In[51]:=
```
SetDirectory["/home/dimas/Física/2 Mestrado/Artigos/JCAP bouncing"]
```

Out[51]=
```
/home/dimas/Física/2 Mestrado/Artigos/JCAP bouncing
```

# I - Symmetric bounce

In[190]:=
```
Clear["Global`*"]
```

In[191]:=
```
a = am * e^(h1 * t^2 / 2)
```

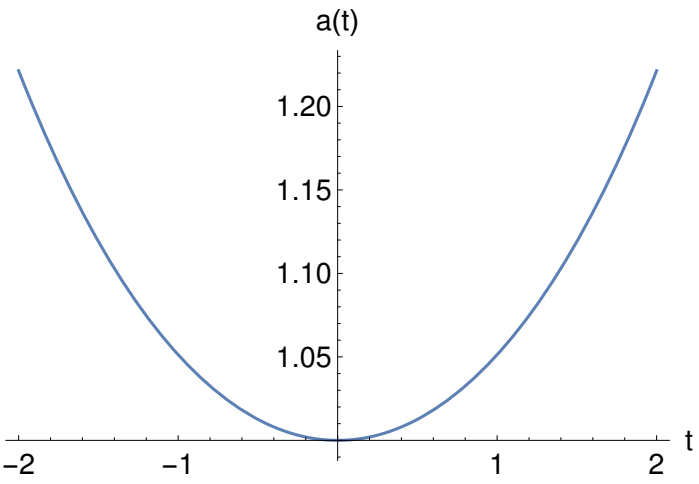
Out[191]=
$$am\, e^{\frac{h1\, t^2}{2}}$$

In[192]:=
```
H = D[a, t] / a
```

Out[192]=
```
h1 t
```

In[193]:=
```
h1 = 0.1; am = 1;
pa = Plot[a, {t, -2, 2}, AxesLabel → {"t", "a(t)"}, LabelStyle → {Black, 15}]
```

Out[193]=




In[194]:=
```
Export["aI.pdf", pa]
```

Out[194]=
```
aI.pdf
```

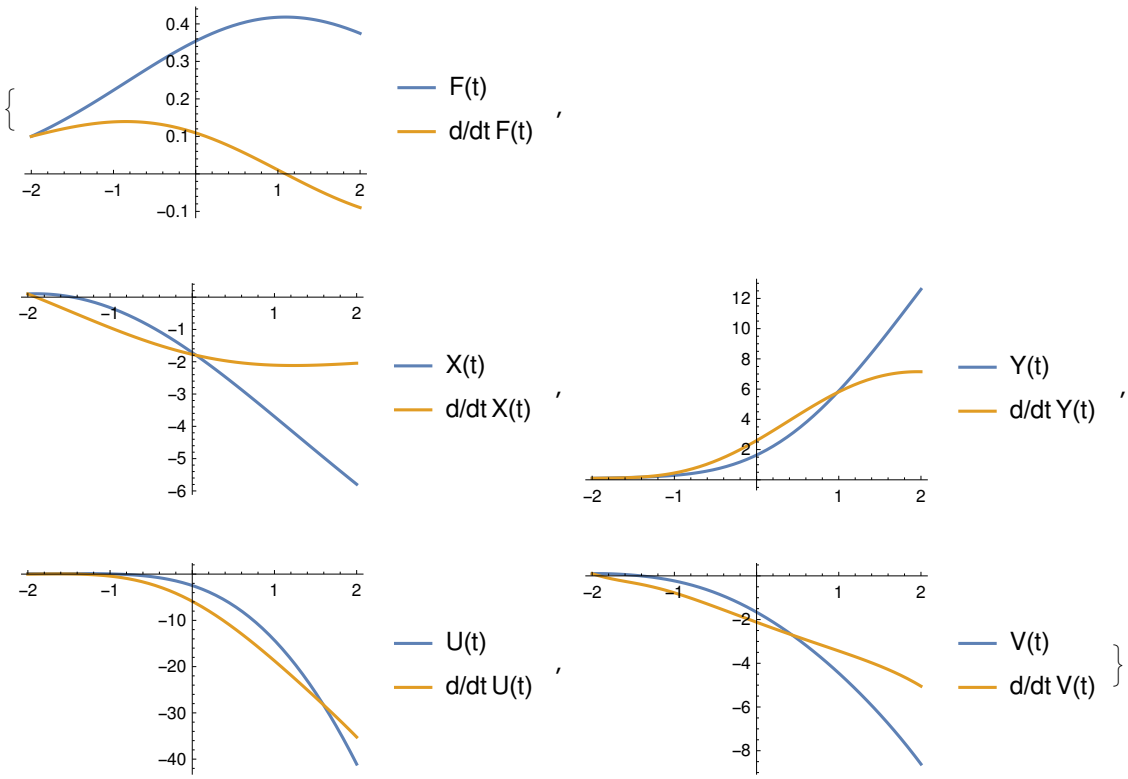
## System of ODE's

In[195]:=
```
simp = Simplify[{2 D[H, t] F[t] + 6 H^2 F[t] + D[F[t], {t, 2}] + 5 H * D[F[t], t] == 0,
    D[X[t], {t, 2}] + 3 H * D[X[t], t] + 6 (D[H, t] + 2 H^2) == 0,
    Y''[t] + 3 H * Y'[t] - D[X[t], t]^2 == 0,
    D[V[t], {t, 2}] + 3 H * D[V[t], t] + 6 (D[H, t] + 2 H^2)
        (D[F[t], t] - D[U[t], t]) / D[Y[t], t] == 0, D[U[t], t] + 2 * V[t] * D[X[t], t] == 0}]
```

Out[195]=
$$\left\{\left(0.2 + 0.06\, t^2\right) F[t] + 0.5\, t\, F'[t] + F''[t] == 0,\right.$$
$$0.6 + 0.12\, t^2 + 0.3\, t\, X'[t] + X''[t] == 0,\ 0.3\, t\, Y'[t] + Y''[t] == X'[t]^2,$$
$$\left.0.3\, t\, V'[t] + \frac{1}{Y'[t]} 6\left(0.1 + 0.02\, t^2\right)\left(F'[t] - U'[t]\right) + V''[t] == 0,\ U'[t] + 2\, V[t]\, X'[t] == 0\right\}$$

In[196]:=
```
Clear[F, X, Y, U, V]; ti = -2; z = 0.1;
sol = NDSolve[{simp, F[ti] == z, F'[ti] == z, X[ti] == z, X'[ti] == z, Y[ti] == z, Y'[ti] == z,
    V[ti] == z, V'[ti] == z, U[ti] == z}, {F[t], X[t], Y[t], U[t], V[t]}, {t, -2, 2}];
L[1] = "F(t)"; L[2] = "X(t)"; L[3] = "Y(t)"; L[4] = "U(t)"; L[5] = "V(t)";
Do[P[i] = sol[[1]][[i]][[2]], {i, 1, 5}]; Do[DP[i] = D[P[i], t], {i, 1, 5}];
Table[Plot[{P[i], DP[i]}, {t, -2, 2}, PlotLegends → {L[i], "d/dt" L[i]}], {i, 1, 5}]
```

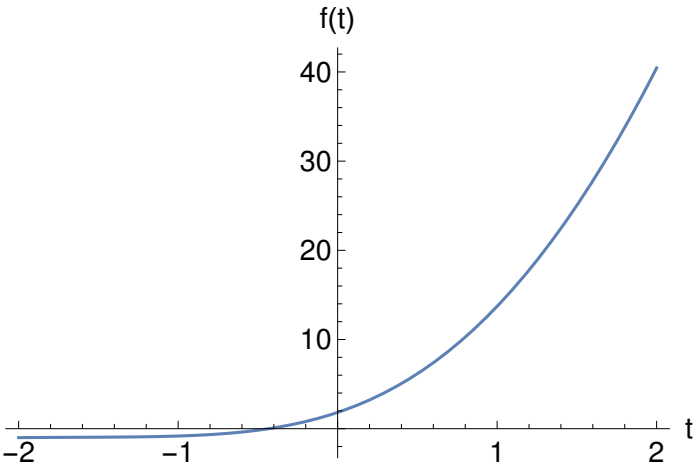
Out[196]=




## Non local distortion function f(Y)

In[197]:=
```
fp = P[1] - P[4] - 1;
pf = Plot[fp, {t, -2, 2}, AxesLabel → {"t", "f(t)"}, LabelStyle → {Black, 15}]
```

Out[197]=

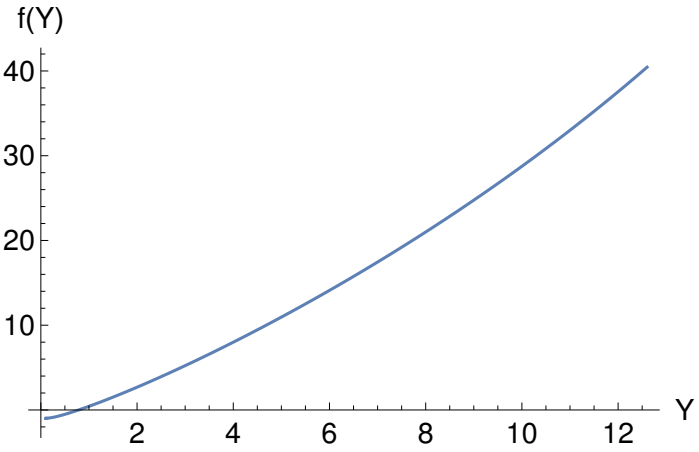


In[198]:=
```
Export["ftI.pdf", pf]
```

Out[198]=
```
ftI.pdf
```

In[199]:=
```
YValues = Table[P[3], {t, -2, 2, 0.005}]; fValues = Table[fp, {t, -2, 2, 0.005}];
data = Table[{YValues[[j]], fValues[[j]]}, {j, 1, 801}];
line = ListLinePlot[data, AxesLabel → {"Y", "f(Y)"}, LabelStyle → {Black, 15}]
```

Out[199]=




In[200]:=
```
Export["fYI.pdf", line]
```

Out[200]=
```
fYI.pdf
```

# II - Oscillatory Bounce

In[179]:=
```
Clear["Global`*"]
```

In[180]:=
```
a = A0 * Sin[k * t]^2 + c
```

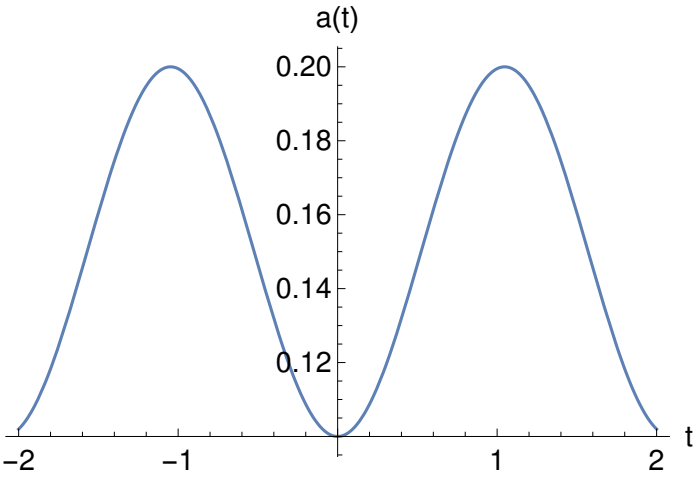
Out[180]=
$$c + A0\ \text{Sin}[k\ t]^2$$

In[181]:=
```
H = D[a, t] / a
```

Out[181]=
$$\frac{2\ A0\ k\ \text{Cos}[k\ t]\ \text{Sin}[k\ t]}{c + A0\ \text{Sin}[k\ t]^2}$$

In[182]:=
```
A0 = 1 / 10; c = 0.1; k = 3 / 2;
pa = Plot[a, {t, -2, 2}, AxesLabel → {"t", "a(t)"}, LabelStyle → {Black, 15}]
```

Out[182]=




In[183]:=
```
Export["aII.pdf", pa]
```

Out[183]=
```
aII.pdf
```

## System of ODE's

In[184]:=
```
simp = Simplify[{2 D[H, t] F[t] + 6 H^2 F[t] + D[F[t], {t, 2}] + 5 H * D[F[t], t] == 0,
    D[X[t], {t, 2}] + 3 H * D[X[t], t] + 6 (D[H, t] + 2 H^2) == 0,
    Y''[t] + 3 H * Y'[t] - D[X[t], t]^2 == 0,
    D[V[t], {t, 2}] + 3 H * D[V[t], t] + 6 (D[H, t] + 2 H^2)
      (D[F[t], t] - D[U[t], t]) / D[Y[t], t] == 0, D[U[t], t] + 2 * V[t] * D[X[t], t] == 0}]
```

Out[184]=
$$\left\{\left((2.25 + 13.5\,\text{Cos}[3\,t] - 6.75\,\text{Cos}[6\,t])\,F[t] + \left(11.25\,\text{Sin}[3\,t] - 1.875\,\text{Sin}[6\,t]\right)F'[t] + (2.375 - 1.5\,\text{Cos}[3\,t] + 0.125\,\text{Cos}[6\,t])\,F''[t]\right) \Big/ \left(1. + \text{Sin}\left[\frac{3\,t}{2}\right]^2\right)^2 == 0,\right.$$

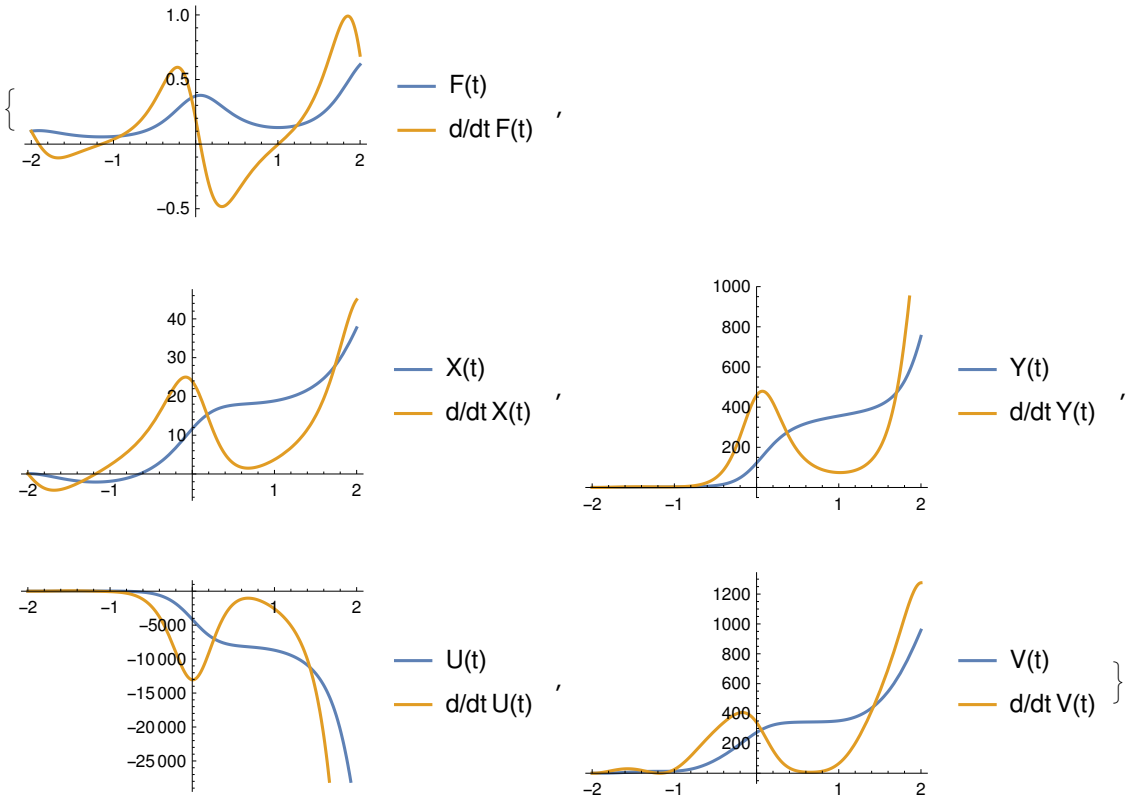
$$\left(40.5\,\text{Cos}[3\,t] - 13.5\,\text{Cos}[6\,t] + \left(6.75\,\text{Sin}[3\,t] - 1.125\,\text{Sin}[6\,t]\right)X'[t] + (2.375 - 1.5\,\text{Cos}[3\,t] + 0.125\,\text{Cos}[6\,t])\,X''[t]\right) \Big/ \left(1. + \text{Sin}\left[\frac{3\,t}{2}\right]^2\right)^2 == 0,$$

$$\frac{4.5\,\text{Sin}[3\,t]\,Y'[t]}{1. + \text{Sin}\left[\frac{3\,t}{2}\right]^2} + Y''[t] == X'[t]^2,\ \frac{1}{\left(1. + \text{Sin}\left[\frac{3\,t}{2}\right]^2\right)^2 Y'[t]}$$

$$\left((40.5\,\text{Cos}[3\,t] - 13.5\,\text{Cos}[6\,t])\,F'[t] + (-40.5\,\text{Cos}[3\,t] + 13.5\,\text{Cos}[6\,t])\,U'[t] + Y'[t]\left(\left(6.75\,\text{Sin}[3\,t] - 1.125\,\text{Sin}[6\,t]\right)V'[t] + (2.375 - 1.5\,\text{Cos}[3\,t] + 0.125\,\text{Cos}[6\,t])\,V''[t]\right)\right) == 0,\ U'[t] + 2\,V[t]\,X'[t] == 0\Big\}$$

In[185]:=
```
Clear[F, X, Y, U, V]; ti = -2; z = 0.1;
sol = NDSolve[{simp, F[ti] == z, F'[ti] == z, X[ti] == z, X'[ti] == z, Y[ti] == z, Y'[ti] == z,
    U[ti] == z, V[ti] == z, V'[ti] == z}, {F[t], X[t], Y[t], U[t], V[t]}, {t, -2, 2}];
L[1] = "F(t)"; L[2] = "X(t)"; L[3] = "Y(t)"; L[4] = "U(t)"; L[5] = "V(t)";
Do[P[i] = sol[[1]][[i]][[2]], {i, 1, 5}]; Do[DP[i] = D[P[i], t], {i, 1, 5}];
Table[Plot[{P[i], DP[i]}, {t, -2, 2}, PlotLegends → {L[i], "d/dt" L[i]}], {i, 1, 5}]
```

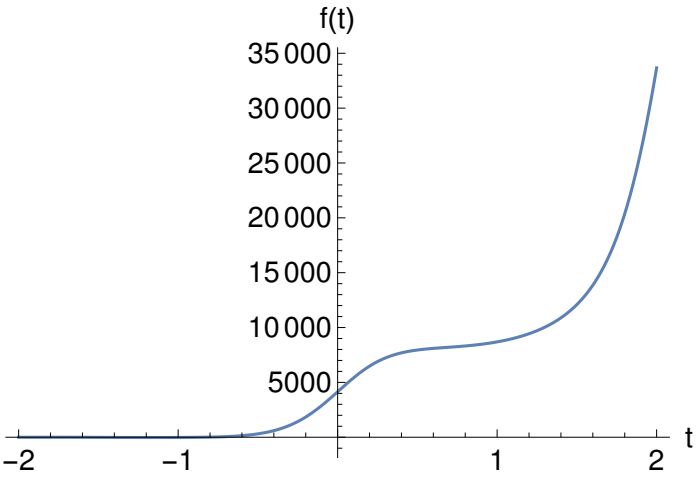
Out[185]=



## Non local distortion function f(Y)

In[186]:=
```
fp = P[1] - P[4] - 1;
pf = Plot[fp, {t, -2, 2}, AxesLabel → {"t", "f(t)"}, LabelStyle → {Black, 15}]
```

Out[186]=

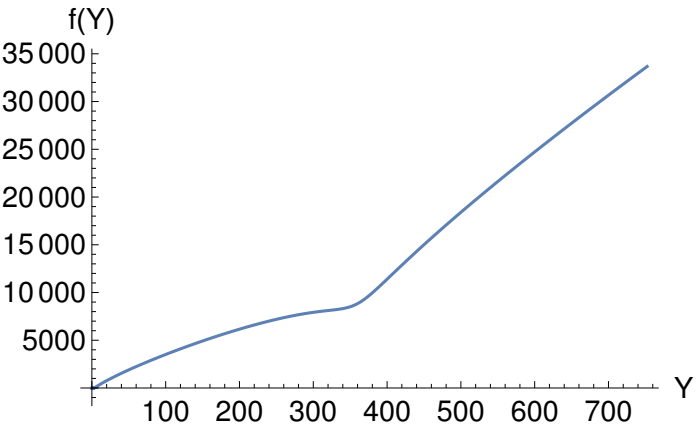



In[187]:=
```
Export["ftII.pdf", pf]
```

Out[187]= ftII.pdf

In[188]:=
```
YValues = Table[P[3], {t, -2, 2, 0.005}]; fValues = Table[fp, {t, -2, 2, 0.005}];
data = Table[{YValues[[j]], fValues[[j]]}, {j, 1, 801}];
line = ListLinePlot[data, AxesLabel → {"Y", "f(Y)"}, LabelStyle → {Black, 15}]
```

Out[188]=




In[189]:=
```
Export["fYII.pdf", line]
```

Out[189]= fYII.pdf

# III - Matter bounce

In[167]:=
```
Clear["Global`*"]
```

In[168]:=
```
a = A0 * (3 / 2 ρ * t^2 + 1) ^ (1 / 3)
```

Out[168]=
$$A0 \left(1 + \frac{3\, t^2\, \rho}{2}\right)^{1/3}$$

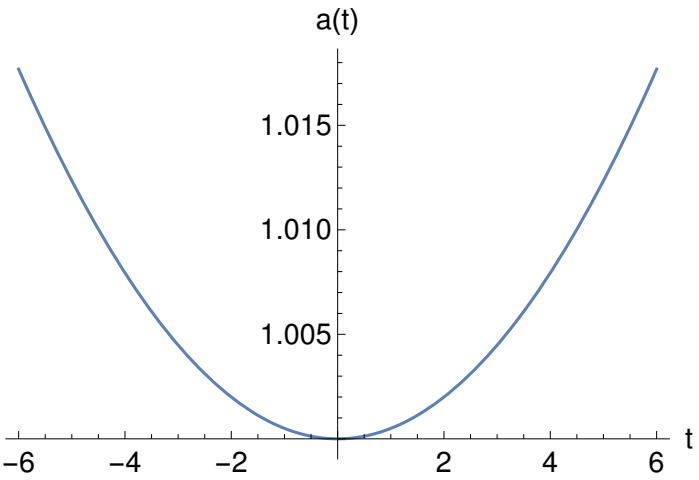
In[169]:=
```
H = D[a, t] / a
```

Out[169]=
$$\frac{t\, \rho}{1 + \frac{3\, t^2\, \rho}{2}}$$

$0 < \rho << 1$ is a critical density from LQC

In[170]:=
```
A0 = 1; ρ = 10^(-3);
PaI = Plot[a, {t, -6, 6}, AxesLabel → {"t", "a(t)"}, LabelStyle → {Black, 15}]
```

Out[170]=




In[171]:=
```
Export["aIII.pdf", PaI]
```

Out[171]=
```
aIII.pdf
```

## System of ODE's

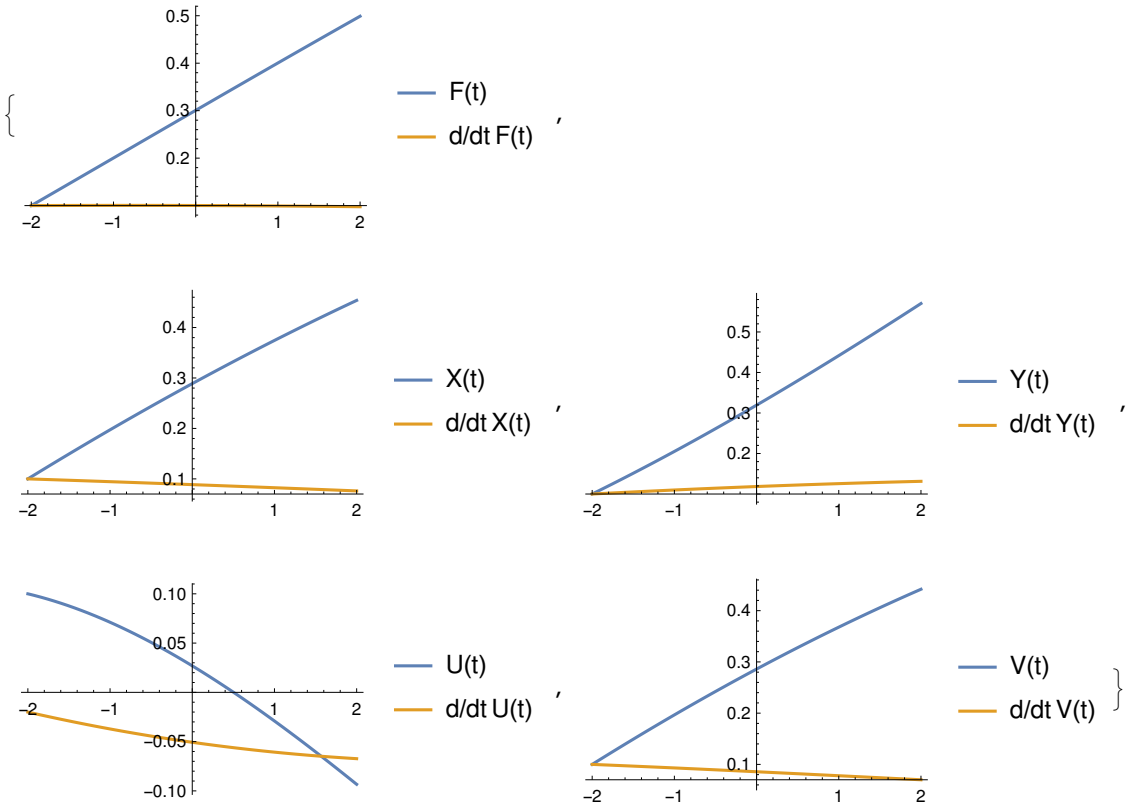
In[172]:=
```
Clear[F, X, U, V, Y]
```

In[173]:=
```
simp = Simplify[{2 D[H, t] F[t] + 6 H^2 F[t] + D[F[t], {t, 2}] + 5 H * D[F[t], t] == 0,
    D[X[t], {t, 2}] + 3 H * D[X[t], t] + 6 (D[H, t] + 2 H^2) == 0,
    Y''[t] + 3 H * Y'[t] - D[X[t], t]^2 == 0,
    D[V[t], {t, 2}] + 3 H * D[V[t], t] + 6 (D[H, t] + 2 H^2)
        (D[F[t], t] - D[U[t], t]) / D[Y[t], t] == 0, D[U[t], t] + 2 * V[t] * D[X[t], t] == 0}]
```

Out[173]=
$$\left\{\left(4\, F[t] + 10\, t\, F'[t] + \left(2000 + 3\, t^2\right) F''[t]\right) \Big/ \left(2000 + 3\, t^2\right) == 0,\right.$$
$$\left(12 \left(2000 + t^2\right) + 6\, t \left(2000 + 3\, t^2\right) X'[t] + \left(2000 + 3\, t^2\right)^2 X''[t]\right) \Big/ \left(2000 + 3\, t^2\right) == 0,$$
$$\frac{6\, t\, Y'[t]}{2000 + 3\, t^2} + Y''[t] == X'[t]^2,\ \left(12 \left(2000 + t^2\right) F'[t] - 12 \left(2000 + t^2\right) U'[t] + \left(2000 + 3\, t^2\right) Y'[t]\right.$$
$$\left.\left.\left(6\, t\, V'[t] + \left(2000 + 3\, t^2\right) V''[t]\right)\right) \Big/ \left(\left(2000 + 3\, t^2\right) Y'[t]\right) == 0,\ U'[t] + 2\, V[t]\, X'[t] == 0\right\}$$

In[174]:=
```
ti = -2; z = 0.1; Clear[F, X, Y, U, V];
sol = NDSolve[{simp, F[ti] == z, F'[ti] == z, X[ti] == z, X'[ti] == z, Y[ti] == z, Y'[ti] == z,
    V[ti] == z, V'[ti] == z, U[ti] == z}, {F[t], X[t], Y[t], U[t], V[t]}, {t, -2, 2}];
L[1] = "F(t)"; L[2] = "X(t)"; L[3] = "Y(t)"; L[4] = "U(t)"; L[5] = "V(t)";
Do[P[i] = sol[[1]][[i]][[2]], {i, 1, 5}]; Do[DP[i] = D[P[i], t], {i, 1, 5}];
Table[Plot[{P[i], DP[i]}, {t, -2, 2}, PlotLegends → {L[i], "d/dt" L[i]}], {i, 1, 5}]
```

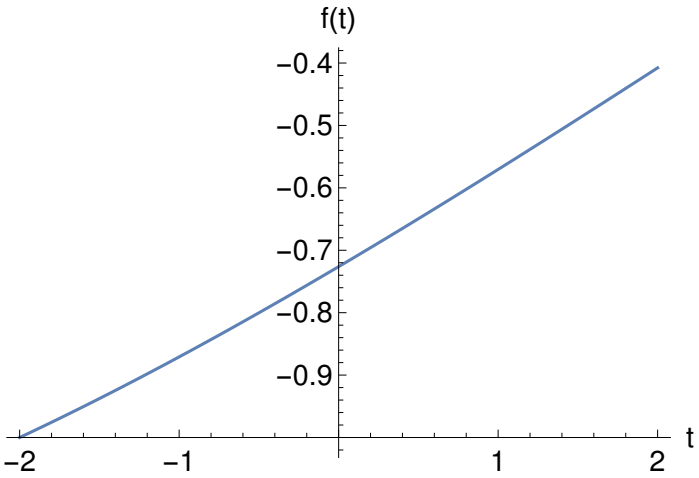
Out[174]=



## Non local distortion function f(Y)

In[175]:=
```
fp = P[1] - P[4] - 1;
pf = Plot[fp, {t, -2, 2}, AxesLabel → {"t", "f(t)"}, LabelStyle → {Black, 15}]
```

Out[175]=

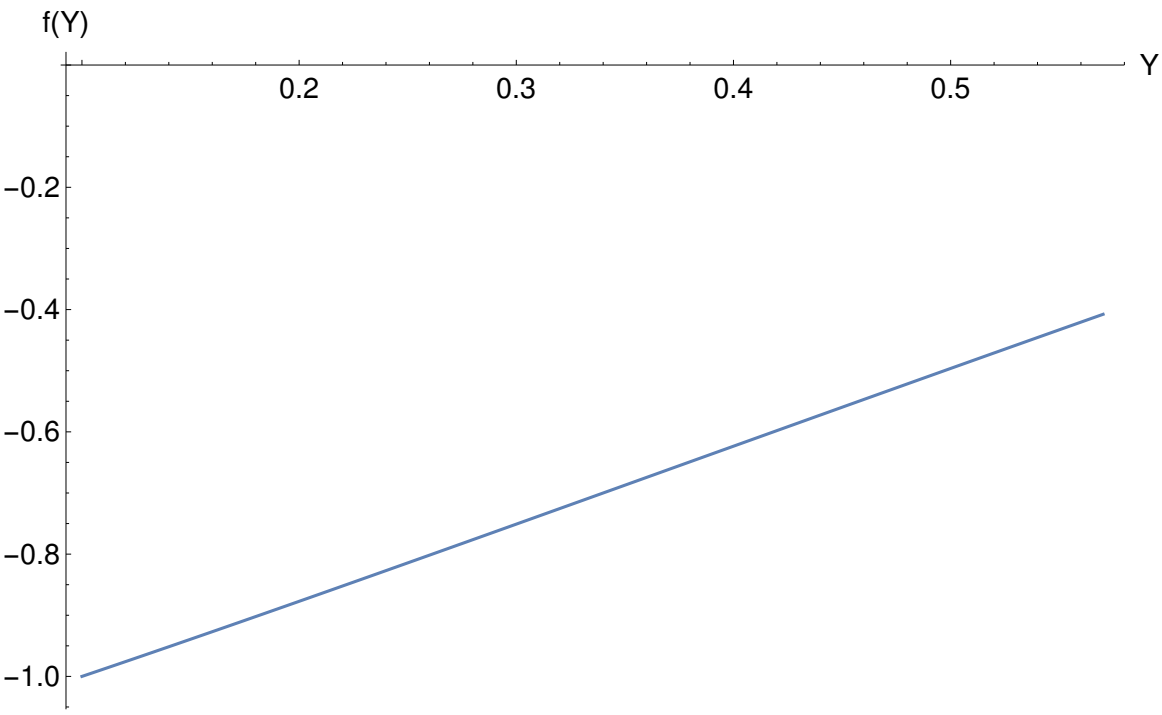



In[176]:=
```
Export["ftIII.pdf", pf]
```

Out[176]=
```
ftIII.pdf
```

In[177]:=
```
YValues = Table[P[3], {t, -2, 2, 0.005}]; fValues = Table[fp, {t, -2, 2, 0.005}];
data = Table[{YValues[[j]], fValues[[j]]}, {j, 1, 801}];
line = ListLinePlot[data, AxesLabel → {"Y", "f(Y)"}, LabelStyle → {Black, 15}]
```

Out[177]=



In[178]:=
```
Export["fYIII.pdf", line]
```
Out[178]= fYIII.pdf

# IV - Singularities cosmologies

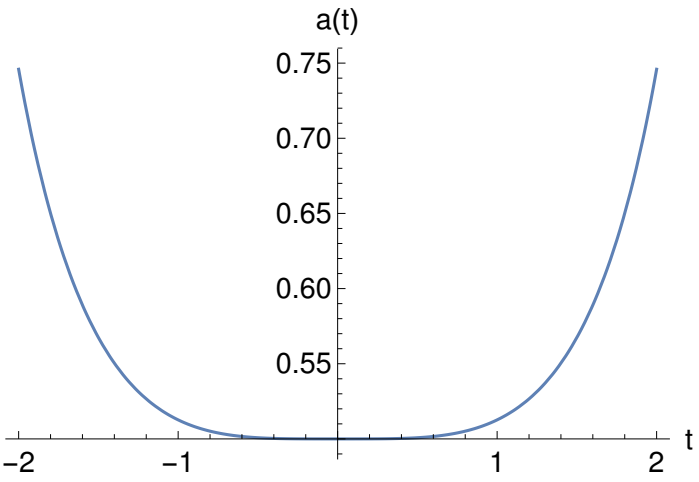
In[85]:=
```
Clear["Global`*"]
```

In[86]:=
```
a = A0 * e^(f0 / (α + 1) * (t - ts)^(α + 1))
```
Out[86]= $A0\, e^{\frac{f0\,(t-ts)^{1+\alpha}}{1+\alpha}}$

In[87]:=
```
H = D[a, t] / a
```
Out[87]= $f0\,(t-ts)^{\alpha}$

## $\alpha > 1$

In[88]:=
```
A0 = 1 / 2; f0 = 1 / 10; α = 3; ts = 0;
pa2 = Plot[{a}, {t, -2, 2}, AxesLabel → {"t", "a(t)"}, LabelStyle → {Black, 15}]
```
Out[88]=




In[89]:=
```
Export["aIV.pdf", pa2]
```
Out[89]= aIV.pdf

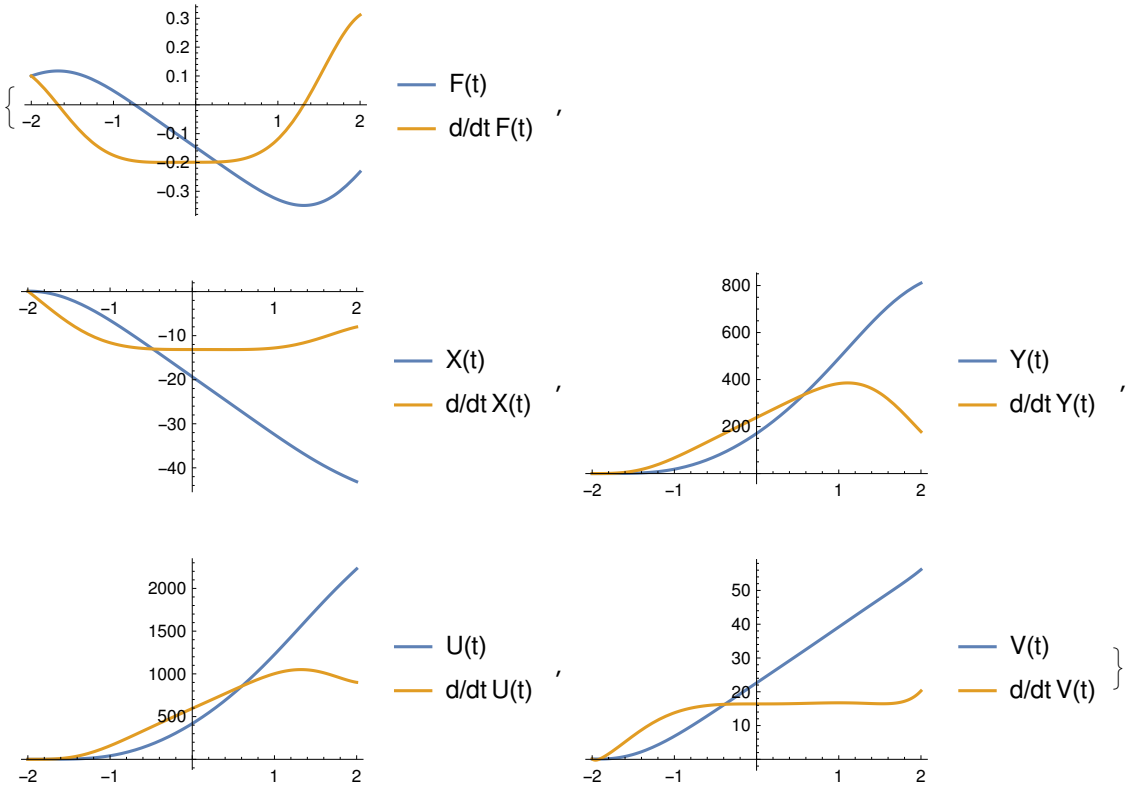
## System of ODE's

In[90]:=
```
simp = Simplify[{2 D[H, t] F[t] + 6 H^2 F[t] + D[F[t], {t, 2}] + 5 H * D[F[t], t] == 0,
    D[X[t], {t, 2}] + 3 H * D[X[t], t] + 6 (D[H, t] + 2 H^2) == 0,
    Y''[t] + 3 H * Y'[t] - D[X[t], t]^2 == 0,
    D[V[t], {t, 2}] + 3 H * D[V[t], t] + 6 (D[H, t] + 2 H^2)
      (D[F[t], t] - D[U[t], t]) / D[Y[t], t] == 0, D[U[t], t] + 2 * V[t] * D[X[t], t] == 0}]
```

Out[90]=
$$\left\{3\,t^2\left(10+t^4\right)F[t]+25\left(t^3F'[t]+2F''[t]\right)==0,\ 90\,t^2+6\,t^6+15\,t^3X'[t]+50X''[t]==0,\ \frac{3}{10}t^3Y'[t]+Y''[t]==X'[t]^2,\ \frac{3}{10}t^3V'[t]+\frac{3\,t^2\left(15+t^4\right)\left(F'[t]-U'[t]\right)}{25\,Y'[t]}+V''[t]==0,\ U'[t]+2\,V[t]\,X'[t]==0\right\}$$

In[91]:=
```
ti = -2; z = 0.1; cc = {F[ti] == z, X[ti] == z, Y[ti] == z, F'[ti] == z, X'[ti] == z,
   Y'[ti] == z, V[ti] == z, V'[ti] == z, U[ti] == z}; Clear[F, X, Y, U, V];
sol = NDSolveValue[{simp, cc}, {F[t], X[t], Y[t], U[t], V[t]}, {t, -2, 2}];
L[1] = "F(t)"; L[2] = "X(t)"; L[3] = "Y(t)"; L[4] = "U(t)"; L[5] = "V(t)";
Do[P[i] = sol[[i]], {i, 1, 5}]; Do[DP[i] = D[P[i], t], {i, 1, 5}];
Table[Plot[{P[i], DP[i]}, {t, -2, 2}, PlotLegends → {L[i], "d/dt" L[i]}], {i, 1, 5}]
```

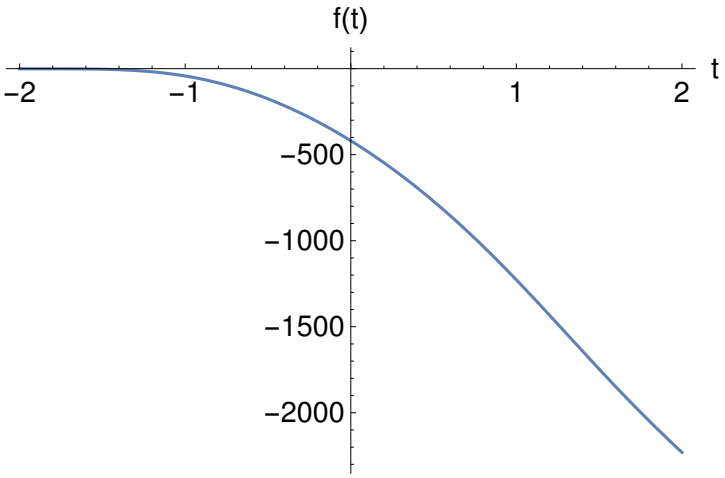
Out[91]=



## Non local distortion function f(Y)

In[92]:=
```
fp = P[1] - P[4] - 1;
pf = Plot[fp, {t, -2, 2}, AxesLabel → {"t", "f(t)"}, LabelStyle → {Black, 15}]
```

Out[92]=

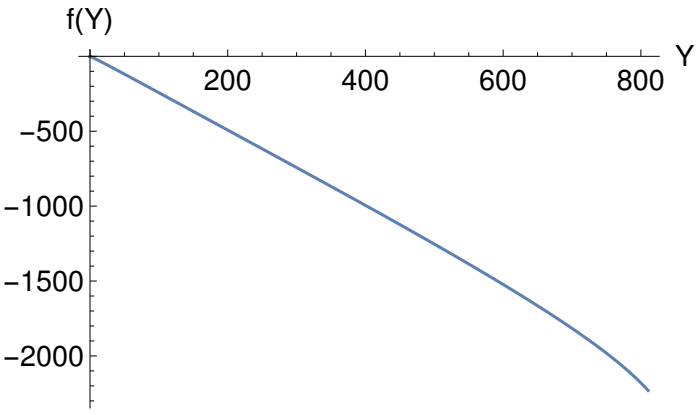



In[93]:=
```
Export["ftIV.pdf", pf]
```

Out[93]= ftIV.pdf

In[94]:=
```
YValues = Table[P[3], {t, -2, 2, 0.005}]; fValues = Table[fp, {t, -2, 2, 0.005}];
data = Table[{YValues[[j]], fValues[[j]]}, {j, 1, 801}];
line = ListLinePlot[data, AxesLabel → {"Y", "f(Y)"}, LabelStyle → {Black, 15}]
```

Out[94]=




In[95]:=
```
Export["fYIV.pdf", line]
```

Out[95]= fYIV.pdf

# V - Pre-inflationary asymmetric bounce

In[343]:=
```
Clear["Global`*"]
```

In[344]:=
```
a = a0 * e^(-Hb^3 * t^3 - Hi^2 t^2 + H0 * t)
```

Out[344]=
$$a0\ e^{H0\ t-Hi^2\ t^2-Hb^3\ t^3}$$

In[345]:=
```
H = D[a, t] / a
```

Out[345]=
$$H0 - 2\ Hi^2\ t - 3\ Hb^3\ t^2$$

In[346]:=
```
Clear[H0, Hb, Hi]
```

In[347]:=
```
Solve[D[a, t] == 0, t]
```

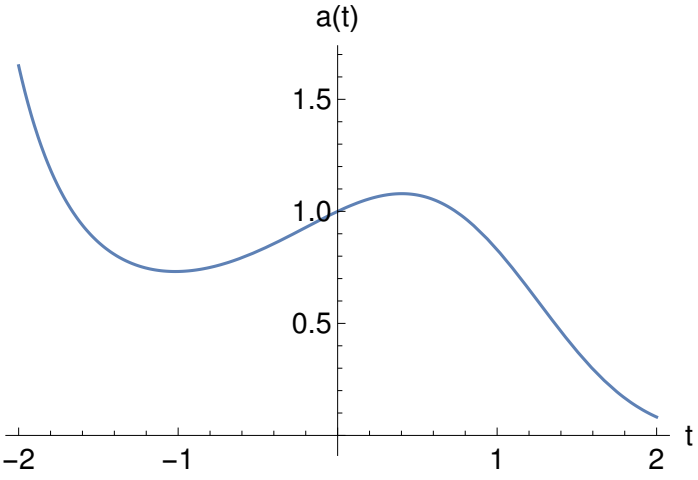
Out[347]=
$$\left\{\left\{t \to \frac{-Hi^2 - \sqrt{3\ H0\ Hb^3 + Hi^4}}{3\ Hb^3}\right\}, \left\{t \to \frac{-Hi^2 + \sqrt{3\ H0\ Hb^3 + Hi^4}}{3\ Hb^3}\right\}\right\}$$

In[348]:=
```
Solve[3 H0 Hb^3 + Hi^4 == 0, H0]
```

Out[348]=
$$\left\{\left\{H0 \to -\frac{Hi^4}{3\ Hb^3}\right\}\right\}$$

In[349]:=
```
H0 = 1 / 3; Hi = 1 / 2; Hb = 11 / 17; a0 = 1;
pa = Plot[a, {t, -2, 2}, AxesLabel → {"t", "a(t)"}, LabelStyle → {Black, 15}]
```

Out[349]=




In[350]:=
```
Export["aV.pdf", pa]
```

Out[350]=
```
aV.pdf
```

## System of ODE's

In[351]:=
```
simp = Simplify[{2 D[H, t] F[t] + 6 H^2 F[t] + D[F[t], {t, 2}] + 5 H * D[F[t], t] == 0,
    D[X[t], {t, 2}] + 3 H * D[X[t], t] + 6 (D[H, t] + 2 H^2) == 0,
    Y''[t] + 3 H * Y'[t] - D[X[t], t]^2 == 0,
    D[V[t], {t, 2}] + 3 H * D[V[t], t] + 6 (D[H, t] + 2 H^2)
        (D[F[t], t] - D[U[t], t]) / D[Y[t], t] == 0, D[U[t], t] + 2 * V[t] * D[X[t], t] == 0}]
```

Out[351]=
$$\left\{2\left(-\frac{1}{2}-\frac{7986\,t}{4913}\right)F[t]+6\left(-\frac{1}{3}+\frac{t}{2}+\frac{3993\,t^2}{4913}\right)^2F[t]+5\left(\frac{1}{3}-\frac{t}{2}-\frac{3993\,t^2}{4913}\right)F'[t]+F''[t]==0,\right.$$

$$6\left(-\frac{1}{2}-\frac{7986\,t}{4913}+2\left(-\frac{1}{3}+\frac{t}{2}+\frac{3993\,t^2}{4913}\right)^2\right)+3\left(\frac{1}{3}-\frac{t}{2}-\frac{3993\,t^2}{4913}\right)X'[t]+X''[t]==0,$$

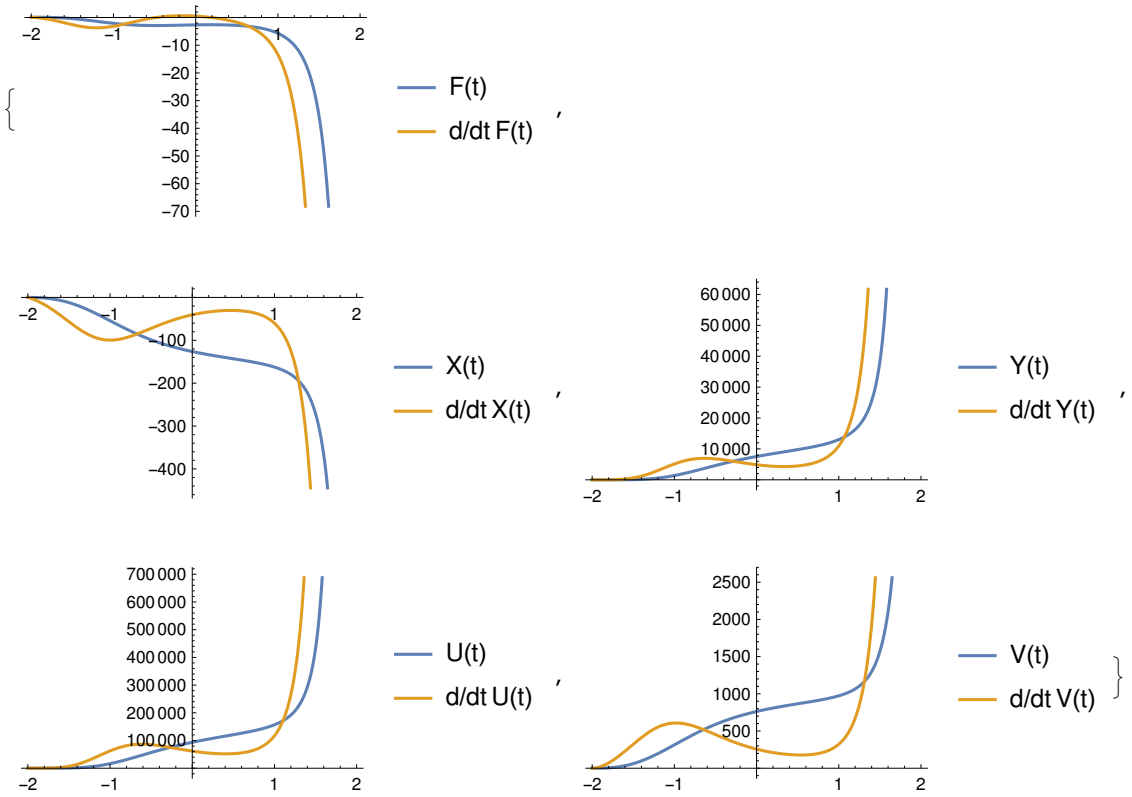
$$\left(1-\frac{3\,t}{2}-\frac{11\,979\,t^2}{4913}\right)Y'[t]+Y''[t]==X'[t]^2,$$

$$3\left(\frac{1}{3}-\frac{t}{2}-\frac{3993\,t^2}{4913}\right)V'[t]+\frac{1}{Y'[t]}6\left(-\frac{1}{2}-\frac{7986\,t}{4913}+2\left(-\frac{1}{3}+\frac{t}{2}+\frac{3993\,t^2}{4913}\right)^2\right)(F'[t]-U'[t])+$$

$$\left.V''[t]==0,\ U'[t]+2\,V[t]\,X'[t]==0\right\}$$

In[352]:=
```
Clear[F, X, Y, U, V]; ti = -2; z = 0.1;
sol = NDSolve[{simp, F[ti] == z, F'[ti] == z, X[ti] == z, X'[ti] == z, Y[ti] == z, Y'[ti] == z,
    V[ti] == z, V'[ti] == z, U[ti] == z}, {F[t], X[t], Y[t], U[t], V[t]}, {t, -2, 2}];
L[1] = "F(t)"; L[2] = "X(t)"; L[3] = "Y(t)"; L[4] = "U(t)"; L[5] = "V(t)";
Do[P[i] = sol[[1]][[i]][[2]], {i, 1, 5}]; Do[DP[i] = D[P[i], t], {i, 1, 5}];
Table[Plot[{P[i], DP[i]}, {t, -2, 2}, PlotLegends → {L[i], "d/dt" L[i]}], {i, 1, 5}]
```

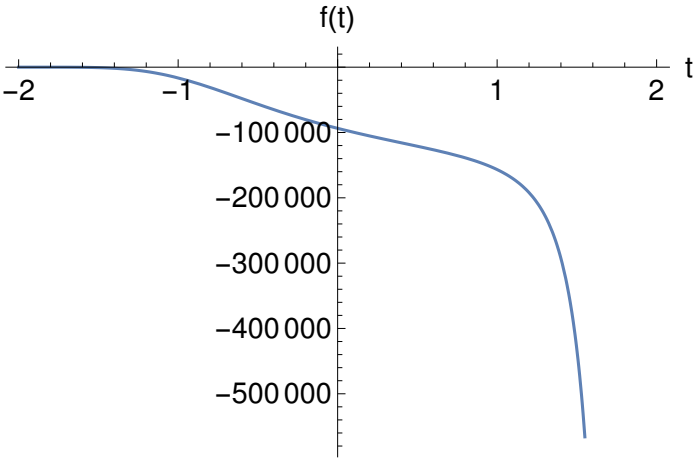
Out[352]=




## Non local distortion function f(Y)

In[353]:=
```
fp = P[1] - P[4] - 1;
pf = Plot[fp, {t, -2, 2}, AxesLabel → {"t", "f(t)"}, LabelStyle → {Black, 15}]
```

Out[353]=

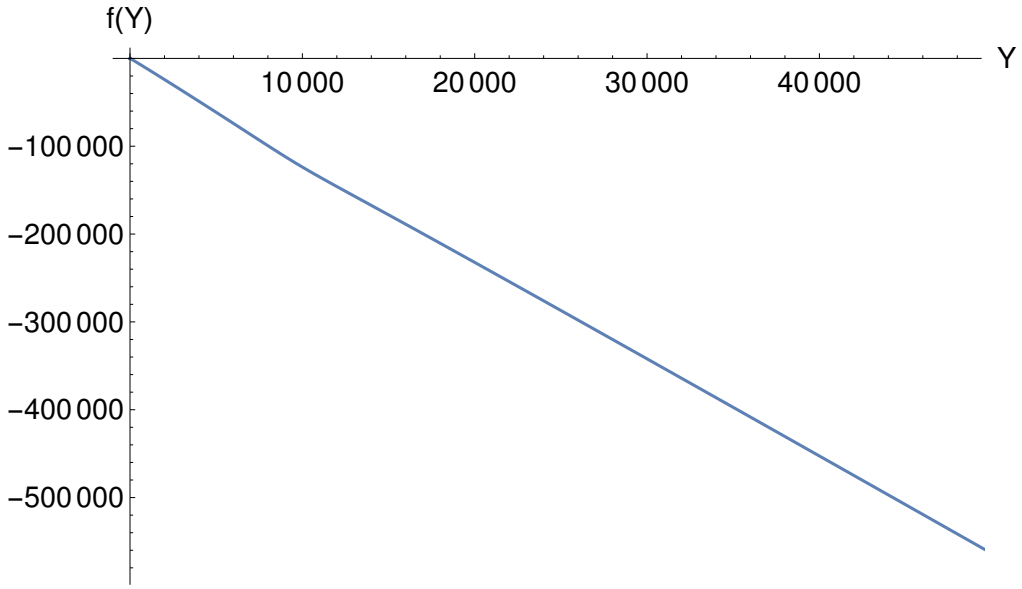


In[354]:=
```
Export["ftV.pdf", pf]
```

Out[354]=
```
ftV.pdf
```

In[355]:=
```
YValues = Table[P[3], {t, -2, 2, 0.005}]; fValues = Table[fp, {t, -2, 2, 0.005}];
data = Table[{YValues[[j]], fValues[[j]]}, {j, 1, 801}];
line = ListLinePlot[data, AxesLabel → {"Y", "f(Y)"}, LabelStyle → {Black, 15}]
```

Out[355]=




In[356]:=
```
Export["fYV.pdf", line]
```

Out[356]=
```
fYV.pdf
```