In [1]:

```
#initialization
import matplotlib.pyplot as plt
import numpy as np

# importing Qiskit
from qiskit import IBMQ, Aer, assemble, transpile
from qiskit import QuantumCircuit, ClassicalRegister, QuantumRegister
from qiskit.providers.ibmq import least_busy

# import basic plot tools
from qiskit.visualization import plot_histogram
from qiskit_textbook.problems import grover_problem_oracle
```

In [2]:

```
def initialize_s(qc, qubits):
    """Apply a H-gate to 'qubits' in qc"""
    for q in qubits:
        qc.h(q)
    return qc
```

In [3]:

```
def diffuser(nqubits):
    qc = QuantumCircuit(nqubits)
    # Apply transformation |s> -> |00..0> (H-gates)
    for qubit in range(nqubits):
        qc.h(qubit)
    # Apply transformation |00..0> -> |11..1> (X-gates)
    for qubit in range(nqubits):
        qc.x(qubit)
    # Do multi-controlled-Z gate
    qc.h(nqubits-1)
    qc.mct(list(range(nqubits-1)), nqubits-1)  # multi-controlled-toffoli
    qc.h(nqubits-1)
    # Apply transformation |11..1> -> |00..0>
    for qubit in range(nqubits):
        qc.x(qubit)
    # Apply transformation |00..0> -> |s>
    for qubit in range(nqubits):
        qc.h(qubit)
    # We will return the diffuser as a gate
    U_s = qc.to_gate()
    U_s.name = "U$_s$"
    return U_s
```

1. grover_problem_oracle(4, variant=2) uses 4 qubits and has 1 solution.

a) How many iterations do we need to have a > 90% chance of measuring this solution?

In [4]:

```
n = 4
qc = QuantumCircuit(n)
oracle1 = grover_problem_oracle(4, variant=2, print_solutions=True)
qc = initialize_s(qc, [0,1,2,3])
# we run two iterations here
qc.append(oracle1, [0,1,2,3])
qc.append(diffuser(n), [0,1,2,3])
qc.append(oracle1, [0,1,2,3])
qc.append(diffuser(n), [0,1,2,3])
```

```python
qc.measure_all()
qc.draw()
```

```
Solutions:
|0011>
```

Out[4]:

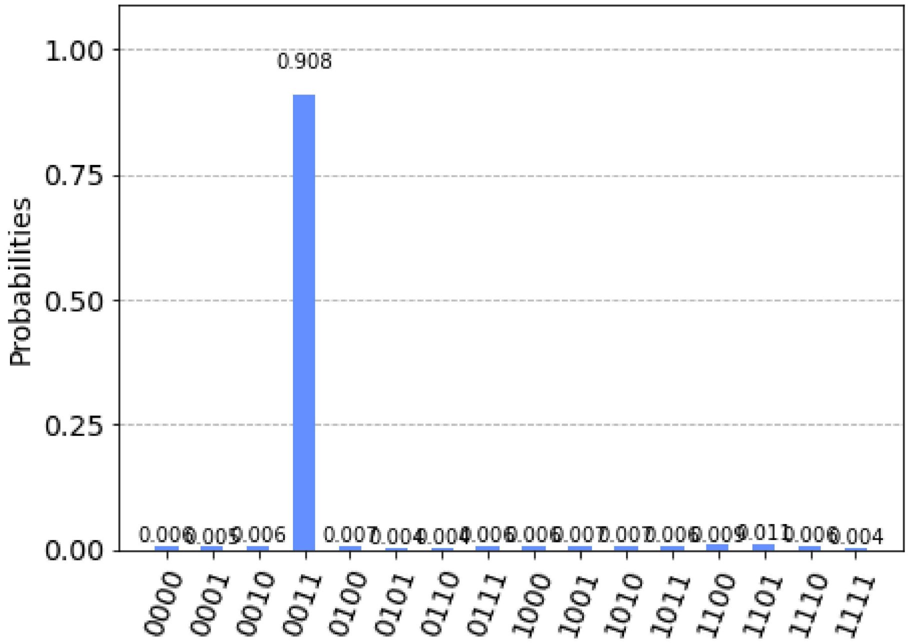
```
        ┌───┐┌──────────────────────┐┌────┐┌──────────────────────┐┌──
   q_0: ┤ H ├┤0                     ├┤0   ├┤0                     ├┤0 
        ├───┤│                      ││    ││                      ││  
   q_1: ┤ H ├┤1                     ├┤1   ├┤1                     ├┤1 
        ├───┤│  Oracle              ││    ││                      ││  
n=4, var=2   │  U$_s$               ││  Oracle
n=4, var=2   │  U$_s$               │»
   q_2: ┤ H ├┤2                     ├┤2   ├┤2                     ├┤2 
        ├───┤│                      ││    ││                      ││  
   q_3: ┤ H ├┤3                     ├┤3   ├┤3                     ├┤3 
        └───┘└──────────────────────┘└────┘└──────────────────────┘└──
meas: 4/══════════════════════════════════════════════════════════════

«
«     q_0: ─░─┤M├────────────
«          ░  └╥┘┌─┐
«     q_1: ─░──╫─┤M├─────────
«          ░  ║ └╥┘┌─┐
«     q_2: ─░──╫──╫─┤M├──────
«          ░  ║  ║ └╥┘┌─┐
«     q_3: ─░──╫──╫──╫─┤M├───
«          ░  ║  ║  ║ └╥┘
«meas: 4/═══════╩══╩══╩══╩═
«               0  1  2  3
```

In [5]:

```python
aer_sim = Aer.get_backend('aer_simulator')
transpiled_qc = transpile(qc, aer_sim)
qobj = assemble(transpiled_qc)
results = aer_sim.run(qobj).result()
counts = results.get_counts()
plot_histogram(counts)
```

Out[5]:



Here we can observe, after 2 iterations we have a 90% chance of measuring the solution state |0011>.

b) Use Grover's algorithm to find this solution state. c. What happens if we apply more

iterations the number we calculated in problem 1a above?Why?

The solution state we can see is |0011> which is also verified below by the oracle. If we apply more iterations we get a higher chance of calculating the correct solution as the amplitude of non-marked states decerase after each iteration and the amplitude of correct solution increases.

In [6]:

```
# Verification of Solution
n = 4
oracle = grover_problem_oracle(n, variant=2, print_solutions = True)  # 0th variant
qc = QuantumCircuit(n)
qc.append(oracle, [0,1,2,3])
```

```
Solutions:
|0011>
```

Out[6]:

```
<qiskit.circuit.instructionset.InstructionSet at 0x1bc8f3ac4f0>
```

2. With 2 solutions and 4 qubits, how many iterations do we need for a >90% chance of measuring a solution? Test your answer using the oracle grover_problem_oracle(4, variant=1) (which has two solutions)

In [23]:

```
n = 4
qc = QuantumCircuit(n)
oracle = grover_problem_oracle(4, variant=1, print_solutions=True)
qc = initialize_s(qc, [0,1,2,3])
# we run two iterations here
qc.append(oracle, [0,1,2,3])
qc.append(diffuser(n), [0,1,2,3])
qc.measure_all()
qc.draw()
```

```
Solutions:
|0111>
|1001>
```

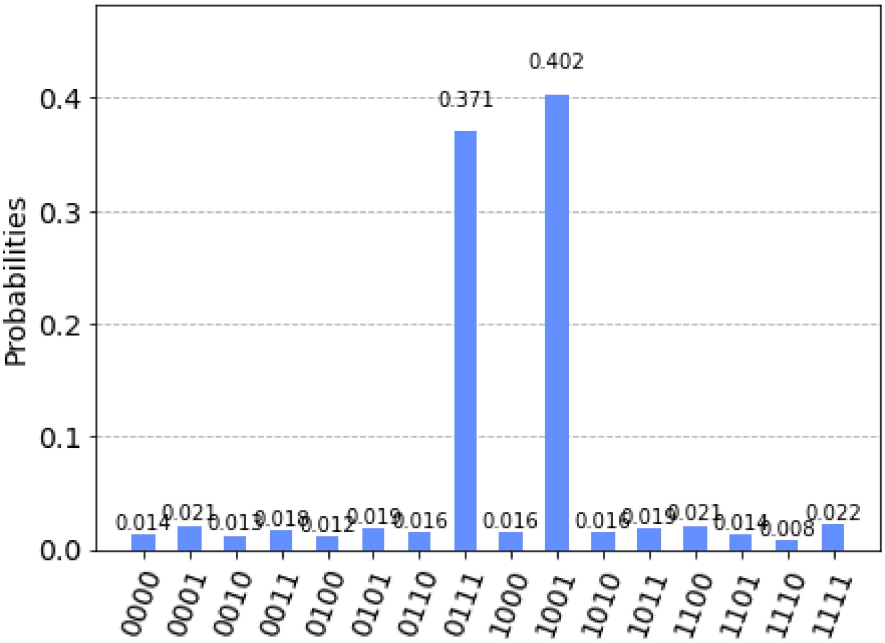
Out[23]:

```
        ┌───┐┌────────────────────┐┌─────────┐ ░ ┌─┐
   q_0: ┤ H ├┤0                   ├┤0        ├─░─┤M├─────────
        ├───┤│                    ││         │ ░ └╥┘┌─┐
   q_1: ┤ H ├┤1                   ├┤1        ├─░──╫─┤M├──────
        ├───┤│    Oracle          ││         │ ░  ║ └╥┘
n=4, var=1 │  U$_s$ │ ░  ‖ ╨┐ ┌
   q_2: ┤ H ├┤2                   ├┤2        ├─░──╫──╫─┤M├───
        ├───┤│                    ││         │ ░  ║  ║ └╥┘┌─┐
   q_3: ┤ H ├┤3                   ├┤3        ├─░──╫──╫──╫─┤M├
        └───┘└────────────────────┘└─────────┘ ░  ║  ║  ║ └╥┘
meas: 4/══════════════════════════════════════════╩══╩══╩══╩═
                                                  0  1  2  3
```

In [24]:

```
aer_sim = Aer.get_backend('aer_simulator')
transpiled_qc = transpile(qc, aer_sim)
qobj = assemble(transpiled_qc)
results = aer_sim.run(qobj).result()
counts = results.get_counts()
plot_histogram(counts)
```

Out[24]:



If we take one iteration then we have a 0.371+0.402 = 0.773 that is around 77% of getting the solutions

In [25]:

```
n = 4
qc = QuantumCircuit(n)
oracle = grover_problem_oracle(4, variant=1, print_solutions=True)
qc = initialize_s(qc, [0,1,2,3])
# we run two iterations here
qc.append(oracle, [0,1,2,3])
qc.append(diffuser(n), [0,1,2,3])
qc.append(oracle, [0,1,2,3])
qc.append(diffuser(n), [0,1,2,3])
qc.measure_all()
qc.draw()
```

```
Solutions:
|0111>
|1001>
```

Out[25]:

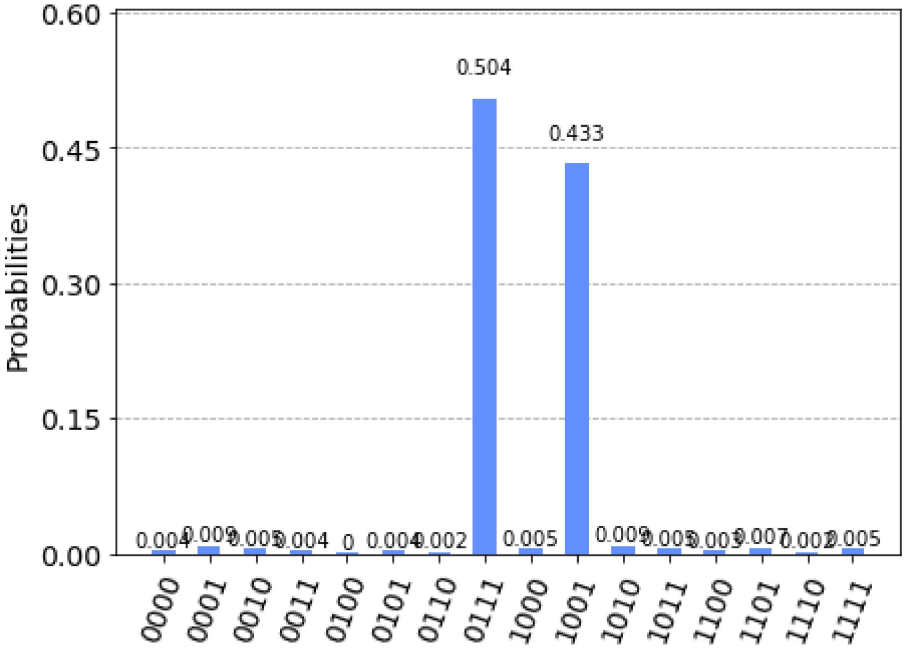
```
     q_0: ┤ H ├┤0                          ├┤0           ├┤0                          ├┤0
     q_1: ┤ H ├┤1                          ├┤1           ├┤1                          ├┤1
               │  Oracle
n=4, var=1 │   U$_s$ ││ Oracle
n=4, var=1 │   U$_s$ │»
     q_2: ┤ H ├┤2                          ├┤2           ├┤2                          ├┤2
     q_3: ┤ H ├┤3                          ├┤3           ├┤3                          ├┤3
meas: 4/═══════════════════════════════════════════════════════════════════════════════

«
«     q_0: ─░─┤M├──────────
«
«     q_1: ─░──╫─┤M├───────
«
«     q_2: ─░──╫──╫─┤M├────
«
«     q_3: ─░──╫──╫──╫─┤M├─
«
«meas: 4/═════╩══╩══╩══╩═
«             0  1  2  3
```

In [26]:

```python
aer_sim = Aer.get_backend('aer_simulator')
transpiled_qc = transpile(qc, aer_sim)
qobj = assemble(transpiled_qc)
results = aer_sim.run(qobj).result()
counts = results.get_counts()
plot_histogram(counts)
```

Out[26]:



If we take two iteration then we have a 0.504+0.433 = 0.937 that is >90% chance of getting the solutions

3

3. Create a function, `grover_solver(oracle, iterations)` that takes as input:

- A Grover oracle as a gate (`oracle`)
- An integer number of iterations (`iterations`)

and returns a `QuantumCircuit` that performs Grover's algorithm on the '`oracle`' gate, with '`iterations`' iterations.

In [74]:

```python
# define the number of qubits needed n
# define your own oracle and the nummber of iterations you need
# this code makes use of just the diffuser function

def grover_oracle(oracle, iterations):
    # applying h gates
    for i in range(n):
        qc.h(i)
    #applying the oracle specified
    a = []
    for i in range(n):
        a.append(i)

    for i in range(iterations):
        qc.append(oracle, a)
        qc.append(diffuser(n), a)

    qc.measure_all()
    return qc
```

Testing

In [75]:

```
n = 4
iterations = 2
oracle = grover_problem_oracle(n, variant=1, print_solutions=True)

qc = QuantumCircuit(n, n)
qc = grover_oracle(oracle, iterations)
qc.draw()
```

```
Solutions:
|0111>
|1001>
```

Out[75]:

```
        ┌───┐┌──────────────────────────┐┌────────────┐┌───────────────────────────┐┌─
   q_0: ┤ H ├┤0                         ├┤0           ├┤0                          ├┤0
        ├───┤│                          ││            ││                           ││
   q_1: ┤ H ├┤1                         ├┤1           ├┤1                          ├┤1
        ├───┤│  Oracle
n=4, var=1 │  U$_s$ ││  Oracle
n=4, var=1 │  U$_s$ │»
   q_2: ┤ H ├┤2                         ├┤2           ├┤2                          ├┤2
        ├───┤│                          ││            ││                           ││
   q_3: ┤ H ├┤3                         ├┤3           ├┤3                          ├┤3
        └───┘└──────────────────────────┘└────────────┘└───────────────────────────┘└─
   c: 4/══════════════════════════════════════════════════════════════════════════════

meas: 4/══════════════════════════════════════════════════════════════════════════════

«           ░ ┌─┐
«   q_0: ───░─┤M├─────────
«           ░ └╥┘┌─┐
«   q_1: ───░──╫─┤M├──────
«           ░  ║ └╥┘┌─┐
«   q_2: ───░──╫──╫─┤M├───
«           ░  ║  ║ └╥┘┌─┐
«   q_3: ───░──╫──╫──╫─┤M├
«           ░  ║  ║  ║ └╥┘
«   c: 4/══════╬══╬══╬══╬═
«              ║  ║  ║  ║
«meas: 4/══════╩══╩══╩══╩═
«              0  1  2  3
```

In [ ]: