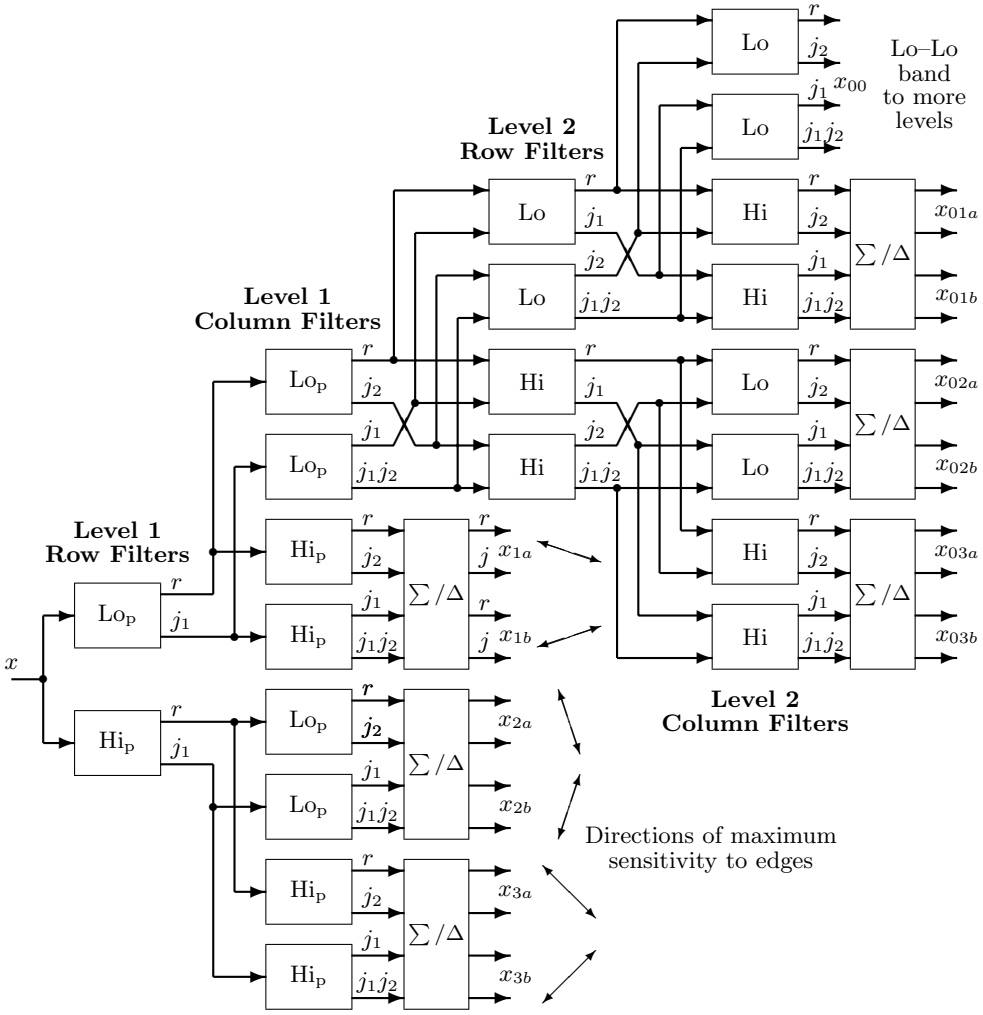

Figure 7. Two levels of the complex wavelet tree for a real two-dimensional input image x, giving six directional bands at each level (the directions are shown for level 1). Components of four-element 'complex' vectors are labelled r, j_1, j_2, j_1j_2.

PR at all levels of the transform. Although such filters can be designed to give PR quite easily at level 1 of the tree by applying the constraint that the reconstructed output signal must be real, a similar constraint cannot be applied at further levels where inputs *and* outputs are complex. For PR below level 1, the set of four filters in figure 1*b* must have a flat overall frequency response. However, this is not possible if all of the filters tend to reject negative frequencies. Hence, a different approach to generating a complex filter tree is needed.

In Kingsbury (1998*a*, *b*), we introduced the DT CWT, which added perfect reconstruction to the other attractive properties of complex wavelets: shift invariance; good directional selectivity; limited redundancy; and efficient order-N computation.

The dual-tree transform was developed by noting that approximate shift invariance can be achieved with a *real* DWT by doubling the sampling rate at each level of the