Invariant information clustering

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Based on https://arxiv.org/abs/1807.06653

Foreword

- We use invariant information clustering (IIC)
- Several SOTA results

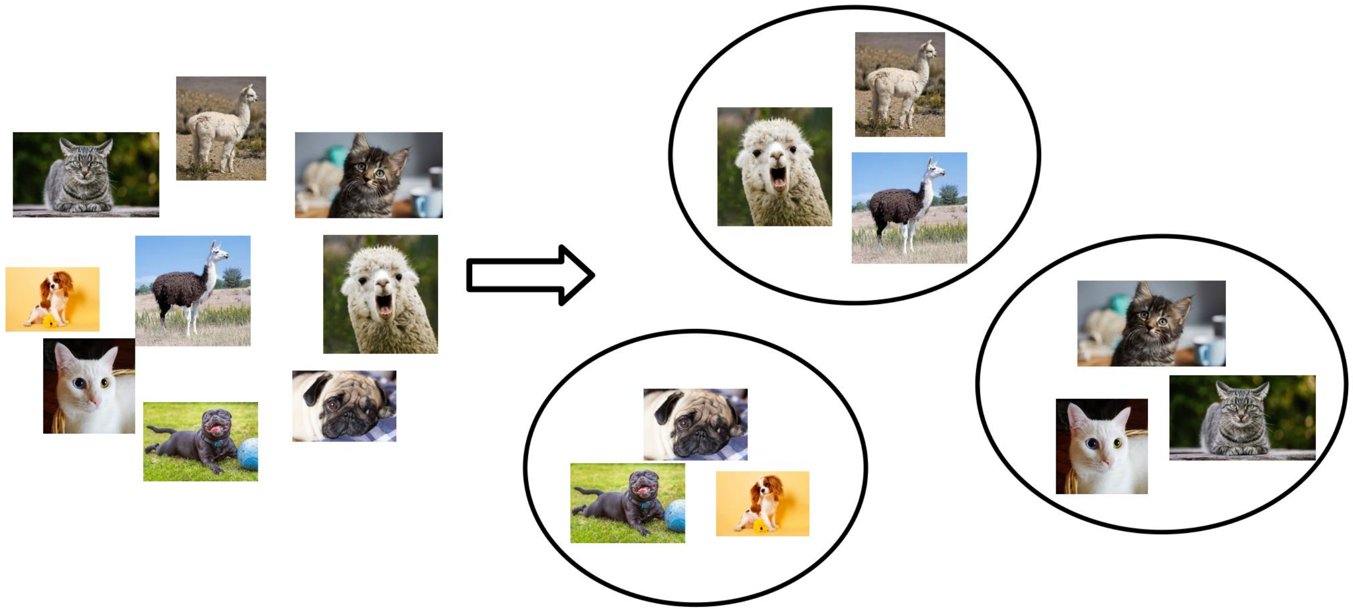
Outline

1. Problem statements
2. Method overview
3. Mutual information loss
4. Couple of tricks
5. Results
6. Summary

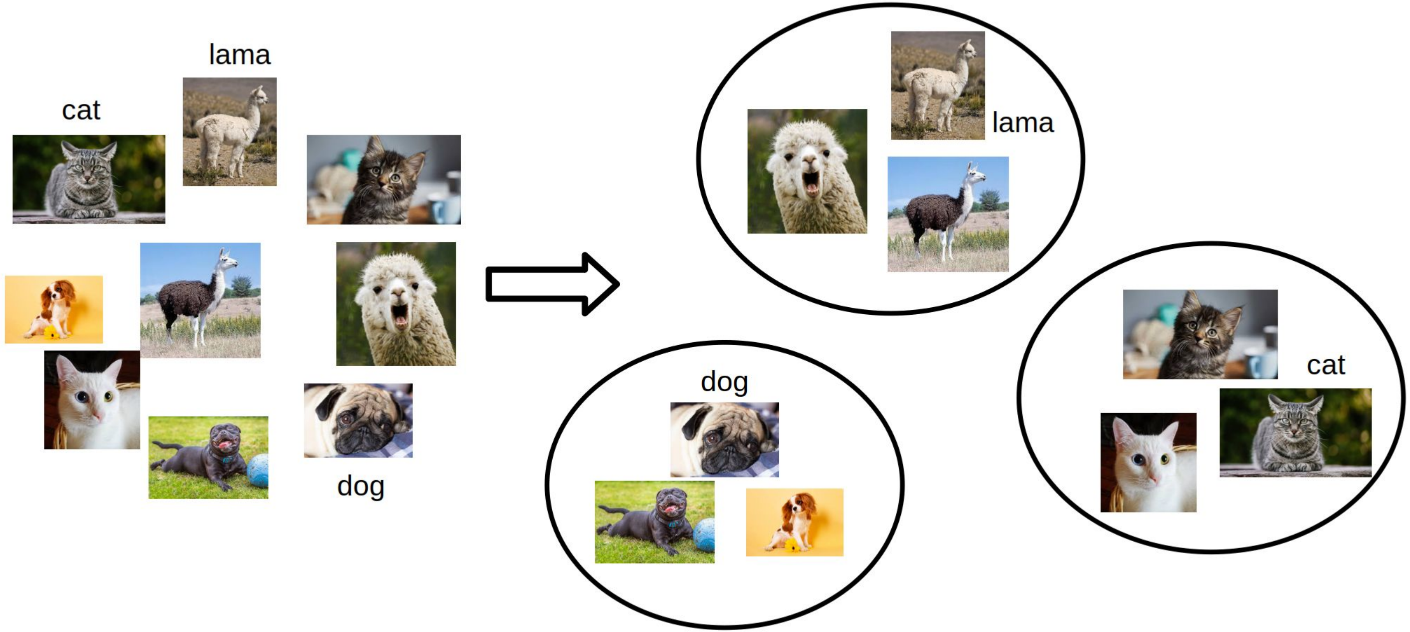
Problem statements

01

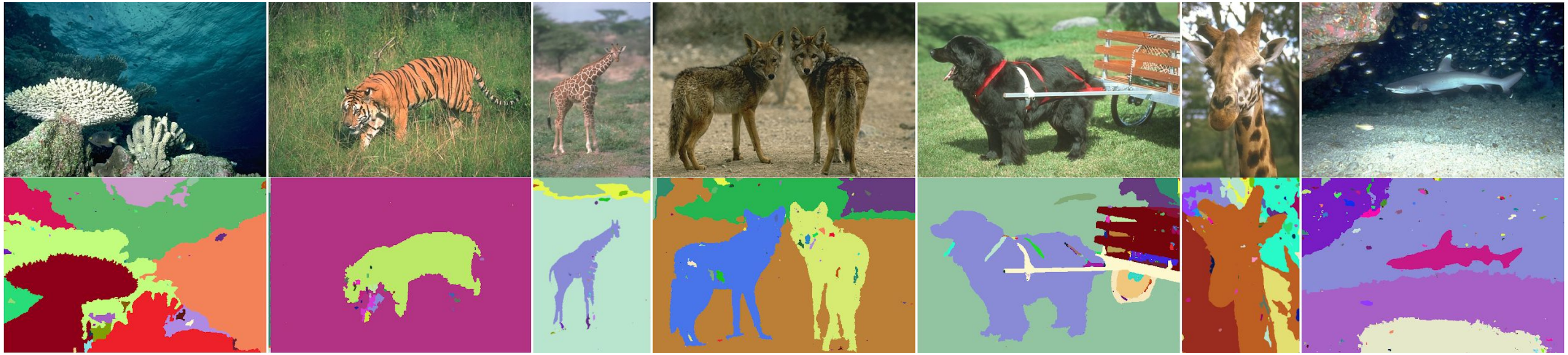
Clustering



Semi-supervised classification

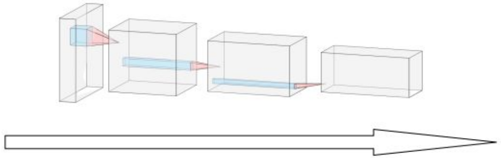


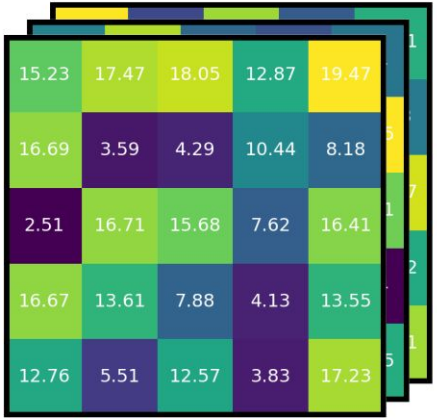
Unsupervised segmentation



Encoder training

Feature
extractor





Input image

Feature maps

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Tasks to consider

- clustering
- semi-supervised classification
- unsupervised segmentation
- encoder training

Method overview

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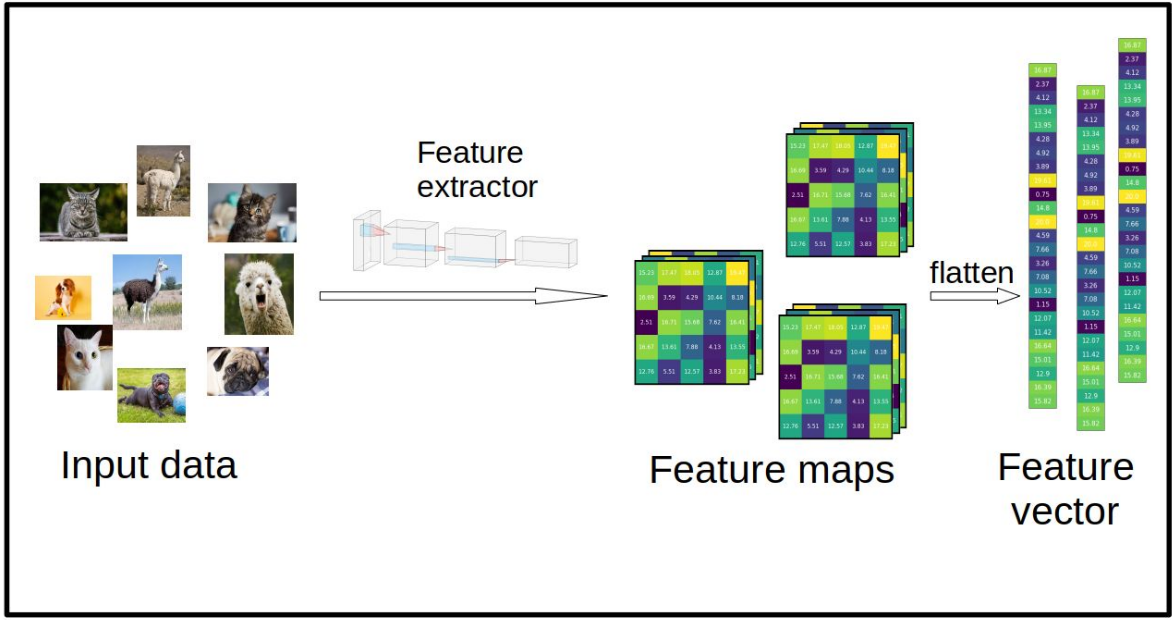
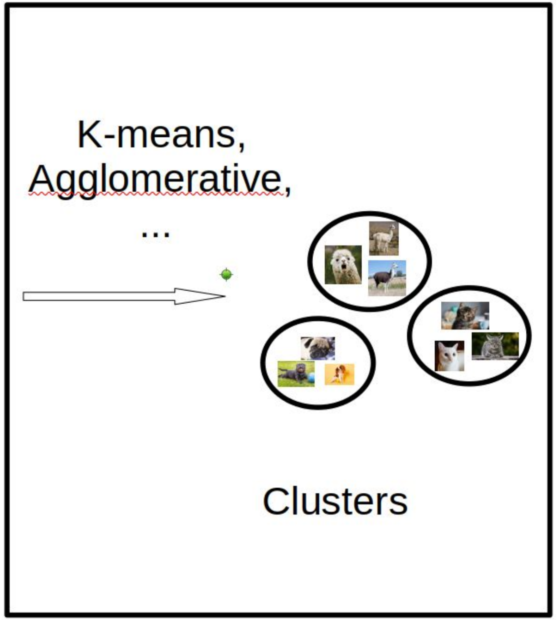
02

2 stage clustering

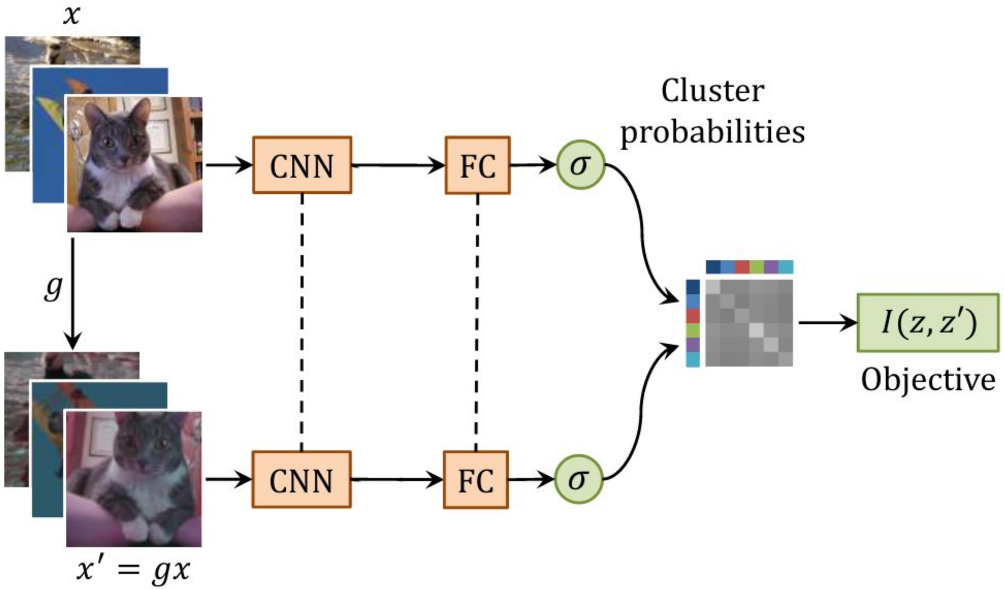




Train feature extractor (i.e. with autoencoder)

Train clusterisator

IIC approach



Mutual information loss

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Mutual information

2 random variables *A* and *B* : values from 0 to N_c

Joint probability distribution: $p_{AB}(a,b)$

The marginals: $p_A(a) = \sum_{b=0}^{N_C} p_{AB}(a,b)$ $\qquad p_B(b) = \sum_{a=0}^{N_C} p_{AB}(a,b)$

Mutual information:

$$\boxed{I(A,B) = \sum_{a=0}^{N_C}\sum_{b=0}^{N_C} p_{AB}(a,b)\log\frac{p_{AB}(a,b)}{p_A(a)p_B(a)}}$$

Mutual information

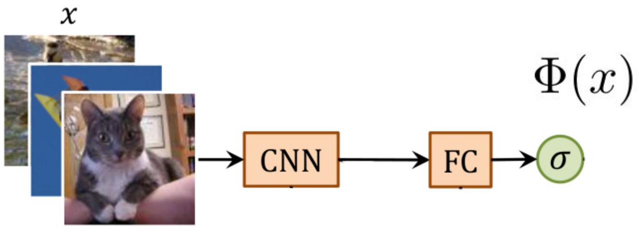
$$I(A,B) = \sum_{a=0}^{N_C} \sum_{b=0}^{N_C} p_{AB}(a,b) \log \frac{p_{AB}(a,b)}{p_A(a) p_B(a)}$$

Properties:

1. Symmetry: $I(A,B) = I(B,A)$
2. $I(A,B) = 0$ if and on if *A* and *B* are independent
3. Non-negative: $I(A,B) > 0$ for any *A* and *B*
4. In some sense maximized when *A* and *B* can be predicted from each other

What are these random variables?

Encoder function: $\Phi(\cdot)$



On original images:

$$\Phi(x_i) = \begin{pmatrix} \Phi_0(x_i) \\ \Phi_1(x_i) \\ \vdots \\ \Phi_{N_C}(x_i) \end{pmatrix}$$

On transformed images:

$$\Phi(gx_i) = \begin{pmatrix} \Phi_0(gx_i) \\ \Phi_1(gx_i) \\ \vdots \\ \Phi_{N_C}(gx_i) \end{pmatrix}$$

Interpretation: $P_{orig}(a|i) = \Phi_a(x_i)$ $P_{trans}(a|i) = \Phi_a(gx_i)$

Joint probabilities estimation

Joints:

$$P_{orig,trans}(a, b) = \frac{1}{N_B} \sum_{i \in batch} \Phi_a(x_i)\Phi_b(gx_i)$$

Symmetrization:

$$P^{sym}_{orig,trans}(a, b) = \frac{1}{2}(P_{orig,trans}(a, b) + P_{orig,trans}(b, a))$$

Mutual information:

$$I(orig, trans) = \sum_{a=0}^{N_C} \sum_{b=0}^{N_C} P_{orig,trans}(a, b) \log \frac{P_{orig,trans}(a, b)}{P_{orig}(a) P_{trans}(b)}$$

Joint probabilities estimation

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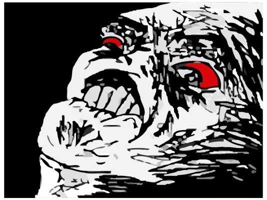
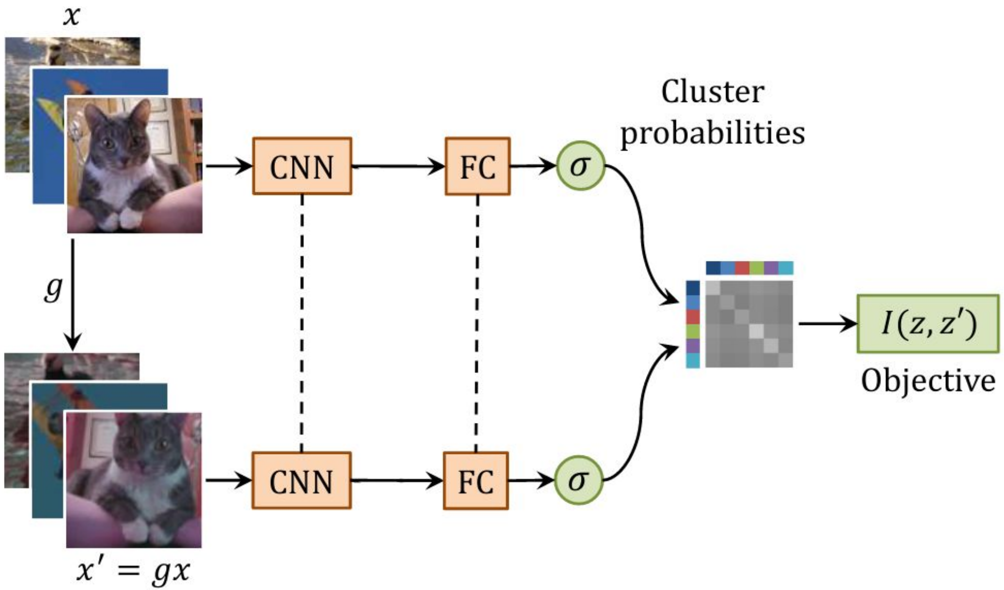
Model assumptions stuff



Joint probabilities estimation

Note: it’s sufficient to estimate joint probabilities



Couple of tricks

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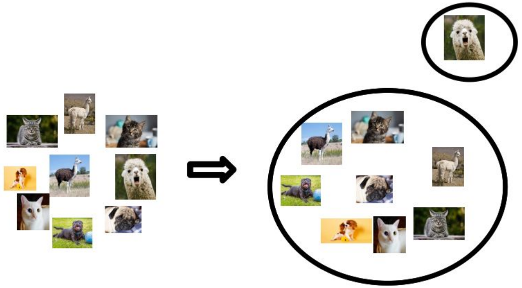
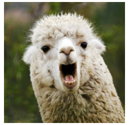
Common issues

Clustering degeneracy:



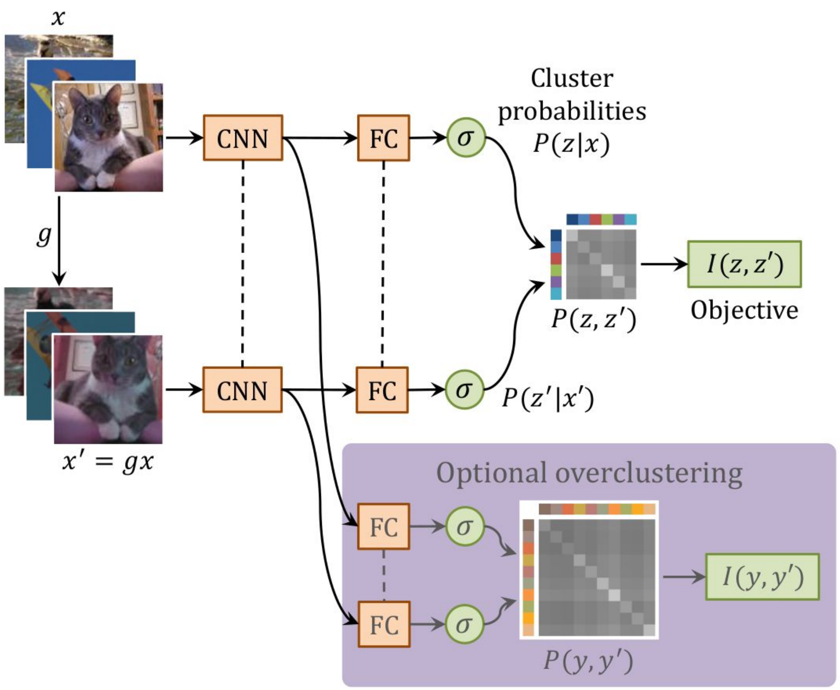
Noisy data:



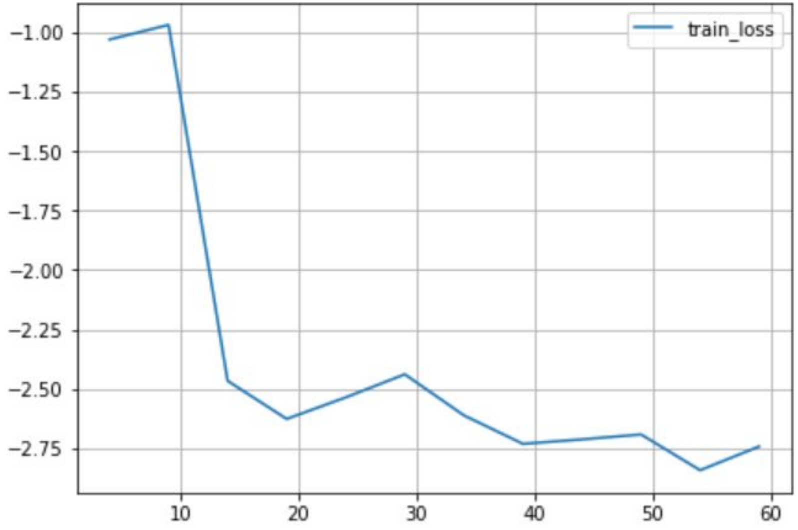
dog

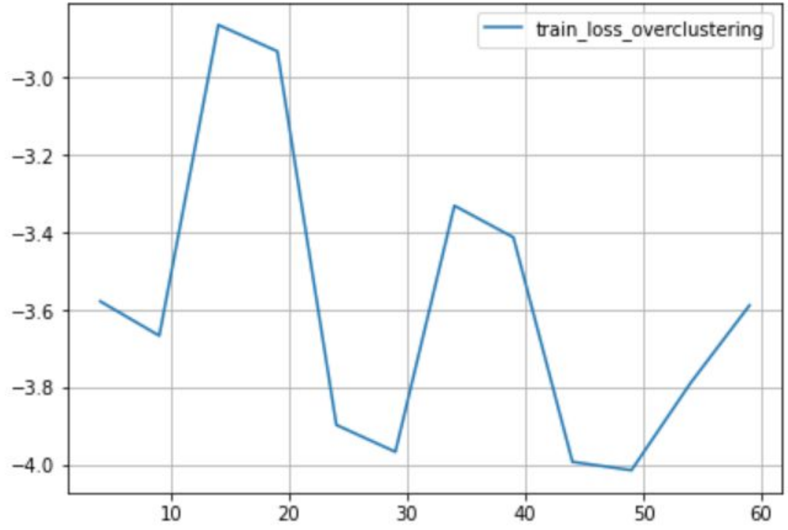
Distractors:

Auxiliary overclustering



Auxiliary overclustering





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Entropy correction

Another form of mutual information: $$\underbrace{I(A,B) = \frac{1}{2}\big(H(A)+H(B)\big)}_{\text{pushes towards spreading}} - \frac{1}{2}\big(H(A|B)+H(B|A)\big)$$

Corrected mutual information: $$I_\lambda(A,B) = I(A,B) + (\lambda - 1)\big(H(A)+H(B)\big)$$

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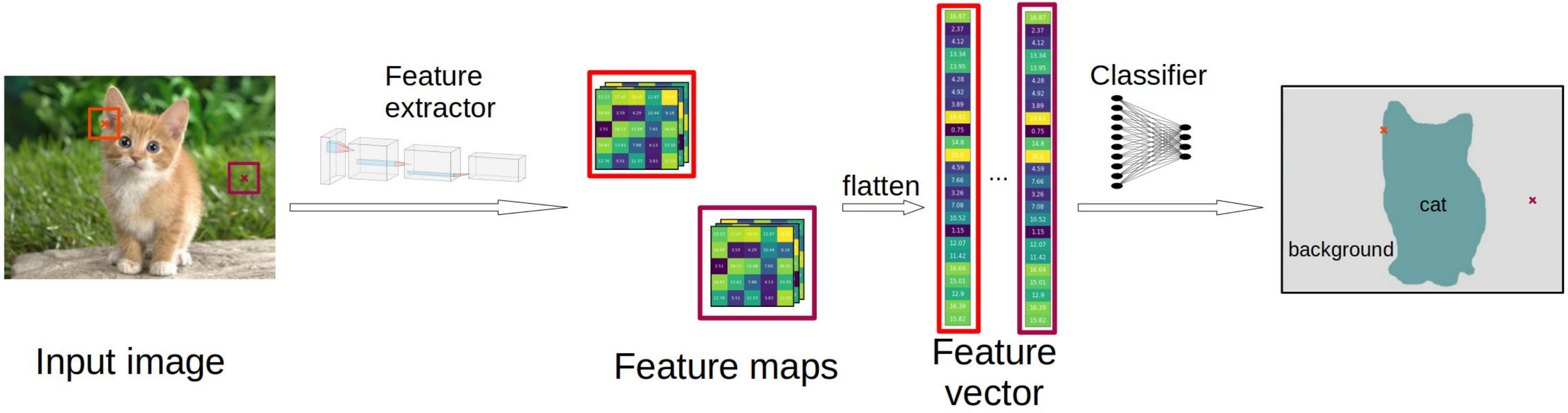
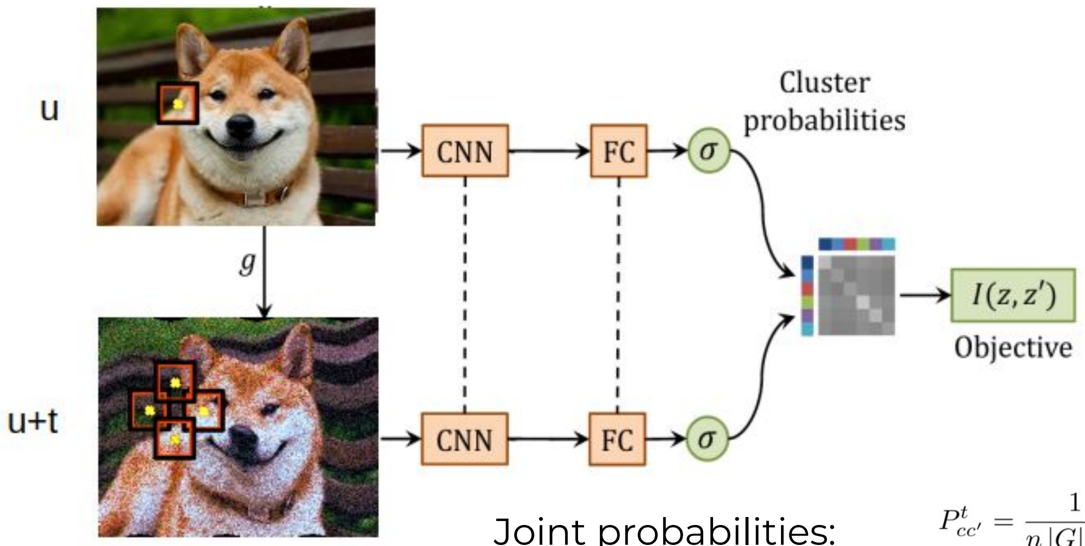
Segmentation



Segmentation



$g \in G$ — number of transforms
$u \in \Omega$ — pixel positions
$t \in T$ — displacements
n — number of images
C — number of classes

Joint probabilities: $$P^t_{cc'} = \frac{1}{n\,|G|\,|\Omega|} \sum_{i=1}^{n} \sum_{g \in G} \sum_{u \in \Omega} P(z = c|x_u) P(z = c'|gx_{u+t})$$

$$I_t = \sum_{c,c'=1}^{C} P^t_{cc'} \ln \frac{P^t_{cc'}}{P^t_c P^t_{c'}}$$

Corrected mutual information: $$I = \frac{1}{|T|} \sum_{t \in |T|} I_t$$

Results

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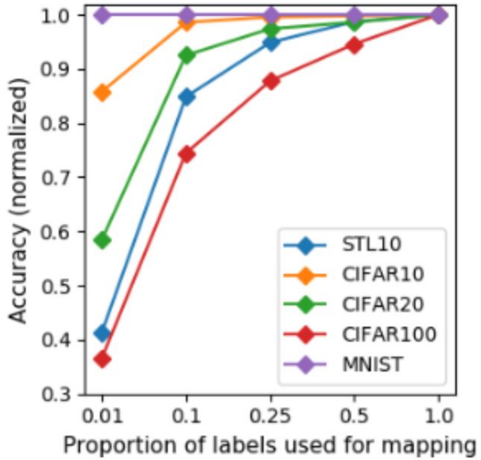
Semi-supervised classification

Accuracy

	STL10
Dosovitskiy 2015 [18]†	74.2
SWWAE 2015 [54]†	74.3
Dundar 2015 [19]	74.1
Cutout* 2017 [15]	$\boxed{87.3}$
Oyallon* 2017 [42]†	76.0
Oyallon* 2017 [42]	87.6
DeepCluster 2018 [7]	73.4⋆
ADC 2018 [24]	56.7⋆
DeepINFOMAX 2018 [27]	77.0
IIC plus finetune†	**79.2**
IIC plus finetune	**88.8**

Table 3: **Fully and semi-supervised classification.** Legend: *Fully supervised method. ⋆Our experiments with authors' code. †Multi-fold evaluation.



Unsupervised segmentation

COCO-Stuff-3 and Postdam-3

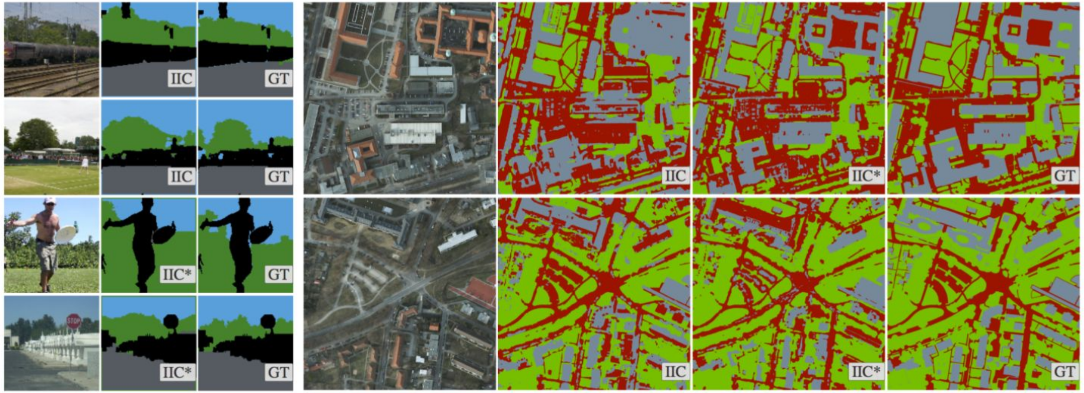


Figure 7: **Example segmentation results (un- and semi-supervised).** Left: COCO-Stuff-3 (non-stuff pixels in black), right: Potsdam-3. Input images, IIC (fully unsupervised segmentation) and IIC* (semi-supervised overclustering) results are shown, together with the ground truth segmentation (GT).

Per-pixel accuracy

	COCO-Stuff-3	COCO-Stuff	Potsdam-3	Potsdam
Random CNN	37.3	19.4	38.2	28.3
K-means [44]†	52.2	14.1	45.7	35.3
SIFT [39]‡	38.1	20.2	38.2	28.5
Doersch 2015 [17]‡	47.5	23.1	49.6	37.2
Isola 2016 [30]‡	$\boxed{54.0}$	$\boxed{24.3}$	$\boxed{63.9}$	$\boxed{44.9}$
DeepCluster 2018 [7]† ‡	41.6	19.9	41.7	29.2
IIC	**72.3**	**27.7**	**65.1**	**45.4**

Table 4: **Unsupervised segmentation.** IIC experiments use a single sub-head. Legend: †Method based on k-means. ‡Method that does not directly learn a clustering function and requires further application of k-means to be used for image clustering.

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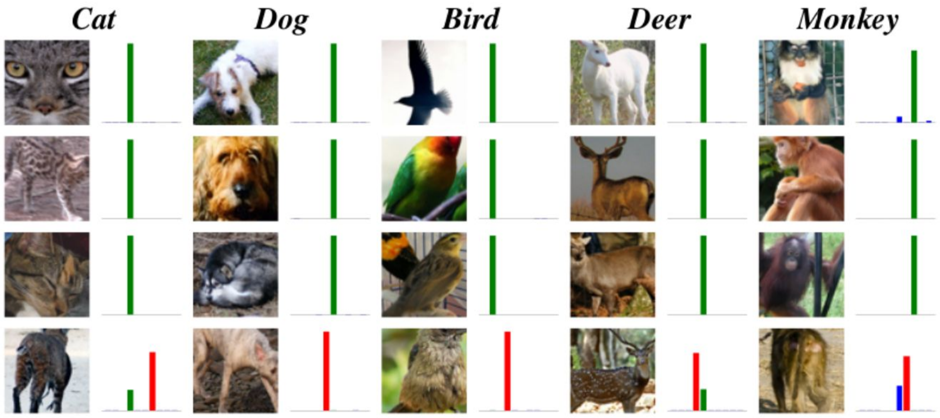
Clustering

Accuracy

	STL10	CIFAR10	CFR100-20	MNIST
Random network	13.5	13.1	5.93	26.1
K-means [53]†	19.2	22.9	13.0	57.2
Spectral clustering [49]	15.9	24.7	13.6	69.6
Triplets [46]‡	24.4	20.5	9.94	52.5
AE [5]‡	30.3	31.4	16.5	81.2
Sparse AE [40]‡	32.0	29.7	15.7	82.7
Denoising AE [48]‡	30.2	29.7	15.1	83.2
Variational Bayes AE [34]‡	28.2	29.1	15.2	83.2
SWWAE 2015 [54]‡	27.0	28.4	14.7	82.5
GAN 2015 [45]‡	29.8	31.5	15.1	82.8
JULE 2016 [52]	27.7	27.2	13.7	96.4
DEC 2016 [51]†	35.9	30.1	18.5	84.3
DAC 2017 [8]	47.0	$\boxed{52.2}$	$\boxed{23.8}$	97.8
DeepCluster 2018 [7]† ‡	33.4⋆	37.4⋆	18.9⋆	65.6 ⋆
ADC 2018 [24]	$\boxed{53.0}$	32.5	16.0⋆	$\boxed{99.2}$
IIC (lowest loss sub-head)	**59.6**	**61.7**	**25.7**	**99.2**
IIC (avg sub-head ± STD)	59.8 ± 0.844	57.6 ± 5.01	25.5 ± 0.462	98.4 ± 0.652

Table 1: **Unsupervised image clustering.** Legend: †Method based on k-means. ‡Method that does not directly learn a clustering function and requires further application of k-means to be used for image clustering. ⋆Results obtained using our experiments with authors' original code.

STL10 dataset



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Overall results

TASK	DATASET	MODEL	METRIC NAME	METRIC VALUE	GLOBAL RANK	RESULT	BENCHMARK
Unsupervised Image Classification	CIFAR-10	IIC	Accuracy	61.7	# 3		Compare
Unsupervised Image Classification	CIFAR-20	IIC	Accuracy	25.7	# 4		Compare
Unsupervised Semantic Segmentation	COCO-Stuff-15	IIC	Accuracy	27.7	# 1		Compare
Unsupervised Semantic Segmentation	COCO-Stuff-3	IIC	Accuracy	72.3	# 1		Compare
Unsupervised MNIST	MNIST	IIC	Accuracy	99.3	# 1		Compare
Unsupervised Image Classification	MNIST	IIC	Accuracy	99.3	# 1		Compare
Unsupervised Semantic Segmentation	Potsdam	IIC	Accuracy	65.1	# 1		Compare
Unsupervised Semantic Segmentation	Potsdam-3	IIC	Accuracy	45.4	# 1		Compare
Unsupervised Image Classification	STL-10	IIC	Accuracy	61.00	# 3		Compare
Image Classification	STL-10	IIC	Percentage correct	88.8	# 37		Compare
Semi-Supervised Image Classification	STL-10	IIC	Accuracy	88.8	# 2		Compare

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Summary

1. simple method with several SOTA results
2. multiple problems
3. can be used as encoder trainer
4. data type agnostic (theoretically)

Links

1. Original paper:
   https://arxiv.org/abs/1807.06653
2. Authors git:
   https://github.com/xu-ji/IIC
3. My tutorial (with colab version):
   https://github.com/vandedok/IIC_tutorial

Thanks for attention!

Questions? Additions? Welcome!

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