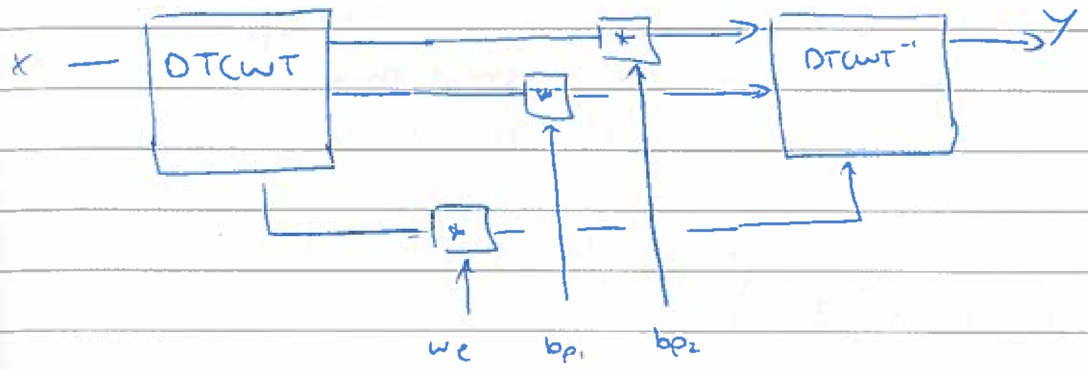
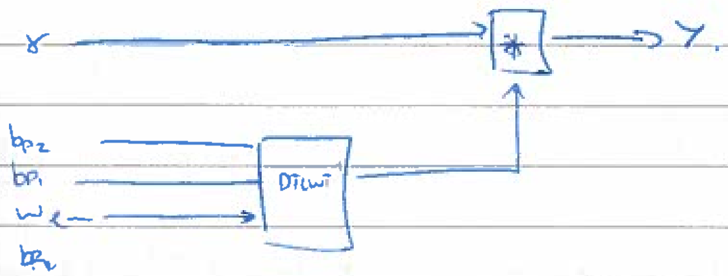
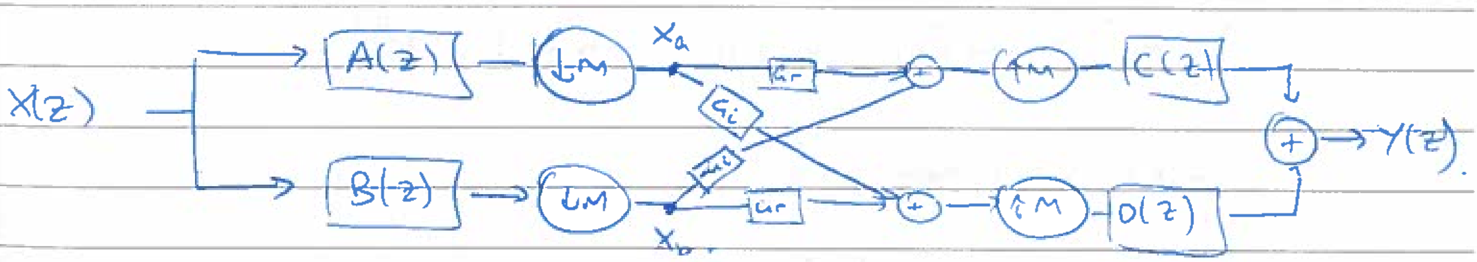
DTCWT convolution

Comparison of Type 2 to Type 3 Layers:





Second type is slightly easier to analyse.
Let us consider a single subband to start.



For second type, there is no input, we only have.

$G_r(z), G_i(z)$

$$H = G_r(z^M)C(z) + G_i(z^M)D(z)$$

$$\boxed{\therefore\; Y(z) = X(z)\left[G_r(z^M)C(z) + G_i(z^M)D(z)\right].}$$

note → there's a mistake in Nick's paper: all the W_M^m terms should be W_M^{-m}. This doesn't affect things

Now, first system:

$$X_a = \frac{1}{M}\sum_m X(W_M^m z^{1/M})A(W_M^m z^{1/M})$$

$$X_b = \frac{1}{M}\sum_m X(W_M^m z^{1/M})B(W_M^m z^{1/M})$$

$$Y_a = X_a G_r - X_b G_i = \frac{1}{M}\sum_m X(W_M^m z^{1/M})\left[A(W_M^m z^{1/M})G_r(z) - B(W_M^m z^{1/M})G_i(z)\right]$$

$$Y_b = X_a G_i + X_b G_r = \frac{1}{M}\sum_m X(W_M^m z^{1/M})\left[A(W_M^m z^{1/M})G_i(z) + B(W_M^m z^{1/M})G_r(z)\right]$$

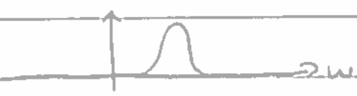
Now $Y = C(z)Y_a(z^M) + D(z)Y_b(z^M)$

$$= G_r(z^M)\left[\frac{1}{M}\sum_m X(W_M^m z)\left[A(W_M^m z)C(z) + B(W_M^m z)D(z)\right]\right]$$

$$+ G_i(z^M)\left[\frac{1}{M}\sum_m X(W_M^m z)\left[A(W_M^m z)D(z) - B(W_M^m z)C(z)\right]\right]$$

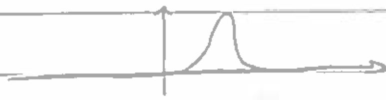
From Nick's 2001 paper:

$$P(z) = \sum_n (p_{nr} + jp_{ni})z^{-n}$$

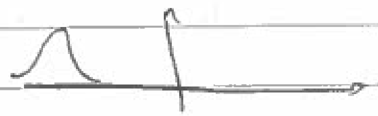


$$P^*(z) = \sum_n (p_{nr} - jp_{ni})z^{-n}$$

$$Q(z) = \sum_n (q_{nr} + jq_{ni})z^{-n}$$



$$Q^*(z) = \sum_n (q_{nr} - jq_{ni})z^{-n}$$

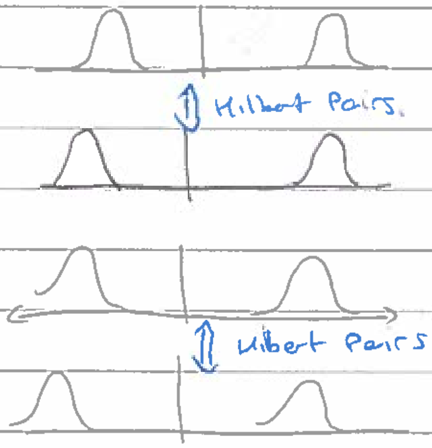


$$A(z) = 2\,\mathrm{Re}[P(z)] = P(z) + P^*(z)$$

$$B(z) = 2\,\mathrm{Im}[P(z)] = -j[P(z) - P^*(z)]$$

$$C(z) = 2\,\mathrm{Re}[Q(z)] = Q(z) + Q^*(z)$$

$$D(z) = -2\,\mathrm{Im}[Q(z)] = j[Q(z) - Q^*(z)]$$



$$\sum_m A(W^m z)C(z) + B(W^m z)D(z) = 2P(z)Q(z) + 2P^*(z)Q^*(z)$$

$$\sum_m A(W^m z)D(z) - B(W^m z)C(z) = j\left(2P(z)Q(z) - 2P^*(z)Q^*(z)\right).$$

So
$$Y = G_r(z^M)\frac{1}{M}X(z)\,2[P(z)Q(z) + P^*(z)Q^*(z)]$$
$$+ G_i(z^M)\frac{1}{M}X(z)\,2j[P(z)Q(z) - P^*(z)Q^*(z)]$$

$$= \frac{2}{M}X(z)\left[G_r(z^M)(PQ + P^*Q^*) + jG_i(z^M)(PQ - P^*Q^*)\right].$$

c.f.

$$Y = X(z)[G_r(z^M)C(z) + G_i(z^M)D(z)]$$

$$= X(z)[G_r(z^M)(Q + Q^*) + jG_i(z^M)(Q - Q^*)].$$

↳ they're ~~ce~~ closer than I thought. Just have to compare

$PQ + P^*Q^*$ & $PQ - P^*Q^*$

to $Q + Q^*$ to. $Q - Q^*$.