In [5]:

```
import matplotlib.pyplot as plt
import numpy as np
from qiskit import QuantumCircuit, Aer, transpile, assemble
from qiskit.visualization import plot_histogram
from math import gcd
from numpy.random import randint
import pandas as pd
from fractions import Fraction
```

1. Modify the circuit above for values of a= 2, 8, 11, 13. What results do you get and why?

In [6]:

```
def c_amod15(a, power):
    """Controlled multiplication by a mod 15"""
    if a not in [2,7,8,11,13]:
        raise ValueError("'a' must be 2,7,8,11 or 13")
    U = QuantumCircuit(4)
    for iteration in range(power):
        if a in [2,13]:
            U.swap(0,1)
            U.swap(1,2)
            U.swap(2,3)
        if a in [7,8]:
            U.swap(2,3)
            U.swap(1,2)
            U.swap(0,1)
        if a == 11:
            U.swap(1,3)
            U.swap(0,2)
        if a in [7,11,13]:
            for q in range(4):
                U.x(q)
    U = U.to_gate()
    U.name = "%i^%i mod 15" % (a, power)
    c_U = U.control()
    return c_U
```

In [7]:

```
def qft_dagger(n):
    """n-qubit QFTdagger the first n qubits in circ"""
    qc = QuantumCircuit(n)
```

```
    # Don't forget the Swaps!
    for qubit in range(n//2):
        qc.swap(qubit, n-qubit-1)
    for j in range(n):
        for m in range(j):
            qc.cp(-np.pi/float(2**(j-m)), m, j)
        qc.h(j)
    qc.name = "QFT†"
    return qc
```

(1) a = 2

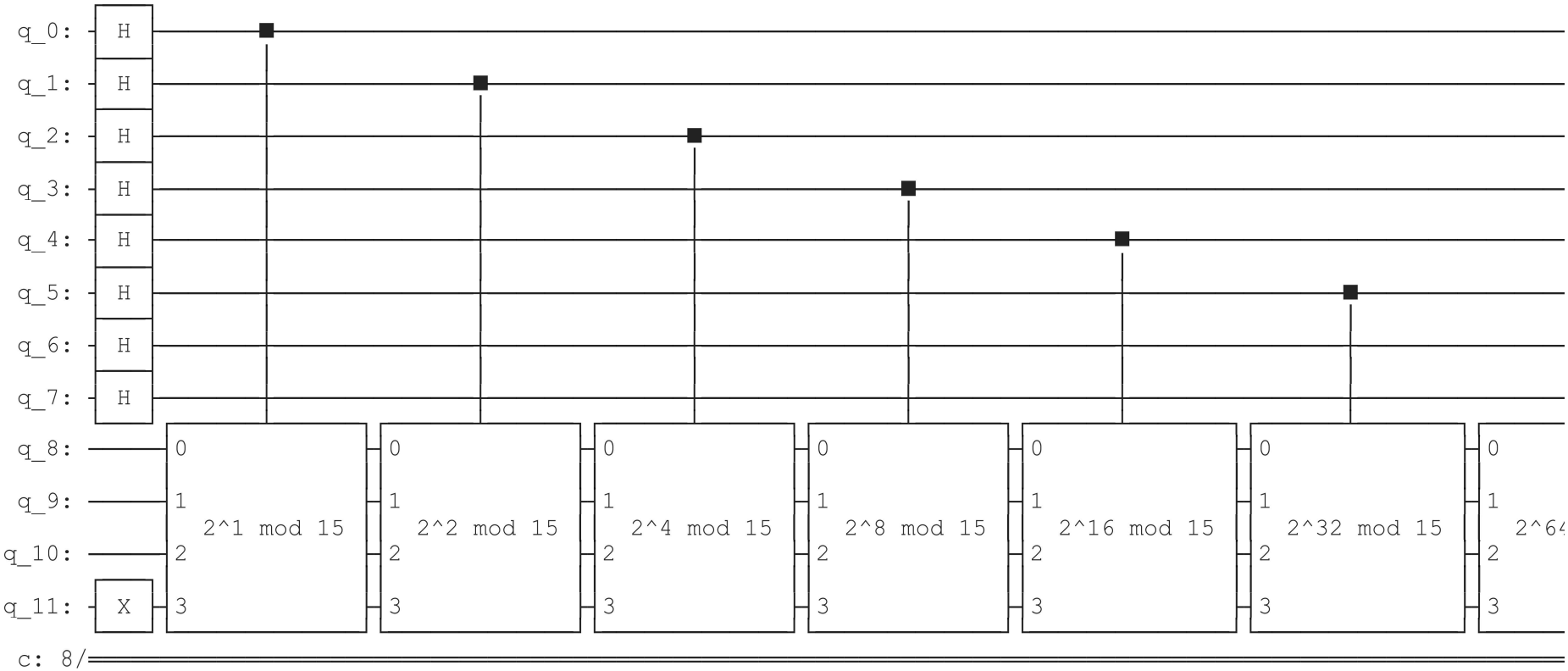
In [8]:

```
# Specify variables
n_count = 8  # number of counting qubits
a = 2
```

In [9]:

```
# Create QuantumCircuit with n_count counting qubits
# plus 4 qubits for U to act on
qc = QuantumCircuit(n_count + 4, n_count)

# Initialize counting qubits
# in state |+>
for q in range(n_count):
    qc.h(q)

# And auxiliary register in state |1>
qc.x(3+n_count)

# Do controlled-U operations
for q in range(n_count):
    qc.append(c_amod15(a, 2**q), # second one is power of 2
              [q] + [i+n_count for i in range(4)]) # i+n_count will be 0+8, 1+8, 2+8, 3+8 where n_count = 8

# Do inverse-QFT
qc.append(qft_dagger(n_count), range(n_count))

# Measure circuit
qc.measure(range(n_count), range(n_count))
qc.draw(fold=-1)  # -1 means 'do not fold'
```

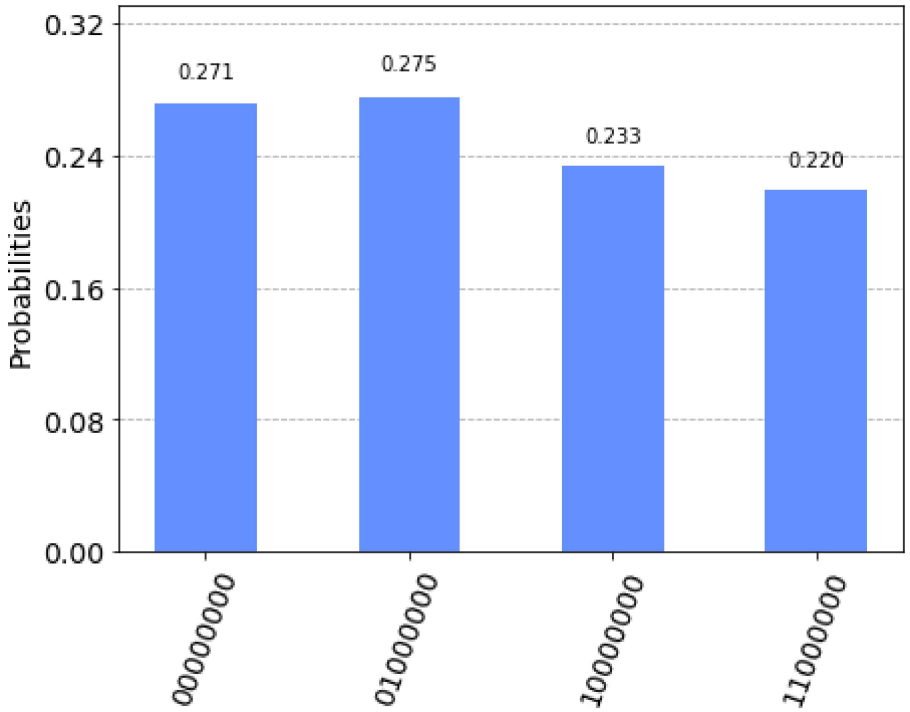
Out[9]:



In [10]:

```
aer_sim = Aer.get_backend('aer_simulator')
t_qc = transpile(qc, aer_sim)
qobj = assemble(t_qc)
results = aer_sim.run(qobj).result()
counts = results.get_counts()
plot_histogram(counts)
```

Out[10]:



In [11]:

```
rows, measured_phases = [], []
for output in counts:
    decimal = int(output, 2)  # Convert (base 2) string to decimal
    phase = decimal/(2**n_count)  # Find corresponding eigenvalue
    measured_phases.append(phase)
    # Add these values to the rows in our table:
    rows.append([f"{output}(bin) = {decimal:>3}(dec)",
                 f"{decimal}/{2**n_count} = {phase:.2f}"])
# Print the rows in a table
headers=["Register Output", "Phase"]
df = pd.DataFrame(rows, columns=headers)
print(df)
```

```
           Register Output           Phase
0  00000000(bin) =   0(dec)    0/256 = 0.00
1  10000000(bin) = 128(dec)  128/256 = 0.50
2  01000000(bin) =  64(dec)   64/256 = 0.25
3  11000000(bin) = 192(dec)  192/256 = 0.75
```

In [12]:

```
rows = []
for phase in measured_phases:
    frac = Fraction(phase).limit_denominator(15)
    rows.append([phase, f"{frac.numerator}/{frac.denominator}", frac.denominator])
# Print as a table
headers=["Phase", "Fraction", "Guess for r"]
df = pd.DataFrame(rows, columns=headers)
print(df)
```

```
   Phase Fraction  Guess for r
0   0.00      0/1            1
1   0.50      1/2            2
2   0.25      1/4            4
3   0.75      3/4            4
```

We can see the result we get is r=4. Hence we can see that 2^4 mod15 = 1

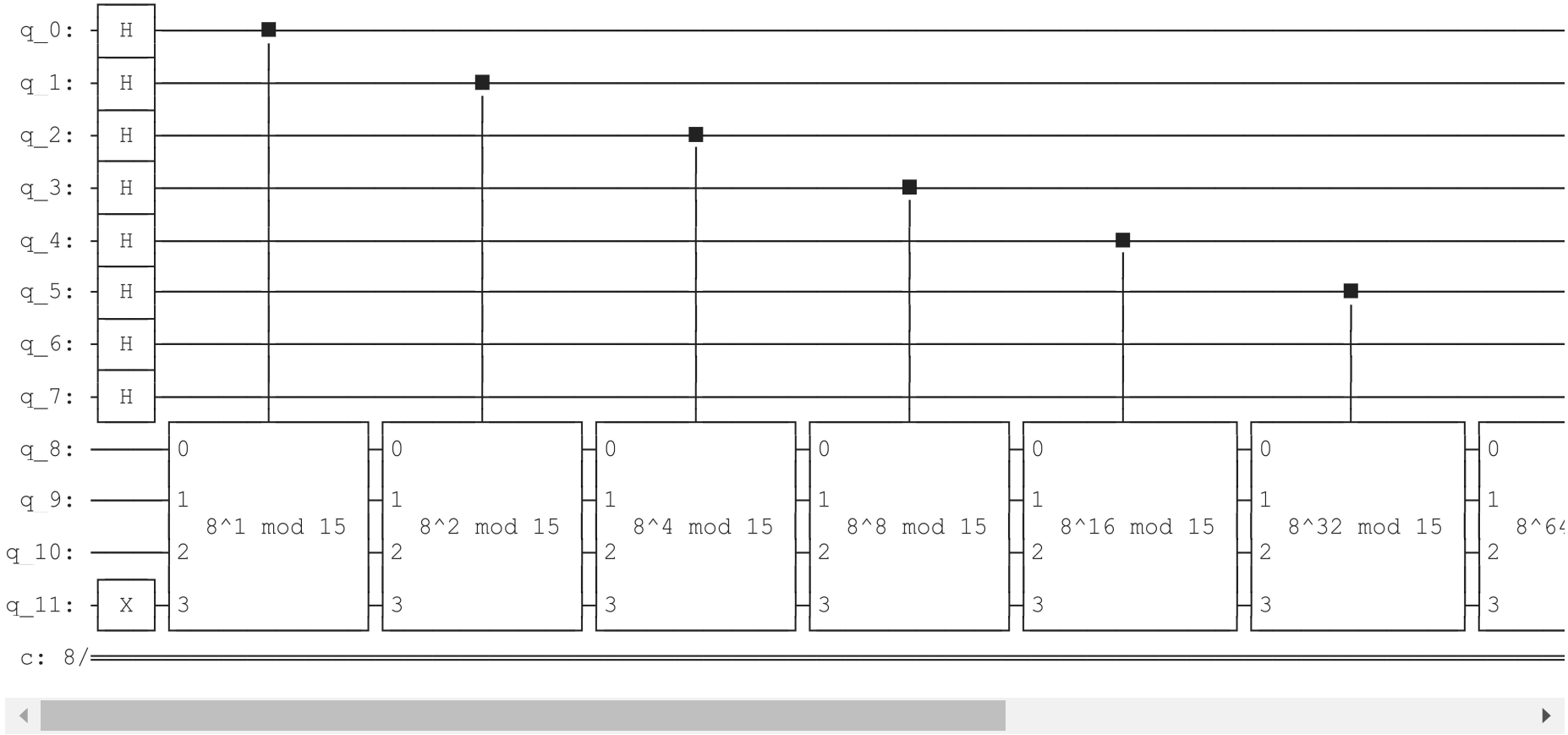
(2) a = 8

In [13]:

```
# Specify variables
n_count = 8  # number of counting qubits
a = 8

# Create QuantumCircuit with n_count counting qubits
# plus 4 qubits for U to act on
qc = QuantumCircuit(n_count + 4, n_count)

# Initialize counting qubits
# in state |+>
for q in range(n_count):
    qc.h(q)

# And auxiliary register in state |1>
qc.x(3+n_count)

# Do controlled-U operations
for q in range(n_count):
    qc.append(c_amod15(a, 2**q), # second one is power of 2
              [q] + [i+n_count for i in range(4)]) # i+n_count will be 0+8, 1+8, 2+8, 3+8 where n_count = 8

# Do inverse-QFT
qc.append(qft_dagger(n_count), range(n_count))
```

```
# Measure circuit
qc.measure(range(n_count), range(n_count))
qc.draw(fold=-1)  # -1 means 'do not fold'
```

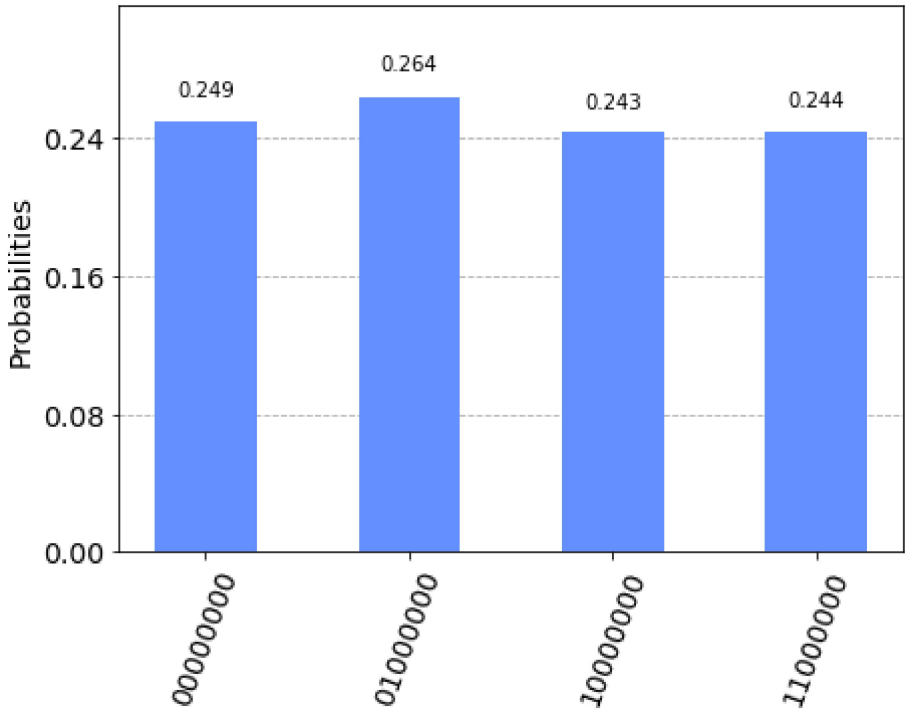
Out[13]:



In [14]:

```
aer_sim = Aer.get_backend('aer_simulator')
t_qc = transpile(qc, aer_sim)
qobj = assemble(t_qc)
results = aer_sim.run(qobj).result()
counts = results.get_counts()
plot_histogram(counts)
```

Out[14]:



In [15]:

```
rows, measured_phases = [], []
for output in counts:
    decimal = int(output, 2)  # Convert (base 2) string to decimal
    phase = decimal/(2**n_count)  # Find corresponding eigenvalue
    measured_phases.append(phase)
    # Add these values to the rows in our table:
    rows.append([f"{output}(bin) = {decimal:>3}(dec)",
                 f"{decimal}/{2**n_count} = {phase:.2f}"])
# Print the rows in a table
headers=["Register Output", "Phase"]
df = pd.DataFrame(rows, columns=headers)
print(df)
```

```
           Register Output           Phase
0  01000000(bin) =  64(dec)   64/256 = 0.25
1  10000000(bin) = 128(dec)  128/256 = 0.50
2  00000000(bin) =   0(dec)    0/256 = 0.00
3  11000000(bin) = 192(dec)  192/256 = 0.75
```

In [16]:

```
rows = []
for phase in measured_phases:
    frac = Fraction(phase).limit_denominator(15)
    rows.append([phase, f"{frac.numerator}/{frac.denominator}", frac.denominator])
# Print as a table
headers=["Phase", "Fraction", "Guess for r"]
df = pd.DataFrame(rows, columns=headers)
print(df)
```

```
   Phase Fraction  Guess for r
0   0.25      1/4            4
1   0.50      1/2            2
2   0.00      0/1            1
3   0.75      3/4            4
```

We can see the result we get is r=4. Hence we can see that 8^4 mod15 = 1

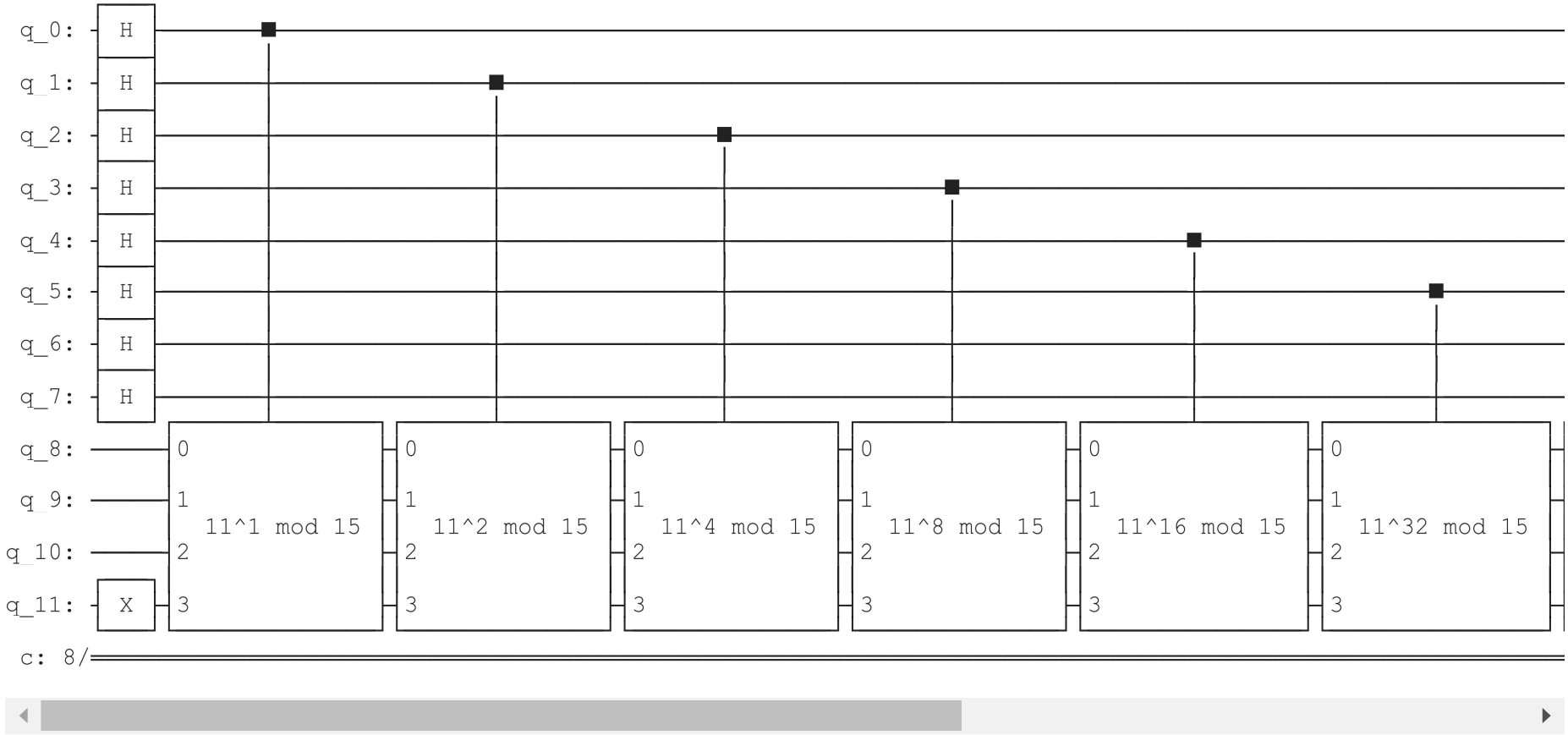
(3) a = 11

In [17]:

```
# Specify variables
n_count = 8  # number of counting qubits
a = 11

# Create QuantumCircuit with n_count counting qubits
# plus 4 qubits for U to act on
qc = QuantumCircuit(n_count + 4, n_count)

# Initialize counting qubits
# in state |+>
for q in range(n_count):
    qc.h(q)

# And auxiliary register in state |1>
qc.x(3+n_count)

# Do controlled-U operations
for q in range(n_count):
    qc.append(c_amod15(a, 2**q), # second one is power of 2
              [q] + [i+n_count for i in range(4)]) # i+n_count will be 0+8, 1+8, 2+8, 3+8 where n_count = 8

# Do inverse-QFT
qc.append(qft_dagger(n_count), range(n_count))
```

```
# Measure circuit
qc.measure(range(n_count), range(n_count))
qc.draw(fold=-1)  # -1 means 'do not fold'
```

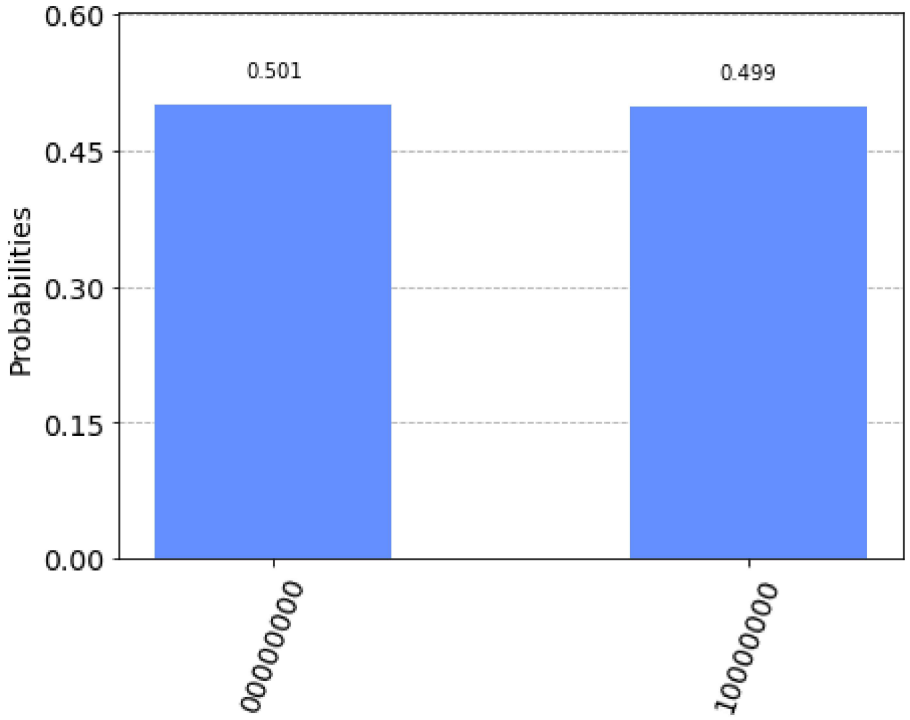
Out[17]:



In [18]:

```
aer_sim = Aer.get_backend('aer_simulator')
t_qc = transpile(qc, aer_sim)
qobj = assemble(t_qc)
results = aer_sim.run(qobj).result()
counts = results.get_counts()
plot_histogram(counts)
```

Out[18]:



In [19]:

```
rows, measured_phases = [], []
for output in counts:
    decimal = int(output, 2)  # Convert (base 2) string to decimal
    phase = decimal/(2**n_count)  # Find corresponding eigenvalue
    measured_phases.append(phase)
    # Add these values to the rows in our table:
    rows.append([f"{output}(bin) = {decimal:>3}(dec)",
                 f"{decimal}/{2**n_count} = {phase:.2f}"])
# Print the rows in a table
headers=["Register Output", "Phase"]
df = pd.DataFrame(rows, columns=headers)
print(df)
```

```
           Register Output           Phase
0  10000000(bin) = 128(dec)  128/256 = 0.50
1  00000000(bin) =   0(dec)    0/256 = 0.00
```

In [20]:

```
rows = []
for phase in measured_phases:
```

```
    frac = Fraction(phase).limit_denominator(15)
    rows.append([phase, f"{frac.numerator}/{frac.denominator}", frac.denominator])
# Print as a table
headers=["Phase", "Fraction", "Guess for r"]
df = pd.DataFrame(rows, columns=headers)
print(df)
```

```
   Phase Fraction  Guess for r
0    0.5      1/2            2
1    0.0      0/1            1
```

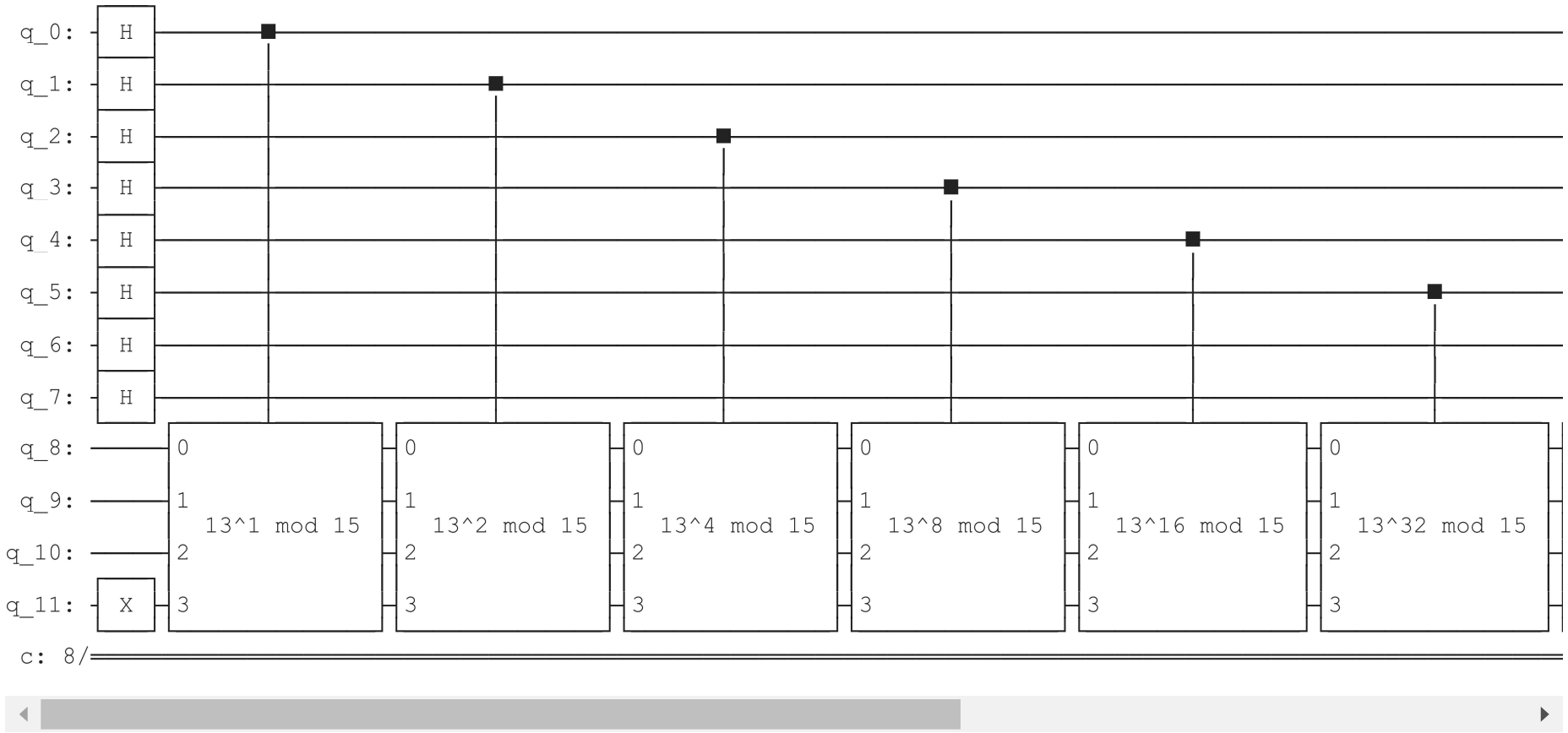
We can see the result we get is r=2. Hence we can see that 11^2 mod15 = 1

(4) a = 13

In [22]:

```
# Specify variables
n_count = 8  # number of counting qubits
a = 13

# Create QuantumCircuit with n_count counting qubits
# plus 4 qubits for U to act on
qc = QuantumCircuit(n_count + 4, n_count)

# Initialize counting qubits
# in state |+>
for q in range(n_count):
    qc.h(q)

# And auxiliary register in state |1>
qc.x(3+n_count)

# Do controlled-U operations
for q in range(n_count):
    qc.append(c_amod15(a, 2**q), # second one is power of 2
              [q] + [i+n_count for i in range(4)]) # i+n_count will be 0+8, 1+8, 2+8, 3+8 where n_count = 8

# Do inverse-QFT
qc.append(qft_dagger(n_count), range(n_count))

# Measure circuit
qc.measure(range(n_count), range(n_count))
qc.draw(fold=-1)  # -1 means 'do not fold'
```

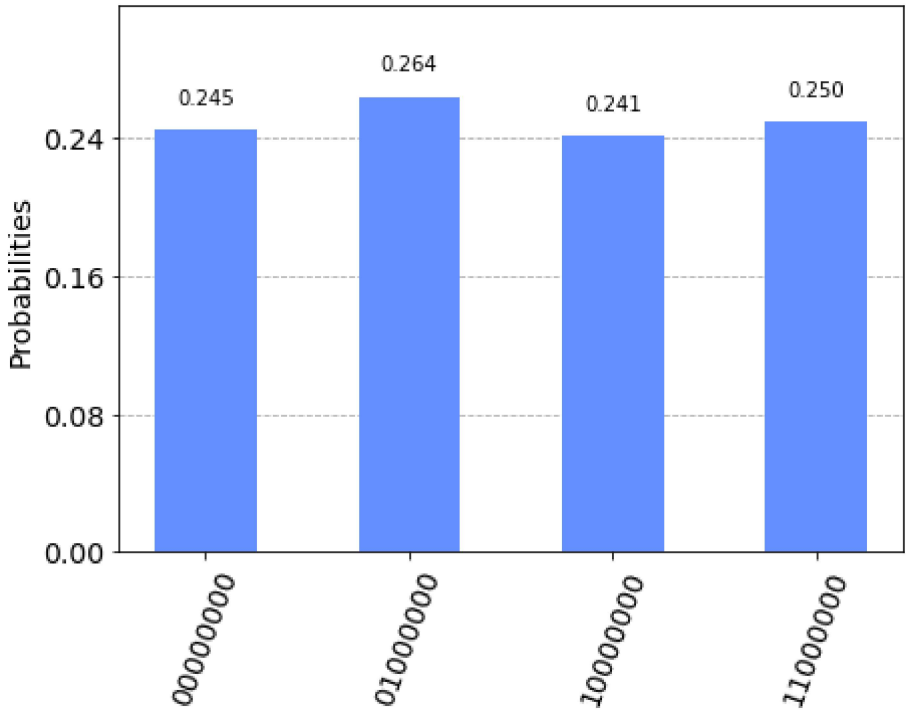
Out[22]:



In [23]:

```
aer_sim = Aer.get_backend('aer_simulator')
t_qc = transpile(qc, aer_sim)
qobj = assemble(t_qc)
results = aer_sim.run(qobj).result()
counts = results.get_counts()
plot_histogram(counts)
```

Out[23]:



In [24]:

```
rows, measured_phases = [], []
for output in counts:
    decimal = int(output, 2)  # Convert (base 2) string to decimal
    phase = decimal/(2**n_count)  # Find corresponding eigenvalue
    measured_phases.append(phase)
    # Add these values to the rows in our table:
    rows.append([f"{output}(bin) = {decimal:>3}(dec)",
                 f"{decimal}/{2**n_count} = {phase:.2f}"])
# Print the rows in a table
headers=["Register Output", "Phase"]
df = pd.DataFrame(rows, columns=headers)
print(df)
```

```
            Register Output           Phase
0  01000000(bin) =  64(dec)   64/256 = 0.25
1  11000000(bin) = 192(dec)  192/256 = 0.75
2  00000000(bin) =   0(dec)    0/256 = 0.00
3  10000000(bin) = 128(dec)  128/256 = 0.50
```

In [25]:

```
rows = []
for phase in measured_phases:
    frac = Fraction(phase).limit_denominator(15)
    rows.append([phase, f"{frac.numerator}/{frac.denominator}", frac.denominator])
# Print as a table
headers=["Phase", "Fraction", "Guess for r"]
df = pd.DataFrame(rows, columns=headers)
print(df)
```

```
   Phase Fraction  Guess for r
0   0.25      1/4            4
1   0.75      3/4            4
2   0.00      0/1            1
3   0.50      1/2            2
```

We can see the result we get is r=4. Hence we can see that 13^4 mod15 = 1

In [ ]: