FLRW and Perturbative Solutions

From Plank (2018)

```
ΩΛ = 0.6847; Ωr = 9.265 * 10 ^ (-5); Ωm = 0.3153; H0 = N[67.4 / 3.09 * 10 ^ (-19)];
```

```
H = H0 * √(Ωr * a[n] ^ (-4) + Ωm * a[n] ^ (-3) + ΩΛ)
```

$$2.18123\times 10^{-18}\sqrt{0.6847+\frac{0.00009265}{a[n]^4}+\frac{0.3153}{a[n]^3}}$$

```
HN = H /. {a[n] -> e^(n)}
```

$$2.18123\times 10^{-18}\sqrt{0.6847+0.00009265\,e^{-4n}+0.3153\,e^{-3n}}$$

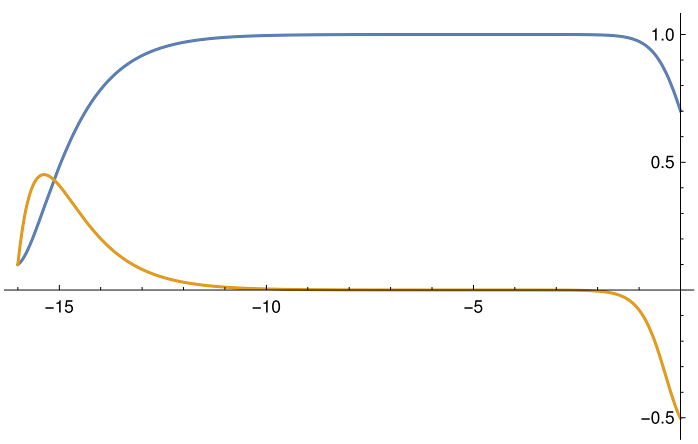
```
ξ = Simplify[D[HN, n] / HN]
```

$$\frac{-0.000270629-0.69074\,e^{n}}{0.000135315+0.460494\,e^{n}+1.\,e^{4n}}$$

Differential equation for F(N)=1+U+f

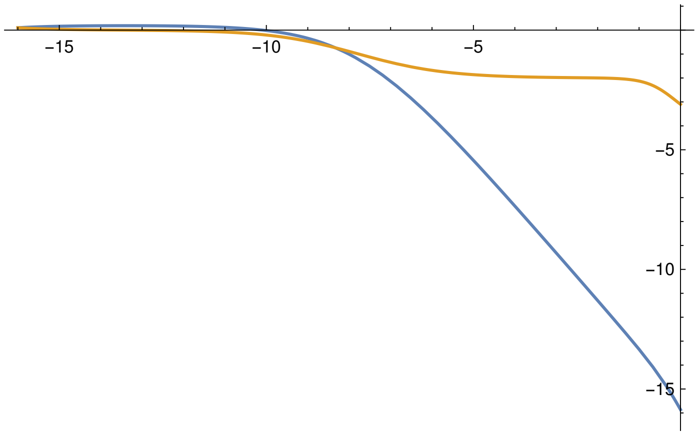
Numerical solution using initial conditions: F(Nini) = 0, where Nini=16 (radiation era)

```
Nin = -16; Clear[F];
Fs = NDSolve[{D[F[n], {n, 2}] + (ξ + 5) D[F[n], n] + (6 + 2 ξ) F[n] == 3 H0^2
      (2 / 3 * Ωr * e^(-4 n) + Ωm * e^(-3 n)) / HN^2, F[Nin] == 0.1, F'[Nin] == 0.1},
   F[n], {n, Nin, 0}]; F = Evaluate[F[n] /. Fs[[1]][[1]]];
DF = D[F, n]; Plot[{F, DF}, {n, 0, Nin}]
```



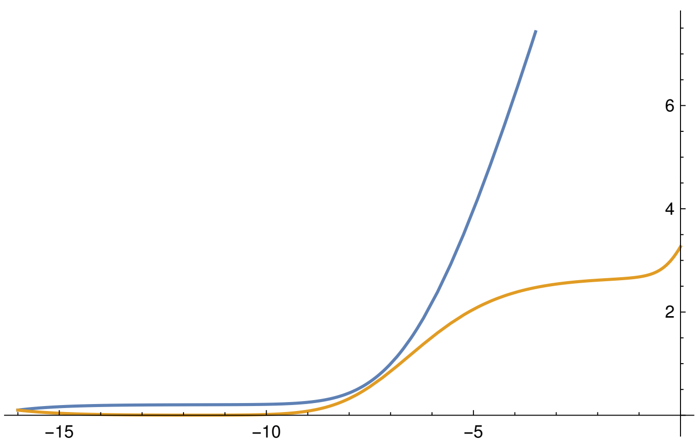
Differential equation for X:

```
Clear[X]; Xs = NDSolve[{D[X[n], {n, 2}] + (ξ + 3) D[X[n], n] == -6 (2 + ξ),
    X[Nin] == 0.1, X'[Nin] == 0.1}, X[n], {n, Nin, 0}];
X2 = Evaluate[X[n] /. Xs[[1]][[1]]]; DX = D[X2, n]; Xe = Plot[{X2, DX}, {n, Nin, 0}]
```



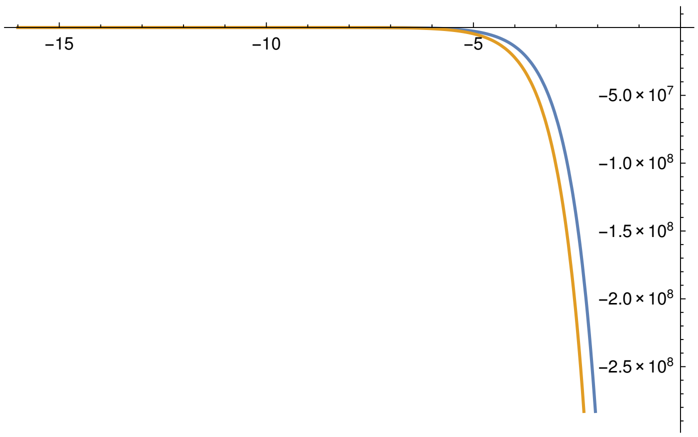
Differential equation for Y:

```
Clear[Y];
Yn = NDSolve[{D[Y[n], {n, 2}] + (ξ + 3) D[Y[n], n] == DX^2, Y[Nin] == 0.1, Y'[Nin] == 0.1},
   Y[n], {n, Nin, 0}]; Y2 = Evaluate[Y[n] /. Yn[[1]][[1]]];
DY = D[Y2, n]; Ye = Plot[{Y2, DY}, {n, Nin, 0}]
```

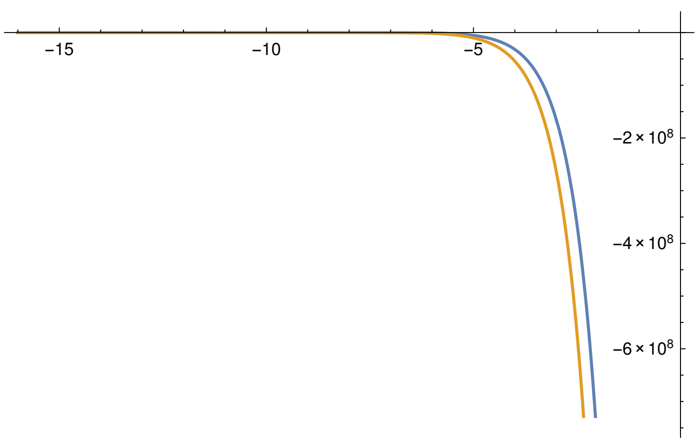


Differential equation for U and V:

```
Vn = NDSolve[{2 D[V[n], {n, 2}] + 2 (ξ + 3) D[V[n], n] + 12 (2 + ξ) (2 DX * V[n] + DF) / DY == 0,
   V[Nin] == 0.1, V'[Nin] == 0.1}, V[n], {n, 0, Nin}];
V2 = Evaluate[{V[n]} /. Vn][[1]][[1]]; DV = D[V2, n];
Plot[{V2, DV}, {n, 0, Nin}]
```

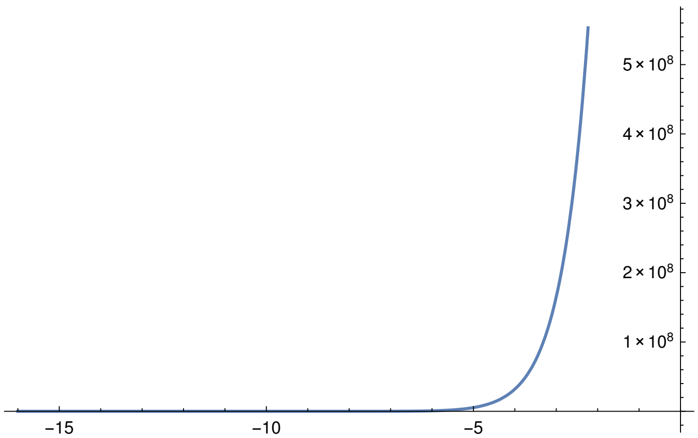



```
Un = NDSolve[{D[U[n], n] + 2 V2 * DX == 0, U[Nin] == 0.1}, U[n], {n, 0, Nin}];
U2 = Evaluate[{U[n]} /. Un][[1]][[1]]; DU = D[U2, n]; Plot[{U2, DU}, {n, 0, Nin}]
```



Non-local distortion function

```
f = F - 1 - U2; Plot[f, {n, Nin, 0}]
```




```
YValues = Table[Y2, {n, Nin, 0, 0.00025}];
```

```
Length[YValues]
```

64 001

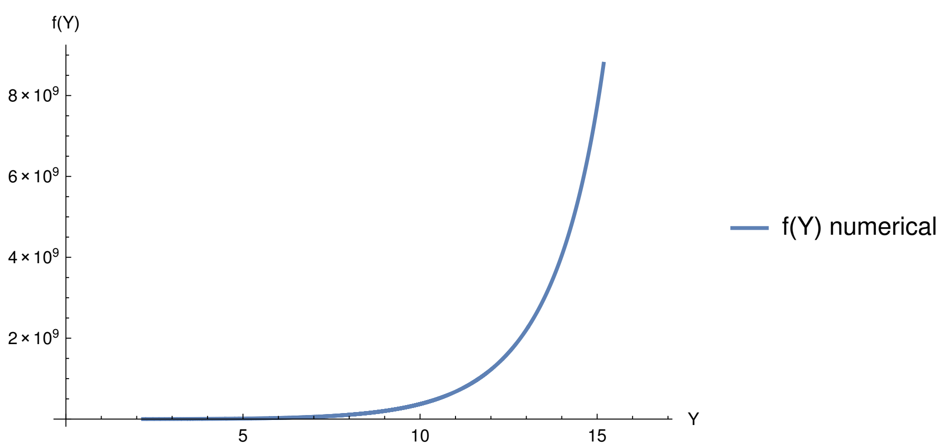
```
fValues = Table[f, {n, Nin, 0, 0.00025}];
```

```
Length[fValues]
```

64 001

```
data = Table[{YValues[[j]], fValues[[j]]}, {j, 40 000, 64 001}];
```

```
L = ListLinePlot[data, AxesLabel → {"Y", "f(Y)"},
  PlotStyle → {Thick}, PlotLegends → {"f(Y) numerical"}]
```

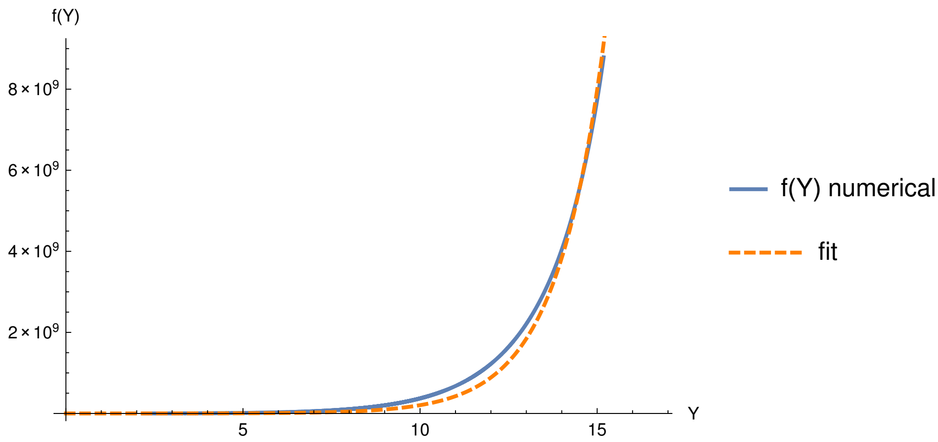


Exponential fit

```
fitv = FindFit[data, e^(δ (Y + β)), {β, δ}, Y]
```

{β → 16.1286, δ → 0.732467}

```
fit = Plot[{((e^(δ (Y + β))) /. Evaluate[fitv])}, {Y, 0, 17},
  PlotStyle → {Orange, Dashed, Thick}, PlotLegends → {"fit"}]; S = Show[L, fit]
```




```
SetDirectory["/media/dimas/06F0EC7FF0EC75F9/Física/3 Doutorado/Papers"]
```

/media/dimas/06F0EC7FF0EC75F9/Física/3 Doutorado/Papers

```
Export["fdeY.pdf", S]
```

fdeY.pdf

```
(e^(δ (Y + β))) /. Evaluate[fitv] // TraditionalForm
```

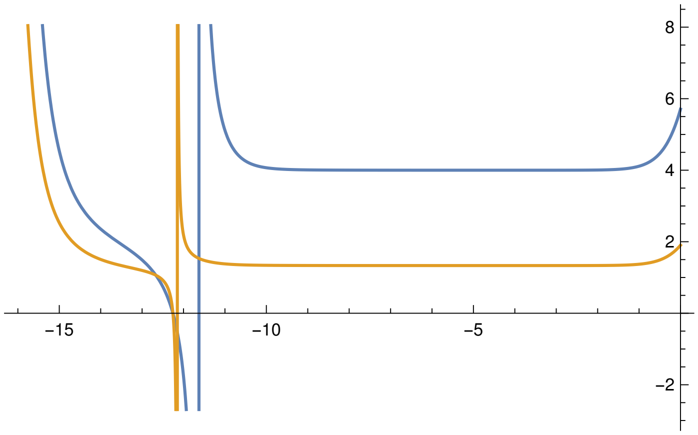
$e^{0.732467 (Y+16.1286)}$

Differential equation for δm

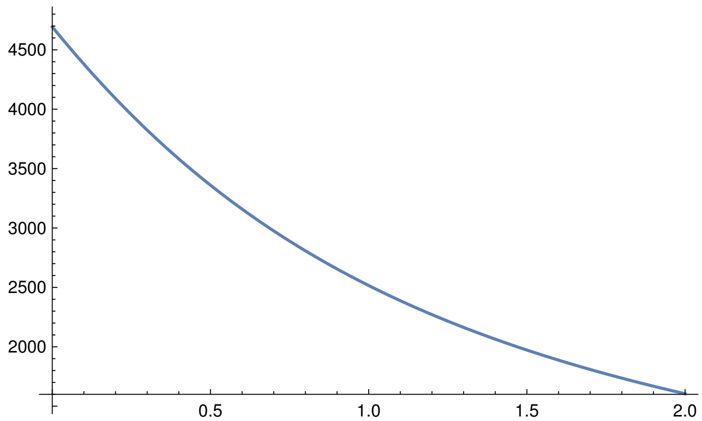
```
δsol = NDSolve[{D[δ[n], {n, 2}] + (2 + ξ) D[δ[n], n] -
     3 / 2 H0^2 / (e^(3 n) HN^2) Ωm ((F + 8 V2) / (F (F + 6 V2))) δ[n] == 0,
   δ[-11] == 0.1, δ'[-11] == 0.1}, δ[n], {n, -11, 0}];
δ2 = Evaluate[{δ[n]} /. δsol][[1]][[1]]
```

InterpolatingFunction[Domain: {{-11., 0.}} Output: scalar][n]

```
Plot[{(F + 8 V2) / (F (F + 2 V2)), (F + 8 V2) / (F (F + 6 V2))}, {n, Nin, 0}]
```




```
d1 = Plot[δ2 /. {n → Log[1 / (1 + z)]}, {z, 0, 2}]
```

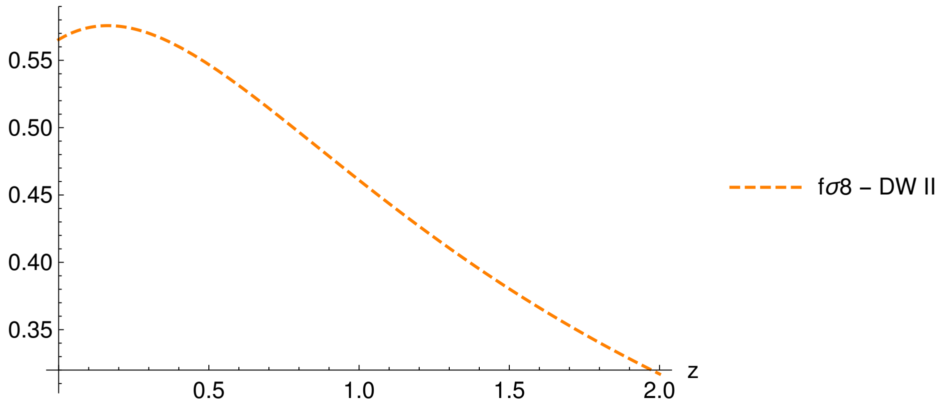


Defining growth rate fr

```
Clear[z]; fr = D[Log[δ2], n]; σ08 = 0.811;
σ8 = σ08 * δ2 / (δ2 /. {n → 0}); fσ8DWII = fr * σ8 /. {n → Log[1 / (1 + z)]}
```

0.000172782 InterpolatingFunction[Domain: {{−11., 0.}} Output: scalar][Log[$\frac{1}{1+z}$]]

```
Pd0 = Plot[{fσ8DWII}, {z, 0, 2}, PlotLegends → {"fσ8 - DW II"},
  LabelStyle → {Black, 12}, AxesLabel → {"z", ""}, PlotStyle → {Orange, Dashed}]
```

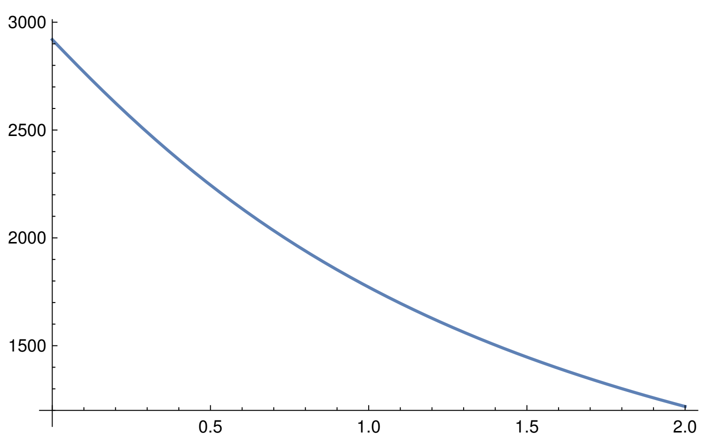


Comparison with ΛCDM

```
δcdm =
 NDSolve[{D[δ[n], {n, 2}] + (2 + ξ) D[δ[n], n] - 3 H0^2 / (2 ⅇ^{3 n} HN^2) Ωm * δ[n] == 0,
   δ[-16] == 0.1, δ'[-16] == 0.1}, δ[n], {n, 0, -16}];
δCDM = Evaluate[{δ[n]} /. δcdm][[1]][[1]]
```

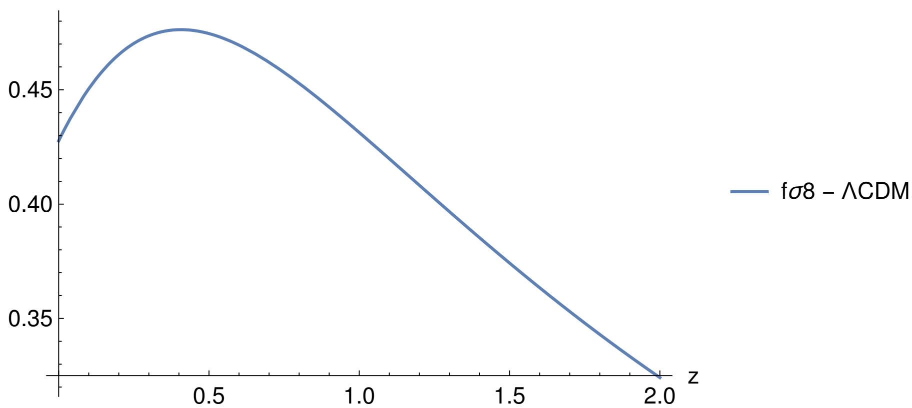
```
InterpolatingFunction[ Domain: {{-16., 0.}} Output: scalar ][n]
```

```
d2 = Plot[δCDM /. {n → Log[1 / (1 + z)]}, {z, 0, 2}]
```

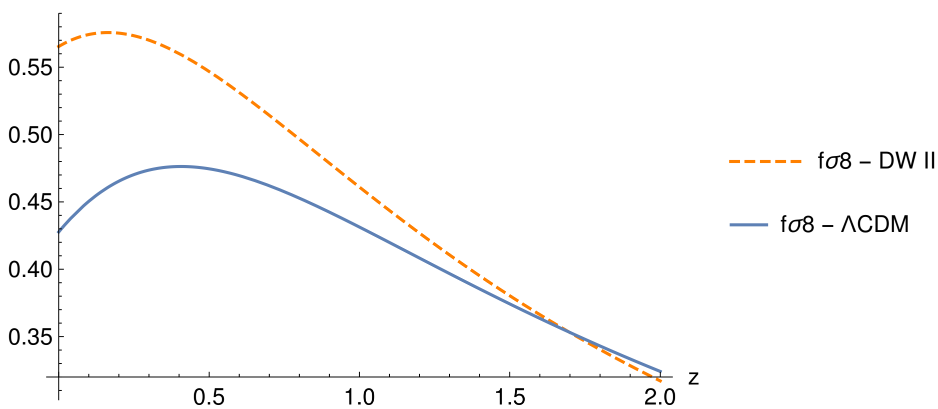


Defining growth rate fr

```
Clear[z]; frCDM = D[Log[δCDM], n]; σ08 = 0.811;
σ8CDM = σ08 * δCDM / (δCDM /. {n → 0}); fσ8CDM = frCDM * σ8CDM /. {n → Log[1 / (1 + z)]};
Pd2 = Plot[{fσ8CDM}, {z, 0, 2}, PlotLegends → {"fσ8 - ΛCDM"},
  LabelStyle → {Black, 12}, AxesLabel → {"z", ""}]
```




```
Show[Pd0, Pd2]
```

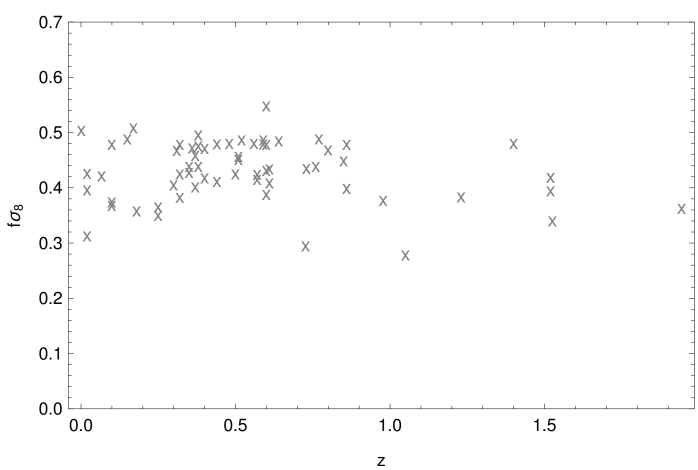



```
Export["fsigma8.pdf", Pd2]
fsigma8.pdf
```

RSD data

```
RSDdata = {{0.35}, {0.44 ± 0.05}, {0.77}, {0.49 ± 0.18}, {0.17}, {0.51 ± 0.06}, {0.02},
   {0.314 ± 0.048}, {0.02}, {0.398 ± 0.065}, {0.25}, {0.3512 ± 0.00583}, {0.37},
   {0.4602 ± 0.0378}, {0.25}, {0.3665 ± 0.0601}, {0.37}, {0.4031 ± 0.0586},
   {0.44}, {0.413 ± 0.08}, {0.6}, {0.39 ± 0.063}, {0.73}, {0.437 ± 0.072}, {0.067},
   {0.423 ± 0.055}, {0.3}, {0.407 ± 0.055}, {0.4}, {0.419 ± 0.041}, {0.5},
   {0.427 ± 0.043}, {0.6}, {0.433 ± 0.067}, {0.8}, {0.47 ± 0.08}, {0.35}, {0.429 ± 0.089},
   {0.18}, {0.36 ± 0.09}, {0.38}, {0.44 ± 0.06}, {0.32}, {0.384 ± 0.095}, {0.32},
   {0.48 ± 0.1}, {0.57}, {0.417 ± 0.045}, {0.15}, {0.49 ± 0.145}, {0.1}, {0.37 ± 0.13},
   {1.4}, {0.482 ± 0.116}, {0.59}, {0.488 ± 0.06}, {0.38}, {0.497 ± 0.045},
   {0.51}, {0.458 ± 0.038}, {0.61}, {0.436 ± 0.034}, {0.38}, {0.477 ± 0.051},
   {0.51}, {0.453 ± 0.05}, {0.61}, {0.41 ± 0.044}, {0.76}, {0.44 ± 0.04}, {1.05},
   {0.28 ± 0.08}, {0.32}, {0.427 ± 0.056}, {0.57}, {0.426 ± 0.029}, {0.727},
   {0.296 ± 0.0765}, {0.02}, {0.428 ± 0.0465}, {0.6}, {0.48 ± 0.12}, {0.86},
   {0.48 ± 0.1}, {0.6}, {0.55 ± 0.12}, {0.86}, {0.4 ± 0.11}, {0.1}, {0.48 ± 0.16},
   {0.001}, {0.505 ± 0.085}, {0.85}, {0.45 ± 0.11}, {0.31}, {0.469 ± 0.098},
   {0.36}, {0.474 ± 0.097}, {0.4}, {0.473 ± 0.086}, {0.44}, {0.481 ± 0.076},
   {0.48}, {0.482 ± 0.067}, {0.52}, {0.488 ± 0.065}, {0.56}, {0.482 ± 0.067},
   {0.59}, {0.481 ± 0.066}, {0.64}, {0.486 ± 0.07}, {0.1}, {0.376 ± 0.038},
   {1.52}, {0.42 ± 0.076}, {1.52}, {0.396 ± 0.076}, {0.978}, {0.379 ± 0.176},
   {1.23}, {0.385 ± 0.099}, {1.526}, {0.342 ± 0.07}, {1.944}, {0.364 ± 0.106}};
```

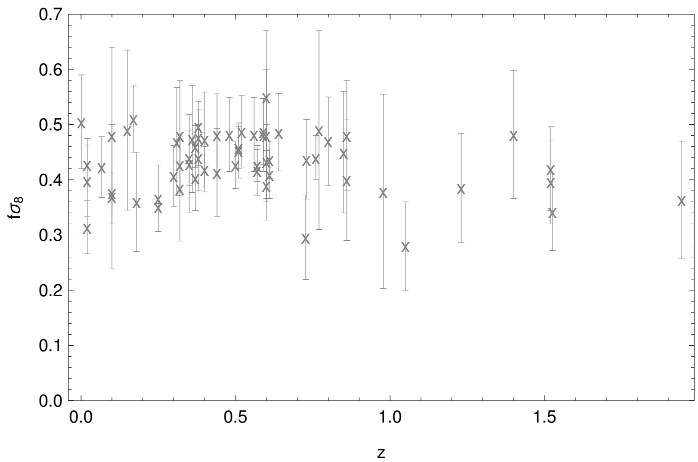
```
Do[z[k] = RSDdata[[(2 k + 1)]][[1]], {k, 0, 62, 1}]; Clear[fσ8];
Do[fσ8[k - 1] = RSDdata[[2 k]][[1]][[1]], {k, 1, 63, 1}];
Do[Errf[k - 1] = RSDdata[[2 k]][[1]][[2]], {k, 1, 63, 1}];
RSDdata2 = Table[{z[j], fσ8[j]}, {j, 0, 125}]; Ebar = Table[{Errf[k]}, {k, 0, 62}];
ListPlot[{RSDdata2}, Sequence[Frame → True, FrameLabel → {"z", Subscript[fσ, 8]},
  PlotMarkers → {"x", 10}], PlotRange → {0, .7}, PlotStyle → {Gray}]
```



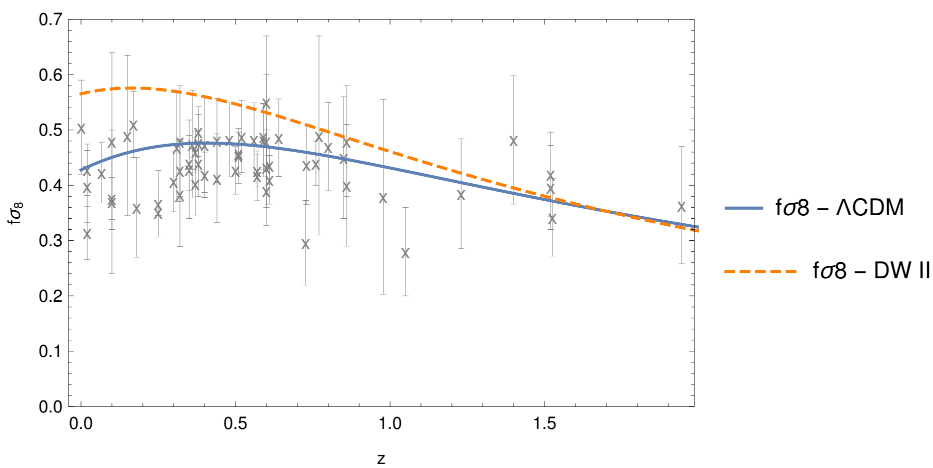
Execute these lines two times!

```
datacom = Table[{RSDdata2[[k + 1]], ErrorBar[Errf[k]]}, {k, 0, 62}];
```

```
Needs["ErrorBarPlots`"];
Pd = ErrorListPlot[datacom, Method → {"OptimizePlotMarkers" → False},
  Sequence[Frame → True, FrameLabel → {"z", Subscript[fσ, 8]},
   PlotMarkers → {"x", 10}], PlotRange → {0, .7}, PlotStyle → {Gray, Thin}]
```




```
Show[Pd, Pd2, Pd0]
```



I - Symmetric bounce

```
Clear["Global`*"]
```

```
SetDirectory["/media/dimas/06F0EC7FF0EC75F9/Física/3 Doutorado/Papers"]
```

```
/media/dimas/06F0EC7FF0EC75F9/Física/3 Doutorado/Papers
```

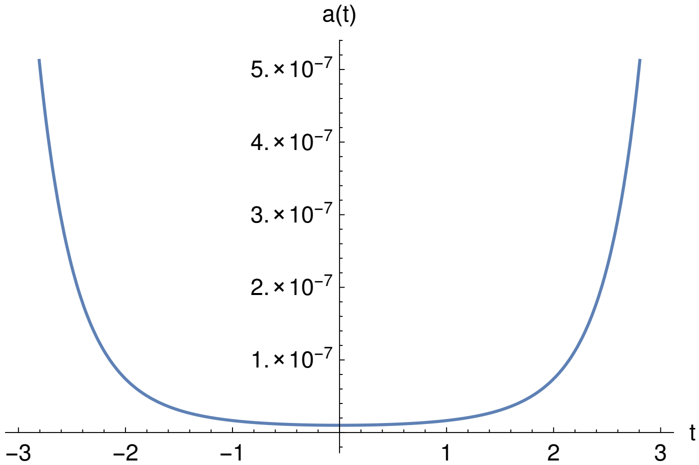
```
a = am * e^ (h1 * t^2 / 2)
```

$am\, e^{\frac{h1\, t^2}{2}}$

```
H = D[a, t] / a
```

```
h1 t
```

```
h1 = 1; am = 10^(-8);
pa = Plot[a, {t, -3, 3}, AxesLabel → {"t", "a(t)"}, LabelStyle → {Black, 12}]
```




```
Export["aI.pdf", pa]
```

```
aI.pdf
```

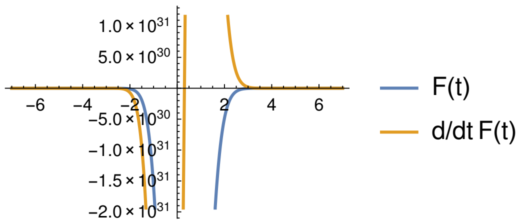
System of ODE’s

```
simp = Simplify[{2 D[H, t] F[t] + 6 H^2 F[t] + D[F[t], {t, 2}] + 5 H * D[F[t], t] == 0,
   D[X[t], {t, 2}] + 3 H * D[X[t], t] + 6 (D[H, t] + 2 H^2) == 0,
   Y''[t] + 3 H * Y'[t] - D[X[t], t]^2 == 0,
   D[V[t], {t, 2}] + 3 H * D[V[t], t] + 6 (D[H, t] + 2 H^2)
      (D[F[t], t] - D[U[t], t]) / D[Y[t], t] == 0, D[U[t], t] + 2 * V[t] * D[X[t], t] == 0}]
```

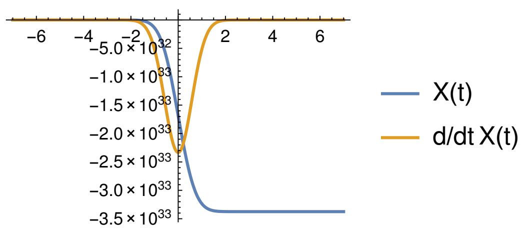
$$\left\{\left(2+6\,t^2\right) F[t] + 5\,t\,F'[t] + F''[t] == 0,\; 6 + 12\,t^2 + 3\,t\,X'[t] + X''[t] == 0,\; X'[t]^2 == 3\,t\,Y'[t] + Y''[t],\; 3\,t\,V'[t] + \frac{6\left(1+2\,t^2\right)\left(F'[t]-U'[t]\right)}{Y'[t]} + V''[t] == 0,\; U'[t] + 2\,V[t]\,X'[t] == 0\right\}$$

```
Clear[F, X, Y, U, V]; ti = -7; x0 = 0.1;
sol = NDSolve[{simp, F[ti] == x0, F'[ti] == x0, X[ti] == x0, X'[ti] == x0,
    Y[ti] == x0, Y'[ti] == x0, V[ti] == x0, V'[ti] == x0, U[ti] == x0},
   {F[t], X[t], Y[t], U[t], V[t]}, {t, -7, 7}]; L[1] = "F(t)";
L[2] = "X(t)"; L[3] = "Y(t)"; L[4] = "U(t)"; L[5] = "V(t)";
Do[P[i] = sol[[1]][[i]][[2]], {i, 1, 5}];
Do[DP[i] = D[P[i], t], {i, 1, 5}];
Table[Plot[{P[i], DP[i]}, {t, -7, 7}, PlotLegends → {L[i], "d/dt" L[i]}], {i, 1, 5}]
```

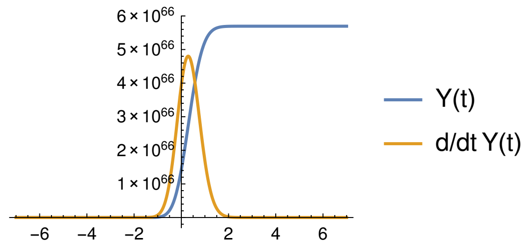
{



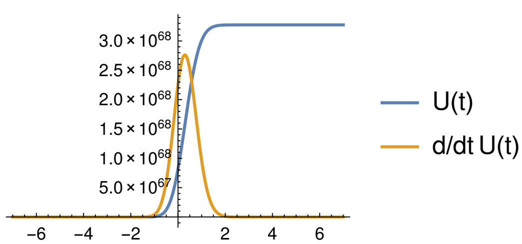
,



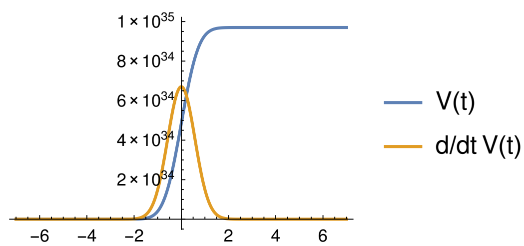
,



,



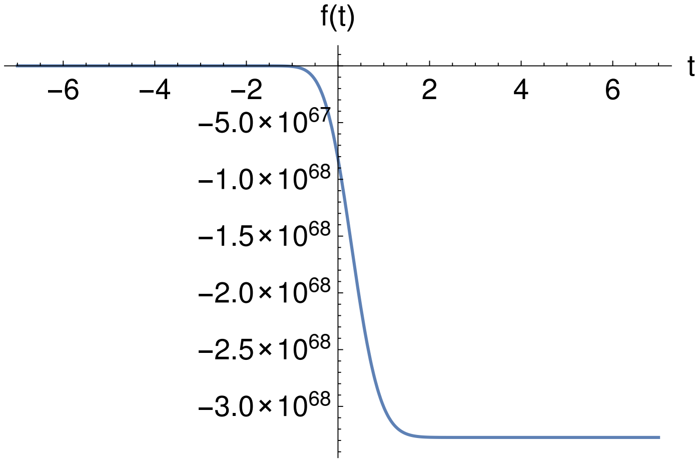
,



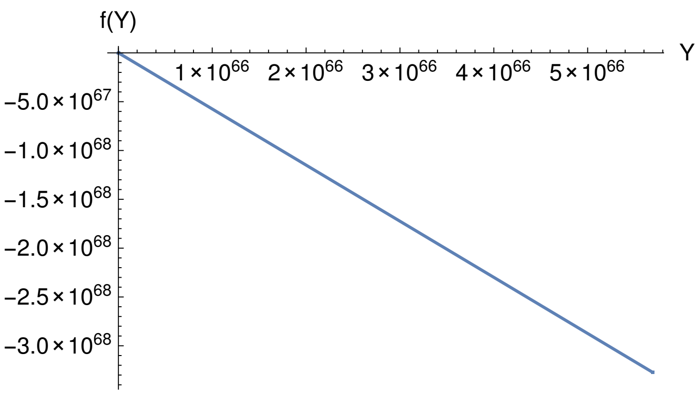
}

Non local distortion function f(Y)

```
fp = P[1] - P[4] - 1;
pf = Plot[fp, {t, -7, 7}, AxesLabel → {"t", "f(t)"}, LabelStyle → {Black, 15}]
```




```
YValues = Table[P[3], {t, -7, 7, 0.005}]; fValues = Table[fp, {t, -7, 7, 0.005}];
data = Table[{YValues[[j]], fValues[[j]]}, {j, 1, 2801}];
line = ListLinePlot[data, AxesLabel → {"Y", "f(Y)"}, LabelStyle → {Black, 12}]
```




```
Export["fYI.pdf", line]
```

fYI.pdf

Differential equation for δm

From Plank (2018)

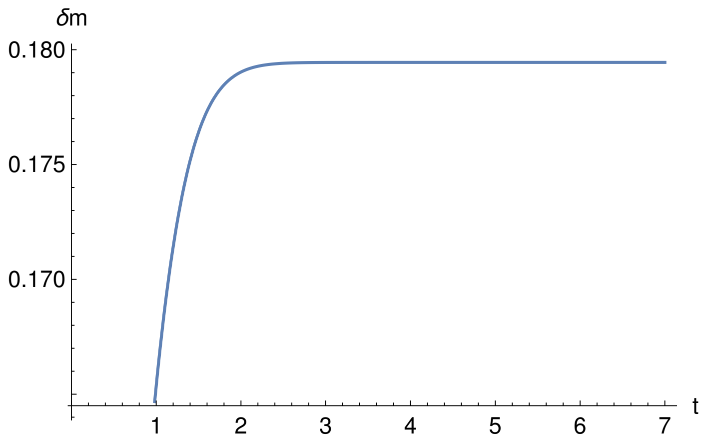
```
ΩΛ = 0.6847; Ωr = 9.265 * 10^(-5); Ωm = 0.3153; H0 = N[67.4 / 3.09 * 10^(-19)];
```

Initial conditions $\delta(t = 0.1) = 0.1$

```
δb = NDSolve[{D[δ[t], {t, 2}] + 2 H * D[δ[t], t] -
     3 H0^2 / (2 a^3 H^2) Ωm (P[1] + 8 P[5]) / (P[1] (P[1] + 2 P[5])) δ[t] == 0,
   δ[0.1] == 0.1, δ'[0.1] == 0.1}, δ[t], {t, 0.1, 7}]
```

{{δ[t] → InterpolatingFunction[Domain: {{0.1, 7.}} Output: scalar][t]}}

```
δb2 = Evaluate[{δ[t]} /. δb][[1]][[1]];
Plot[δb2, {t, 0.1, 7}, LabelStyle → {Black, 12}, AxesLabel → {"t", "δm"}]
```




```
Solve[a == 1 / (1 + z), t]
```

$$\left\{\left\{t \to \text{ConditionalExpression}\left[-\sqrt{2}\,\sqrt{2\,i\,\pi\,C[1] + \text{Log}\left[\frac{100\,000\,000}{1+z}\right]},\ C[1] \in \text{Integers}\right]\right\},\ \left\{t \to \text{ConditionalExpression}\left[\sqrt{2}\,\sqrt{2\,i\,\pi\,C[1] + \text{Log}\left[\frac{100\,000\,000}{1+z}\right]},\ C[1] \in \text{Integers}\right]\right\}\right\}$$

```
Solve[a == 1 / (1 + z), z]
```

$$\left\{\left\{z \to -1 + 100\,000\,000\, e^{-\frac{t^2}{2}}\right\}\right\}$$

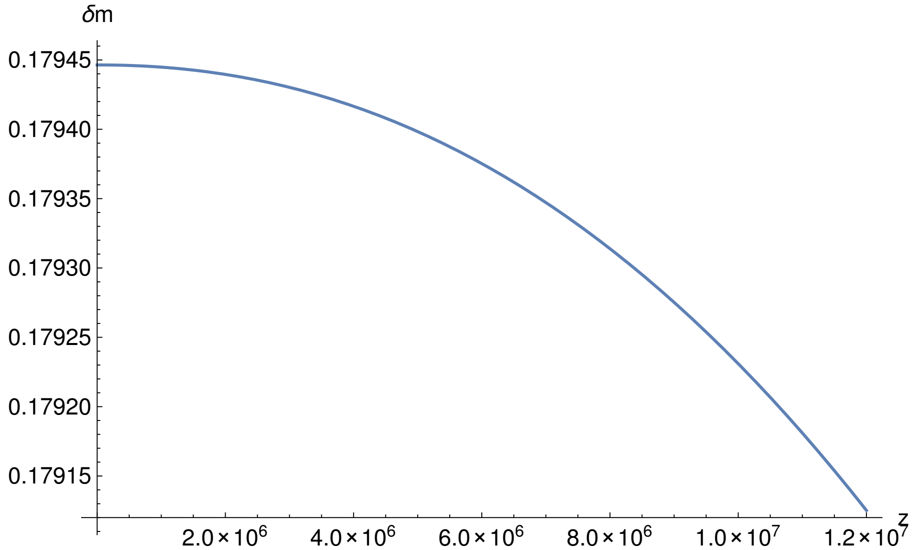
$$\text{N}\left[\left(-1 + 100\,000\,000\, e^{-\frac{t^2}{2}}\right) /.\ \{t \to 2\}\right]$$

1.35335×10^{7}

$$\text{N}\left[\left(-1 + 100\,000\,000\, e^{-\frac{t^2}{2}}\right) /.\ \{t \to 0.1\}\right]$$

9.95012×10^{7}

$$\texttt{p}\delta = \texttt{Plot}\left[\delta\texttt{b2} \;/.\; \left\{t \to \sqrt{2}\,\sqrt{\texttt{Log}\left[\frac{100\,000\,000}{1+z}\right]}\right\},\right.$$
$$\left.\{z, 0.5, 1.2 * 10\wedge 7\}, \texttt{LabelStyle} \to \{\texttt{Black}, 12\}, \texttt{AxesLabel} \to \{\texttt{"z"}, \texttt{"}\delta\texttt{m"}\}\right]$$




```
Export["delta-m.pdf", pδ]
```

```
delta-m.pdf
```

$$\texttt{dndt} = \texttt{D}\left[\texttt{Log}\left[\frac{e^{\frac{t^2}{2}}}{100\,000\,000}\right], t\right]$$

```
t
```

$$\texttt{N}\left[\sqrt{2}\,\sqrt{+\texttt{Log}[100\,000\,000]}\right]$$

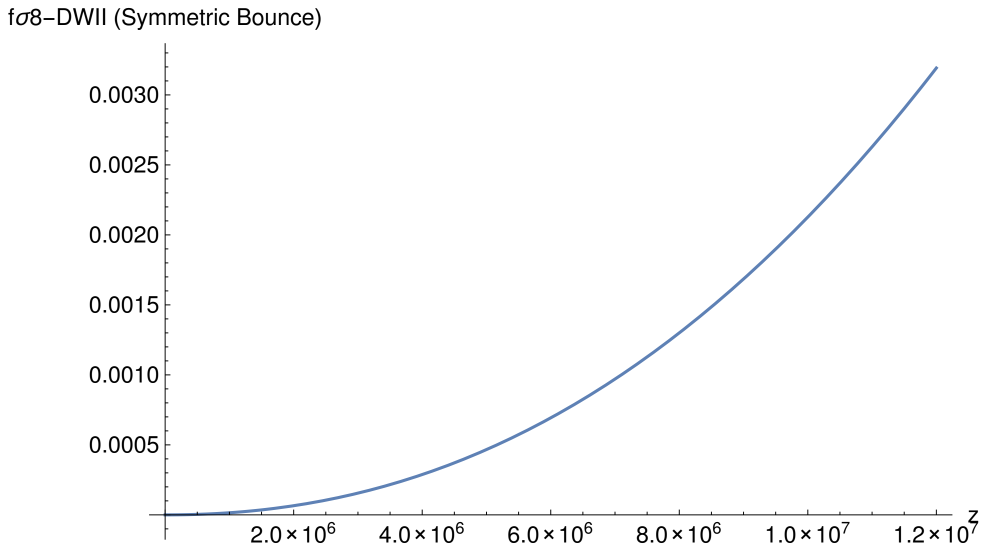
```
6.06971
```

Defining growth rate

```
fr = 1 / t * D[Log[δb2], t]; σ08 = 0.811; σ8 = σ08 * δb2 / (δb2 /. {t → 6}); fσ8DWIIb = fr * σ8
```

$$\frac{4.51946\ \texttt{InterpolatingFunction}\left[\;\text{Domain: }\{\{0.1, 7.\}\}\ \text{Output: scalar}\;\right][t]}{t}$$

```
Plot[fσ8DWIIb /. {t → √2 √(Log[100 000 000/(1 + z)])}, {z, 0.5, 1.2 * 10^7},
 AxesLabel → {"z", "fσ8-DWII (Symmetric Bounce)"}, LabelStyle → {Black, 12}]
```



II - Oscillatory Bounce

```
Clear["Global`*"]
```

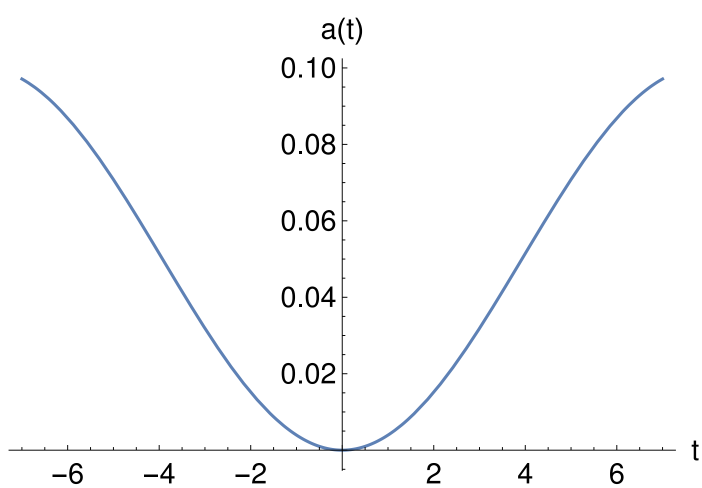
```
a = A0 * Sin[k * t]^2 + c
```

$c + A0\ \mathrm{Sin}[k\,t]^2$

```
H = D[a, t] / a
```

$$\frac{2\ A0\ k\ \mathrm{Cos}[k\,t]\ \mathrm{Sin}[k\,t]}{c + A0\ \mathrm{Sin}[k\,t]^2}$$

```
A0 = 1 / 10; c = 10^(-8); k = 3 / 15;
pa = Plot[a, {t, -7, 7}, AxesLabel → {"t", "a(t)"}, LabelStyle → {Black, 15}]
```



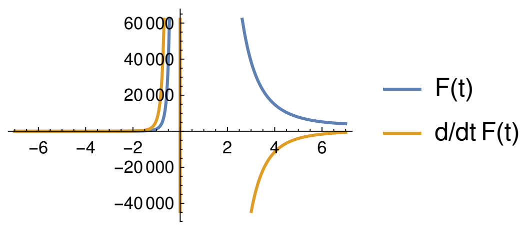
```
Export["aII.pdf", pa]
```

```
aII.pdf
```

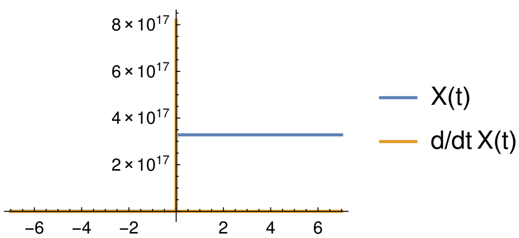
System of ODE's

```
simp = Simplify[{2 D[H, t] F[t] + 6 H^2 F[t] + D[F[t], {t, 2}] + 5 H * D[F[t], t] == 0,
    D[X[t], {t, 2}] + 3 H * D[X[t], t] + 6 (D[H, t] + 2 H^2) == 0,
    Y''[t] + 3 H * Y'[t] - D[X[t], t]^2 == 0, D[V[t], {t, 2}] + 3 H * D[V[t], t] +
      6 (D[H, t] + 2 H^2) (D[F[t], t] - D[U[t], t]) / D[Y[t], t] == 0,
    D[U[t], t] + 2 * V[t] * D[X[t], t] == 0}];
```

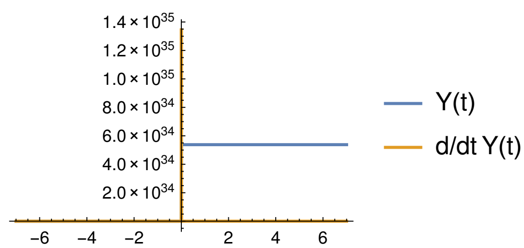
```
Clear[F, X, Y, U, V]; ti = -7; z = 0.1;
sol = NDSolve[{simp, F[ti] == z, F'[ti] == z, X[ti] == z, X'[ti] == z, Y[ti] == z, Y'[ti] == z,
   U[ti] == z, V[ti] == z, V'[ti] == z}, {F[t], X[t], Y[t], U[t], V[t]}, {t, -7, 7}];
L[1] = "F(t)"; L[2] = "X(t)"; L[3] = "Y(t)"; L[4] = "U(t)"; L[5] = "V(t)";
Do[P[i] = sol[[1]][[i]][[2]], {i, 1, 5}]; Do[DP[i] = D[P[i], t], {i, 1, 5}];
Table[Plot[{P[i], DP[i]}, {t, -7, 7}, PlotLegends → {L[i], "d/dt" L[i]}], {i, 1, 5}]
```

{


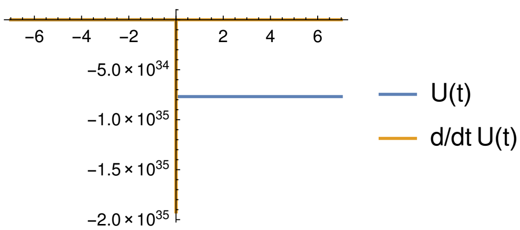
,



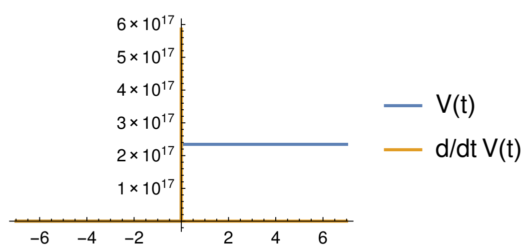
,



,



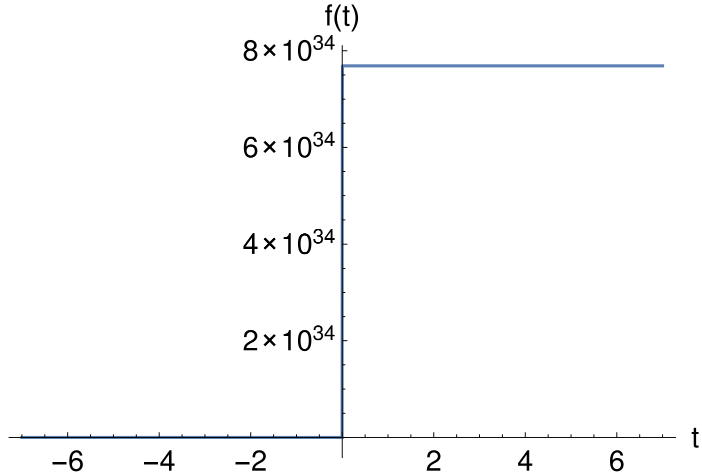
,



}

Non local distortion function f(Y)

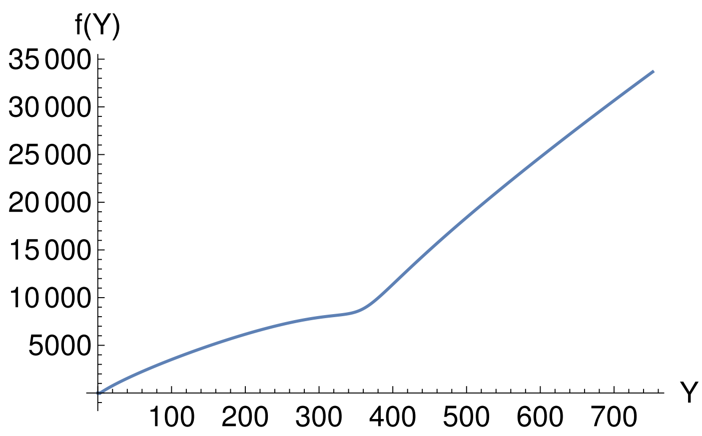
```
fp = P[1] - P[4] - 1;
pf = Plot[fp, {t, -7, 7}, AxesLabel → {"t", "f(t)"}, LabelStyle → {Black, 15}]
```




```
Export["ftII.pdf", pf]
```

ftII.pdf

```
YValues = Table[P[3], {t, -2, 2, 0.005}]; fValues = Table[fp, {t, -2, 2, 0.005}];
data = Table[{YValues[[j]], fValues[[j]]}, {j, 1, 801}];
line = ListLinePlot[data, AxesLabel → {"Y", "f(Y)"}, LabelStyle → {Black, 15}]
```




```
Export["fYII.pdf", line]
```

fYII.pdf

III - Matter bounce

IV - Singularities cosmologies

V - Pre-inflationary asymmetric bounce