# Case Study 14.1: Decline in expected earthquake numbers with magnitude

Tou Ohone Andate - staff number 1234567

## Problem

The Gutenberg-Richter law says that the expected frequency of earthquakes decreases multiplicatively with their magnitude. The formula is

$$\log_{10} N = a - bM,$$

where $N$ is the expected number of earthquakes of magnitude $M$ or more on the Richter scale. Here, $a$ and $b$ are unknown parameters.

After applying a healthy dash of calculus, this formula can be re-expressed in a form that is more familiar to us

$$E[Y|x] = \exp(\beta_0 + \beta_1 x),$$

where $Y$ is the number of number of earthquakes of magnitude between $x - \delta$ and $x + \delta$.[1]

The variables of interest were:

- `Magnitude`: The strength of a recorded earthquake.
- `Locn`: A two-level factor which describe the location of the earthquakes.
  - It has two levels Southern California ("SC") and Washington ("WA").
- `Freq`: The number of earthquakes recorded at `Locn` with magnitude `Magnitude`.

### Question of Interest

The research question is to quantify the rate of decrease in earthquake frequency (with increasing magnitude) in both California and Washington states, and to assess whether these rates are the same.

### Read in and Inspect the Data

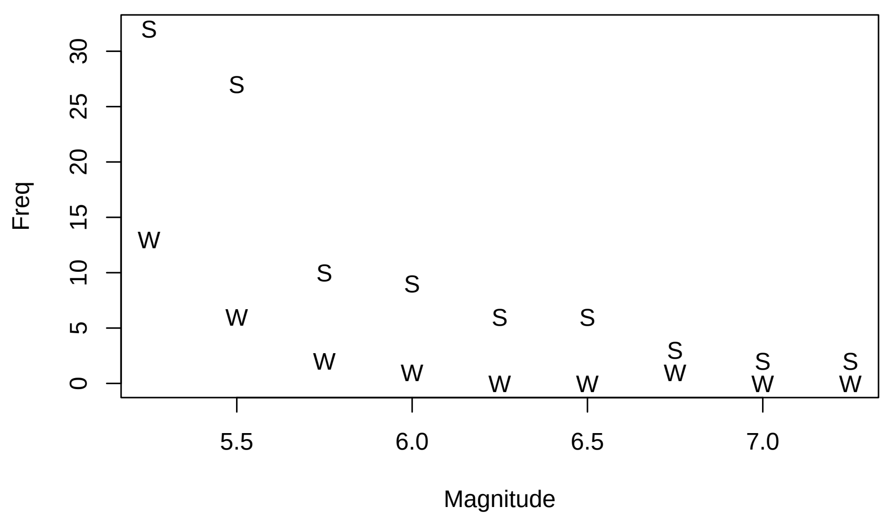
```
Quakes.df = read.table("EarthquakeMagnitudes.txt", header = TRUE)
Quakes.df$Locn=as.factor(Quakes.df$Locn)
Quakes.df
```

```
##    Locn Magnitude Freq
## 1    SC      5.25   32
## 2    SC      5.50   27
## 3    SC      5.75   10
```

[1] In the above formula, $\beta_0$ and $\beta_1$ depend on $a$, $b$ and $\delta$ in a complicated way that we are not going to concern ourselves with.

```
## 4    SC      6.00    9
## 5    SC      6.25    6
## 6    SC      6.50    6
## 7    SC      6.75    3
## 8    SC      7.00    2
## 9    SC      7.25    2
## 10   WA      5.25   13
## 11   WA      5.50    6
## 12   WA      5.75    2
## 13   WA      6.00    1
## 14   WA      6.25    0
## 15   WA      6.50    0
## 16   WA      6.75    1
## 17   WA      7.00    0
## 18   WA      7.25    0
```

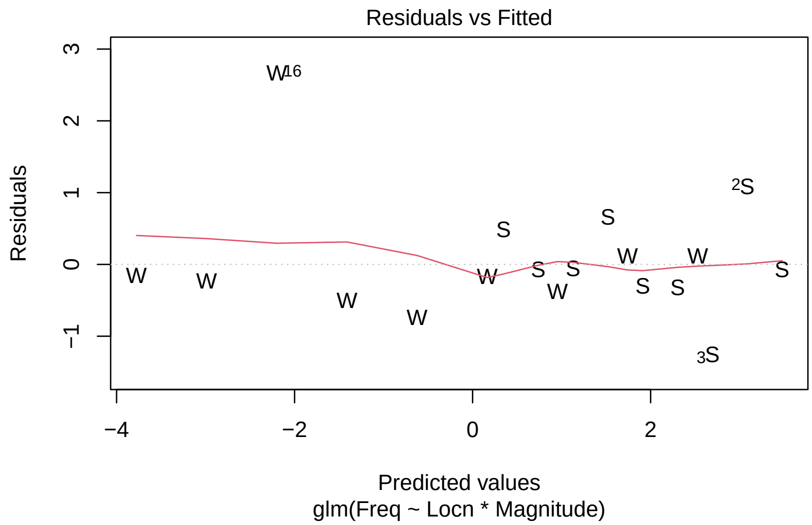
```
plot(Freq ~ Magnitude, data = Quakes.df, pch = substr(Locn, 1, 1))
```



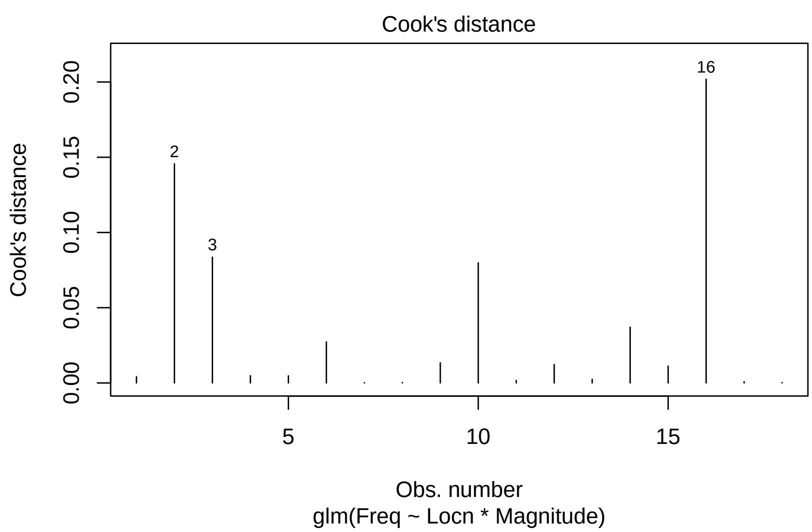

The plot is consistent with an exponential decreasing relationship between frequency and magnitude. It is unclear whether the multiplicative rate of decay is the same in the two locations.

### Model Building and Check Assumptions

```
Quake.gfit = glm(Freq ~ Locn * Magnitude, family = poisson, data = Quakes.df)
plot(Quake.gfit, which = 1, pch = substr(Quakes.df$Locn, 1, 1))
```




```r
plot(Quake.gfit, which = 4)
```




Observation 16 has a largish residual, but as this observation has a very small fitted value of $\mu$ this can be ignored.

```r
summary(Quake.gfit)
```

```
##
## Call:
## glm(formula = Freq ~ Locn * Magnitude, family = poisson, data = Quakes.df)
##
## Deviance Residuals:
##     Min       1Q   Median       3Q      Max
```

```
## -1.3261  -0.3225  -0.1172   0.1241   1.6190
##
## Coefficients:
##                   Estimate Std. Error z value Pr(>|z|)
## (Intercept)        11.6923     1.1762   9.941  < 2e-16 ***
## LocnWA              7.3923     3.9500   1.871   0.0613 .
## Magnitude          -1.5648     0.2055  -7.616 2.61e-14 ***
## LocnWA:Magnitude  -1.5884     0.7199  -2.206   0.0274 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for poisson family taken to be 1)
##
##     Null deviance: 176.1767  on 17  degrees of freedom
## Residual deviance:   8.2295  on 14  degrees of freedom
## AIC: 65.11
##
## Number of Fisher Scoring iterations: 5
```

```
1 - pchisq(8.23, 14)
```

```
## [1] 0.8770025
```

```
exp(confint(Quake.gfit)[3, ]) # Multiplicative annual change in CA
```

```
## Waiting for profiling to be done...
```

```
##     2.5 %    97.5 %
## 0.1374743 0.3082437
```

```
100 * (1 - exp(confint(Quake.gfit)[3, ])) # Percentage annual decrease in CA
```

```
## Waiting for profiling to be done...
```

```
##    2.5 %   97.5 %
## 86.25257 69.17563
```

### Additional Output with `WA` as the baseline Level

```
Quakes.df$Locn = relevel(Quakes.df$Locn, ref = "WA")
Quake2.gfit = glm(Freq ~ Locn * Magnitude, family = poisson, data = Quakes.df)
exp(confint(Quake2.gfit)[3, ]) # Multiplicative annual change in WA
```

```
## Waiting for profiling to be done...
```

```
##       2.5 %      97.5 %
## 0.009077661 0.140175445
```

```r
100 * (1 - exp(confint(Quake2.gfit)[3, ])) # Percentage annual decrease in WA
```

```
## Waiting for profiling to be done...
```

```
##    2.5 %   97.5 %
## 99.09223 85.98246
```

## Methods and Assumption Checks

Since the response variable, earthquake frequency, is a count, we have fitted a generalised linear model with a Poisson response distribution. We have two explanatory variables: Magnitude (Numeric) and Location (Categorical). The scatterplot of magnitude vs frequency shows an exponentially decreasing trend for both locations. A Poisson model with interaction between magnitude and location was fitted. It shows that the interaction between magnitude and location is significant ($P\text{-}value \approx 0.03$).

All model assumptions were satisfied. We can trust the results from this model.

Our final model is

$$\log(\mu_i) = \beta_0 + \beta_1 \times Locn.WA_i + \beta_2 \times Magnitude_i + \beta_3 \times Locn.WA_i \times Magnitude_i,$$

where $\mu_i$ is the mean number of earthquakes with magnitude $i$, and $Locn.Wa_i$ is 1 if the earthquake was in Washington and 0 otherwise. The number of earthquakes (with magnitude $Magnitude_i$ at location $Locn.Wa_i$) has a Poisson distribution with mean $\mu_i$.

## Executive Summary

The research question is to quantify the rate of decrease in earthquake frequency (with increasing magnitude) in both California and Washington states, and to assess whether these rates are the same.

The rate of decline in the frequency of earthquakes (with increasing magnitude) is more rapid in Washington than California.

In Washington, there is a 86.0 to 99.0% drop in the expected number of earthquakes for a one unit increase in their magnitude on the Richter scale.

In California, the decrease is between 69.2 to 86.3%.