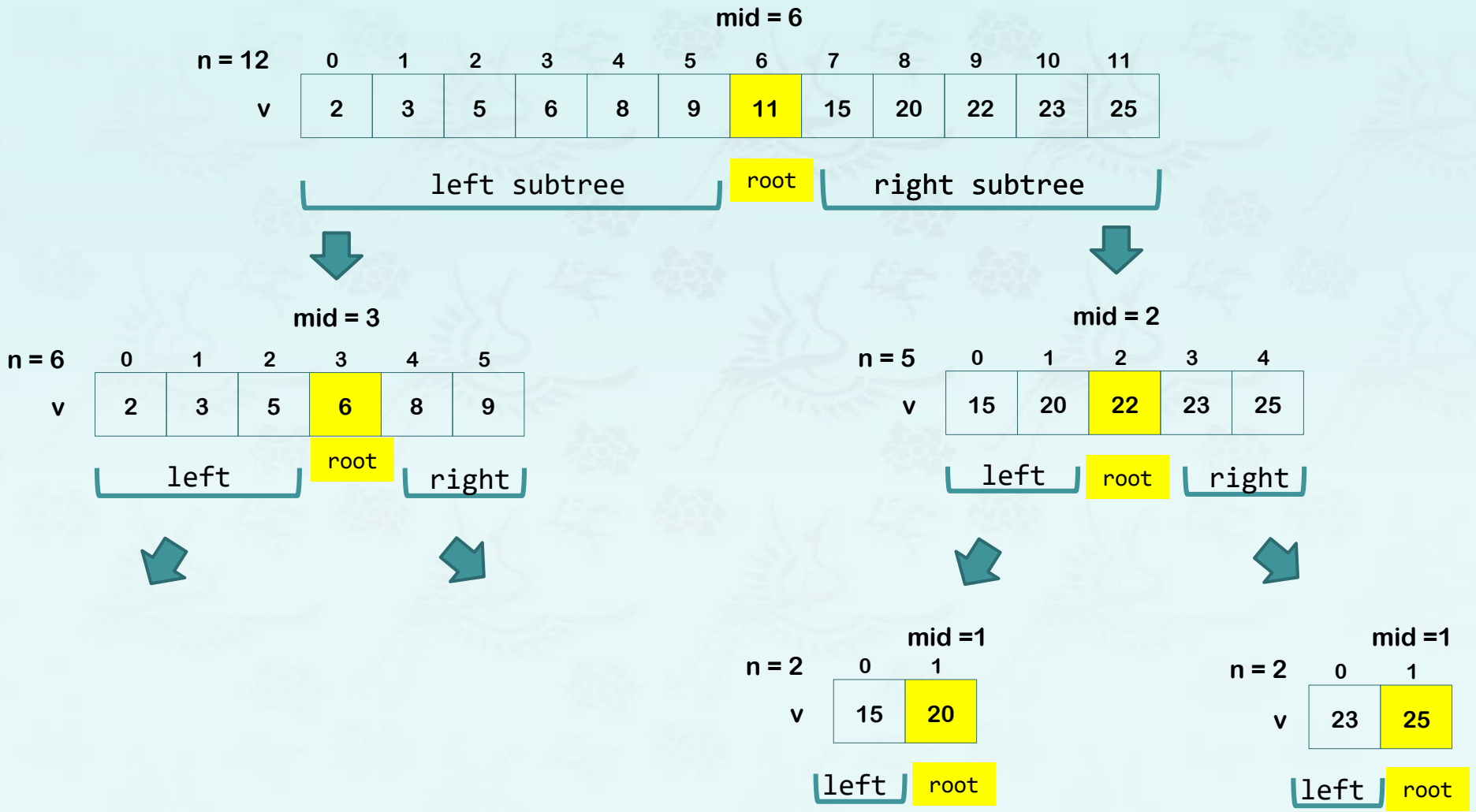




Building AVL tree from BST in O(n) – Review



Building AVL tree from BST in O(n) – Review



Building AVL tree from BST in O(n) – Review

```
// rebuilds an AVL tree with a list of keys sorted.
// v – an array of keys sorted, n – the array size

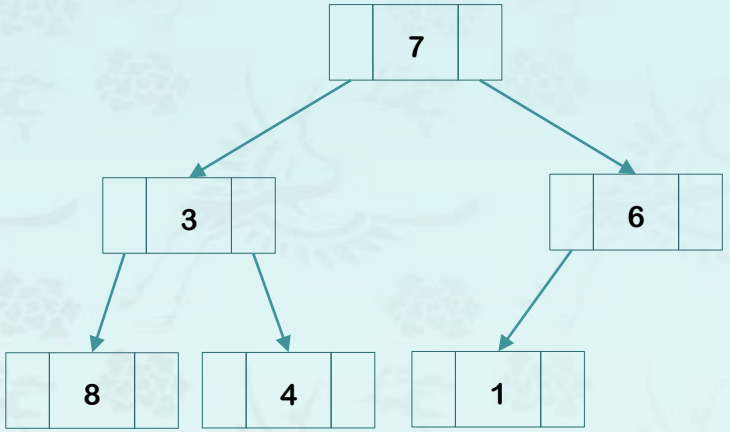
tree buildAVL(int* v, int n) {
  if (n <= 0) return nullptr;

  int mid = n / 2;
  return root;
}
```

```
tree rebalanceTree(tree root) {
  if (root == nullptr) return nullptr;
  vector<int> v;        // get keys sorted
  inorder(root, v);     // O(n)
  clear(root);

  tree new_root = buildAVL(v.data(), v.size()); // O(n)
  return new_root;
}
```

heapprint() – convert heap to tree using recursion



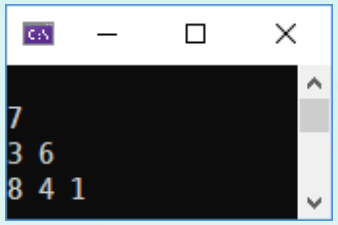
Create a recursive function that creates a binary tree from an int array. This function takes an int array, starting index, and size of the array and returns the root as shown below:

```
tree _buildBT(int *nodes, int i, int n) {
```

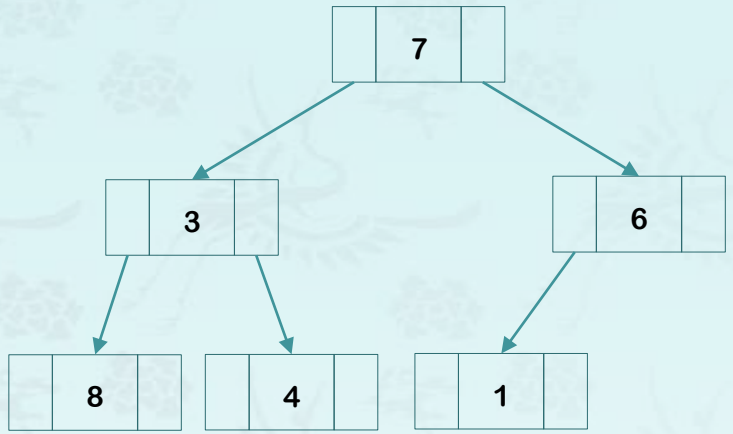
1. If `i > n`, return nulltptr – terminate condition
2. Create the **tree (root) node** with **nodes[i]**).
   A. Invoke _buildBT() for all its left children (or `i * 2`).
      Set its return to the left child of the root.
   B. Invoke _buildBT() for all its right children (or `i * 2 + 1`).
      Set its return to the right child of the root.
3. return root

}

heapprint() – convert heap to tree using recursion

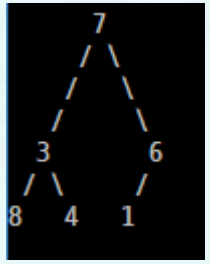


hp->nodes[] hp->N

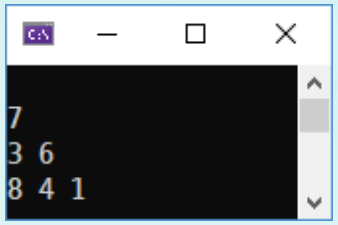


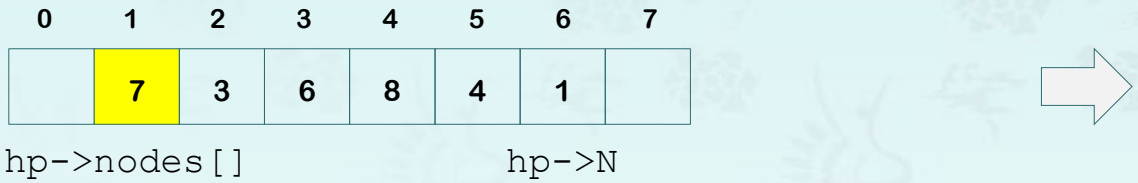
```
tree _buildBT(int *nodes, int i, int n) {


  return  root;
}
```



heapprint() – convert heap to tree using recursion





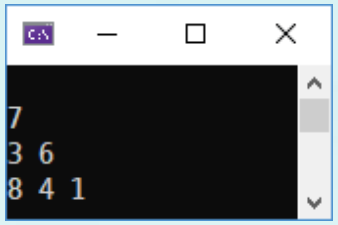

```
tree _buildBT(int *nodes, int i, int n) {
  if (i > n) return nullptr;

  tree root = new TreeNode{ nodes[i] };
  root->left  = _buildBT(nodes, i * 2, n);
  root->right = _buildBT(nodes, i * 2 + 1, n);
  return  root;
}
```

```
tree _buildBT(*nodes, i=1, n=6) { }
```

```
void heapprint(heap p) {
  if (empty(p)) return;
  tree root = _buildBT(p->nodes, 1, size(p));
  treeprint(root);
}
```

heapprint() – convert heap to tree using recursion

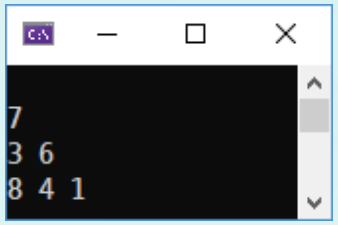


```
tree _buildBT(int *nodes, int i, int n) {
  if (i > n) return nullptr;

  tree root = new TreeNode{ nodes[i] };
  root->left  = _buildBT(nodes, i * 2, n);
  root->right = _buildBT(nodes, i * 2 + 1, n);
  return  root;
}
```

```
tree _buildBT(*nodes, i=1, n=6) { }
```

heapprint() – convert heap to tree using recursion



0	1	2	3	4	5	6	7
	7	3	6	8	4	1	

hp->nodes[] hp->N

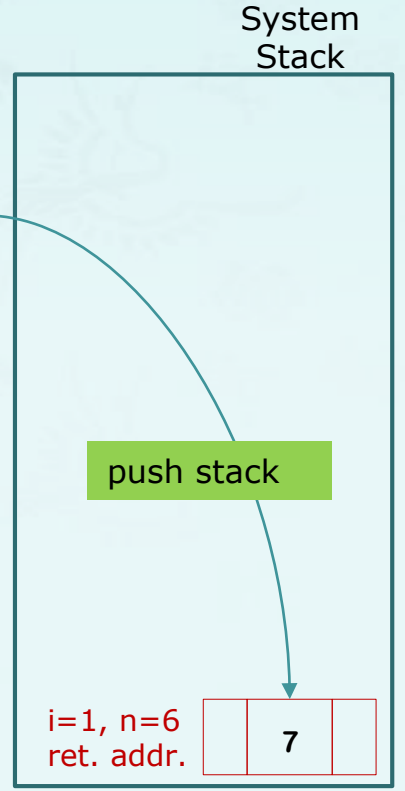
```
tree _buildBT(int *nodes, int i, int n) {
  if (i > n) return nullptr;

  tree root = new TreeNode{ nodes[i] };
  root->left  = _buildBT(nodes, i * 2, n);
  root->right = _buildBT(nodes, i * 2 + 1, n);
  return  root;
}
```

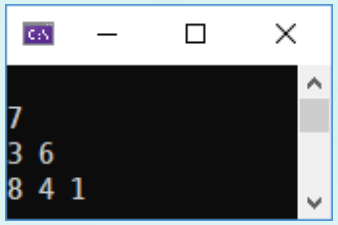
ret. addr. root=7

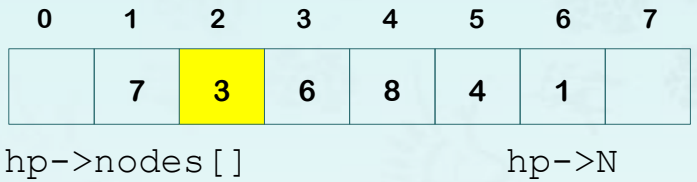
```
tree _buildBT(*nodes, i=1, n=6) { }
```

```
tree _buildBT(*nodes, i=2, n=6) { }
```



heapprint() – convert heap to tree using recursion

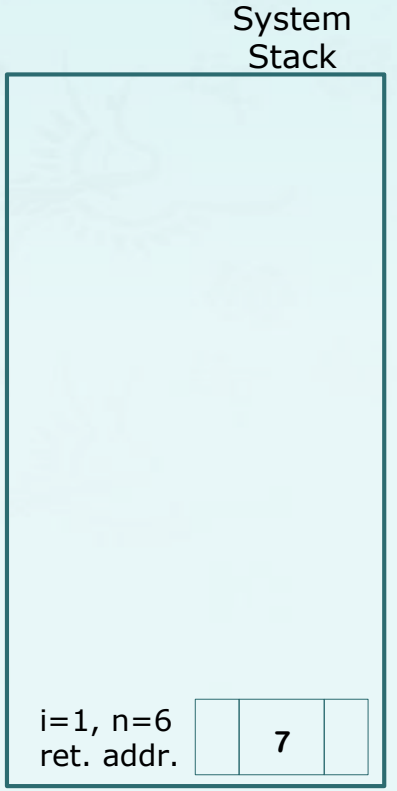




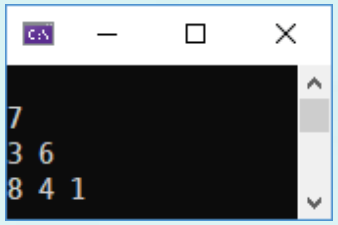

```
tree _buildBT(int *nodes, int i, int n) {
  if (i > n) return nullptr;

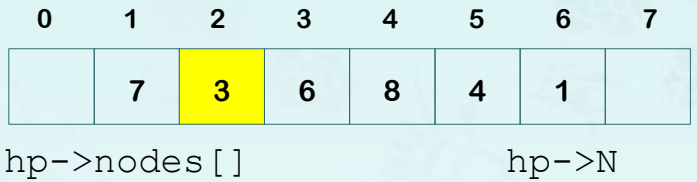
  tree root = new TreeNode{ nodes[i] };
  root->left  = _buildBT(nodes, i * 2, n);
  root->right = _buildBT(nodes, i * 2 + 1, n);
  return  root;
}
```

```
tree _buildBT(*nodes, i=2, n=6) { }
```



heapprint() – convert heap to tree using recursion



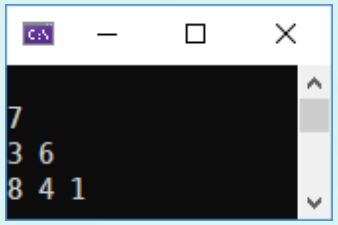



```
tree _buildBT(int *nodes, int i, int n) {
  if (i > n) return nullptr;

  tree root = new TreeNode{ nodes[i] };
  root->left  = _buildBT(nodes, i * 2, n);
  root->right = _buildBT(nodes, i * 2 + 1, n);
  return  root;
}
```

```
tree _buildBT(*nodes, i=2, n=6) { }
```

heapprint() – convert heap to tree using recursion



0	1	2	3	4	5	6	7
	7	3	6	8	4	1	

hp->nodes[] hp->N

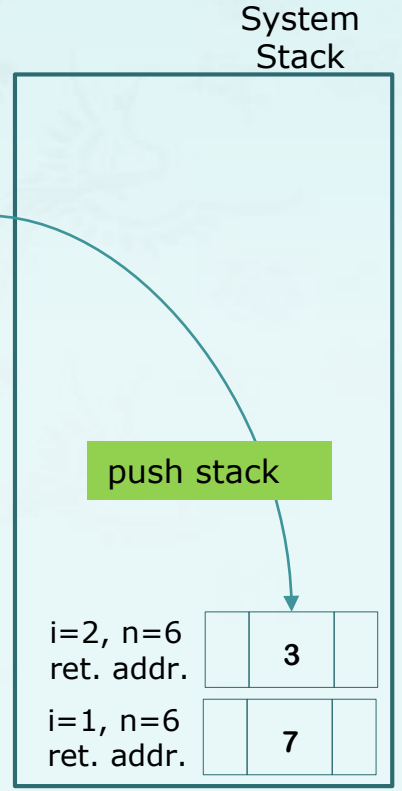
```
tree _buildBT(int *nodes, int i, int n) {
  if (i > n) return nullptr;

  tree root = new TreeNode{ nodes[i] };
  root->left  = _buildBT(nodes, i * 2, n);
  root->right = _buildBT(nodes, i * 2 + 1, n);
  return  root;
}
```

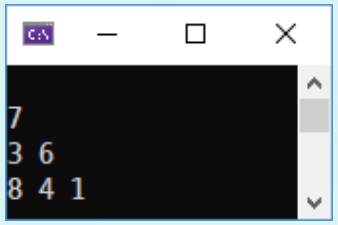
ret. addr. root(3)


```
tree _buildBT(*nodes, i=2, n=6) { }
```

```
tree _buildBT(*nodes, i=4, n=6) { }
```



heapprint() – convert heap to tree using recursion

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tree _buildBT(int *nodes, int i, int n) {
  if (i > n) return nullptr;

  tree root = new TreeNode{ nodes[i] };
  root->left  = _buildBT(nodes, i * 2, n);
  root->right = _buildBT(nodes, i * 2 + 1, n);
  return  root;
}
```
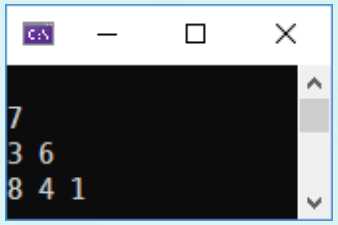
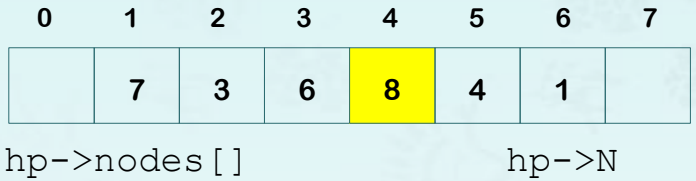

```
tree _buildBT(*nodes, i=4, n=6) { }
```

heapprint() – convert heap to tree using recursion

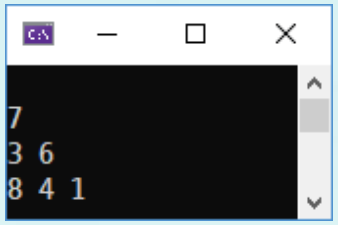


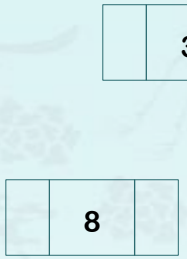



```
tree _buildBT(int *nodes, int i, int n) {
  if (i > n) return nullptr;

  tree root = new TreeNode{ nodes[i] };
  root->left  = _buildBT(nodes, i * 2, n);
  root->right = _buildBT(nodes, i * 2 + 1, n);
  return  root;
}
```

```
tree _buildBT(*nodes, i=4, n=6) { }
```

heapprint() – convert heap to tree using recursion



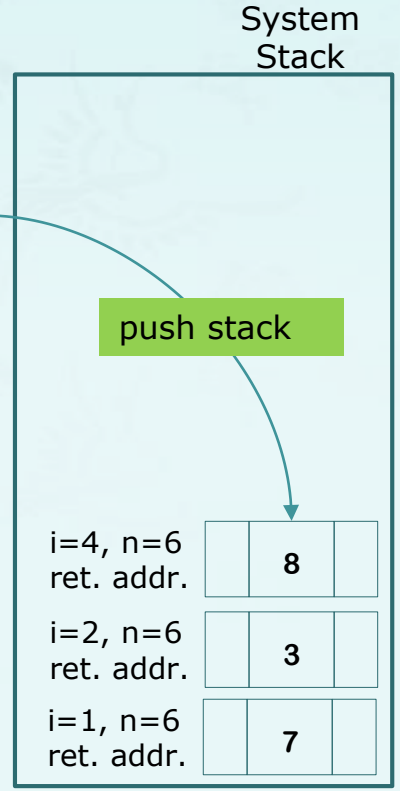



```
tree _buildBT(int *nodes, int i, int n) {
  if (i > n) return nullptr;

  tree root = new TreeNode{ nodes[i] };
  root->left  = _buildBT(nodes, i * 2, n);
  root->right = _buildBT(nodes, i * 2 + 1, n);
  return  root;
}
```

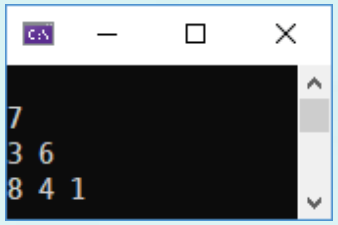
ret. addr. root(8)

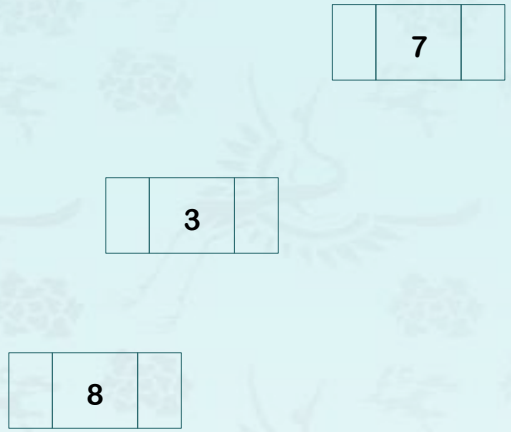
```
tree _buildBT(*nodes, i=4, n=6) { }
```

```
tree _buildBT(*nodes, i=8, n=6) { }
```



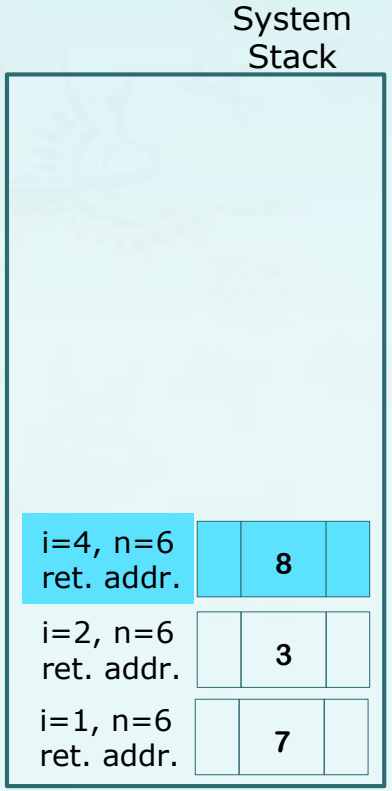
heapprint() – convert heap to tree using recursion



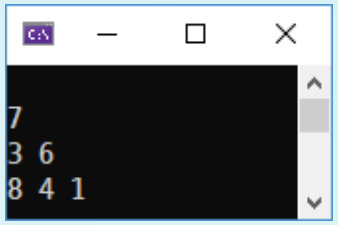



```
tree _buildBT(int *nodes, int i, int n) {
  if (i > n) return nullptr;

  tree root = new TreeNode{ nodes[i] };
  root->left  = _buildBT(nodes, i * 2, n);
  root->right = _buildBT(nodes, i * 2 + 1, n);
  return  root;
}
```

```
tree _buildBT(*nodes, i=8, n=6) { }
```



heapprint() – convert heap to tree using recursion



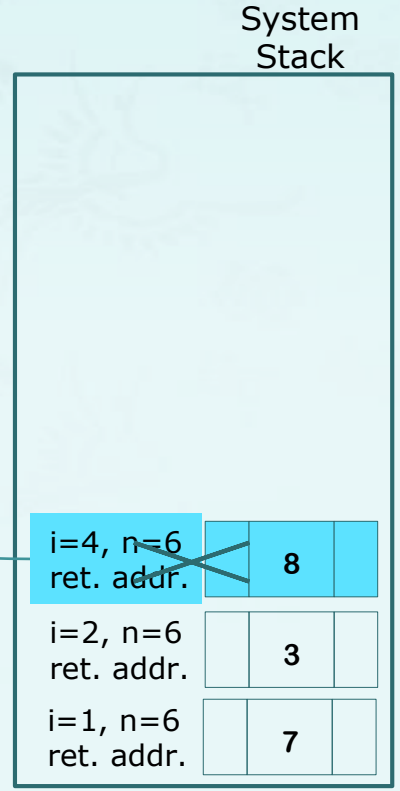
0	1	2	3	4	5	6	7
	7	3	6	8	4	1	

hp->nodes[] hp->N

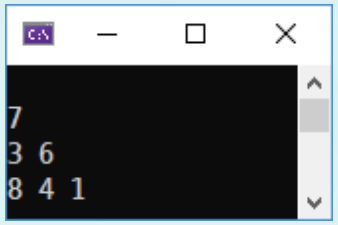

```
tree _buildBT(int *nodes, int i, int n) {
  if (i > n) return nullptr;

  tree root = new TreeNode{ nodes[i] };
  root->left  = _buildBT(nodes, i * 2, n);
  root->right = _buildBT(nodes, i * 2 + 1, n);
  return  root;
}
```

```
i=4, n=6, root(8), root->left = nullptr
```

pop stack



heapprint() – convert heap to tree using recursion

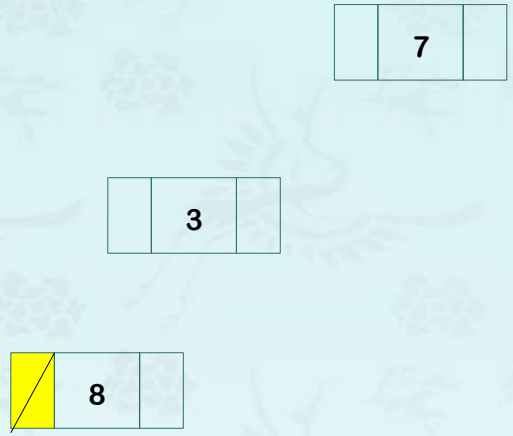


0	1	2	3	4	5	6	7
	7	3	6	8	4	1	

hp->nodes[] hp->N

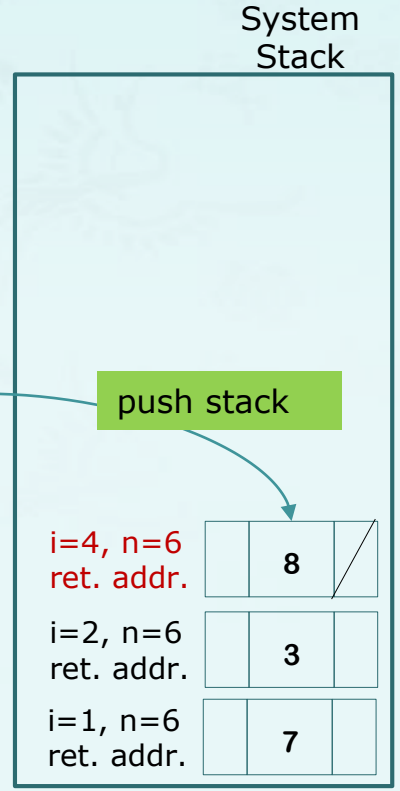
```
tree _buildBT(int *nodes, int i, int n) {
  if (i > n) return nullptr;

  tree root = new TreeNode{ nodes[i] };
  root->left  = _buildBT(nodes, i * 2, n);
  root->right = _buildBT(nodes, i * 2 + 1, n);
  return  root;
}
```

ret. addr. root(8)

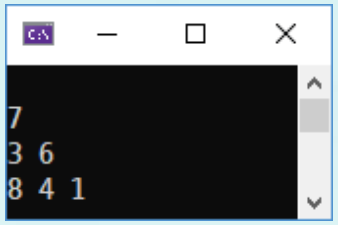


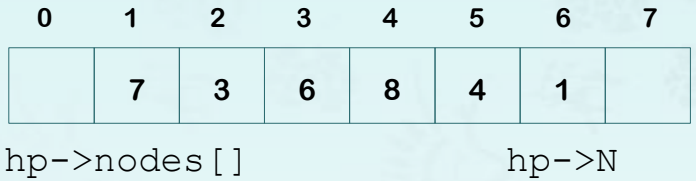
i=4, n=6, root(8), root->left = nullptr

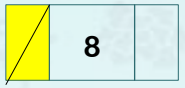
tree _buildBT(*nodes,i=4*2+1,n=6){}



heapprint() – convert heap to tree using recursion



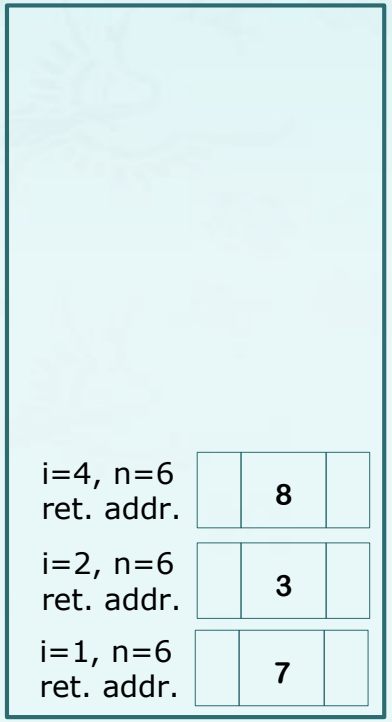




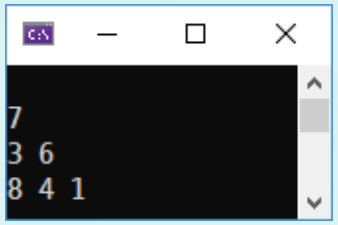

```
tree _buildBT(int *nodes, int i, int n) {
  if (i > n) return nullptr;

  tree root = new TreeNode{ nodes[i] };
  root->left  = _buildBT(nodes, i * 2, n);
  root->right = _buildBT(nodes, i * 2 + 1, n);
  return  root;
}
```

```
tree _buildBT(*nodes, i=9, n=6) { }
```

System Stack



heapprint() – convert heap to tree using recursion



0	1	2	3	4	5	6	7
	7	3	6	8	4	1	

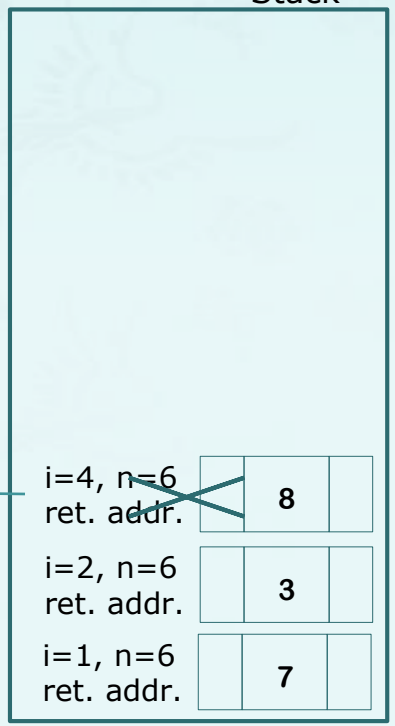
hp->nodes[] hp->N


```
tree _buildBT(int *nodes, int i, int n) {
  if (i > n) return nullptr;

  tree root = new TreeNode{ nodes[i] };
  root->left  = _buildBT(nodes, i * 2, n);
  root->right = _buildBT(nodes, i * 2 + 1, n);
  return  root;
}
```

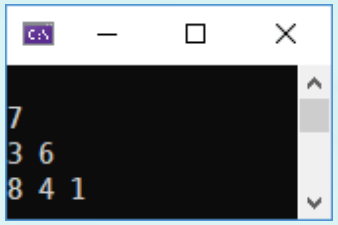
i=4, n=6, root(8), root->right = nullptr

pop stack

System Stack

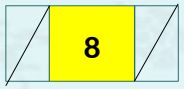


heapprint() – convert heap to tree using recursion



0	1	2	3	4	5	6	7
	7	3	6	8	4	1	

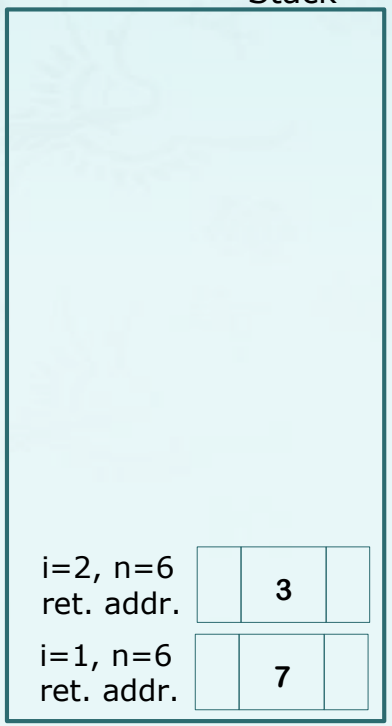
hp->nodes[] hp->N



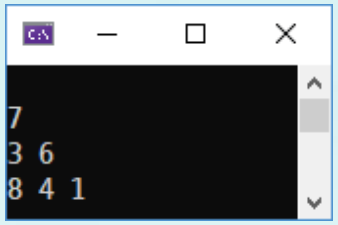

```
tree _buildBT(int *nodes, int i, int n) {
  if (i > n) return nullptr;

  tree root = new TreeNode{ nodes[i] };
  root->left  = _buildBT(nodes, i * 2, n);
  root->right = _buildBT(nodes, i * 2 + 1, n);
  return  root;
}
```

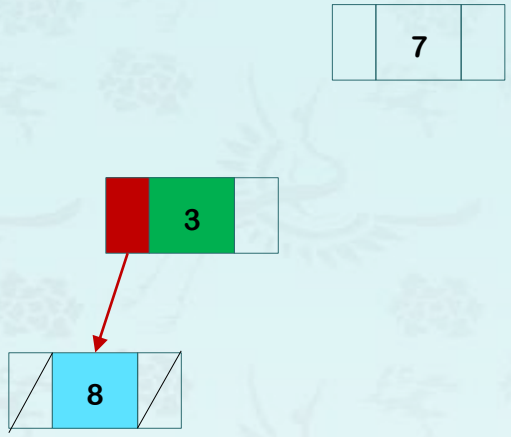
```
i=4, n=6, root(8), root->right = nullptr
return root(8)
```

System Stack



heapprint() – convert heap to tree using recursion

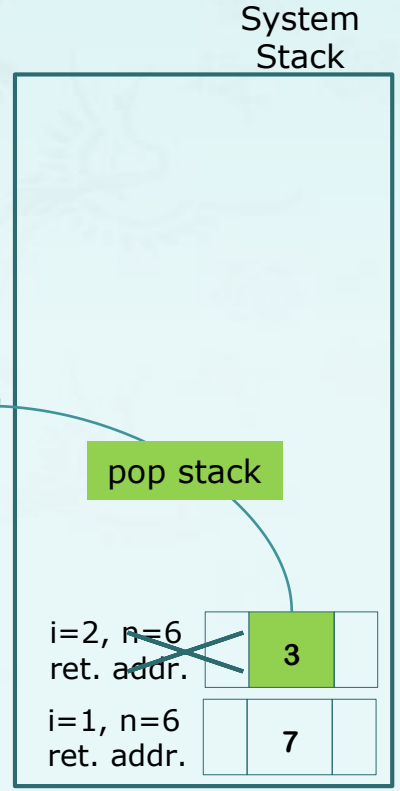




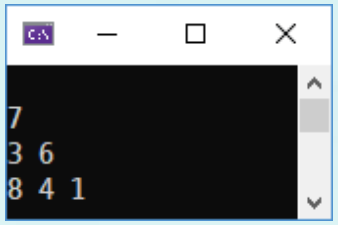

```
tree _buildBT(int *nodes, int i, int n) {
  if (i > n) return nullptr;

  tree root = new TreeNode{ nodes[i] };
  root->left  = _buildBT(nodes, i * 2, n);
  root->right = _buildBT(nodes, i * 2 + 1, n);
  return  root;
}
```

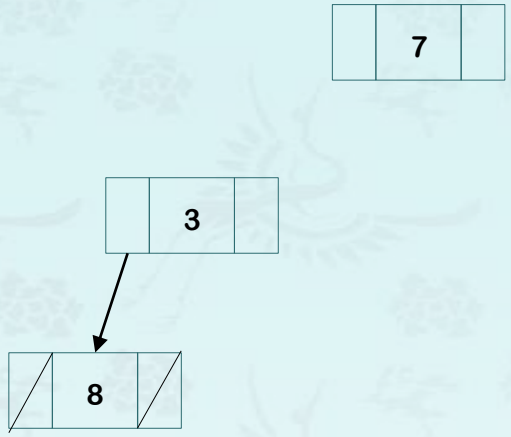
```
i=2, n=6, root(3) root->left = root(8)
```

```
return root(8)
```



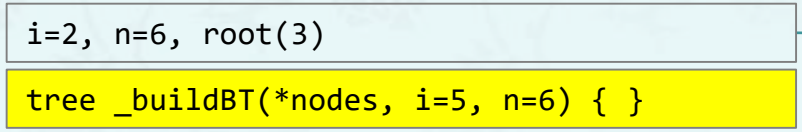
heapprint() – convert heap to tree using recursion

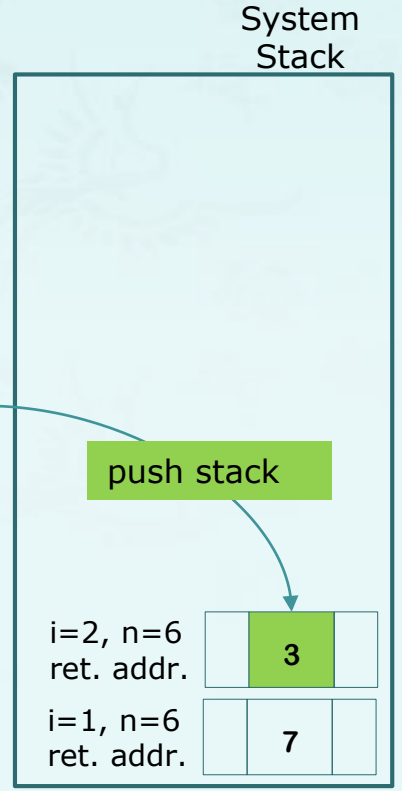




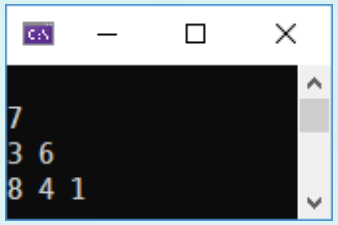

```
tree _buildBT(int *nodes, int i, int n) {
  if (i > n) return nullptr;

  tree root = new TreeNode{ nodes[i] };
  root->left  = _buildBT(nodes, i * 2, n);
  root->right = _buildBT(nodes, i * 2 + 1, n);
  return  root;
}
```

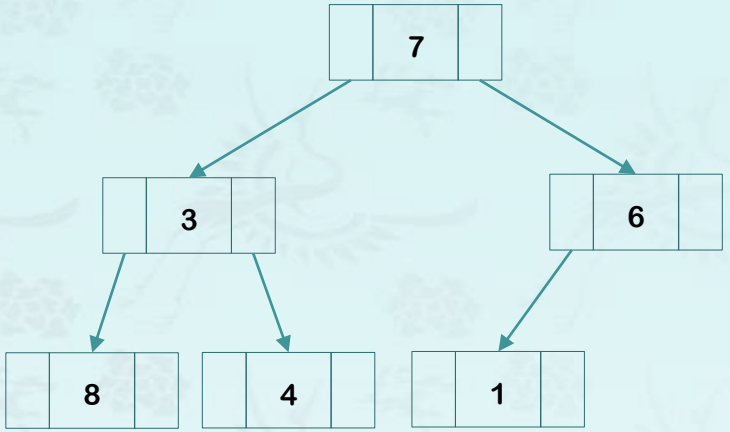
ret. addr. root(3)



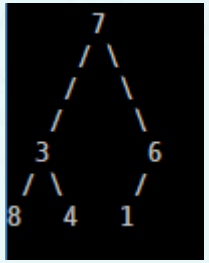


heapprint() – convert heap to tree using recursion






```
tree _buildBT(int *nodes, int i, int n) {
  if (i > n) return nullptr;

  tree root = new TreeNode{ nodes[i] };
  root->left  = _buildBT(nodes, i * 2, n);
  root->right = _buildBT(nodes, i * 2 + 1, n);
  return  root;
}
```



heap

- *complete binary tree (review)*
- *heap and priority queues (Chapter 9)*
- *binary heap and min-heap*
- *max-heap demo*
- *max-heap coding*
- *heapsort (Chapter 7)*