

# Chapter 1

## A Learnable ScatterNet: Locally Invariant Convolutional Layers

In this chapter we explore tying together the ideas from Scattering Transforms and Convolutional Neural Networks (CNN) for Image Analysis by proposing a learnable ScatterNet. The work presented in ?? implies that while the Scattering Transform has been a promising start in using complex wavelets in image understanding tasks, there is something missing from them. To address this, we propose a learnable ScatterNet by building it with our proposed “Locally Invariant Convolutional Layers”.

Previous attempts at tying ScatterNets together with CNNs in hybrid networks [1]–[3] have tended to keep the two parts separate, with the ScatterNet forming a fixed front end and the CNN forming a learned backend. We instead look at adding learning between scattering orders, as well as adding learned layers before the ScatterNet.

We do this by adding a second stage after a scattering order, which mixes output activations together in a learnable way. The flexibility of the mixing we introduce allows us to build a layer that acts as a Scattering Layer with no learning, or as one that acts more as a convolutional layer with a controlled number of input and output channels, or more interestingly, as a hybrid between the two.

Our experiments show that these locally invariant layers can improve accuracy when added to either a CNN or a ScatterNet. We also discover some surprising results in that the ScatterNet may be best positioned after one or more layers of learning rather than at the front of a neural network.

In ?? we briefly review convolutional layers and scattering layers before introducing our learnable scattering layers in ?. In [section 1.4](#) we describe how we implement our proposed layer, and present some experiments we have run in [section 1.5](#) and then draw conclusions about how these new ideas might improve neural networks in the future.

### 1.1 Related Work

There has been several similar works that look into designing new convolutional layers by separating them into two stages — a first stage that performs a non-standard filtering process, and a second stage that combines the first stage into single activations. The inception layer [4] by Szegedy *et*

*al.* does this by filtering with different kernel sizes in the first stage, and then combining with a  $1 \times 1$  convolution in the second stage. Ioannou *et al.* also do something similar by making a first stage with horizontal and vertical filters, and then combining in the second stage again with a  $1 \times 1$  convolution[5]. But perhaps the most similar works are those that use a first stage with fixed filters, combining them in a learned way in the second stage. Of particular note are:

- “Local Binary Convolutional Neural Networks” [6]. This paper builds a first stage with a small  $3 \times 3$  kernel filled with zeros, and randomly insert plus and minus ones into it. This builds a very crude spatial differentiator in random directions. The output of the first stage is then passed through a sigmoid nonlinearity before being mixed with a  $1 \times 1$  convolution. The imposed structure on the first stage was found to be a good regularizer and prevented overfitting, and the combination of the mixing in the second layer allowed for a powerful and expressive layer, with performance near that of a regular CNN layer.
- “DCFNet: Deep Neural Network with Decomposed Convolutional Filters” [7]. This paper decomposes convolutional filters as linear combinations of Fourier Bessel and random bases. The first stage projects the inputs onto the chosen basis, and the second stage learns how to mix these projections with a  $1 \times 1$  convolution. Unlike [6] this layer is purely linear. The supposed advantage being that the basis can be truncated to save parameters and make the input less susceptible to high frequency variations. The work found that this layer had marginal benefits over regular CNN layers in classification, but had improved stability to noisy inputs.

## 1.2 Recap of Useful Terms

### 1.2.1 Convolutional Layers

Let the output of a CNN at layer  $l$  be:

$$x^{(l)}(c, \mathbf{u}), \quad c \in \{0, \dots, C_l - 1\}, \mathbf{u} \in \mathbb{R}^2$$

where  $c$  indexes the channel dimension, and  $\mathbf{u}$  is a vector of coordinates for the spatial position. Of course,  $\mathbf{u}$  is typically sampled on a grid, but we keep it continuous to more easily differentiate between the spatial and channel dimensions. A typical convolutional layer in a standard CNN (ignoring the bias term) is:

$$y^{(l+1)}(f, \mathbf{u}) = \sum_{c=0}^{C_l-1} x^{(l)}(c, \mathbf{u}) * h_f^{(l)}(c, \mathbf{u}) \quad (1.2.1)$$

$$x^{(l+1)}(f, \mathbf{u}) = \sigma\left(y^{(l+1)}(f, \mathbf{u})\right) \quad (1.2.2)$$

where  $h_f^{(l)}(c, \mathbf{u})$  is the  $f$ th filter of the  $l$ th layer (i.e.  $f \in \{0, \dots, C_{l+1} - 1\}$ ) with  $C_l$  different point spread functions.  $\sigma$  is a non-linearity such as the ReLU, possibly combined with scaling such as batch normalization. The convolution is done independently for each  $c$  in the  $C_l$  channels and the resulting outputs are summed together to give one activation map. This is repeated  $C_{l+1}$  times to give  $\left\{x^{(l+1)}(f, \mathbf{u})\right\}_{f \in \{0, \dots, C_{l+1} - 1\}, \mathbf{u} \in \mathbb{R}^2}$

### 1.2.2 Wavelet Transforms

The 2-D wavelet transform is done by convolving the input with a mother wavelet dilated by  $2^j$  and rotated by  $\theta$ :

$$\psi_{j,\theta}(\mathbf{u}) = 2^{-j} \psi(2^{-j} R_{-\theta} \mathbf{u}) \quad (1.2.3)$$

where  $R$  is the rotation matrix,  $1 \leq j \leq J$  indexes the scale, and  $1 \leq k \leq K$  indexes  $\theta$  to give  $K$  angles between 0 and  $\pi$ . We copy notation from [8] and define  $\lambda = (j, k)$  and the set of all possible  $\lambda$ s is  $\Lambda$  whose size is  $|\Lambda| = JK$ . The wavelet transform, including lowpass, is then:

$$Wx(c, \mathbf{u}) = \{x(c, \mathbf{u}) * \phi_J(\mathbf{u}), x(c, \mathbf{u}) * \psi_\lambda(\mathbf{u})\}_{\lambda \in \Lambda} \quad (1.2.4)$$

### 1.2.3 Scattering Transforms

Taking the modulus of the wavelet coefficients removes the high frequency oscillations of the output signal while preserving the energy of the coefficients over the frequency band covered by  $\psi_\lambda$ . This is crucial to ensure that the scattering energy is concentrated towards zero-frequency as the scattering order increases, allowing sub-sampling. We define the wavelet modulus propagator to be:

$$\tilde{W}x(c, \mathbf{u}) = \{x(c, \mathbf{u}) * \phi_J(\mathbf{u}), |x(c, \mathbf{u}) * \psi_\lambda(\mathbf{u})|\}_{\lambda \in \Lambda} \quad (1.2.5)$$

Let us call these modulus terms  $U[\lambda]x = |x * \psi_\lambda|$  and define a path as a sequence of  $\lambda$ s given by  $p = (\lambda_1, \lambda_2, \dots, \lambda_m)$ . Further, define the modulus propagator acting on a path  $p$  by:

$$U[p]x = U[\lambda_m] \cdots U[\lambda_2] U[\lambda_1] x \quad (1.2.6)$$

$$= |\cdots |x * \psi_{\lambda_1}| * \psi_{\lambda_2}| \cdots * \psi_{\lambda_m}| \quad (1.2.7)$$

These descriptors are then averaged over the window  $2^J$  by a scaled lowpass filter  $\phi_J = 2^{-J} \phi(2^{-J} \mathbf{u})$  giving the ‘invariant’ scattering coefficient

$$S[p]x(\mathbf{u}) = U[p]x * \phi_J(\mathbf{u}) \quad (1.2.8)$$

If we define  $p + \lambda = (\lambda_1, \dots, \lambda_m, \lambda)$  then we can combine Equation 1.2.5 and Equation 1.2.6 to give:

$$\tilde{W}U[p]x = \{S[p]x, U[p + \lambda]x\}_\lambda \quad (1.2.9)$$

Hence we iteratively apply  $\tilde{W}$  to all of the propagated  $U$  terms of the previous layer to get the next order of scattering coefficients and the new  $U$  terms.

The resulting scattering coefficients have many nice properties, one of which is stability to diffeomorphisms (such as shifts and warping). From [9], if  $\mathcal{L}_\tau x = x(\mathbf{u} - \tau(\mathbf{u}))$  is a diffeomorphism which is bounded with  $\|\nabla \tau\|_\infty \leq 1/2$ , then there exists a  $K_L > 0$  such that:

$$\|S\mathcal{L}_\tau x - Sx\| \leq K_L P H(\tau) \|x\| \quad (1.2.10)$$

where  $P = \text{length}(p)$  is the scattering order, and  $H(\tau)$  is a function of the size of the displacement, derivative and Hessian of  $\tau$ ,  $H\tau$  [9]:

$$H(\tau) = 2^{-J} \|\tau\|_\infty + \|\nabla \tau\|_\infty \max\left(\log \frac{\|\Delta \tau\|_\infty}{\|\nabla \tau\|_\infty}, 1\right) + \|H\tau\|_\infty \quad (1.2.11)$$

### 1.3 Locally Invariant Layer

We propose to mix the terms at the output of each wavelet modulus propagator  $\tilde{W}$ . The second term in  $\tilde{W}$ , the  $U$  terms are often called ‘covariant’ terms but in this work we will call them locally invariant, as they tend to be invariant up to a scale  $2^j$ . We propose to mix the locally invariant terms  $U$  and the lowpass terms  $S$  with learned weights  $a_{f,\lambda}$  and  $b_f$ . For example, consider a wavelet modulus propagator [Equation 1.2.5](#), and let the input to it be  $x^{(l)}$ . Our proposed output is then:

$$\begin{aligned} y^{(l+1)}(f, \mathbf{u}) &= \sum_{\gamma} \sum_{c=0}^{C-1} |x^{(l)}(c, \mathbf{u}) * \psi_{\gamma}(\mathbf{u})| a_{f,\lambda}(c) \\ &+ \left( \sum_{c=0}^{C-1} x^{(l)}(c, \mathbf{u}) * \phi_J(\mathbf{u}) \right) b_f(c) \end{aligned} \quad (1.3.1)$$

Returning to [Equation 1.2.5](#), we define a new index variable  $\gamma$  such that:

$$\tilde{W}[\gamma]x(c, \mathbf{u}) = \begin{cases} x^{(l)}(c, \mathbf{u}) * \phi_J(\mathbf{u}) & \text{if } \gamma = 1 \\ |x^{(l)}(c, \mathbf{u}) * \psi_{\gamma}(\mathbf{u})| & \text{if } \gamma \in \{2, \dots, JK + 1\} \end{cases} \quad (1.3.2)$$

We do the same for the weights  $a, b$  by defining  $\tilde{a}_f = \{b_f, a_{f,\lambda}\}_{\gamma}$  and  $\tilde{a}_f[\gamma] = b_f$  if  $\gamma = 1$  and  $\tilde{a}_f[\gamma] = a_{f,\lambda}$  if  $\gamma \in \{2, \dots, JK + 1\}$ .

We further define  $q = (c, \gamma) \in Q$  to combine the channel and index terms. Then we can index both the weights and the wavelet modulus operator by:

$$\tilde{W}x[q] = \tilde{W}[\gamma]x(c, \mathbf{u}) \quad (1.3.3)$$

$$\tilde{a}_f[q] = \tilde{a}_f[\gamma](c) \quad (1.3.4)$$

This simplifies [Equation 1.3.1](#) to be:

$$z^{(l+1)}(q, \mathbf{u}) = \tilde{W}x^{(l)}[q] = \tilde{W}[\gamma]x^{(l)}(c, \mathbf{u}) \quad (1.3.5)$$

$$y^{(l+1)}(f, \mathbf{u}) = \sum_{q \in Q} z^{(l+1)}(q, \mathbf{u}) \tilde{a}_f(q) \quad (1.3.6)$$

or in matrix form with  $A_{f,q} = \tilde{a}_f(q)$

$$Y^{(l+1)}(\mathbf{u}) = AZ^{(l+1)}(\mathbf{u}) \quad (1.3.7)$$

This is very similar to the standard convolutional layer from [Equation 1.2.1](#), except we have replaced the previous layer’s  $x$  with intermediate coefficients  $z$  (with  $|Q| = (JK + 1)C$  channels), and the convolutions of [Equation 1.2.1](#) have been replaced by a matrix multiply (which can also be seen as a  $1 \times 1$  convolutional layer). We can then apply [Equation 1.2.2](#) to [Equation 1.3.6](#) to get the next layer’s output (or equivalently, the next order scattering coefficients).

[Figure 1.1](#) shows a block diagram for [Equations 1.3.2 – 1.3.6](#).

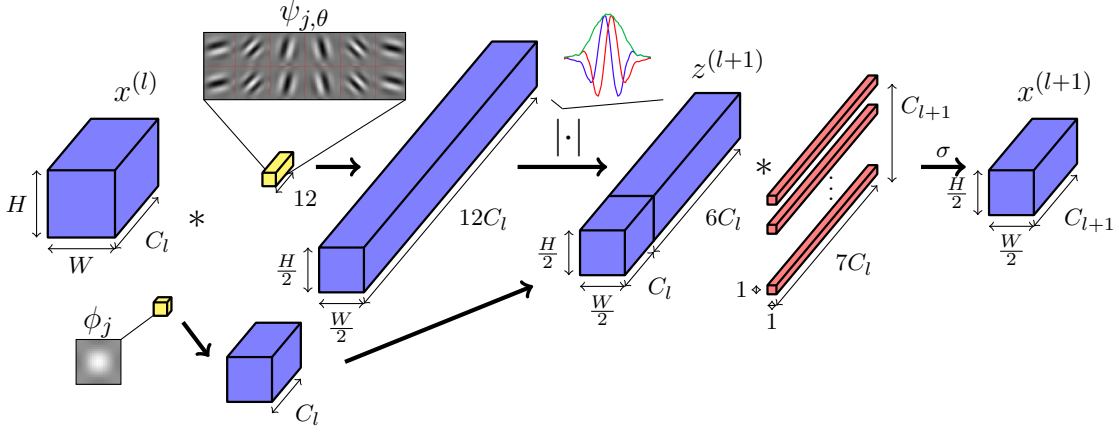


Figure 1.1: Block Diagram of Proposed Invariant Layer for  $J = 1$ . Activations are shaded blue, fixed parameters yellow and learned parameters red. Input  $x^{(l)} \in \mathbb{R}^{C_l \times H \times W}$  is filtered by real and imaginary oriented wavelets and a lowpass filter and is downsampled. The channel dimension increases from  $C_l$  to  $(2K + 1)C_l$ , where the number of orientations is  $K = 6$ . The real and imaginary parts are combined by taking their magnitude (an example of what this looks like in 1D is shown above the magnitude operator) - the components oscillating in quadrature are combined to give  $z^{(l+1)}$ . The resulting activations are concatenated with the lowpass filtered activations, mixed across the channel dimension, and then passed through a nonlinearity  $\sigma$  to give  $x^{(l+1)}$ . If the desired output spatial size is  $H \times W$ ,  $x^{(l+1)}$  can be bilinearly upsampled paying only a few multiplies per pixel.

### 1.3.1 Properties

#### Recovering the original ScatterNet Design

The first thing to note is that with careful choice of  $A$  and  $\sigma$ , we can recover the original translation invariant ScatterNet [1], [8]. If  $C_{l+1} = (JK + 1)C_l$  and  $A$  is the identity matrix  $I_{C_{l+1}}$ , we remove the mixing and then  $y^{(l+1)} = \tilde{W}x$ .

Further, if  $\sigma = \text{ReLU}$  as is commonly the case in training CNNs, it has no effect on the positive locally invariant terms  $U$ . It will affect the averaging terms if the signal is not positive, but this can be dealt with by adding a channel dependent bias term  $\alpha_c$  to  $x^{(l)}$  to ensure it is positive. This bias term will not affect the propagated signals as  $\int \alpha_c \psi_\lambda(\mathbf{u}) d\mathbf{u} = 0$ . The bias can then be corrected by subtracting  $\alpha_c \|\phi_J\|_2$  from the averaging terms after taking the ReLU, then  $x^{(l+1)} = \tilde{W}x$ .

This makes one layer of our system equivalent to a first order scattering transform, giving  $S_0$  and  $U_1$  (invariant to input shifts of  $2^1$ ). Repeating the same process for the next layer again works, as we saw in Equation 1.2.6, giving  $S_1$  and  $U_2$  (invariant to shifts of  $2^2$ ). If we want to build higher invariance, we can continue or simply average these outputs with an average pooling layer.

#### Flexibility of the Layer

Unlike a regular ScatterNet, we are free to choose the size of  $C_{l+1}$ . This means we can set  $C_{l+1} = C_l$  as is commonly the case in a CNN, and make a convolutional layer from mixing the locally invariant

terms.

### Stability to Noise and Deformations

Let us define the action of our layer on the scattering coefficients to be  $Vx$ . We would like to find a bound on  $\|V\mathcal{L}_\tau x - Vx\|$ . To do this, we note that the mixing is a linear operator and hence is Lipschitz continuous. The authors in [7] find constraints on the mixing weights to make them non-expansive (i.e. Lipschitz constant 1). Further, the ReLU is non-expansive meaning the combination of the two is also non-expansive, so  $\|V\mathcal{L}_\tau x - Vx\| \leq \|S\mathcal{L}_\tau x - Sx\|$ , and ?? holds.

## 1.4 Implementation Details

Again, we use the DTCWT[10] for our wavelet filters  $\psi_{j,\theta}$  due to their fast implementation with separable convolutions which we will discuss more in subsection 1.4.2. There are two side effects of this choice. The first is that the number of orientations of wavelets is restricted to  $K = 6$ . The second is that we naturally downsample the output activations by a factor of  $2^j$  in each direction for each scale  $j$ . This represents the source of the invariance in our layer. If we do not wish to downsample the output (say to make the layer fit in a larger network), we can bilinearly interpolate the output of our layer. This is computationally cheap to do on its own, but causes the next layer’s computation to be higher than necessary (there will be no energy for frequencies higher than  $f_s/4$ ).

In all our experiments we set  $J = 1$  for each invariant layer, meaning we can mix the lowpass and bandpass coefficients at the same resolution. Figure 1.1 shows how this is done. Note that setting  $J = 1$  for a single layer does not restrict us from having  $J > 1$  for the entire system, as if we have a second layer with  $J = 1$  after the first, including downsampling ( $\downarrow$ ), we would have:

$$(((x * \phi_1) \downarrow 2) * \psi_{1,\theta}) \downarrow 2 = (x * \psi_{2,\theta}) \downarrow 4 \quad (1.4.1)$$

### 1.4.1 Memory Cost

A standard convolutional layer with  $C_l$  input channels,  $C_{l+1}$  output channels and kernel size  $L$  has  $L^2 C_l C_{l+1}$  parameters.

The number of learnable parameters in each of our proposed invariant layers with  $J = 1$  and  $K = 6$  orientations is:

$$\#params = (JK + 1)C_l C_{l+1} = 7C_l C_{l+1} \quad (1.4.2)$$

The spatial support of the wavelet filters is typically  $5 \times 5$  pixels or more, and we have reduced  $\#params$  to less than  $3 \times 3$  per filter, while producing filters that are significantly larger than this.

### 1.4.2 Computational Cost

A standard convolutional layer with kernel size  $L$  needs  $L^2 C_{l+1}$  multiplies per input pixel (of which there are  $C_l \times H \times W$ ).

As mentioned in subsection 1.4.1, we use the DTCWT for our complex, shift invariant wavelet decomposition. We use the open source Pytorch implementation of the DTCWT[11] as it can run on GPUs and has support for backpropagating gradients.

There is an overhead in doing the wavelet decomposition for each input channel. A regular discrete wavelet transform (DWT) with filters of length  $L$  will have  $2L(1 - 2^{-2J})$  multiplies for a

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**Algorithm 1** Locally Invariant Convolutional Layer forward pass

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```

1: procedure LOCALINV( $x, A$ )
2:    $yl, yh \leftarrow \text{DTCWT}(x^l, \text{nlevels} = 1)$  ▷  $yh$  has 6 orientations and is complex
3:    $U \leftarrow \text{COMPLEXMAG}(yh)$ 
4:    $yl \leftarrow \text{AVGPOOL}2 \times 2(yl)$  ▷ Downsample and recentre lowpass to match  $U$  size
5:    $Z \leftarrow \text{CONCATENATE}(yl, U)$  ▷ concatenated along the channel dimension
6:    $Y \leftarrow AZ$  ▷ Mix
7:   save  $Z$  ▷ For the backwards pass
8:   return  $Y$ 
9: end procedure

```

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$J$  scale decomposition. A DTCWT has 4 DWTs for a 2-D input, so its cost is  $8L(1 - 2^{-2J})$ , with  $L = 6$  a common size for the filters. It is important to note that unlike the filtering operation, this does not scale with  $C_{l+1}$ , the end result being that as  $C_{l+1}$  grows, the cost of  $C_l$  forward transforms is outweighed by that of the mixing process.

Because we are using a decimated wavelet decomposition, the sample rate decreases after each wavelet layer. The benefit of this is that the mixing process then only works on one quarter the spatial size after one first scale and one sixteenth the spatial after the second scale. Restricting ourselves to  $J = 1$  as we mentioned in [section 1.4](#), the computational cost is then:

$$\underbrace{\frac{7}{4}C_{l+1}}_{\text{mixing}} + \underbrace{36}_{\text{DTCWT}} \quad \text{multiplies per input pixel} \quad (1.4.3)$$

In most CNNs,  $C_{l+1}$  is several dozen if not several hundred, which makes [Equation 1.4.3](#) significantly smaller than  $L^2C_{l+1} = 9C_{l+1}$  multiplies for  $3 \times 3$  convolutions.

### 1.4.3 Forward and Backward Algorithm

There are several hyperparameters to choose in our layer, in particular:

- The number of output channels  $C_{l+1}$ . This may be restricted by the architecture.
- The variance of the weight initialization for the mixing matrix  $A$ .
- The optimizer parameters such as learning rate, momentum and weight decay.

These are also considerations for a regular CNN layer as well, so we can use previous techniques to guide our choices here. We will explore the effects of these choices in a later section. Assuming we have already chosen hyperparameters then the forward and backward algorithms can be computed with [Algorithms 1](#) and [2](#) respectively.

## 1.5 Experiments

In this section we examine the effectiveness of our invariant layer by testing its performance on the well known datasets (listed in the order of increasing difficulty):

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**Algorithm 2** Locally Invariant Convolutional Layer backward pass

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```
1: procedure LOCALINVBWD( $\frac{\partial L}{\partial Y}$ ,  $A$ )
2:   load  $Z$ 
3:    $\frac{\partial L}{\partial A} \leftarrow \frac{\partial L}{\partial Y} Z^T$ 
4:    $\Delta Z \leftarrow A^T \frac{\partial L}{\partial Y}$ 
5:    $\Delta y_l, \Delta U \leftarrow \text{UNSTACK}(\Delta Z)$ 
6:    $\Delta y_l \leftarrow \text{AVGPOOL2x2BWD}(\Delta y_l)$ 
7:    $\Delta y_h \leftarrow \text{COMPLEXMAGBWD}(\Delta U)$ 
8:    $\frac{\partial L}{\partial x} \leftarrow \text{DTCWTBWD}(\Delta y_l, \Delta y_h)$ 
9:   return  $\frac{\partial L}{\partial x}, \frac{\partial L}{\partial A}$ 
10: end procedure
```

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- MNIST: 10 classes, 6000 images per class,  $28 \times 28$  pixels per image.
- CIFAR-10: 10 classes, 5000 images per class,  $32 \times 32$  pixels per image.
- CIFAR-100: 100 classes, 500 images per class,  $32 \times 32$  pixels per image.
- Tiny ImageNet[12]: 200 classes, 500 images per class,  $64 \times 64$  pixels per image.

Our experiment code is available at [https://github.com/fbcotter/invariant\\_convolution](https://github.com/fbcotter/invariant_convolution).

### 1.5.1 Layer Introduction with MNIST

To begin our experiments on the proposed locally invariant layer, we must look at the previously mentioned hyperparameters such as how to initialize the mixing matrix  $A$ , how fast we should update the parameters, and whether or not we should use momentum. On top of this, how well does the layer scale with the number of output channels  $C_{l+1}$ ?

To achieve this, we build a very simple four layer network on MNIST. The first two layers are our proposed invariant layer, the third layer is a random projection layer that unravels the invariant layer output and projects down to the number of classes (10). This layer is not learned, so does not add any power to the classifier. The fourth and final layer is a small learned fully connected layer, that takes the 10 outputs from the projection layer and outputs 10 new values. This is to facilitate swapping of the values from the previous layer.

On this network we search over a range of learning rates, momenta, weight decay and the variance of the initialized weights in the mixing matrix. We do a grid search over values, but use Hyperband [13] to schedule early stopping of poor runs. We use Tune [14] to do a parallel distributed search over the parameter space. For reference, we also train a system that uses standard convolutional blocks for the first two layers.

Finally, we use fANOVA [15] analysis to find the importance of these hyperparameters to our layer, comparing to a standard layer.

**Not sure how to report results.** The takeaway message I want to give here is that I found out that the most important hyperparameters were the learning rate and the momentum. We need the learning rate to be larger than for the conv case, about 0.5 is a good start. Momentum should be slightly smaller, with the best value at 0.25. The initialization scheme we found to be the best for the weights was a uniform method based on [16]:



$$A_{ij} \sim U \left[ -\sqrt{\frac{6}{C_l + C_{l+1}}}, \sqrt{\frac{6}{C_l + C_{l+1}}} \right] \quad (1.5.1)$$

### 1.5.2 Layer Comparison with CIFAR and Tiny ImageNet

To compare our proposed locally invariant layer (inv) to a regular convolutional layer (conv), we build a simple yet powerful reference network based on VGG. It has six convolutional layers for CIFAR and eight layers for Tiny ImageNet as shown in [Table 1.1](#). The initial number of channels  $C$  we use is 64. Despite this simple design, this reference architecture achieves competitive performance for the three datasets.

This network is optimized with stochastic gradient descent with momentum. The initial learning rate is 0.5, momentum is 0.85, batch size  $N = 128$  and weight decay is  $10^{-4}$ . For CIFAR-10/CIFAR-100 we scale the learning rate by a factor of 0.2 after 60, 80 and 100 epochs, training for 120 epochs in total. For Tiny ImageNet, the rate change is at 18, 30 and 40 epochs (training for 45 in total).

Table 1.1: Base Architecture used for Convolution Layer comparison tests. <sup>†</sup>indicates only used in Tiny ImageNet experiments.

Name	Output Size
convA	$C \times H \times W$
convB	$C \times H \times W$
convC	$2C \times H/2 \times W/2$
convD	$2C \times H/2 \times W/2$
convE	$4C \times H/4 \times W/4$
convF	$4C \times H/4 \times W/4$
convG <sup>†</sup>	$8C \times H/8 \times W/8$
convH <sup>†</sup>	$8C \times H/8 \times W/8$
fc	num classes

We perform an ablation study where we progressively swap out convolutional layers for invariant layers keeping the input and output activation sizes the same. As there are 6 layers (or 8 for Tiny ImageNet), there are too many permutations to list the results for swapping out all layers for our locally invariant layer, so we restrict our results to swapping 1 or 2 layers. [Table 1.2](#) reports the top-1 classification accuracies for CIFAR-10, CIFAR-100 and Tiny ImageNet. In addition to testing on the full datasets we report results for a reduced training set size. In the table, ‘invX’ means that the ‘convX’ layer from [Table 1.1](#) is replaced with an invariant layer.

Interestingly, we see improvements when one or two invariant layers are used near the start of a system, but not for the first layer.

### 1.5.3 Network Comparison

In the previous section, we examined how the locally invariant layer performs when directly swapped in for a convolutional layer in an architecture designed for convolutional layers. In this section, we look at how it performs in a Hybrid ScatterNet-like [\[1\]](#), [\[2\]](#), network.

Table 1.2: Results for testing VGG like architecture with convolutional and invariant layers on several datasets. An architecture with ‘invX’ means the equivalent convolutional layer ‘convX’ from Table 1.1 was swapped for our proposed layer.

Trainset Size	CIFAR-10		CIFAR-100		Tiny ImgNet	
	10k	50k	10k	50k	20k	100k
ref	84.4	91.9	53.2	70.3	37.4	59.1
invA	82.8	91.3	48.4	69.5	33.2	57.7
invB	84.8	91.8	54.8	70.7	36.9	59.5
invC	85.3	92.3	54.2	71.2	37.1	59.8
invD	83.8	91.2	51.2	70.1	37.6	59.3
invE	83.3	91.6	50.3	70.0	37.8	59.4
invF	82.1	90.5	47.6	68.9	34.0	57.8
invA, invB	83.1	90.5	49.8	68.4	35.0	57.9
invB, invC	83.8	91.2	50.6	69.1	34.6	57.5
invC, invD	85.1	92.1	54.3	70.1	<b>37.9</b>	59.0
invD, invE	80.2	89.1	49.0	67.3	33.9	57.5
invA, invC	82.8	90.7	49.5	69.6	34.0	56.9
invB, invD	<b>85.4</b>	<b>92.7</b>	<b>54.6</b>	<b>71.3</b>	<b>37.9</b>	59.8
invC, invE	84.8	91.8	53.5	70.9	37.6	<b>60.2</b>

To build the second order ScatterNet, we stack two invariant layers on top of each other. For 3 input channels, the output of these layers has  $3(1 + 6 + 6 + 36) = 147$  channels at  $1/16$  the spatial input size. We then use 4 convolutional layers, similar to convC to convF in Table 1.1 with  $C = 96$ . In addition, we use dropout after these later convolutional layers with drop probability  $p = 0.3$ .

We compare a ScatterNet with no learning in between scattering orders (ScatNet A) to one with our proposal for a learned mixing matrix  $A$  (ScatNet B). Finally, we also test the hypothesis seen from subsection 1.5.2 about putting conv layers before an inv layer, and test a version with a small convolutional layer before ScatNets A and B, taking the input from 3 to 16 channels, and call these ScatNet architectures ScatNet C and D respectively.

See Table 1.3 for results from these experiments. It is clear from the improvements that the mixing layer helps the Scattering front end. Interestingly, ScatNet C and D further improve on the A and B versions (albeit with a larger parameter and multiply cost than the mixing operation). This reaffirms that there may be benefit to add learning before as well as inside the ScatterNet.

For comparison, we have also listed the performance of other architectures as reported by their authors in order of increasing complexity. Our proposed ScatNet D achieves comparable performance with the the All Conv, VGG16 and FitNet architectures. The Deep[17] and Wide[18] ResNets perform best, but with very many more multiplies, parameters and layers.

ScatNets A to D with 6 layers like convC to convG from Table 1.1 after the scattering, achieve 58.1, 59.6, 60.8 and 62.1% top-1 accuracy on Tiny ImageNet. As these have more parameters and multiplies from the extra layers we exclude them from Table 1.3.

Table 1.3: Hybrid ScatterNet top-1 classification accuracies on CIFAR-10 and CIFAR-100.  $N_l$  is the number of learned convolutional layers, #param is the number of parameters, and #mults is the number of multiplies per  $32 \times 32 \times 3$  image. An asterisk indicates that the value was estimated from the architecture description.

	$N_l$	#param	#mults	CIFAR-10	CIFAR-100
ScatNet A	4	2.6M	165M	89.4	67.0
ScatNet B	6	2.7M	167M	91.1	70.7
ScatNet C	5	3.7M	251M	91.6	70.8
ScatNet D	7	4.3M	294M	93.0	73.5
All Conv[19]	8	1.4M	281M*	92.8	66.3
VGG16[20]	16	138M*	313M*	91.6	-
FitNet[21]	19	2.5M	382M	91.6	65.0
ResNet-1001[17]	1000	10.2M	4453M*	95.1	77.3
WRN-28-10[18]	28	36.5M	5900M*	96.1	81.2

## 1.6 Conclusion

In this work we have proposed a new learnable scattering layer, dubbed the locally invariant convolutional layer, tying together ScatterNets and CNNs. We do this by adding a mixing between the layers of ScatterNet allowing the learning of more complex shapes than the ripples seen in [22]. This invariant layer can easily be shaped to allow it to drop in the place of a convolutional layer, theoretically saving on parameters and computation. However, care must be taken when doing this, as our ablation study showed that the layer only improves upon regular convolution at certain depths. Typically, it seems wise to interleave convolutional layers and invariant layers.

We have developed a system that allows us to pass gradients through the Scattering Transform, something that previous work has not yet researched. Because of this, we were able to train end-to-end a system that has a ScatterNet surrounded by convolutional layers and with our proposed mixing. We were surprised to see that even a small convolutional layer before Scattering helps the network, and a very shallow and simple Hybrid-like ScatterNet was able to achieve good performance on CIFAR-10 and CIFAR-100.

There is still much research to do - why does the proposed layer work best near, but not at the beginning of deeper networks? Why is it beneficial to precede an invariant layer with a convolutional layer? Can we combine invariant layers in Residual style architectures? The work presented here is still nascent but we hope that it will stimulate further interest and research into both ScatterNets and the design of convolutional layers in CNNs.

## 1.7 More to Come

Need to look at what the first layer of the ScatNet D system looks like.

Need to do some analysis on the gradient of a magnitude operation (not previously done).

Can look at the structure of the A matrix.

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