Geometrical Optics and Optical Ray Tracing using the Python Programming Language

Dewi Gould
CID: 01051830
Word Count: 1790

February 10, 2017

1 Abstract

The aim of this project was to design and write an 3-dimensional optical ray tracer in Python using object-oriented programming. The ray-tracer was tested and verified before being used to investigate the imaging performance of single lenses and finally to optimise the design of a biconvex lens. The model was found to perform as expected for spherical refracting and reflecting surfaces, while also generating the expected results for a planoconvex lens. It was found that the planoconvex configuration with the curved surface facing the input produced rays with significantly less spread around the paraxial focus point than the configuration with the plane surface facing the input. An optimizer was used to find the optimal configuration of a lens with two spherical surfaces. The RMS deviation of rays from their paraxial focal point was calculated in several cases to find whether or not the optical system was diffraction limited.

2 Introduction and Theory

Geometrical optics describes light propagation in terms of rays. This formulation can be mathematically derived from wave optics and the wave equation in the small wavelength limit. An optical ray defines the direction of light propagation and is normal to the wavefront. In a sequential ray tracer, an optical system is modelled by propagating rays using the principles of geometrical optics. The refraction of a ray at the interface between two dielectric media, such as the surface of a lens, is mathematically governed by Snell's Law. In 3-dimensions, Snell's Law is a vector equation relating the incoming ray direction $\hat{\boldsymbol{s}}_{\boldsymbol{1}}$, outgoing ray direction $\hat{\boldsymbol{s}}_{\boldsymbol{2}}$, and surface normal $\hat{\boldsymbol{n}}$[1].

$$\hat{\boldsymbol{s}}_{\boldsymbol{2}} = \frac{n_1}{n_2}(\hat{\boldsymbol{n}} \times (-\hat{\boldsymbol{n}} \times \hat{\boldsymbol{s}}_{\boldsymbol{1}})) - \hat{\boldsymbol{n}}\sqrt{1 - \left(\frac{n_1}{n_2}\right)^2 (\hat{\boldsymbol{n}} \times \hat{\boldsymbol{s}}_{\boldsymbol{1}}) \cdot (\hat{\boldsymbol{n}} \times \hat{\boldsymbol{s}}_{\boldsymbol{1}})} \tag{1}$$

where n_1 and n_2 are the refractive indices of the two media respectively. Propagation then continues to the next surface and so on, until the ray reaches the output of the system.

The reflection of a ray off a reflecting surface is also determined by a vector equation:

$$\hat{\boldsymbol{s_2}} = \hat{\boldsymbol{s_1}} - 2\hat{\boldsymbol{n}}(\hat{\boldsymbol{s_1}} \cdot \hat{\boldsymbol{n}}) \tag{2}$$

where the same vector notation as above for incident and reflected ray directions and the surface normal is used.

The resolution of an optical imaging system can be limited by a number of factors including misalignment and imperfections in the lens. However, there is a fundamental maximum to the resolution of an optical system due to diffraction, called the diffraction limit. For a simple lens with focal length f, this limit is defined as[2]:

$$\text{Diffraction scale} = \frac{\lambda f}{D} \tag{3}$$

where λ is the wavelength of the incoming rays and D is the aperture diameter, or in the case of this project, the diameter of the incident beam. The limit has a theoretical basis, but can also be defined geometrically as the smallest separation at which you could discern two points of light from one another. Assuming no other limiting factors, this limit can be found using the 'Rayleigh Criterion'[3].

Parallel incident rays can be used to represent an incident plane wave, corresponding to light from a point object at infinity. In an ideal imaging system, all such rays would meet at a point in the output focal plane after propagating through an ideal lens. In reality, however, this is not the case because of spherical aberration. This is where rays deviate from the ideal behaviour by focusing at a point longitudinally displaced from the paraxial focus. As the distance of the input ray from the optical axis increases, defined as the line passing through the centre of curvature of a lens and parallel to the axis of symmetry, this effect becomes more pronounced. The paraxial focus is defined as the ideal focus in the limit of a narrow input beam. If a screen is placed at the ideal paraxial focus, the image will appear as a bright spot from the rays in the paraxial region and a symmetrical halo from the marginal rays - those further away from the optical axis. For light not entering parallel to the optical axis there is also a transverse or lateral spherical aberration. The spreading effect of spherical aberration can be easily seen using a spot plot in a plane perpendicular to the ray direction placed at the paraxial focus. The spread of the positions around the paraxial focus is called the geometrical focus, and is calculated using the root-mean-square (RMS) deviation of ray positions from the optical axis at the paraxial focus[4].

Therefore, the validity of any optical ray tracer can be determined by comparing the geometrical focus with the theoretical diffraction limit. It is apparent that if the observed geometrical focus is smaller than the diffraction limit, then the model is not valid for that particular case.

3 Method

An object-oriented approach was used to design the model. Optical rays and optical elements were considered as objects. A 'ray' was represented as a series of points that describe its path, and a series of corresponding directions. In order to propagate the ray through optical elements, such as lenses, methods were written to find both the intercept of the ray with a given surface and the effect of the surface on the ray, such as reflection or refraction.

A model capable of propagating a 'bundle' of rays of a given radius through any number of different spherical lenses was created. The model was tested using several simple spherical and planar lenses. The RMS deviation was calculated by propagating the rays to an output plane at the paraxial focal point, and then calculating the RMS deviation of each point from the optical axis.

The following convention was used throughout this project. '$z0$' is the intercept of the surface with the optical axis, and the curvature 'c' is a signed quantity with magnitude (Radius of Curvature)$^{-1}$. A positive curvature corresponds to the centre of curvature lying at $z > z0$, and vice verse for a negative curvature. A curvature of zero corresponds to a plane surface.

4 Results and Evaluation

Reflection

The ray tracer was tested for both a convex and concave reflecting surface. The model performed as expected (see *Figs.1,2,3*) with convergence in the convex case and divergence in the concave case. The close-up in *Fig.2* shows an expected spherical aberration pattern - a distinct variation of focal point position as the distance of the incident ray from the optical axis increases. By varying the radius of the incident beam, it was found that this spreading becomes larger for large radii.

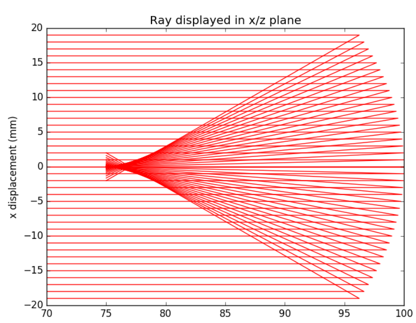


Figure 1: Concave Reflection

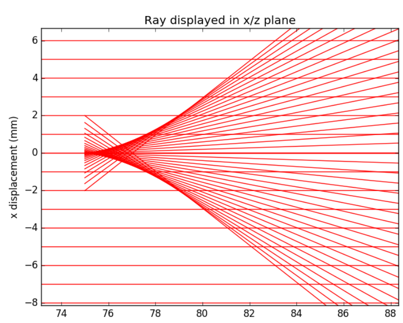


Figure 2: Paraxial focal point in *Fig.1*

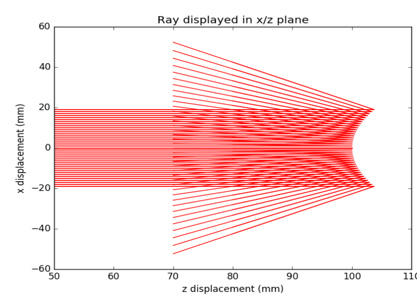


Figure 3: Convex Reflection

Refraction

The ray tracer was also tested with two simple optical systems, comprising of either a convex or concave spherical refracting lens and an output plane (see *Fig.4*). These initial tests generated the expected results, that the rays would converge towards a focus in the convex case and diverge away from the optical axis in the concave case.

One of the most basic type of lens is the planoconvex lens; comprising of two surfaces, one planar and one convex. This was modelled in two separate cases: the first with the convex surface facing the input, and the second with the planar surface facing the input (see *Figs.5,6*). In each case, an estimate of the paraxial focus was found using a ray very close to the optical axis ($0.1mm$). An output plane at this focus was then built for each case and the geometrical spot size was calculated for a range of beam diameters.

(a) Ray Path for simple convex lens, showing expected convergence towards a focal point

(b) Ray Path for simple concave lens, showing expected divergence of rays

Figure 4: Optical rays propagated through a convex and concave lens

Fig.5 shows the spot plot for the planoconvex lens in the first configuration. This spot plot shows that all the rays are focused very near to the paraxial focus. This is further verified upon examination of *Fig.6(a)* which shows the ray spreading along the optical axis to be very minimal. On the other hand, *Fig.6(b)* represents the rays after propagation through the second configuration. This plot shows that the rays focus at a wide range of different points. The conclusion from these results was that the spread in the focal length for the configuration with the plane surface facing the input is significantly larger than that of the other other configuration; a result confirmed by comparison of RMS deviation values for the different cases.

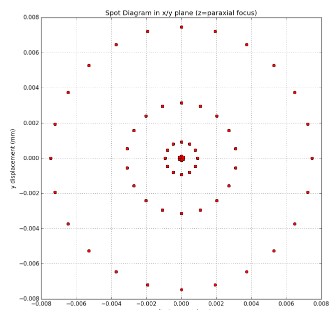
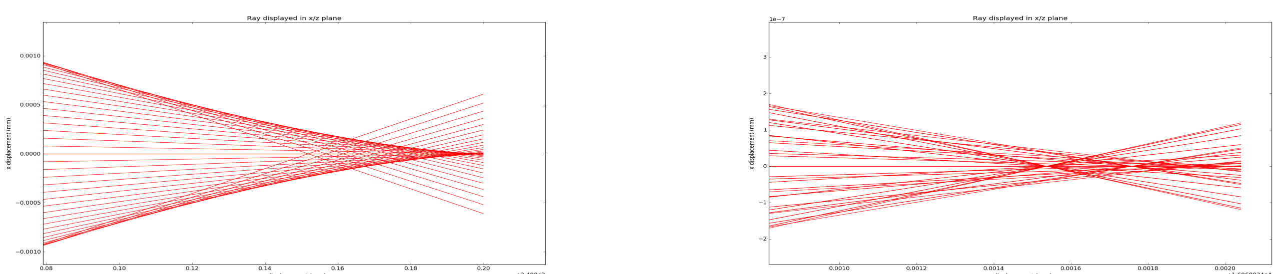


Figure 5: Spot spot for planoconvex lens with curved surface facing input, RMS Deviation= $1.8 \times 10^{-5} m$



(a) Planconvex lens with curved surface facing the input, minimal deviation from paraxial focus

(b) Planoconvex lens with planar surface facing the input, significant deviation from paraxial focus

Figure 6: Ray paths for the two configurations of a simple planoconvex lens

Various RMS deviation values were calculated for the 'correct' configuration of the planoconvex lens, for varying incident ray diameters. These values were then compared to the diffraction limit ($7.2 \mu m$) calculated using a wavelength of $588 nm$. The configurations with RMS less than this limit are said to be 'diffraction limited' the rays are focussed to a point smaller than the theoretical limit.

Spherical Aberration and Lens Optimization

To further test the validity of the ray tracer, a plot of geometrical focus against initial beam radius was generated for a simple planoconvex lens in order to find the 'cut-off' radius at which the model fails to represent reality. For the following example: $\lambda = 588 nm$, $c1 = 0.02/mm$ and $c2 = 0/mm$, where $c1$ and $c2$ are the curvatures of the two surfaces of the lens.

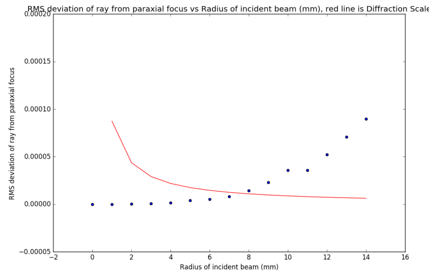


Figure 7: RMS Deviation from Paraxial Focus vs Radius of Incident beam (Planoconvex Lens with curved surface facing input with $0.02/mm$ curvature)

Fig.7 shows that for this particular configuration, incident beams of radius less than $8mm$ converge to a geometrical focus smaller than the calculated diffraction limit, and therefore the model is not valid below this point. Above this point, *Fig.7* demonstrates that the model generates the sensible results that RMS Deviation of the beam increases with incident beam radius.

Although the planoconvex lens performs well when the correct way round, it is not the optimal configuration for a singlet lens. The best form is a lens with two spherical surfaces, with curvatures chosen to optimise the performance of the lens for a given object and image distance[4]. In order to calculate these optimal curvatures the optimizer $fmin$ from *scipy.optimize* was used. A function was written which took two curvatures as parameters and returned an RMS spot radius as an optimization metric.

For two spherical surfaces, separated along the optical axis by $5mm$, the optimal curvatures were found to be $c_1 = 2.627^{-05}/mm$ and $c_2 = -1.8098^{-05}/mm$. From the model's calculations it was observed that for these two values, the RMS deviation $(8 \times 10^{-11} mm)$ was significantly smaller than the diffraction limit $(1.25mm)$. This is represented graphically in *Fig.8*. Hence the system is diffraction limited. Therefore this optimization function is not particularly useful, it minimizes the RMS deviation without constraint. There is also a source of error in this value, given that *Spyder* may not be able to efficiently and correctly handle numbers as small as that calculated for RMS.

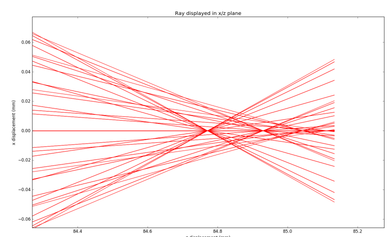


Figure 8: Minimal Spherical Refraction observed for optimized optical system

When comparing *Fig.8* with the similar looking *Fig.6(b)*, it is clear that the optimization has performed its desired function, the effect of spherical aberration has been drastically reduced with the caveat that the system is diffraction limited.

5 Conclusion

The aim of this project was to design, built and test a 3-dimensional optical ray tracer. By testing the model using reflecting and refracting surfaces with known features, it was possible to examine whether or not the model was producing sensible results. The relationship between RMS deviation and the diffraction limit was explored for a number of refracting surfaces in order to examine when the model is valid. The spherical lens optimizer proved useful in the fact that it successfully minimized the RMS deviation of rays from the paraxial focal point, however it did not take the diffraction limit into account and therefore generated an idealised optical system which would not be physically appropriate.

6 Bibliography

[1] http://www.starkeffects.com/snells-law-vector.shtml
[2] Second Year Laboratory and Computing Documentation, (Blackboard)
[3] http://hyperphysics.phy-astr.gsu.edu/hbase/phyopt/diflim.html
[4] https://folk.uio.no/walmann/Publications/Master/node8.html