

1. Review: Polynomials, Exponentials, Logarithms, and their Derivatives

a. Polynomials

$p(n) = \sum_{i=0}^d a_i n^i$ is called a polynomial in n of degree d where $a_d \neq 0$.

A polynomial is asymptotically positive if and only if $a_d > 0$, and we have $p(n) = \Theta(n^d)$.

b. Exponentials

a^n is the basic form of an exponential, where a is called base.

$$(a^n)^m = a^{mn} = (a^m)^n.$$

For all real constants a and b such that $a > 1$,

$$\lim_{x \rightarrow \infty} \frac{n^b}{a^n} = 0$$

from which we can conclude that any exponential function with a base strictly greater than 1 grows faster than any polynomial function.

c. Logarithms

$\lg n = \log_2 n$; $\ln n = \log_e n$ (natural logarithm).

For all real $a > 0$, $b > 0$, $c > 0$, and n , we have the following properties:

i. $a = b^{\log_b a}$

ii. $\log_c(ab) = \log_c a + \log_c b$

iii. $\log_b a^n = n \log_b a$

iv. $\log_b a = \frac{\log_c a}{\log_c b}$

d. Derivatives

$$f(x) = ax^n, \quad \frac{df}{dx} = anx^{n-1}.$$

$$f(x) = a^x, \quad \frac{df}{dx} = a^x \ln a.$$

$$f(x) = \ln x, \quad \frac{df}{dx} = \frac{1}{x}, \quad x > 0; \quad f(x) = \log_a x, \quad \frac{df}{dx} = \frac{1}{x \ln a}, \quad x > 0.$$

2. Asymptotic Notation: O , Ω , Θ

Example: We have $f(n) = \lg n$, $g(n) = \ln n$, what is their relation represented in asymptotic notation? There are two ways.

a. By definition

b. By using limits (L'Hopital's rule)

3. Solving Recurrence Relation

Recall the recurrence relation of merge sort: $T(n) = 2T(\frac{n}{2}) + n - 1, n \geq 2$.

a. Substitution Method

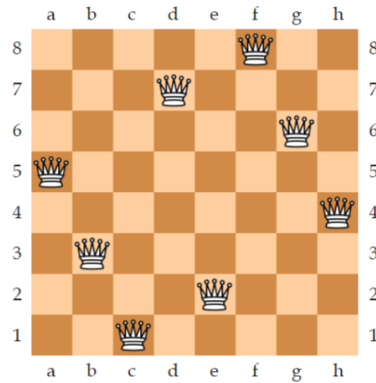
b. Master Theorem

Followup: Which one should we use?

1. Substitution always works.
2. Master Theorem works only when $T(n) = aT(\frac{n}{b}) + f(n)$. Think about $T(n) = 2T(n - 1) + n$.

4. Divide and Conquer: n -Queens Problem

You are asked to place queens so that no two queens are in the same row, column or diagonal on a n -by- n chessboard. Figure 1 is an example.



Devise an algorithm which takes n as input and outputs all distinct placement of n queens on an n -by- n chessboard. Analyze its running time.

Hint: If the first queen is placed at (i, j) , where can't any of the other queens be placed?