

EHT AND BLACK HOLE FACT SHEET

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Units

In $G = c = 1$ units, masses and lengths and times are all the same.

$$\text{Mass of Sun} = M_{\odot} \approx 2 \cdot 10^{30} \text{ kg} \approx 1.5 \text{ km} \approx 5 \mu\text{s}$$

Astronomical distances are typically measured in AU, parsec, or light-year, depending on context

$$1 \text{ AU} \approx 1.5 \cdot 10^8 \text{ km} \quad (\text{Pluto is at } \sim 40 \text{ AU})$$

$$1 \text{ pc} \approx 3 \text{ ly} \approx 3 \cdot 10^{13} \text{ km} \quad (\text{distance to Alpha Cen is } 1.3 \text{ pc})$$

Angles can be measured in radians (2π around a circle), or arcseconds ($1/3600^{\text{th}}$ of a degree)

$$1 \text{ rad} \approx 206265 \text{ arcsec}$$

Black hole basics

The Schwarzschild radius is the characteristic size of a black hole, and it is *linear* with the BH’s mass (contrast with constant density objects).

$$R_s = 2 \frac{GM}{c^2} \quad (\text{in } G = c = 1 \text{ units, simply } R_s = 2M)$$

The event horizon of a non-spinning BH is located at the Schwarzschild radius (everything below will be for a non-spinning BH). Nothing can return from the event horizon, not even light.

Objects can orbit a BH without being sucked in as long as they are not too close. The innermost stable circular orbit (ISCO) for matter is at

$$R_{\text{ISCO}} = 6M$$

Because black holes are the curvature of spacetime’s geometry, light gets “deflected” as it moves around the BH. The most light-deflection possible is if light goes in an orbit. Light can only have unstable orbits, at the “photon sphere” or “light ring”

$$R_{\text{ph}} = 3M$$

This orbit is unstable: A light ray that orbits just outside R_{ph} will spiral out to infinity where we can see it. But because of light deflection, it *looks* like it came from a larger radius, the critical impact parameter or “capture radius”

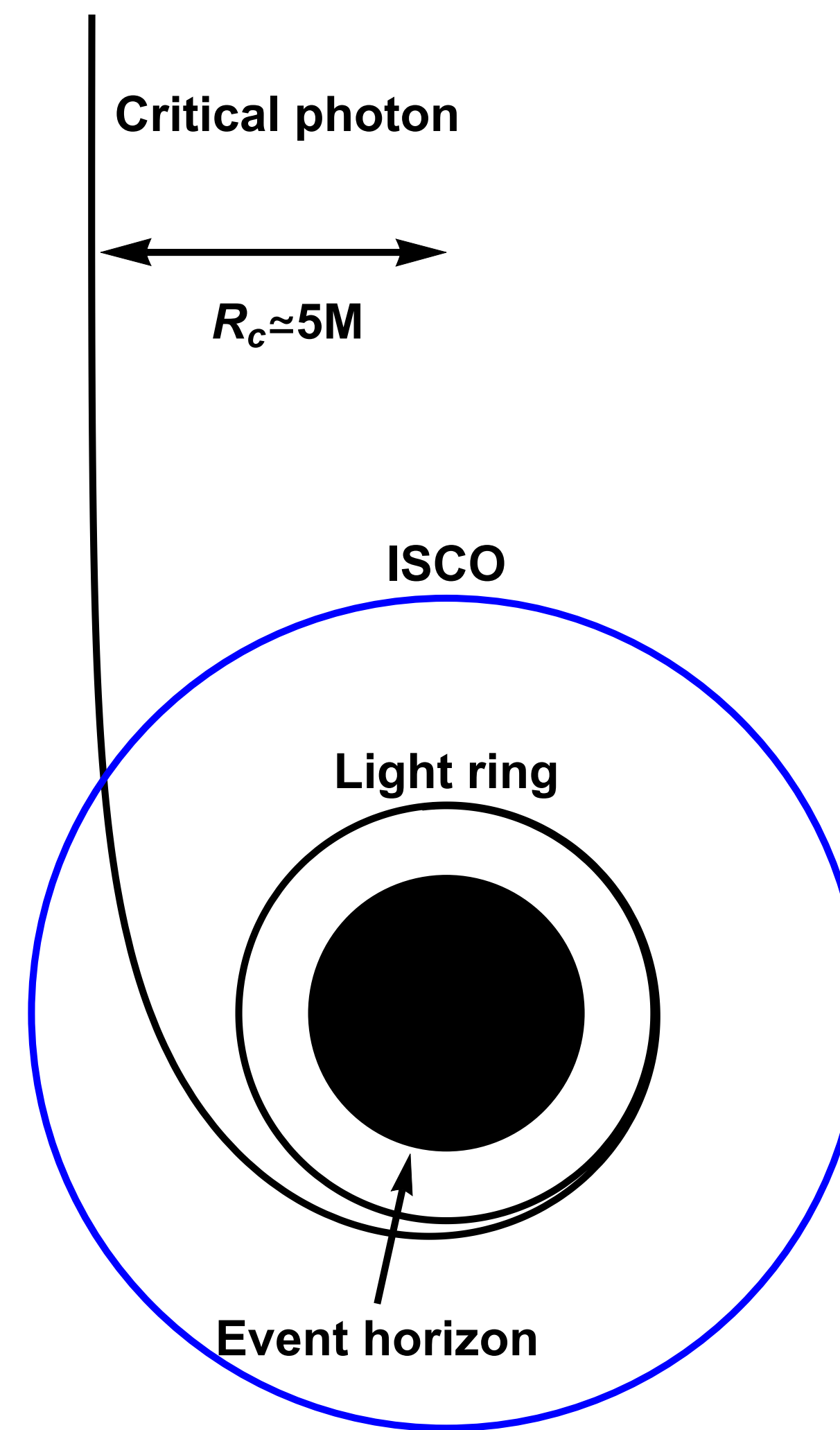
$$R_c = \sqrt{27}M \approx 5M$$

Black holes themselves don’t emit any light. They must be detected by their influence on nearby matter and light, most commonly infalling matter in an “accretion disk.” Matter slowly spirals inward (because of hydrodynamics, not because the BH is “sucking”) until it gets to about R_{ISCO} , and then plunges toward and through the event horizon. This matter gets hotter and brighter as it gets closer in; most of the emission comes from near R_{ISCO} . The time for matter to orbit at R_{ISCO} sets a characteristic time for imaging.

Even though most of the light is emitted near R_{ISCO} , light rays can skim the photon orbit and come back out; they seem to originate from R_{ph} , meaning they *appear* as a ring with radius R_c . This is the **black hole shadow**. Almost no light comes from inside R_c .

The shadow of M87 A*.

Credit: Event Horizon Telescope Collaboration



Imaging

Suppose you try to image a BH with a telescope of diameter D_{tel} , using light of wavelength λ . The telescope resolution is given by *Rayleigh’s criterion*,

$$\theta_{\text{res}} \approx \frac{\lambda}{D_{\text{tel}}}$$

If a BH is at a very large distance D_{BH} , and the shadow has diameter $d_s = 2R_c \approx 10GM/c^2$, the shadow will subtend an angle

$$\theta_s \approx \frac{d_s}{D_{\text{BH}}} \approx \frac{10M}{D_{\text{BH}}}$$

If you want to be able to tell there is a shadow, you need the telescope’s angular resolution θ_{res} to be (at least) a few times smaller than the BH’s shadow angle θ_s .

If you use $\lambda = 1.3\text{mm}$ radio waves, and make the whole Earth your telescope, $D_{\text{tel}} \approx 1.3 \cdot 10^4 \text{ km}$, your limit will be

$$\theta_{\text{res}} \approx 10^{-10} \text{ rad} \approx 20 \mu\text{arcsec}$$

EHT targets

Sgr A*

Sgr A* is the name of the supermassive black hole (SMBH) in our own Milky Way galaxy. It has a mass of

$$M_{\text{Sgr A}^*} \approx 4 \cdot 10^6 M_{\odot} \approx 6 \cdot 10^6 \text{ km} \approx 20 \text{ s}$$

The separation between Earth and Sgr A* is

$$D_{\text{Sgr A}^*} \approx 7.8 \text{ kpc} \approx 2.5 \cdot 10^{17} \text{ km}$$

The expected shadow size is

$$\theta_{\text{Sgr A}^*} \approx \frac{10M_{\text{Sgr A}^*}}{D_{\text{Sgr A}^*}} \approx 2.4 \cdot 10^{-10} \text{ rad} \approx 50 \mu\text{arcsec}$$

This is a bit bigger than θ_{res} for the Earth at 1.3 mm wavelength.

M87 A*

M87 A* is the SMBH in the galaxy M87, which is in the Virgo cluster. It is roughly a thousand times more massive and thus bigger than Sgr A*. However it is also roughly a thousand times farther away, so it is roughly the same angle on the sky.

$$M_{\text{M87 A}^*} \approx 6.5 \cdot 10^9 M_{\odot} \approx 10^{10} \text{ km} \approx 66 \text{ AU} \approx 9 \text{ hr}$$

$$D_{\text{M87 A}^*} \approx 16.8 \text{ Mpc} \approx 5 \cdot 10^{20} \text{ km} \approx 55 \text{ million lyr}$$

$$\theta_{\text{M87 A}^*} \approx \frac{10M_{\text{M87 A}^*}}{D_{\text{M87 A}^*}} \approx 2.4 \cdot 10^{-10} \text{ rad} \approx 40 \mu\text{arcsec}$$

This is barely a bit bigger than θ_{res} for the Earth at 1.3 mm wavelength.