input:

- I: (M, N)
- p: (M, N)
- size = 2 * r + 1

output:

- q: (M, N)

$$\bar{I}(i,j) = E[I](i,j) = \frac{\sum\limits_{\substack{\Delta x=-r\to r\\ \Delta y=-r\to r}} I(i+\Delta x, j+\Delta y)}{[2r+1]^2}$$

$$\bar{p}(i,j) = E[p](i,j) = \frac{\sum\limits_{\substack{\Delta x=-r\to r\\ \Delta y=-r\to r}} p(i+\Delta x, j+\Delta y)}{[2r+1]^2}$$

and

$$E[I^2](i,j) = \frac{\sum\limits_{\substack{\Delta x=-r\to r\\ \Delta y=-r\to r}} I^2(i+\Delta x, j+\Delta y)}{[2r+1]^2}$$

$$E[I\cdot p](i,j) = \frac{\sum\limits_{\substack{\Delta x=-r\to r\\ \Delta y=-r\to r}} I\cdot p(i+\Delta x, j+\Delta y)}{[2r+1]^2}$$

then **goal 1**:

$$\sigma^2[I](i,j) = E[I^2](i,j) - E^2[I](i,j)$$
$$\text{cov}[Ip](i,j) = E[Ip](i,j) - E[I](i,j)E[p](i,j)$$

Define

$$a(i,j) \equiv \frac{\text{cov}[Ip](i,j)}{\sigma^2[I](i,j)+\epsilon}$$
$$b(i,j) \equiv E[p](i,j) - \frac{\text{cov}[Ip](i,j)}{\sigma^2[I](i,j)+\epsilon}E[I](i,j)$$

calc

$$\bar{a}(i,j) = E[a](i,j) = \frac{\sum\limits_{\substack{\Delta x=-r\to r\\ \Delta y=-r\to r}} a(i+\Delta x, j+\Delta y)}{[2r+1]^2}$$

$$\bar{b}(i,j) = E[b](i,j) = \frac{\sum\limits_{\substack{\Delta x=-r\to r\\ \Delta y=-r\to r}} b(i+\Delta x, j+\Delta y)}{[2r+1]^2}$$

In the end **goal 2**:

$$q(i,j) = E[a](i,j) \times I(i,j) + E[b](i,j)$$