

13. Advanced Rendering

Overview

- Ray Tracing
- Radisoity
- Parallel Rendering
- Reading: ANG ch. 13

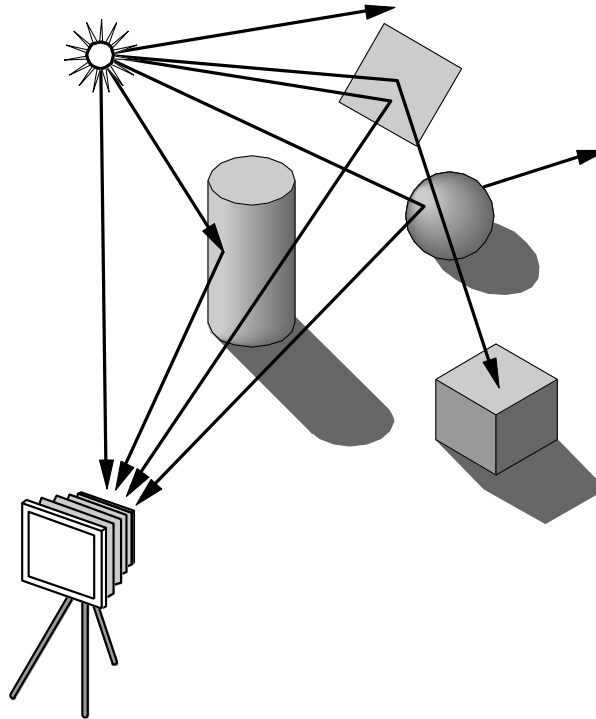
Ray Tracing

Introduction

- OpenGL is based on a pipeline model in which primitives are rendered one at time
 - No shadows (except by tricks or multiple renderings)
 - No multiple reflections
- Global approaches
 - Rendering equation
 - Ray tracing
 - Radiosity

Ray Tracing

- Follow rays of light from a point source
- Can account for reflection and transmission

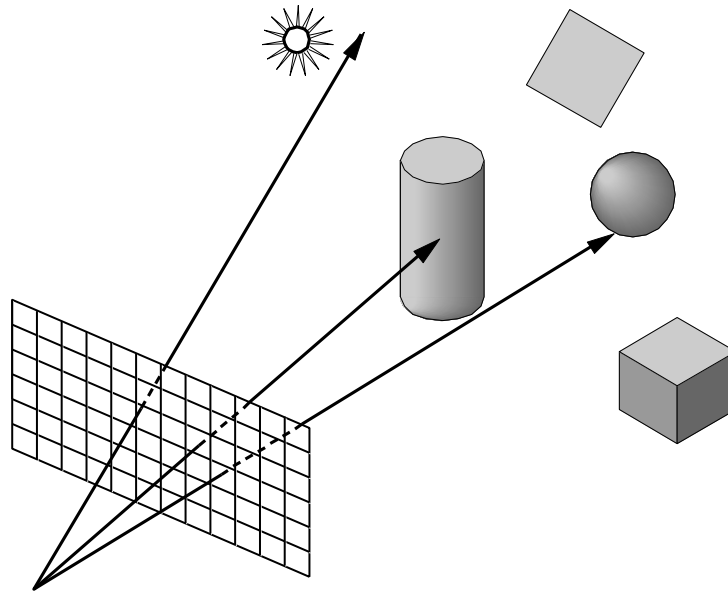


Computation

- Should be able to handle all physical interactions
- Ray tracing paradigm is not computational
- Most rays do not affect what we see
- Scattering produces many (infinite) additional rays
- Alternative: ray casting

Ray Casting

- Only rays that reach the eye matter
- Reverse direction and cast rays
- Need at least one ray per pixel

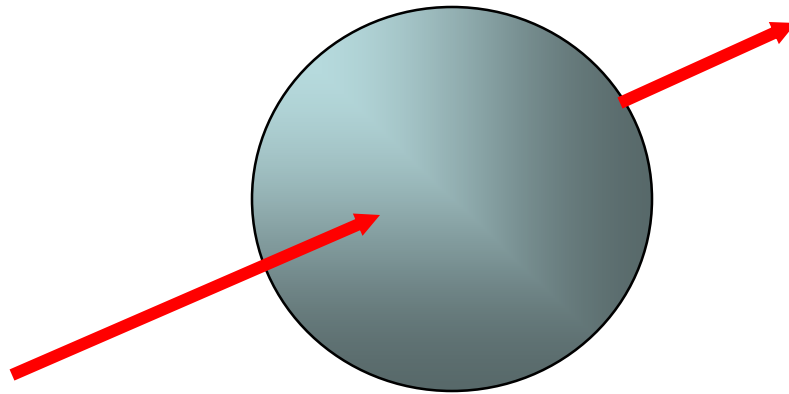


Ray Casting Quadrics

- Ray casting has become the standard way to visualize quadrics which are implicit surfaces in CSG systems
- Constructive Solid Geometry
 - Primitives are solids
 - Build objects with set operations
 - Union, intersection, set difference

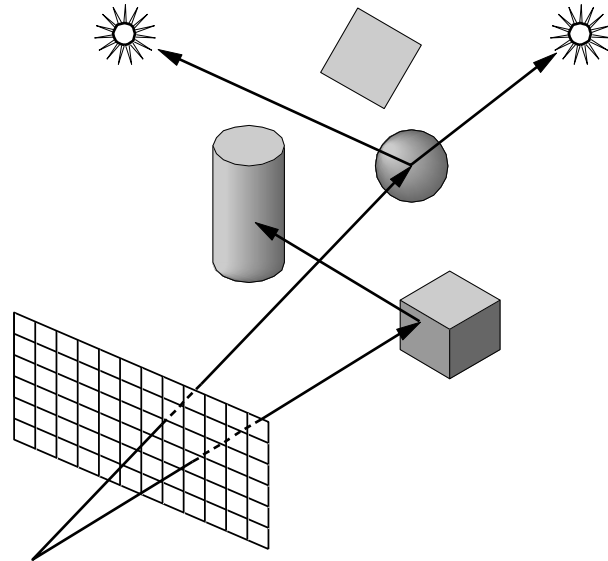
Ray Casting a Sphere

- Ray is parametric
- Sphere is quadric
- Resulting equation is a scalar quadratic equation which gives entry and exit points of ray (or no solution if ray misses)



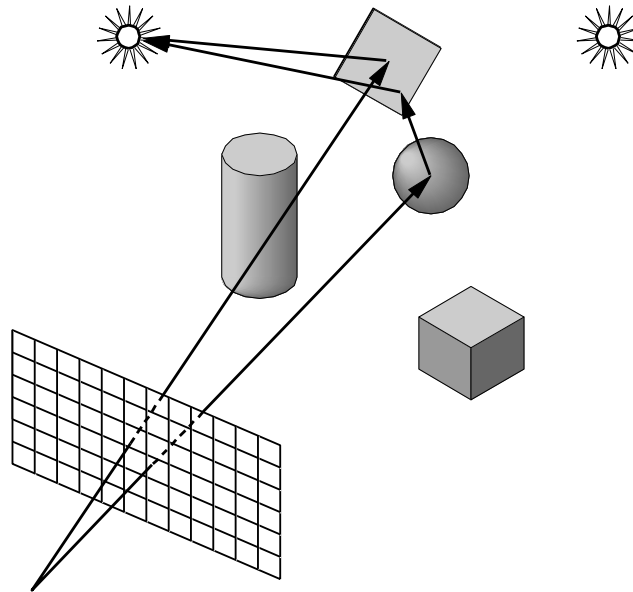
Shadow Rays

- Even if a point is visible, it will not be lit unless we can see a light source from that point
- Cast shadow or feeler rays

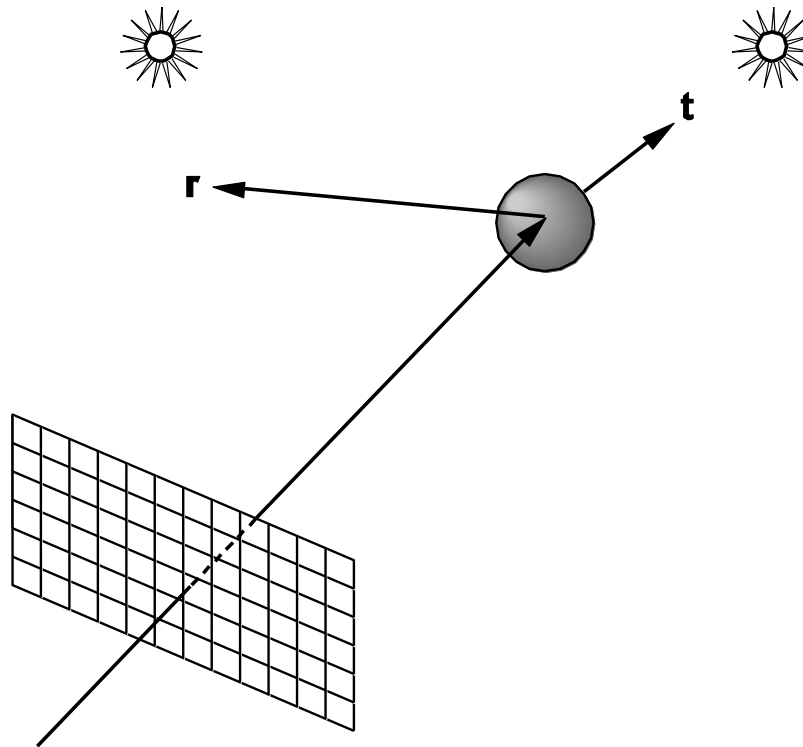


Reflection

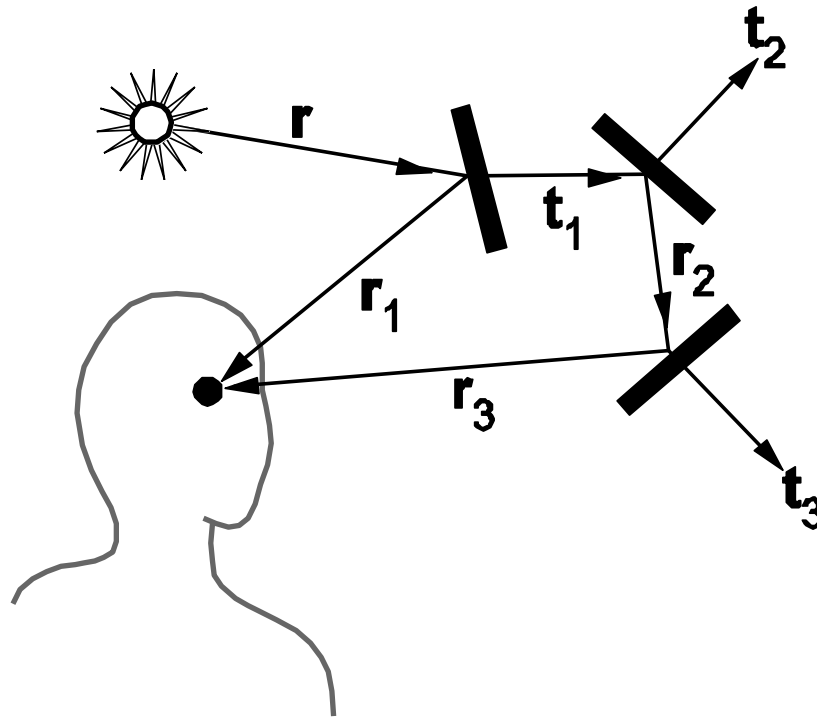
- Must follow shadow rays off reflecting or transmitting surfaces
- Process is recursive



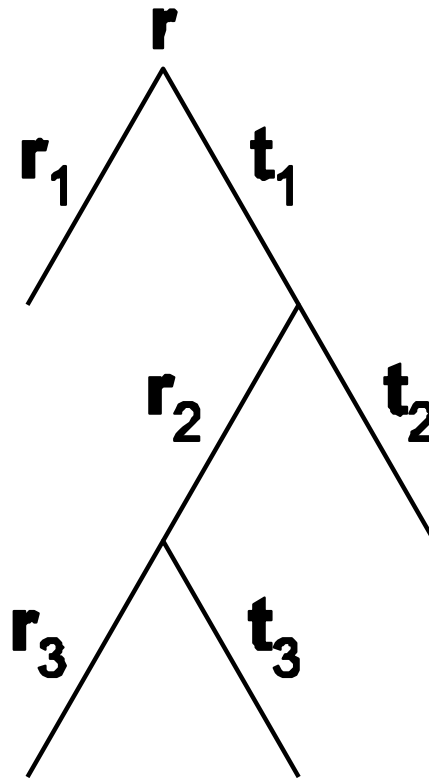
Reflection and Transmission



Ray Trees



Ray Tree



Diffuse Surfaces

- Theoretically the scattering at each point of intersection generates an infinite number of new rays that should be traced
- In practice, we only trace the transmitted and reflected rays but use the Phong model to compute shade at point of intersection
- Radiosity works best for perfectly diffuse (Lambertian) surfaces

Building a Ray Tracer

- Best expressed **recursively**
- Can remove recursion later
- Image based approach
 - For each ray
- Find intersection with closest surface
 - Need whole object database available
 - Complexity of calculation limits object types
- Compute lighting at surface
- Trace reflected and transmitted rays

When to stop

- Some light will be absorbed at each intersection
 - Track amount left
- Ignore rays that go off to infinity
 - Put large sphere around problem
- Count steps

Recursive Ray Tracer

```
color c = trace(point p, vector  
    d, int step)  
{  
    color local, reflected,  
    transmitted;  
    point q;  
    normal n;  
    if(step > max)  
return(background_color);
```

Recursive Ray Tracer

```
q = intersect(p, d, status);  
if(status==light_source)  
    return(light_source_color);  
if(status==no_intersection)  
    return(background_color);
```

```
n = normal(q);  
r = reflect(q, n);  
t = transmit(q,n);
```

Recursive Ray Tracer

```
local = phong(q, n, r);  
reflected = trace(q, r, step+1);  
transmitted = trace(q, t,  
    step+1);  
  
return(local+reflected+  
    transmitted);
```

Computing Intersections

- Implicit Objects
 - Quadrics
- Planes
- Polyhedra
- Parametric Surfaces

Implicit Surfaces

Ray from \mathbf{p}_0 in direction \mathbf{d}

$$\mathbf{p}(t) = \mathbf{p}_0 + t \mathbf{d}$$

General implicit surface

$$f(\mathbf{p}) = 0$$

Solve scalar equation

$$f(\mathbf{p}(t)) = 0$$

General case requires numerical methods

Quadrics

General quadric can be written as

$$\mathbf{p}^T \mathbf{A} \mathbf{p} + \mathbf{b}^T \mathbf{p} + c = 0$$

Substitute equation of ray

$$\mathbf{p}(t) = \mathbf{p}_0 + t \mathbf{d}$$

to get quadratic equation

Sphere

$$(\mathbf{p} - \mathbf{p}_c) \cdot (\mathbf{p} - \mathbf{p}_c) - r^2 = 0$$

$$\mathbf{p}(t) = \mathbf{p}_0 + t \mathbf{d}$$

$$\mathbf{p}_0 \cdot \mathbf{p}_0 t^2 + 2 \mathbf{p}_0 \cdot (\mathbf{d} - \mathbf{p}_0) t + (\mathbf{d} - \mathbf{p}_0) \cdot (\mathbf{d} - \mathbf{p}_0) - r^2 = 0$$

Planes

$$\mathbf{p} \cdot \mathbf{n} + c = 0$$

$$\mathbf{p}(t) = \mathbf{p}_0 + t \mathbf{d}$$

$$t = -(\mathbf{p}_0 \cdot \mathbf{n} + c) / \mathbf{d} \cdot \mathbf{n}$$

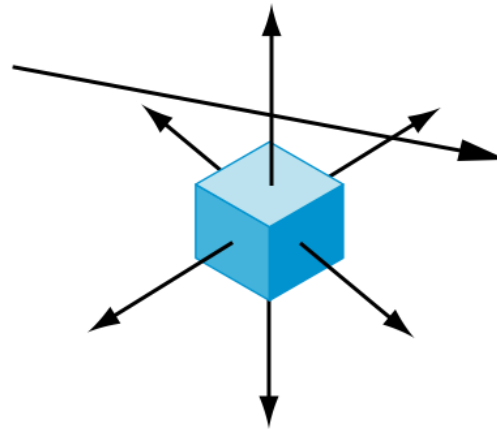
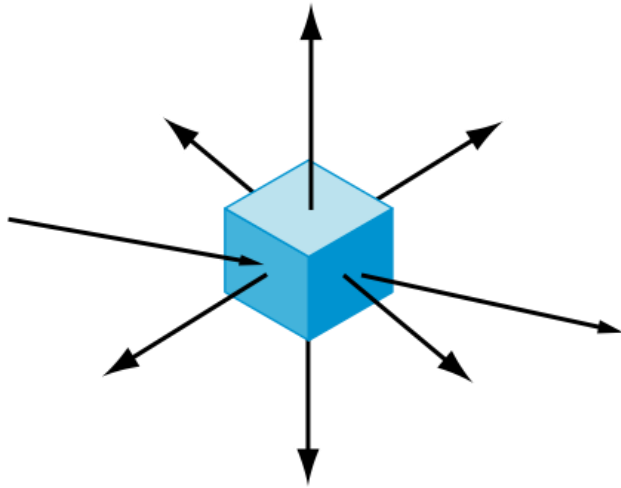
Polyhedra

- Generally we want to intersect with closed objects such as polygons and polyhedra rather than planes
- Hence we have to worry about inside/outside testing
- For convex objects such as polyhedra there are some fast tests

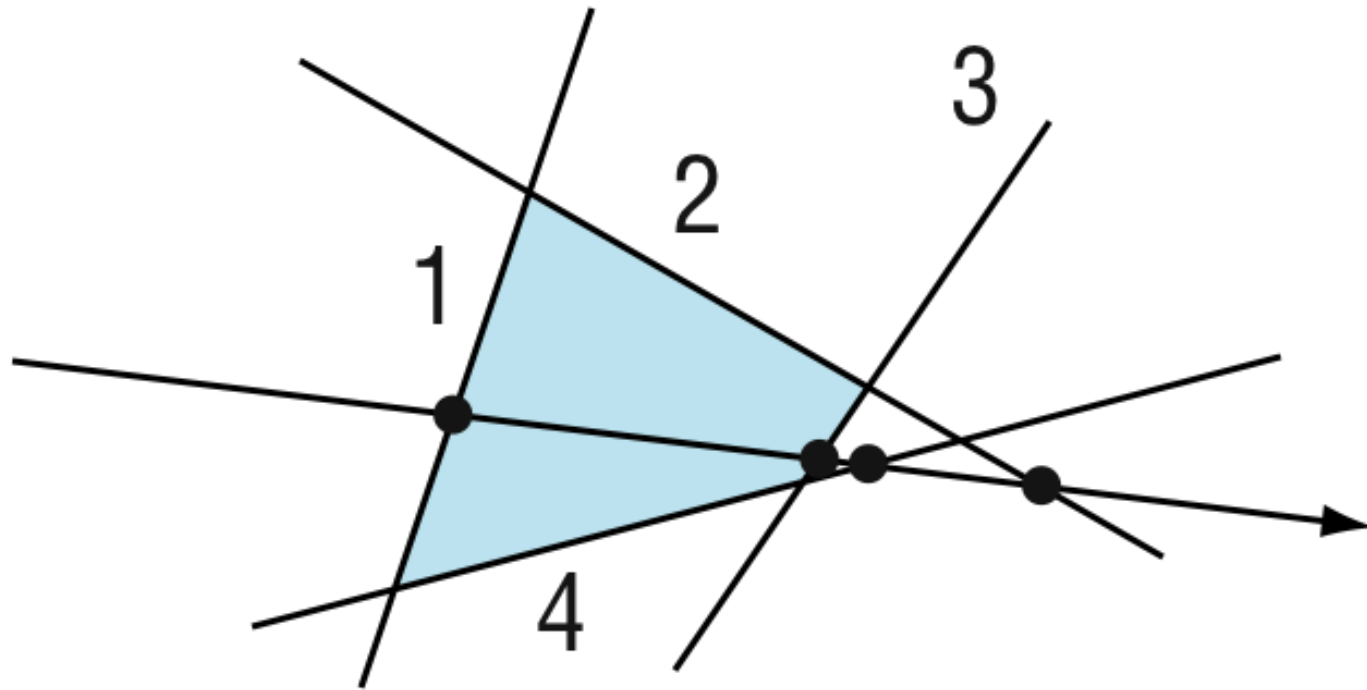
Ray Tracing Polyhedra

- If ray enters an object, it must enter a front facing polygon and leave a back facing polygon
- Polyhedron is formed by intersection of planes
- Ray enters at furthest intersection with front facing planes
- Ray leaves at closest intersection with back facing planes
- If entry is further away than exit, ray must miss the polyhedron

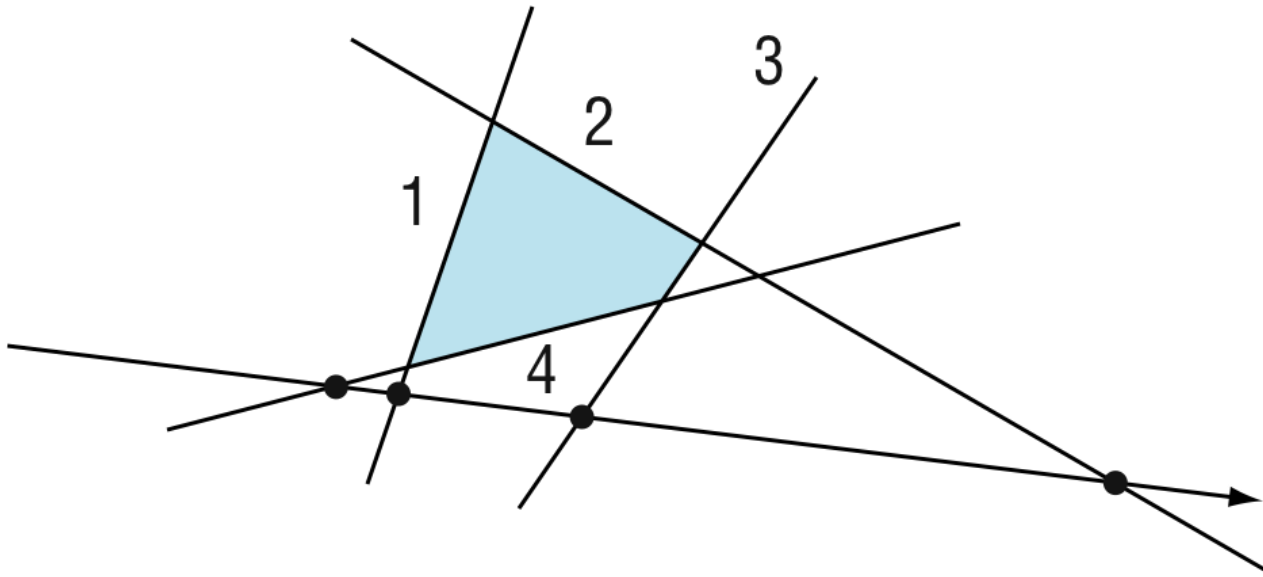
Ray Tracing Polyhedra



Ray Tracing a Polygon



Ray Tracing a Polygon



Radisoity

Introduction

- Ray tracing is best with many highly specular surfaces
 - Not characteristic of real scenes
- Rendering equation describes general shading problem
- Radiosity solves rendering equation for perfectly diffuse surfaces

Terminology

- Energy \sim light (incident, transmitted)
 - Must be conserved
- Energy flux = luminous flux = power = energy/unit time
 - Measured in **lumens**
 - Depends on wavelength so we can integrate over spectrum using **luminous efficiency curve** of sensor
- Energy density (Φ) = energy flux/unit area

Terminology

Intensity ~ brightness

- Brightness is perceptual

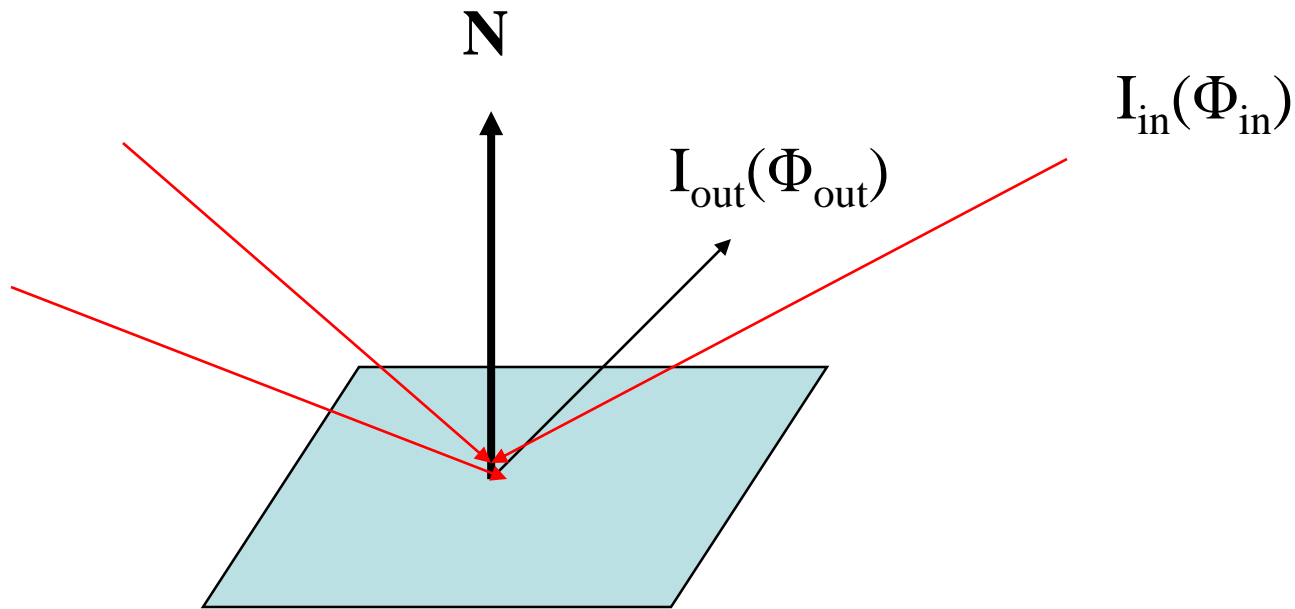
= flux/area-solid angle = power/unit projected area per solid angle

- Measured in **candela**

$$\Phi = \int \int I \, dA \, d\omega$$

Rendering Eqn (Kajiya)

- Consider a point on a surface



Rendering Equation

- Outgoing light is from two sources
 - Emission
 - Reflection of incoming light
- Must integrate over all incoming light
 - Integrate over hemisphere
- Must account for foreshortening of incoming light

Rendering Equation

$$I_{\text{out}}(\Phi_{\text{out}}) = E(\Phi_{\text{out}}) + \int_{2\pi} R_{\text{bd}}(\Phi_{\text{out}}, \Phi_{\text{in}}) I_{\text{in}}(\Phi_{\text{in}}) \cos \theta \, d\omega$$

emission

angle between normal and Φ_{in}

bidirectional reflection coefficient

Note that angle is really two angles in 3D and wavelength is fixed

Rendering Equation

- Rendering equation is an energy balance
 - Energy in = energy out
- Integrate over hemisphere
- Fredholm integral equation
 - Cannot be solved analytically in general
- Various approximations of R_{bd} give standard rendering models
- Should also add an occlusion term in front of right side to account for other objects blocking light from reaching surface

Another version

Consider light at a point \mathbf{p} arriving from \mathbf{p}'

$$i(\mathbf{p}, \mathbf{p}') = v(\mathbf{p}, \mathbf{p}')(\epsilon(\mathbf{p}, \mathbf{p}') + \int \rho(\mathbf{p}, \mathbf{p}', \mathbf{p}'') i(\mathbf{p}', \mathbf{p}'') d\mathbf{p}'')$$

occlusion = 0 or $1/d^2$

emission from \mathbf{p}' to \mathbf{p}

light reflected at \mathbf{p}' from all
points \mathbf{p}'' towards \mathbf{p}

Radiosity

- Consider objects to be broken up into flat patches (which may correspond to the polygons in the model)
- Assume that patches are perfectly diffuse reflectors
- Radiosity = flux = energy/unit area/ unit time leaving patch

Notation

n patches numbered 1 to n

b_i = radiosity of patch i

a_i = area patch i

total intensity leaving patch $i = b_i a_i$

$e_i a_i$ = emitted intensity from patch i

ρ_i = reflectivity of patch i

f_{ij} = form factor = fraction of energy leaving patch j that reaches patch i

Radiosity Equation

energy balance

$$b_i a_i = e_i a_i + \rho_i \sum f_{ji} b_j a_j$$

reciprocity

$$f_{ij} a_i = f_{ji} a_j$$

radiosity equation

$$b_i = e_i + \rho_i \sum f_{ij} b_j$$

Matrix Form

$$\mathbf{b} = [b_i]$$

$$\mathbf{e} = [e_i]$$

$$\mathbf{R} = [r_{ij}] \quad r_{ij} = \rho_i \text{ if } i \neq j \quad r_{ii} = 0$$

$$\mathbf{F} = [f_{ij}]$$

Matrix Form

$$\mathbf{b} = \mathbf{e} - \mathbf{RFb}$$

formal solution

$$\mathbf{b} = [\mathbf{I} - \mathbf{RF}]^{-1} \mathbf{e}$$

Not useful since n is usually very large

Alternative: use observation that F is sparse

We will consider determination of form factors later

Solving the Radiosity Equation

For sparse matrices, iterative methods usually require only $O(n)$ operations per iteration

Jacobi's method

$$\mathbf{b}^{k+1} = \mathbf{e} - \mathbf{R}\mathbf{F}\mathbf{b}^k$$

Gauss-Seidel: use immediate updates

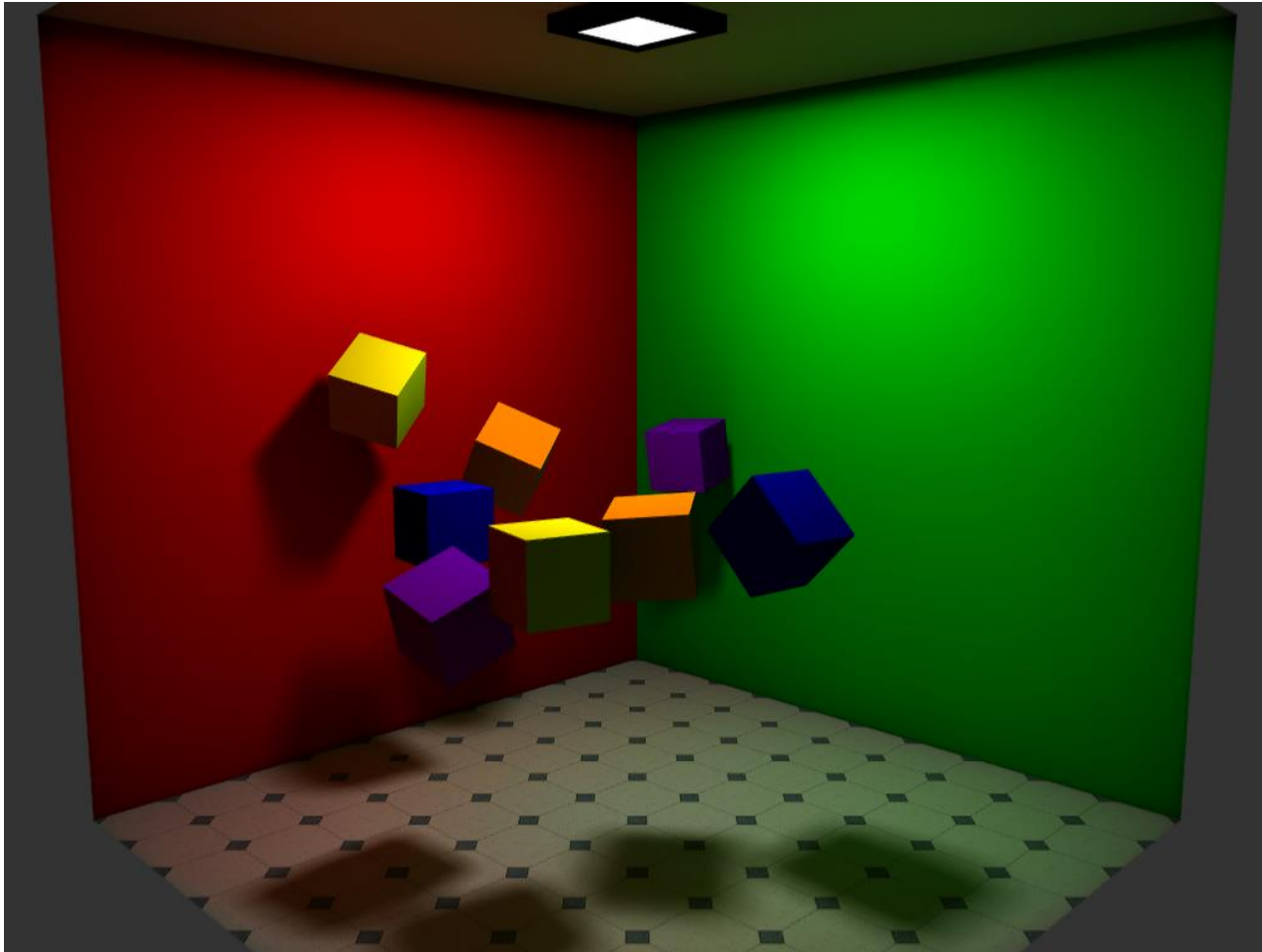
Series Approximation

$$1/(1-x) = 1 + x + x^2 + \dots$$

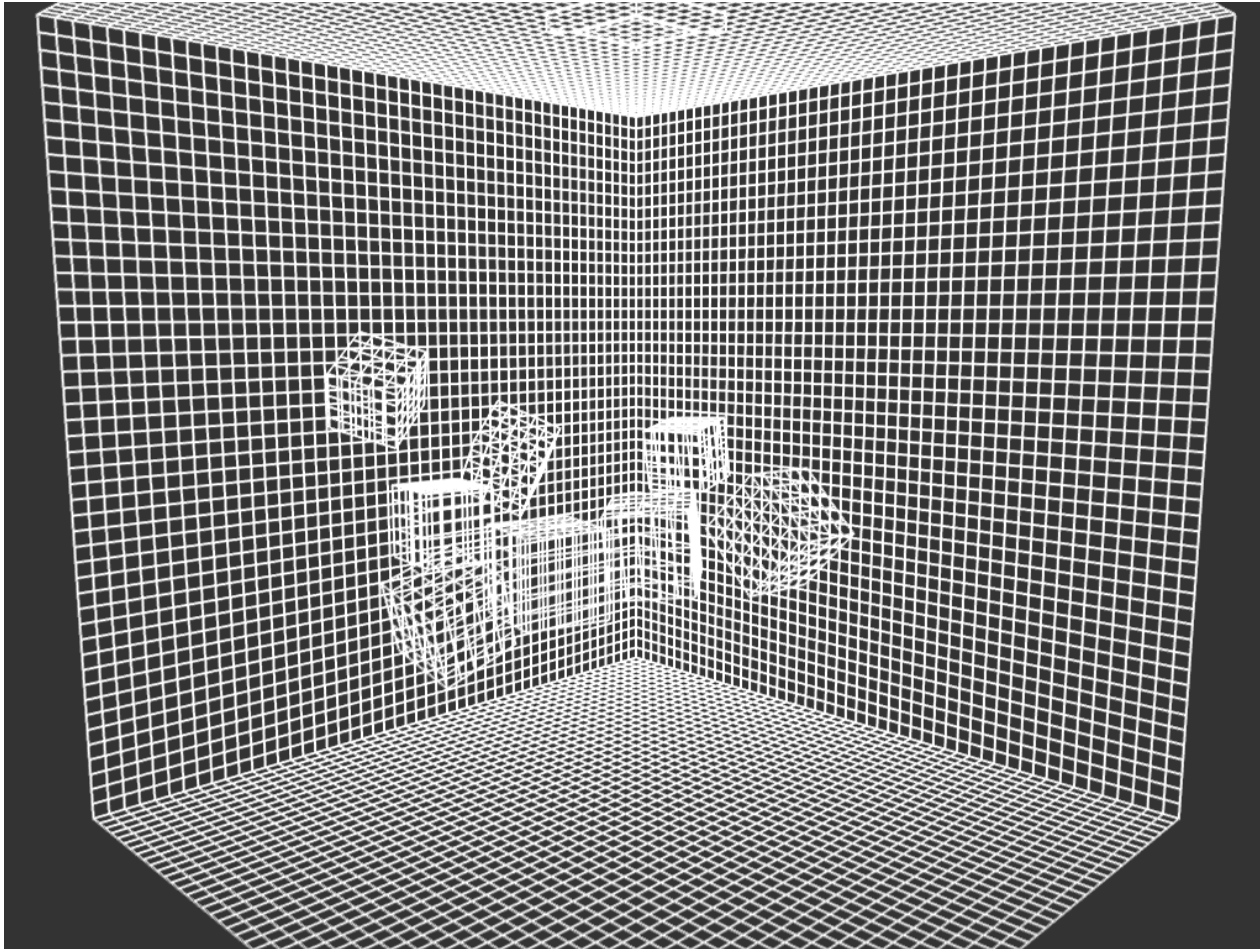
$$[\mathbf{I} - \mathbf{RF}]^{-1} = \mathbf{I} + \mathbf{RF} + (\mathbf{RF})^2 + \dots$$

$$\mathbf{b} = [\mathbf{I} - \mathbf{RF}]^{-1} \mathbf{e} = \mathbf{e} + \mathbf{RFe} + (\mathbf{RF})^2 \mathbf{e} + \dots$$

Rendered Image

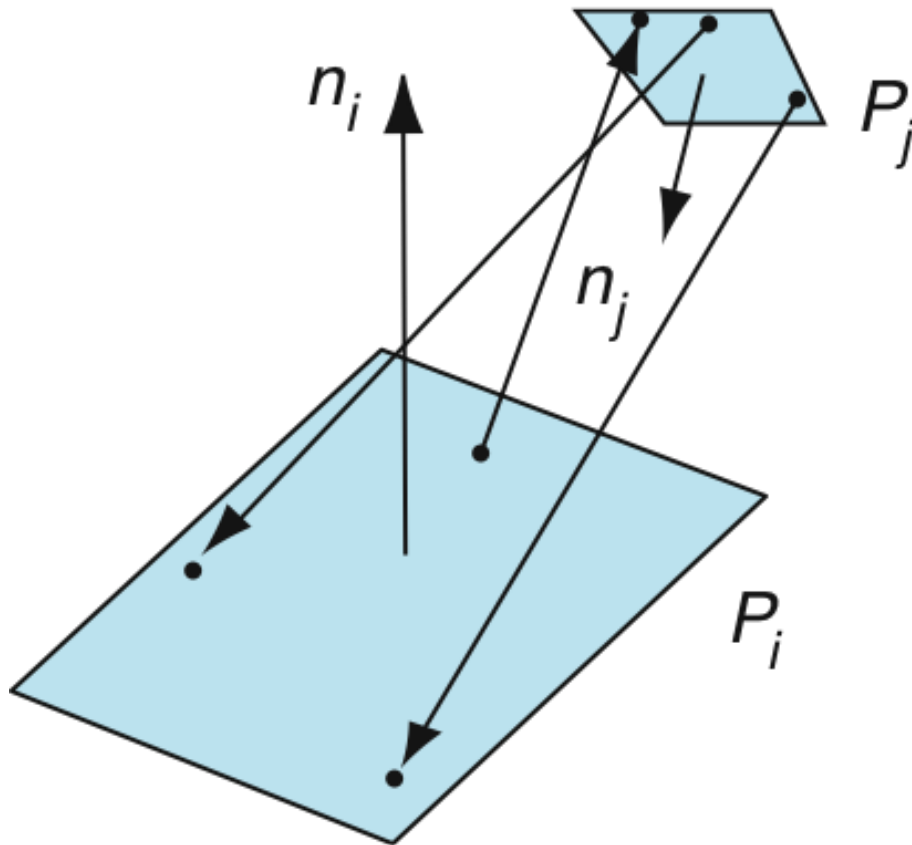


Patches

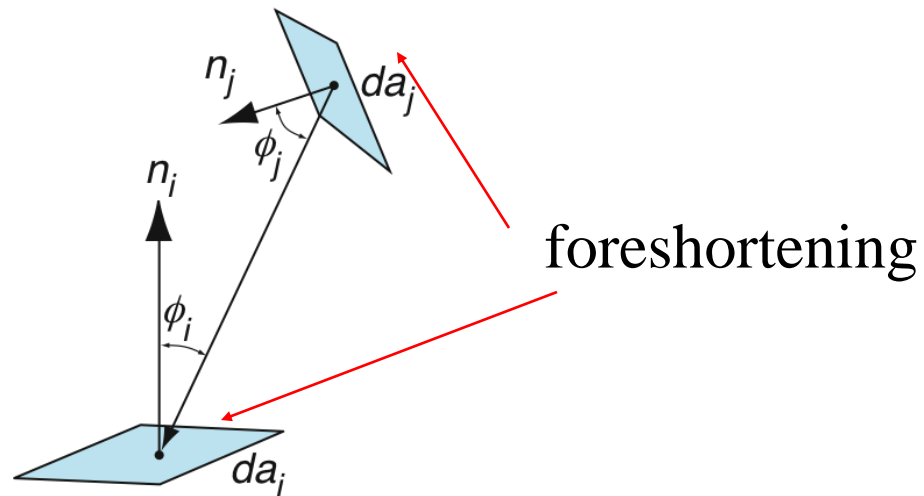


Computing Form Factors

- Consider two flat patches



Using Differential Patches



Form Factor Integral

$$f_{ij} = (1/a_i) \int_{a_i} \int_{a_i} (o_{ij} \cos \theta_i \cos \theta_j / \pi r^2) da_i da_j$$

occlusion



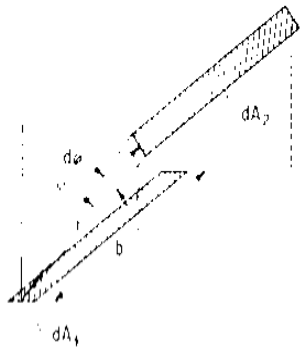
foreshortening of patch i

foreshortening of patch j

Solving the Integral

- There are very few cases where the integral has a (simple) closed form solution
 - Occlusion further complicates solution
- Alternative is to use numerical methods
- Two step process similar to texture mapping
 - Hemisphere
 - Hemicube

Form Factor Examples 1



Strip of finite length b and of differential width, to differential strip of same length on parallel generating line.

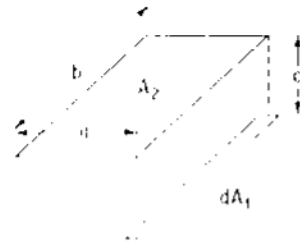
$$dF_{dA_1 \rightarrow dA_2} = \frac{\cos \phi}{\pi} d\phi \tan^{-1} \frac{b}{r}$$



Plane element dA_1 to plane parallel rectangle; normal to element passes through corner of rectangle.

$$X = \frac{a}{c} \quad Y = \frac{b}{c}$$

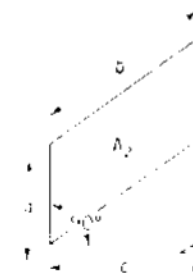
$$F_{dA_1 \rightarrow A_2} = \frac{1}{2\pi} \left[\frac{X}{\sqrt{1+X^2}} \tan^{-1} \frac{Y}{\sqrt{1+X^2}} + \frac{Y}{\sqrt{1+Y^2}} \tan^{-1} \frac{X}{\sqrt{1+Y^2}} \right]$$



Strip element to rectangle in plane parallel to strip; strip is opposite one edge of rectangle.

$$X = \frac{a}{c} \quad Y = \frac{b}{c}$$

$$F_{dA_1 \rightarrow A_2} = \frac{1}{\pi Y} \left[\sqrt{1+Y^2} \tan^{-1} \frac{X}{\sqrt{1+Y^2}} - \tan^{-1} X + \frac{XY}{\sqrt{1+X^2}} \tan^{-1} \frac{Y}{\sqrt{1+X^2}} \right]$$



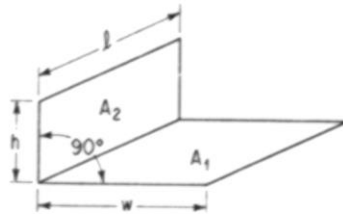
Plane element dA_1 to rectangle in plane 90° to plane of element.

$$X = \frac{a}{b} \quad Y = \frac{c}{b}$$

$$F_{dA_1 \rightarrow A_2} = \frac{1}{2\pi} \left[\tan^{-1} \frac{1}{Y} - \frac{Y}{\sqrt{X^2+Y^2}} \tan^{-1} \frac{1}{\sqrt{X^2+Y^2}} \right]$$

Form Factor Examples 2

12



Two finite rectangles of same length, having one common edge, and at an angle of 90° to each other.

$$H = \frac{h}{l} \quad W = \frac{w}{l}$$

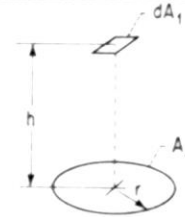
$$F_{1-2} = \frac{1}{\pi W} \left(W \tan^{-1} \frac{1}{W} + H \tan^{-1} \frac{1}{H} - \sqrt{H^2 + W^2} \tan^{-1} \frac{1}{\sqrt{H^2 + W^2}} \right. \\ \left. + \frac{1}{2} \ln \left\{ \left[\frac{(1 + W^2)(1 + H^2)}{(1 + W^2 + H^2)} \right] \left[\frac{W^2(1 + W^2 + H^2)}{(1 + W^2)(W^2 + H^2)} \right]^{W^2} \left[\frac{H^2(1 + H^2 + W^2)}{(1 + H^2)(H^2 + W^2)} \right]^{H^2} \right\} \right)$$

13



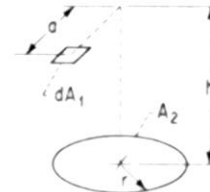
Infinitely long enclosure formed by three plane areas.

$$F_{1-2} = \frac{A_1 + A_2 - A_3}{2A_1}$$



Plane element dA_1 to circular disk in plane parallel to element; normal to element passes through center of disk.

$$F_{d1-2} = \frac{r^2}{h^2 + r^2}$$

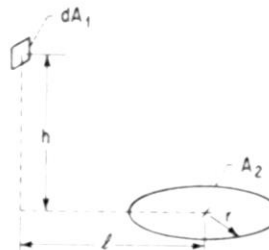


Plane element dA_1 to circular disk in plane parallel to element.

$$H = \frac{h}{a} \quad R = \frac{r}{a}$$

$$Z = 1 + H^2 + R^2$$

$$F_{d1-2} = \frac{1}{2} \left(1 - \frac{1 + H^2 - R^2}{\sqrt{Z^2 - 4R^2}} \right)$$



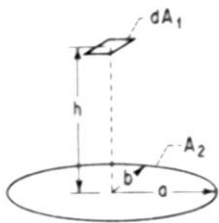
Plane element dA_1 to circular disk; planes containing element and disk intersect at 90° .

$$H = \frac{h}{l} \quad R = \frac{r}{l}$$

$$Z = 1 + H^2 + R^2$$

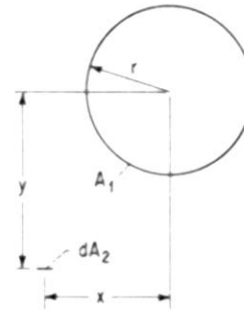
$$F_{d1-2} = \frac{H}{2} \left(\frac{Z}{\sqrt{Z^2 - 4R^2}} - 1 \right)$$

Form Factor Examples 3



Plane element dA_1 to elliptical plate in plane parallel to element; normal to element passes through center of plate.

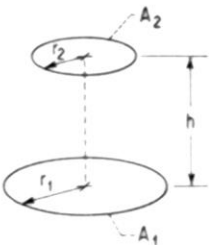
$$F_{d1-2} = \frac{ab}{\sqrt{(h^2 + a^2)(h^2 + b^2)}}$$



Strip element dA_2 of any length to infinitely long cylinder.

$$X = \frac{x}{r} \quad Y = \frac{y}{r}$$

$$F_{d2-1} = \frac{Y}{X^2 + Y^2}$$

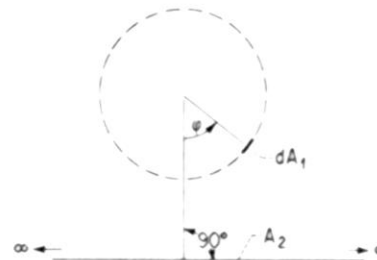


Parallel circular disks with centers along the same normal.

$$R_1 = \frac{r_1}{h} \quad R_2 = \frac{r_2}{h}$$

$$X = 1 + \frac{1 + R_2^2}{R_1^2}$$

$$F_{1-2} = \frac{1}{2} \left[X - \sqrt{X^2 - 4 \left(\frac{R_2}{R_1} \right)^2} \right]$$



Element of any length on cylinder to plane of infinite length and width.

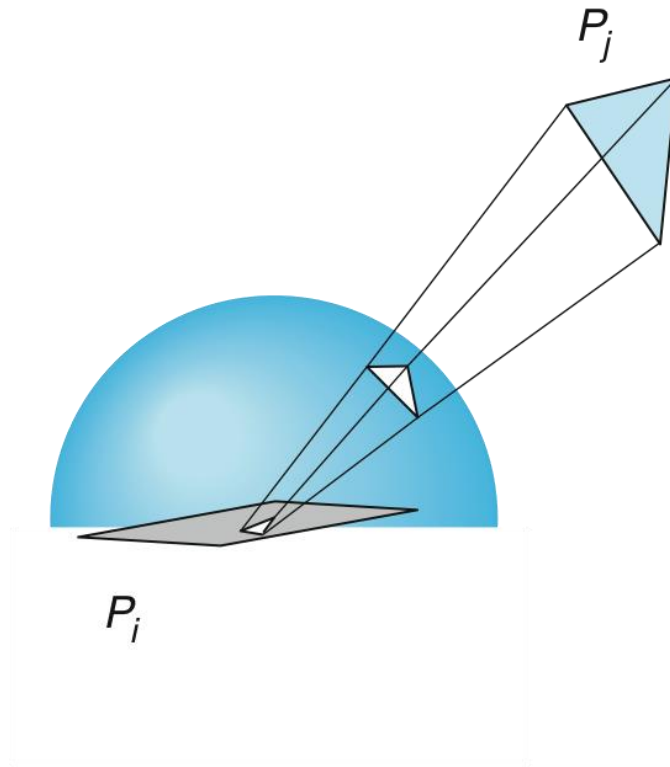
$$F_{d1-2} = \frac{1}{2}(1 + \cos \varphi)$$

Hemisphere

- Use illuminating hemisphere
- Center hemisphere on patch with normal pointing up
- Must shift hemisphere for each point on patch

Hemisphere

- Projecting patch on a hemisphere

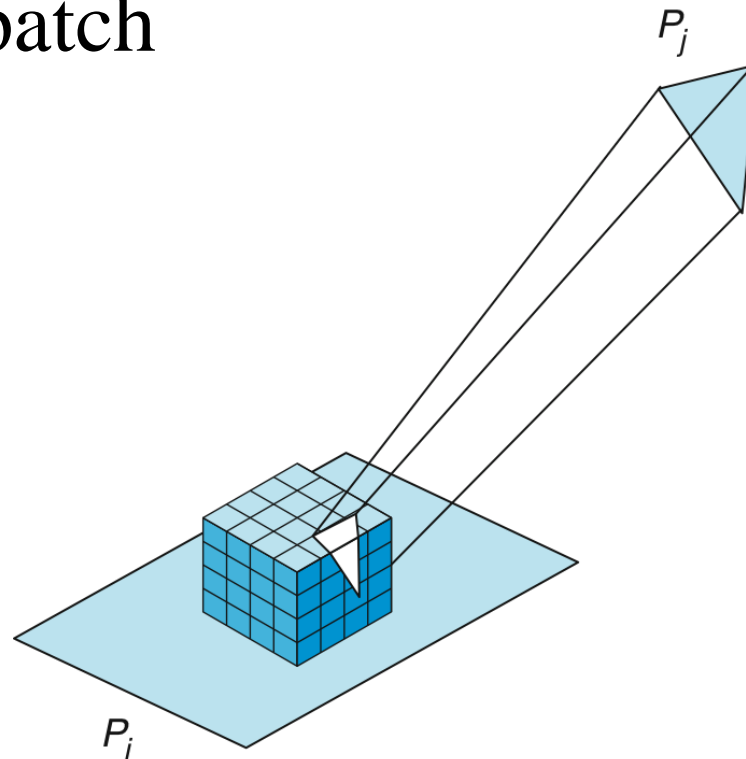


Hemicube

- Easier to use a hemicube instead of a hemisphere
- Rule each side into “pixels”
- Easier to project on pixels which give *delta form factors* that can be added up to give desired form factor
- To get a delta form factor we need only cast a ray through each pixel

Hemicube

- Projecting patch on a hemicube and onto another patch



Instant Radiosity

- Want to use graphics system if possible
- Suppose we make one patch emissive
- The light from this patch is distributed among the other patches
- Shade of other patches \sim form factors
- Must use multiple OpenGL point sources to approximate a uniformly emissive patch

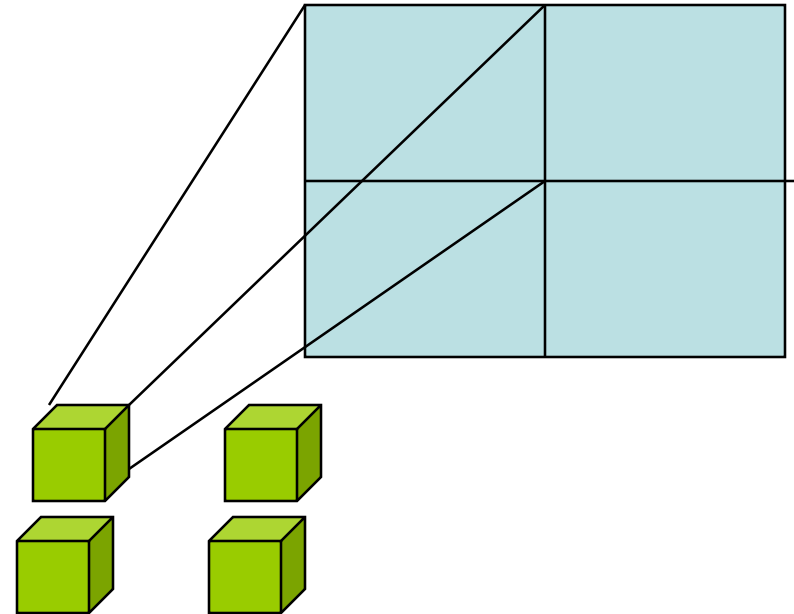
Parallel Rendering

Introduction

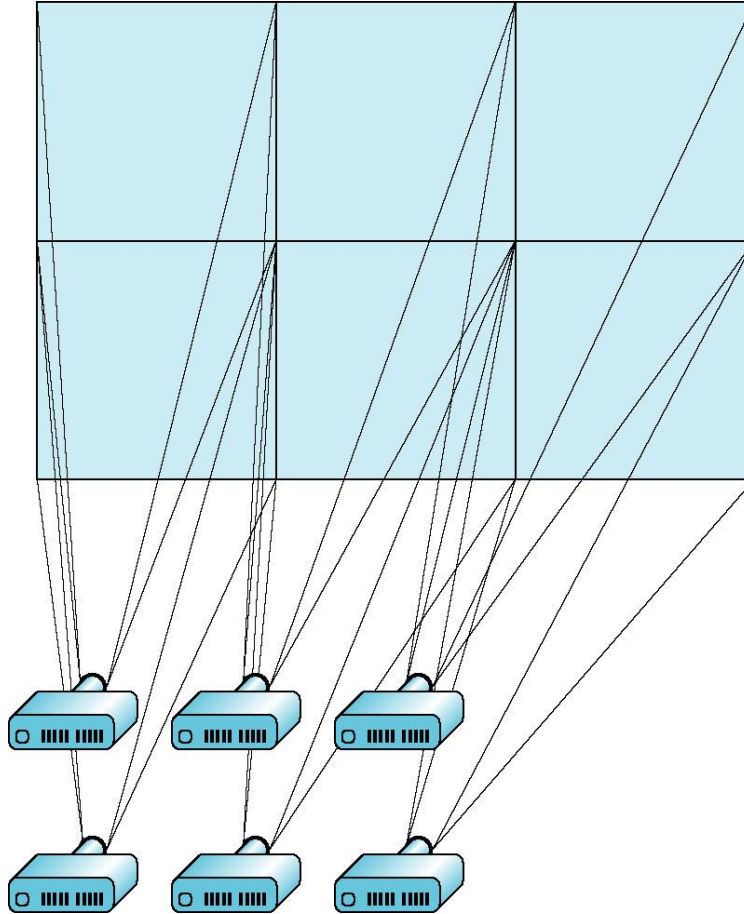
- In many situations, a standard rendering pipeline might not be sufficient
 - Need higher resolution display
 - More primitives than one pipeline can handle
- Want to use commodity components to build a system that can render in parallel
- Use standard network to connect

Power Walls

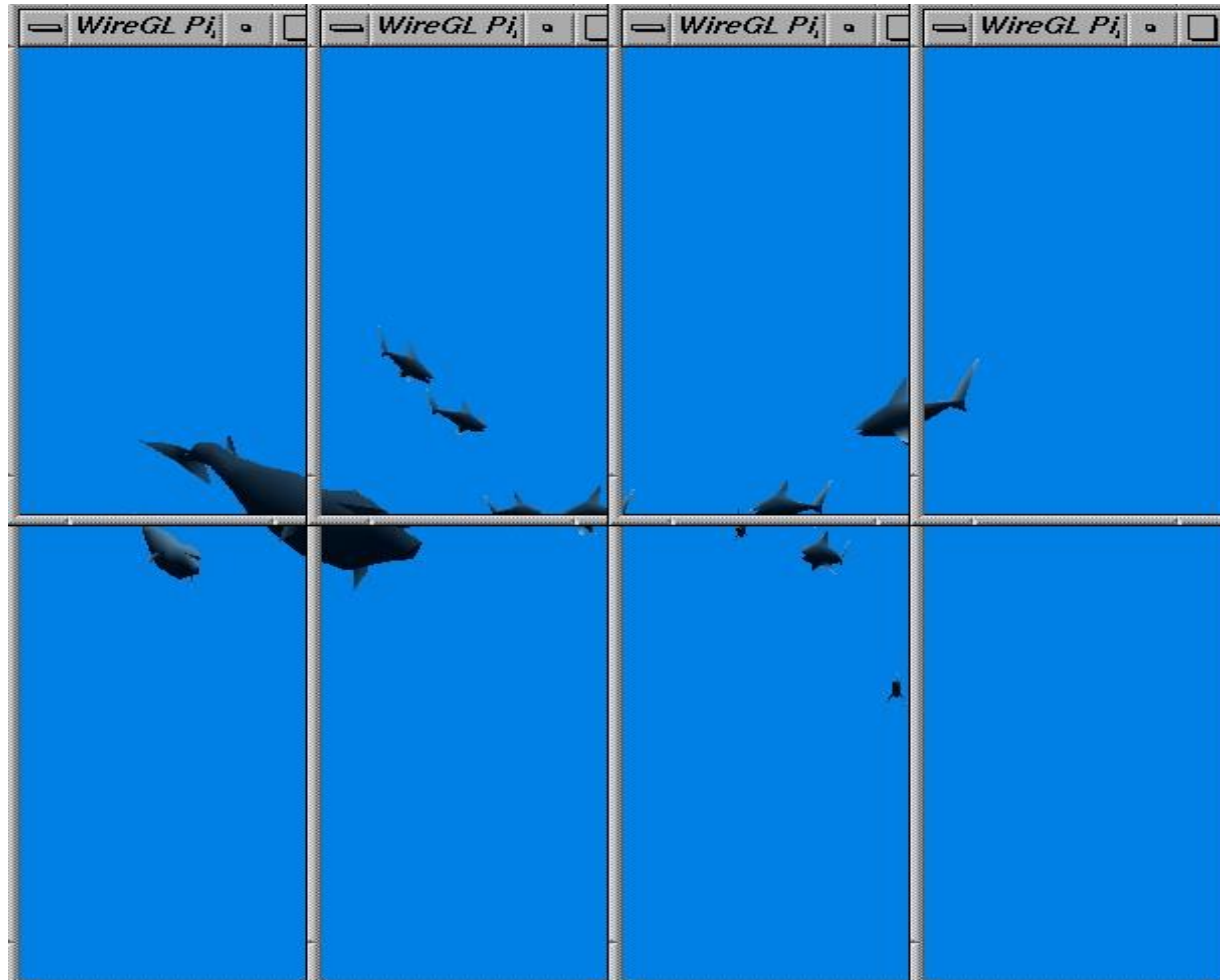
- Where do we display large data sets?
 - CRTs have low resolution (1 Mpixel)
 - LCD panels improving but still expensive
 - Need resolution comparable to data set to see detail
 - CT/MRI/MEG
 - Ocean data
- Solution?
 - Multiple projectors
 - Commodity
 - High-end
 - See IEEE CG & A (July)



Power wall using six projectors



Tiled Display



CS Power Wall

- 6 dual processor Intellestations
- G Force 3 Graphics cards
- 6 commodity projectors (1024 x 768)
- Gigabit ethernet
- Back projected screen
- Shared facility with scalable system group
 - Investigate OS and network issues

CS Power Wall



CS Power Wall

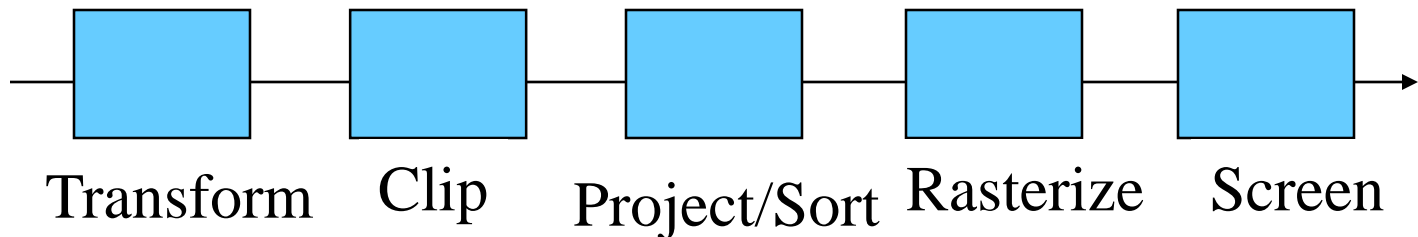


Power Wall

- Inexpensive solution but there are some problems
 - Color matching
 - Vignetting
 - Alignment
 - Overlap areas
 - Synching
 - Dark field

Graphics Architectures

- Pipeline Architecture
 - SGI Geometry Engine
 - Geometry passes through pipeline
 - Hardware for
 - clipping
 - transformations
 - texture mapping



Building Blocks

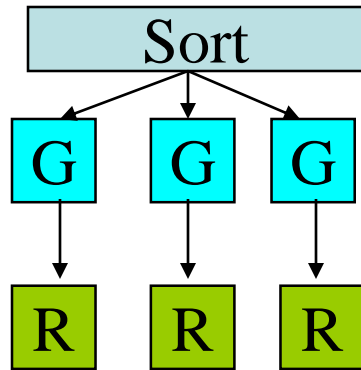
- Graphics processors consist of geometric blocks and rasterizers
- Geometric units: transformations, clipping, lighting
- Rasterization: scan conversion, shading
- Parallelize by using multiple blocks
- Where to do depth check?



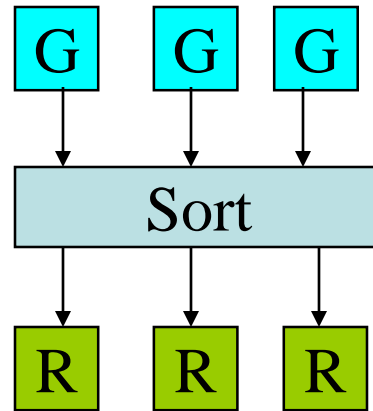
Sorting Paradigm

- We can categorize different ways of interconnecting blocks using a sorting paradigm: each projector is responsible for one area of the screen. Hence, we must sort the primitives and assign them to the proper projector
- Algorithms can be categorized by where this sorting occurs

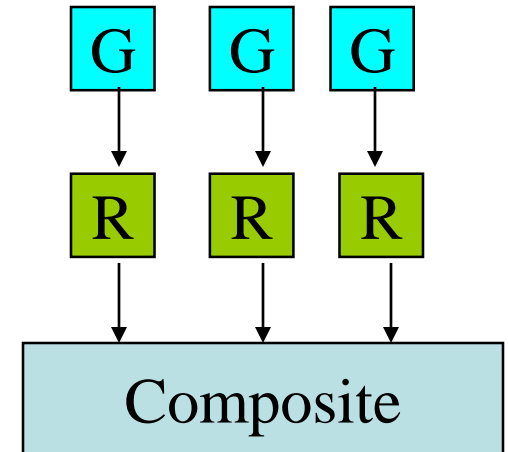
Three Rendering Methods



Sort-First Rendering



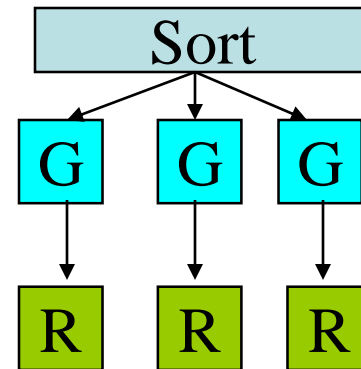
Sort-Middle Rendering



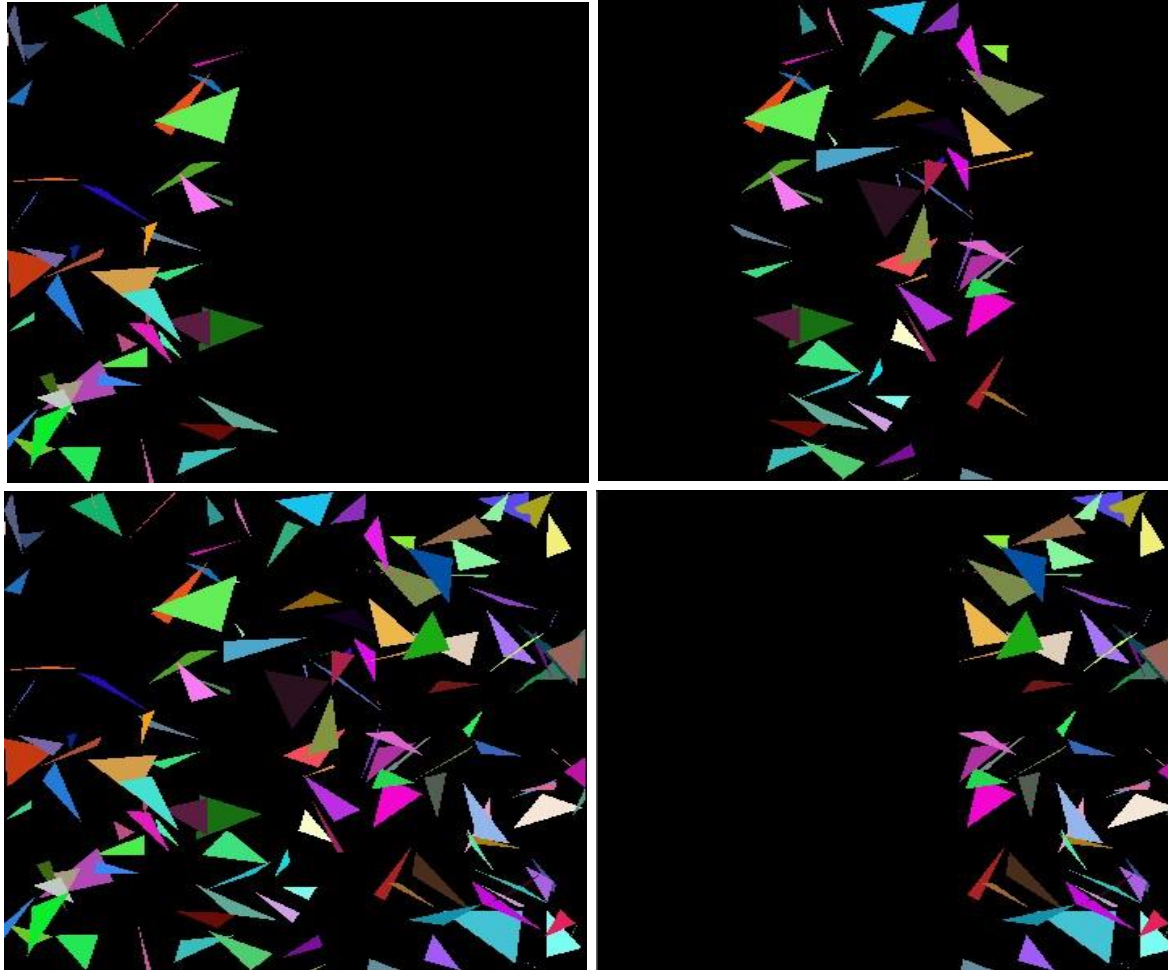
Sort-Last Rendering

Sort First

- Each rasterization unit assigned to an area of the screen
- Each geometric unit coupled to its own rasterizer
- Must sort primitives first
- Can use commodity cards

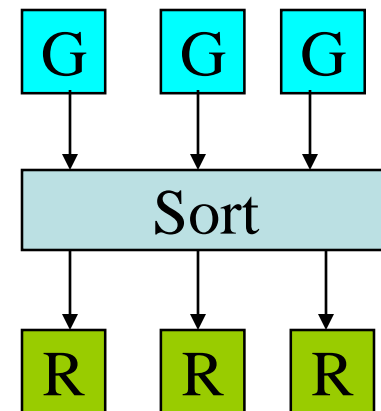


Sort-First Rendering for a Random Triangles Application



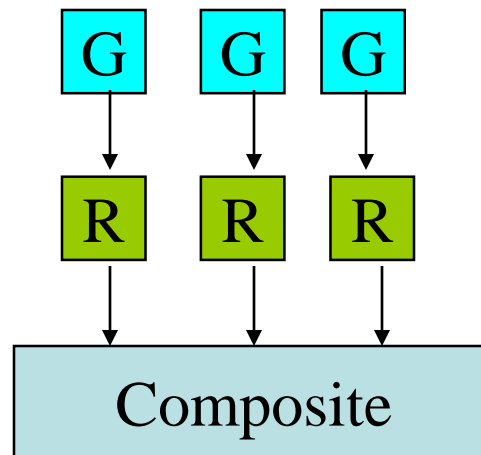
Sort Middle

- Geometric units and rasterization units decoupled
- Each geometric unit can be assigned any group of objects
- Each rasterizer is assigned to an area of the screen
- Must sort between stages



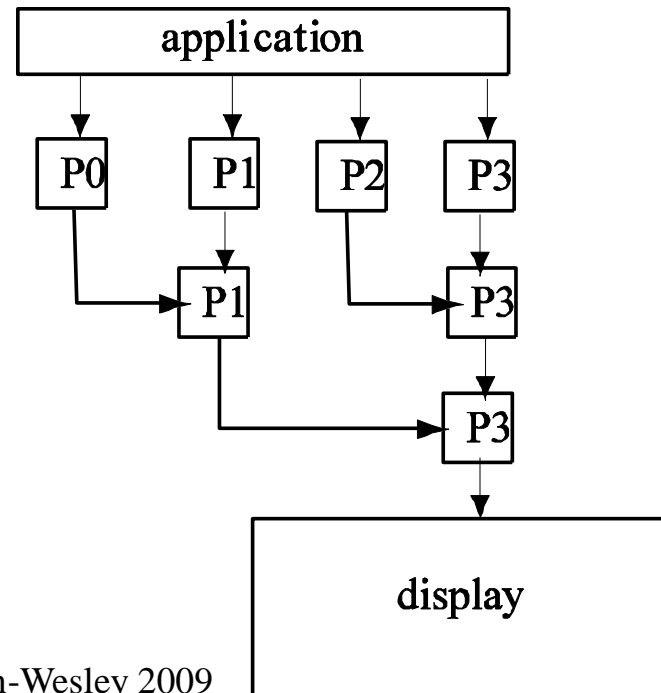
Sort Last

- Couple rasterizers and geometric units
- Assign objects to geometric units to load balance or via application
- Composite results at end



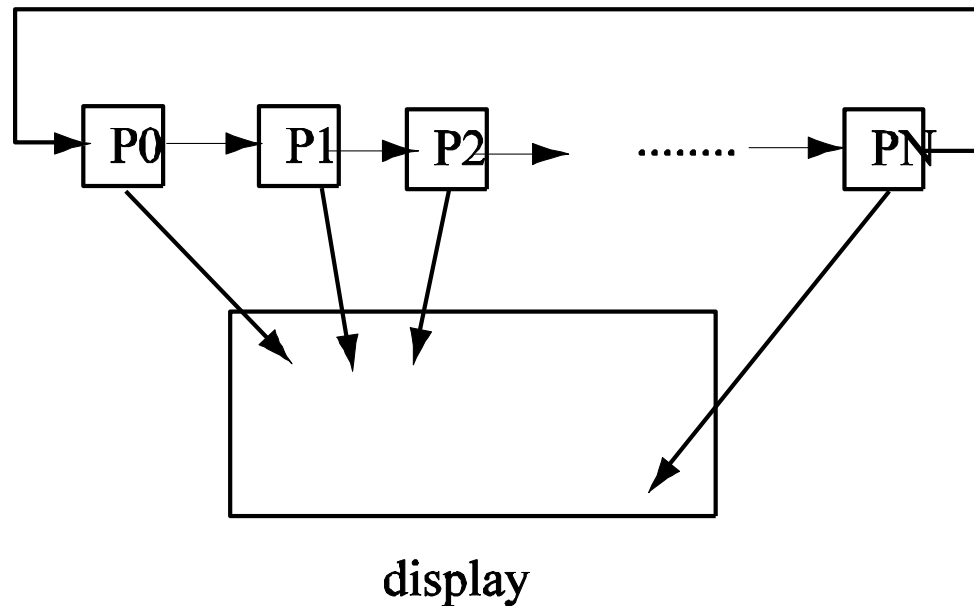
Tree Compositing

- Composite in pairs
- Send color and depth buffers
- Each time half processors become idle

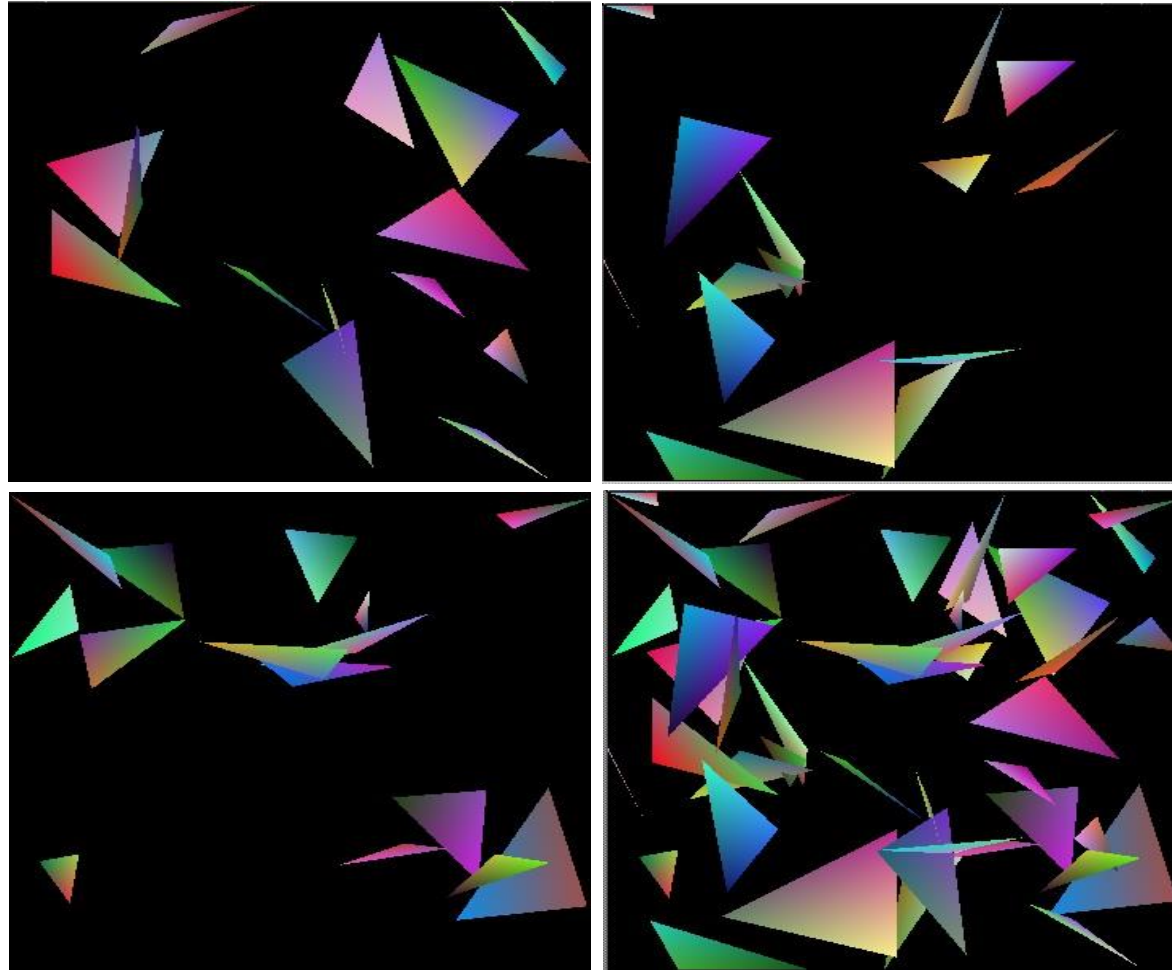


Binary Swap Compositing

- Each processor responsible for one part of display
- Pass data to right n times



Sort-Last Rendering for a Random Triangles Application



Comparison

- Sort first
 - Appealing but hard to implement
- Sort middle
 - Used in hardware pipelines
 - More difficult to implement with add-on commodity cards
- Sort last
 - Easy to implement with a compositing stage
 - High network traffic

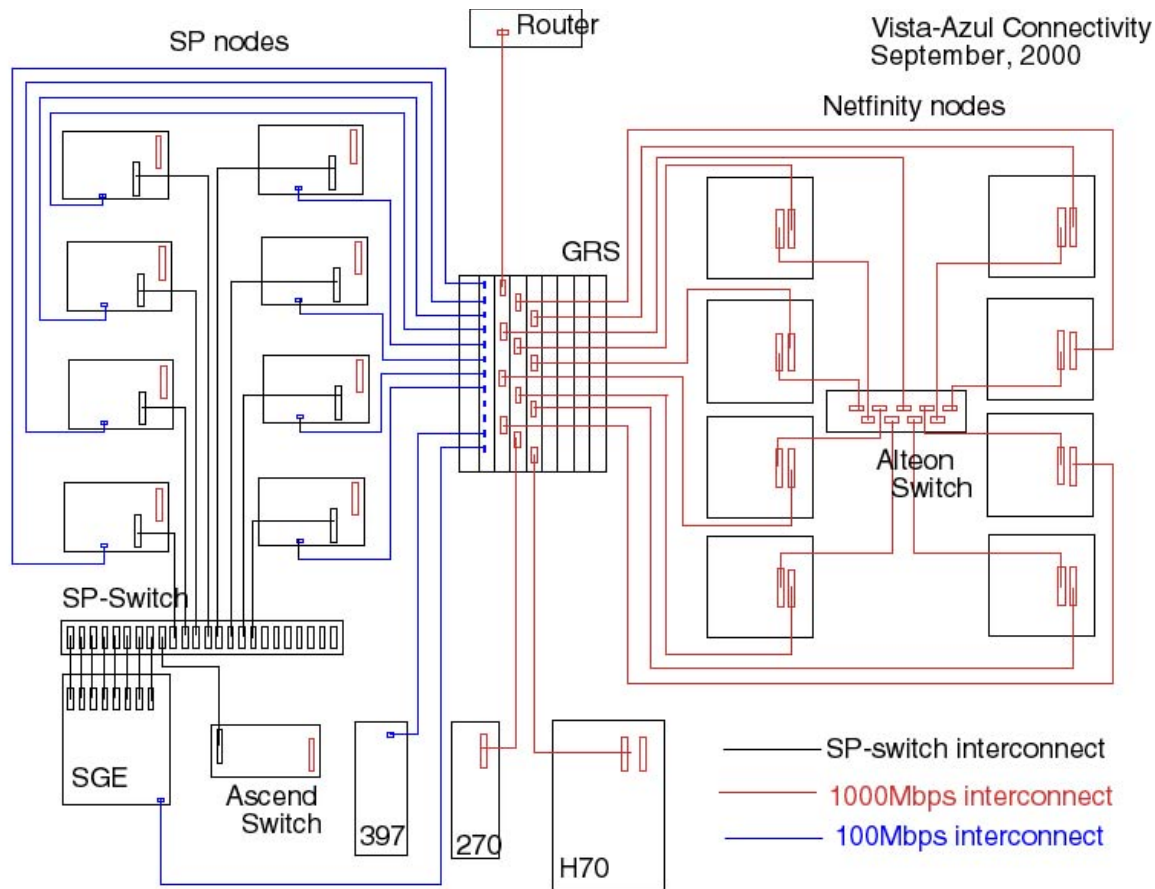
Mapping to Clusters

- Different architectures
 - Shared vs distributed memory
 - Communication overhead
 - Parallel vs distributed algorithms
- Easy to do sort last
- Must evaluate communication cost
- Standard visualization strategies are incorrect if transparency used

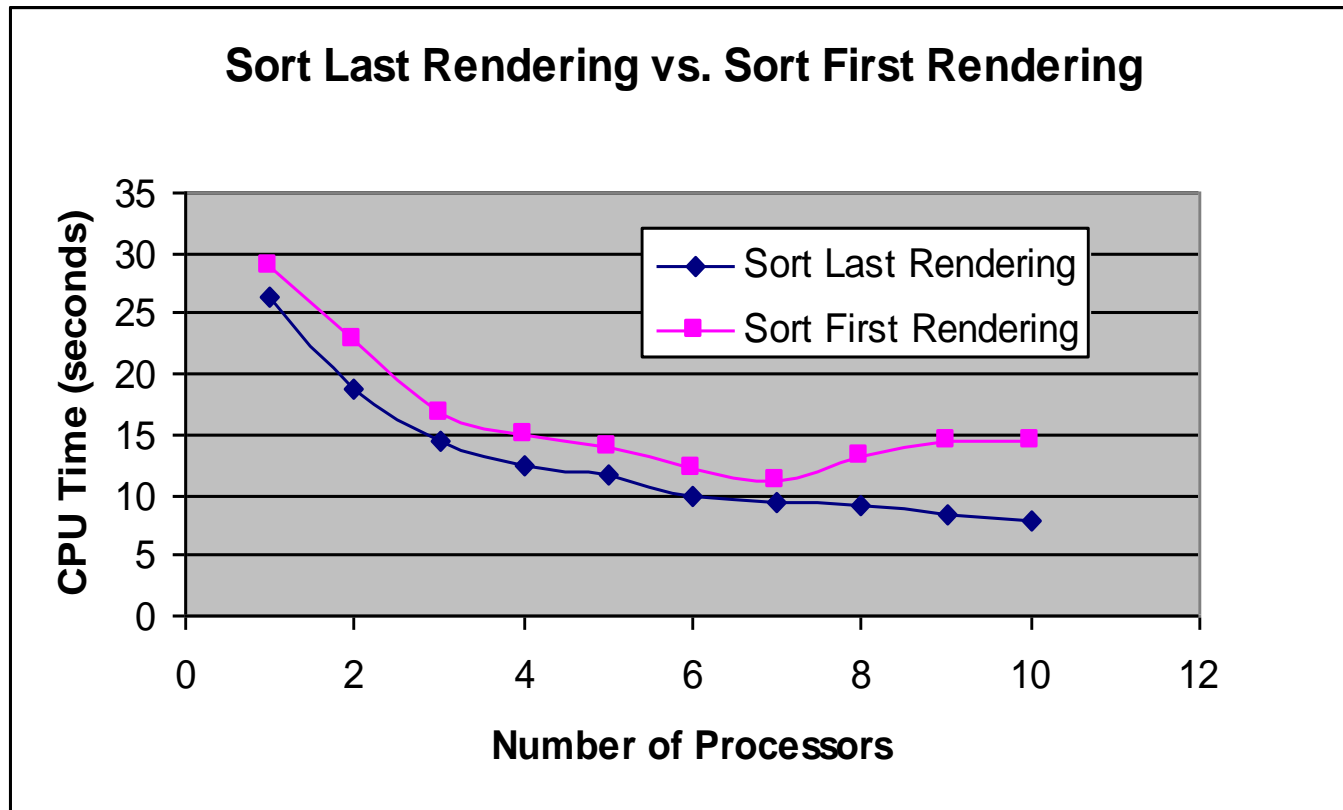
Vista Azul

- Experimental architecture from IBM donated to AHPCC
- Half Intel nodes, half AIX nodes
- Only one (PCI) graphics card per four processors
- Contained a Scalable Graphics Engine (SGE): high speed-high resolution color buffer that is accessible by all processors

Vista Azul



Comparison Between Sort-First and Sort-Last

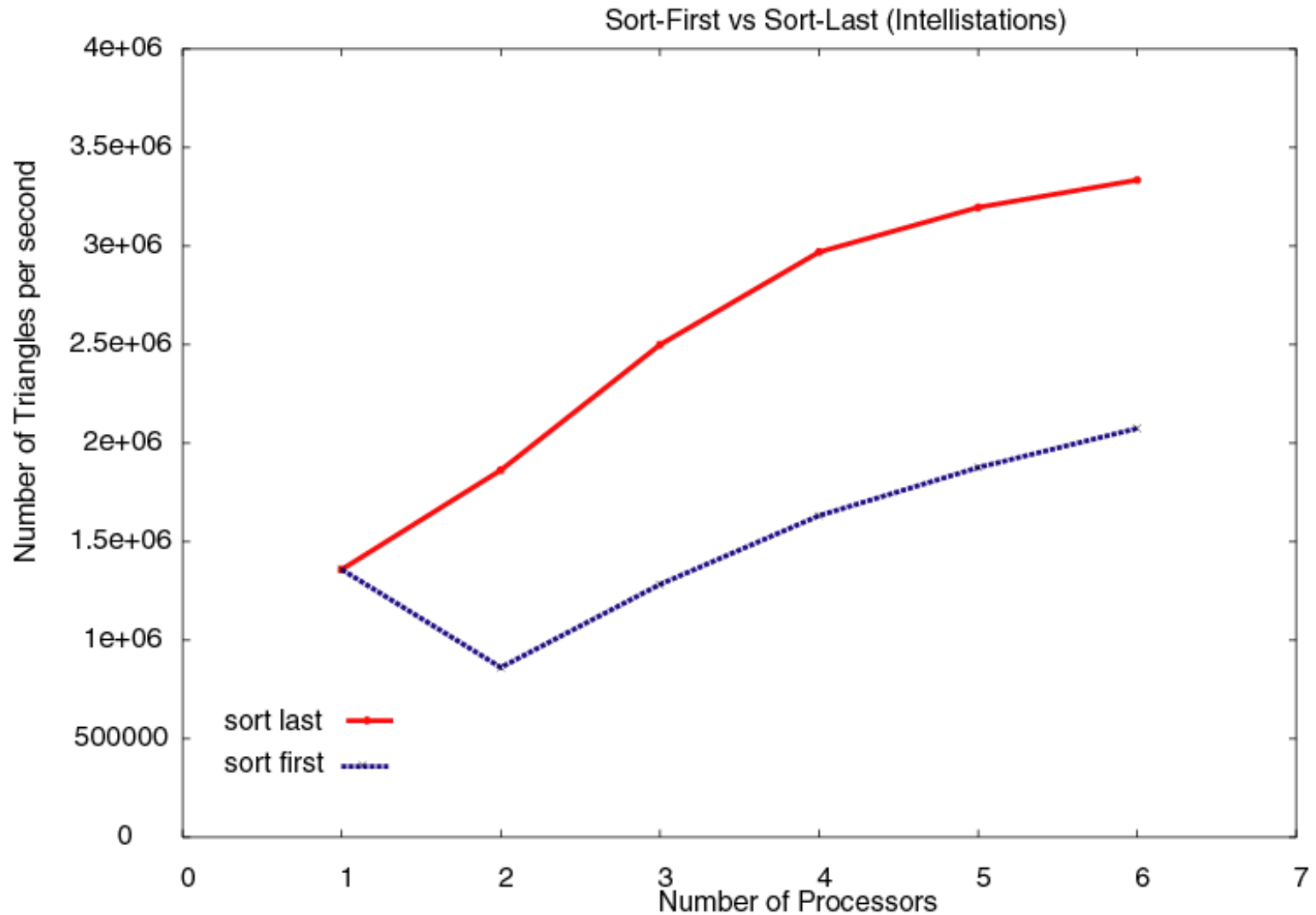


Performance on a PC Cluster

- Following experiments were done by Ye Cong on the CS cluster
 - 6 Intellestations
 - Gigabit Ethernet
 - GeForce 3 graphics
- Show the effect of network

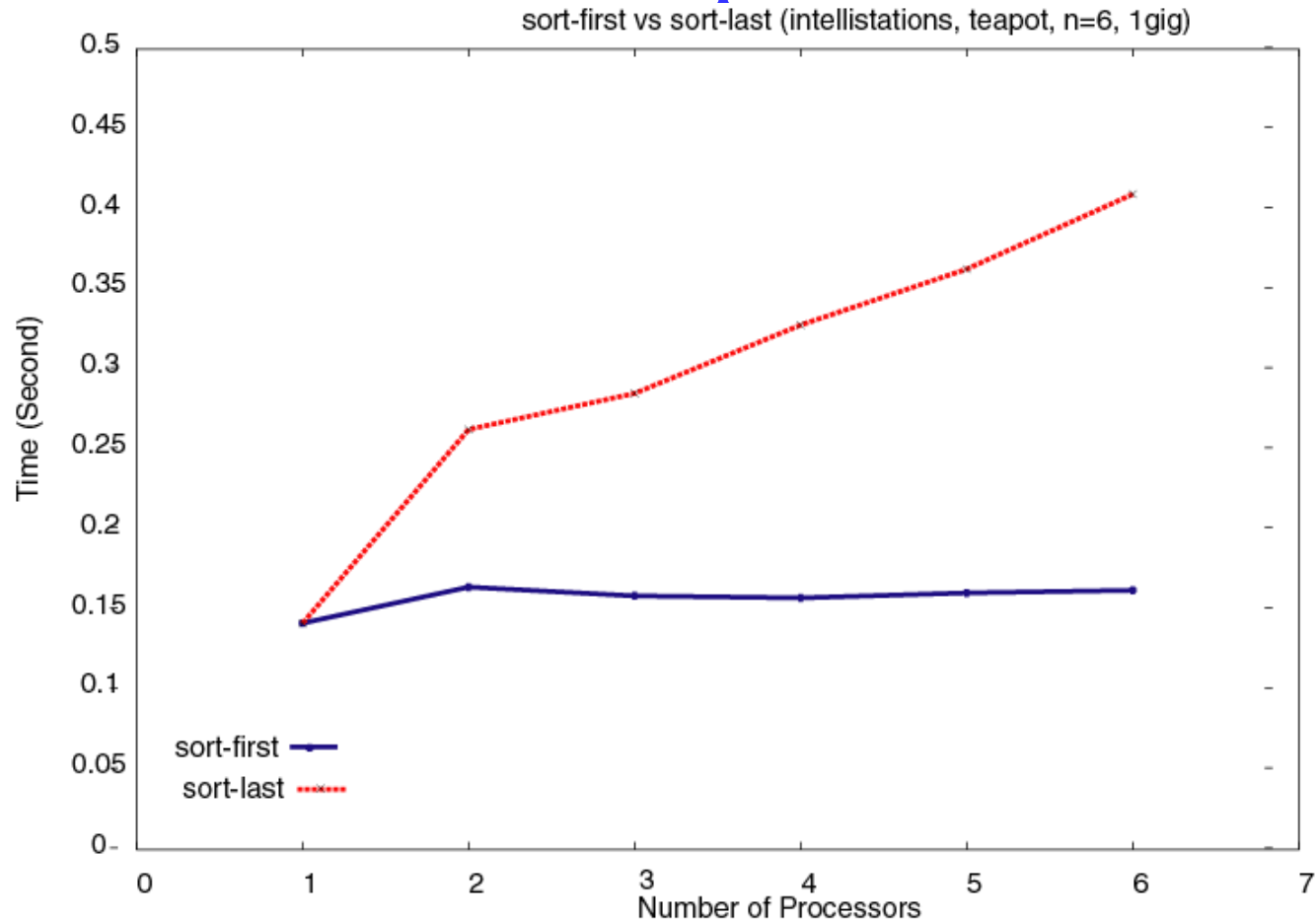
Sort-First vs Sort Last

Random Triangles

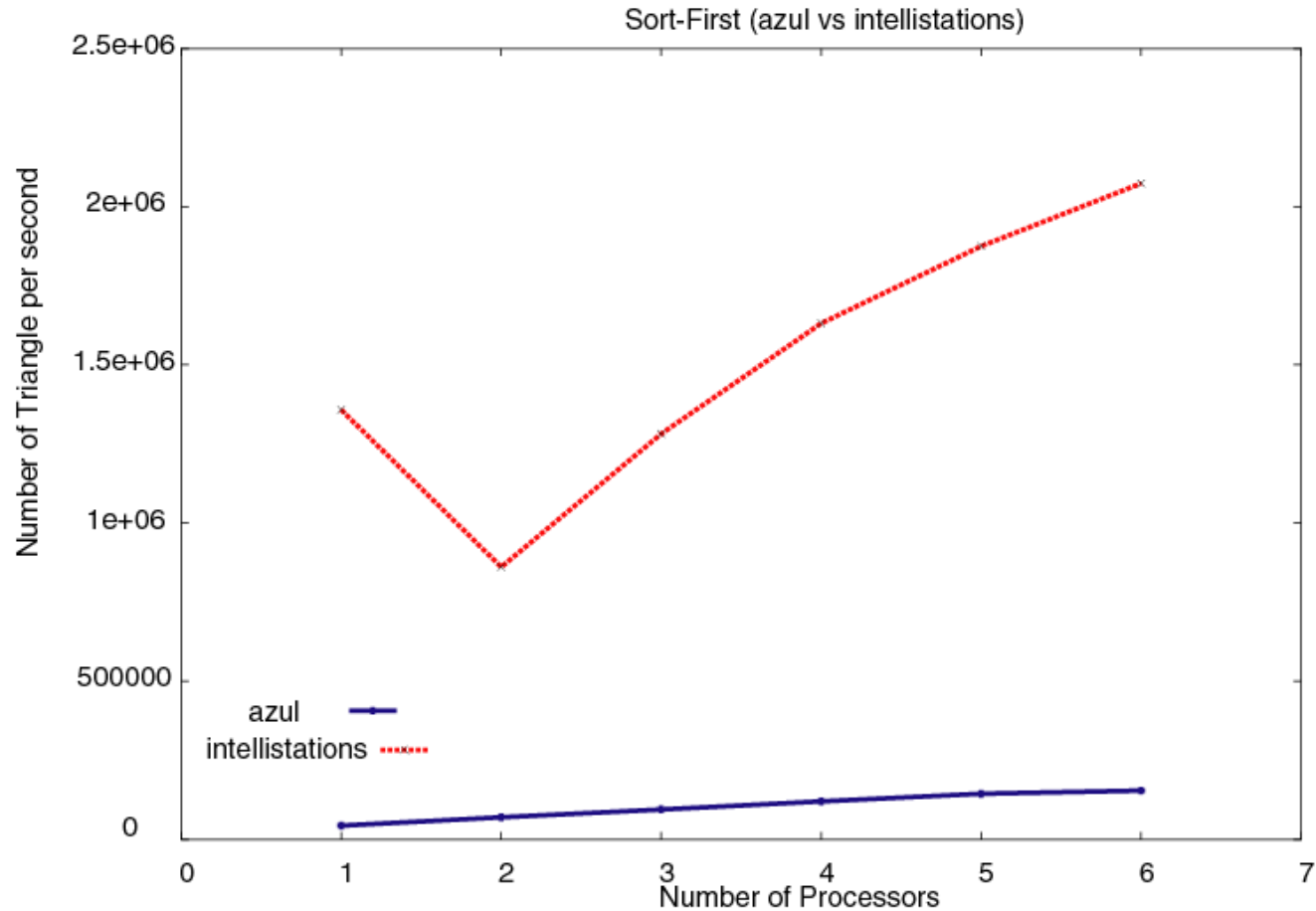


Sort First vs Sort Last

Teapot



Azul vs Intellistations



Software for Parallel Rendering

- Write your own sort-first sort-last
- WireGL/Chromium (Stanford)
- Embed inside package (VTK)

WireGL: A Distributed Graphics System

- A software-based parallel rendering system that unifies the rendering power of a collection of cluster nodes
- Scalability is achieved by integrating parallel applications into its sort-first parallel renders
- Each node in the cluster can be either a rendering client or a rendering server
- Clients submit OpenGL commands concurrently to servers, which render the final physical image

Chromium

- Successor to WireGL
- Allows both sort first and sort last rendering
- Implemented on CS cluster
- Most of gain in performance is because Chromium and WireGL can group state-changing commands separately from rendering commands

Chromium vs Sort First

Sort-First Marching Cube(my-sf vs chromium, Tri=2250818)

