Case Study 6.4: Clouding seeding

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Problem

The data in this question come from an experiment to see whether cloud-seeding works. Cloud-seeding is the process of firing silver nitrate ($AgNO_3$) into the clouds. Water coalesces around the silver nitrate particles, hopefully getting large enough to precipitate (rain). The measurements in this study are in acre-feet. One acre-foot is the amount of water it takes to fill one acre uniformly to the depth of one foot. One acre-foot is about 1.2 million litres of water (1233481.85532 litres to be exact). This data set is included in the `s20x` library.

The variables of interest are:

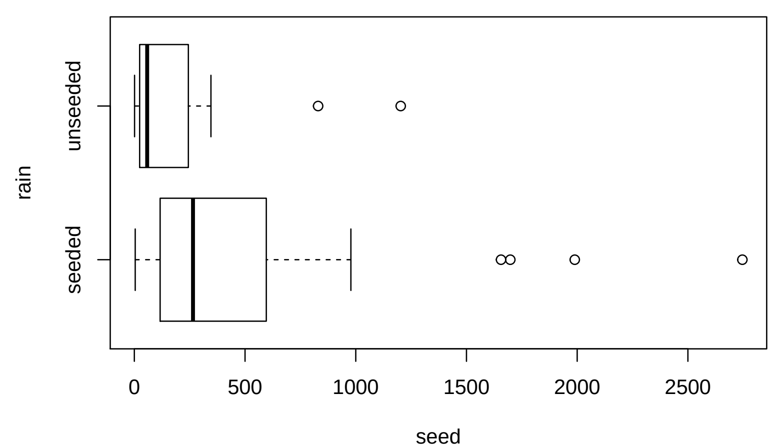
- `rain`: total rainfall.
- `seed`: is a factor with two levels, `seeded` and `unseeded`.

Question of Interest

We wish to see whether cloud seeding produces more rain. Also, what is the typical rainfall from seeded and unseeded clouds?

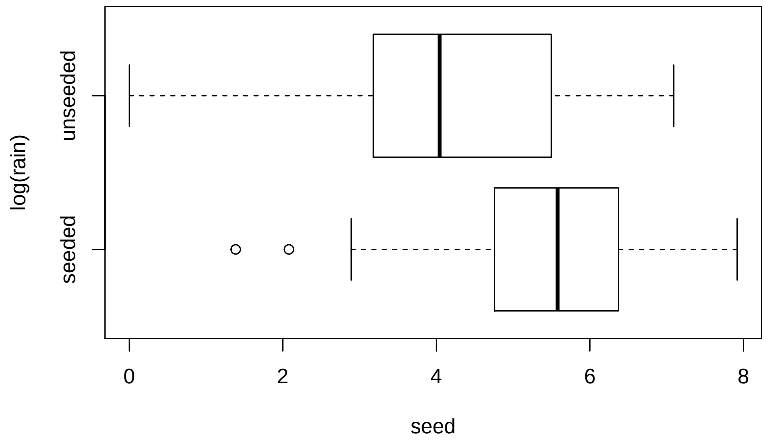
Read in and Inspect the Data

```
data(rain.df)
boxplot(rain ~ seed, data = rain.df, horizontal = TRUE)
```



Looks like our old right-skewed friend again. Maybe, just maybe, a log-transformation would work. Let's try it.

```
boxplot(log(rain) ~ seed, data = rain.df, horizontal = TRUE)
```



Yup. Looks much better, and we have preliminary evidence that the seeding seems to work. Should we be worried about equality of variance?

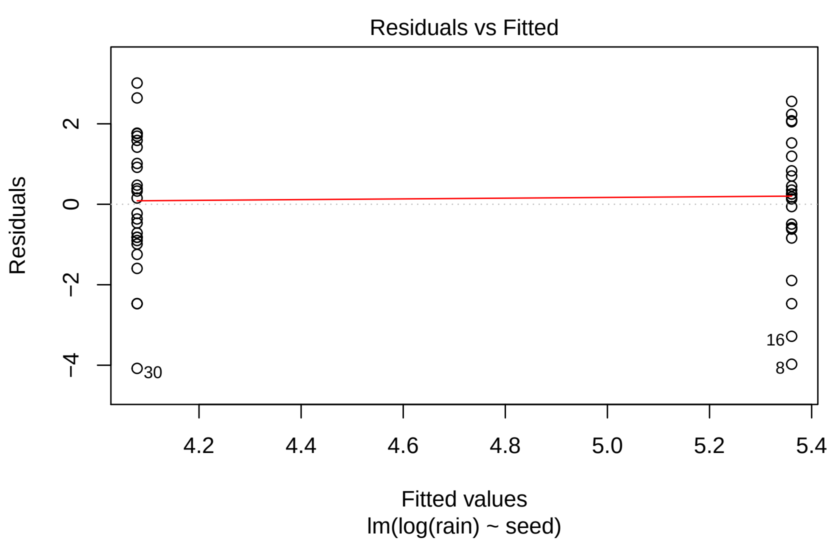
```
summaryStats(log(rain) ~ seed, data = rain.df)
```

```
##          Sample Size     Mean   Median  Std Dev Midspread
## seeded            24 5.360901 5.579017 1.665145  1.518865
## unseeded          26 4.078695 4.042127 1.661585  2.198249
```

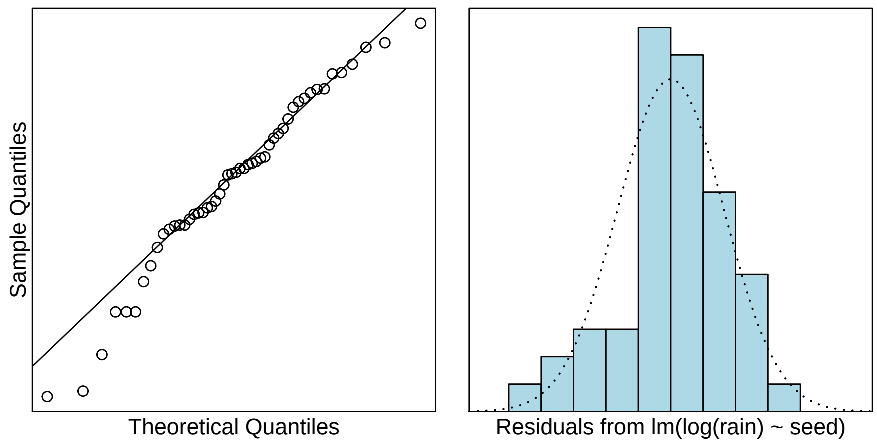
No. On the log-scale, the standard deviations are nearly identical, and the midspreads are well below a factor of two different. Let's go ahead and fit the model.

Model Building and Check Assumptions

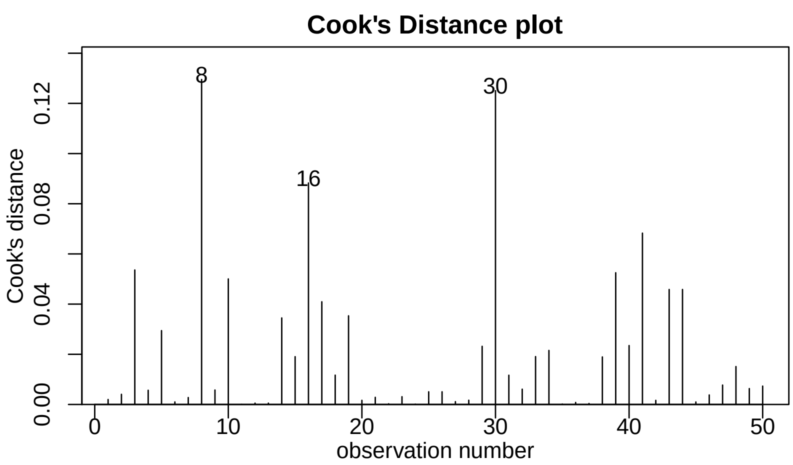
```
rain.fit  = lm(log(rain) ~ seed, data = rain.df)
plot(rain.fit, which = 1)
```



```r
normcheck(rain.fit)
```




```r
cooks20x(rain.fit)
```




```r
summary(rain.fit)
```

```
##
## Call:
## lm(formula = log(rain) ~ seed, data = rain.df)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -4.0787 -0.8206  0.1679  1.1496  3.0139
##
## Coefficients:
```

```
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)    5.3609     0.3395  15.790  < 2e-16 ***
## seedunseeded  -1.2822     0.4708  -2.723  0.00899 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.663 on 48 degrees of freedom
## Multiple R-squared:  0.1338, Adjusted R-squared:  0.1158
## F-statistic: 7.416 on 1 and 48 DF,  p-value: 0.008985
```

```
# Let's refit the model so that `unseeded` is the base level.
rain.df$seed = relevel(rain.df$seed, ref = "unseeded")
rain.fit.2 = lm(log(rain) ~ seed, data = rain.df)
summary(rain.fit.2)
```

```
##
## Call:
## lm(formula = log(rain) ~ seed, data = rain.df)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -4.0787 -0.8206  0.1679  1.1496  3.0139
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   4.0787     0.3262  12.504  < 2e-16 ***
## seedseeded    1.2822     0.4708   2.723  0.00899 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.663 on 48 degrees of freedom
## Multiple R-squared:  0.1338, Adjusted R-squared:  0.1158
## F-statistic: 7.416 on 1 and 48 DF,  p-value: 0.008985
```

```
exp(confint(rain.fit.2))
```

```
##                 2.5 %     97.5 %
## (Intercept) 30.656034 113.813484
## seedseeded   1.398702   9.289329
```

```
100 * (exp(confint(rain.fit.2)) - 1)
```

```
##                  2.5 %     97.5 %
## (Intercept) 2965.60341 11281.3484
## seedseeded    39.87024   828.9329
```

Confidence Interval Output

```
pred.df = data.frame(seed = c("unseeded", "seeded"))
exp(predict(rain.fit.2, pred.df, interval = "confidence"))
```

```
##         fit       lwr      upr
## 1  59.06835  30.65603 113.8135
## 2 212.91668 107.58209 421.3853
```

Methods and Assumption Checks

The boxplots of `log(rain)` showed that the groups were comparable. So, we fitted a linear model to explain `log(rain)` with the explanatory factor `seed`.

All model assumptions were satisified.

Our final model is

$$log(Rain_i) = \beta_0 + \beta_1 \times Seed.Seeded_i + \epsilon_i,$$

where $\epsilon_i \sim iid\ N(0, \sigma^2)$. Here $Seed.Seeded_i = 1$ if the observation was done in a seeded environment, otherwise it is zero.

Our model describes 13% of the variability in the logged measurements of total rainfall.

Executive Summary

We wish to see whether cloud seeding produces more rain.

There is strong evidence cloud-seeding works ($P\text{-}value < 0.01$).

We estimate that the median rain from the seeded clouds is about 40% to 830% higher than that from unseeded clouds.

The unseeded clouds produce median rainfall of 30.7 to 113.8 acre-feet, whereas for the seeded clouds it is 107.6 to 421.4 acre-feet of rain.