Discussion 4

1. Divide and Conquer: Chip Testing (Textbook Problem 4-5)

Professor Bai has n identical integrated-circuit chips that are capable of testing each other. A good chip always reports accurately whether the other chip is good or bad, but the professor cannot trust the answer of a bad chip. Thus, we have four different outcomes:

Chip A says	Chip B says	Conclusion
B is good	A is good	both are good or both are bad
B is good	A is bad	at least one is bad
B is bad	A is good	at least one is bad
B is bad	A is bad	at least one is bad

Show that the good chips can be identified with $\Theta(n)$ pairwise tests, **assuming** that more than $n/2$ of the chips are good. Give and solve the recurrence that describes the number of tests.

Hint: First think about how to find a **single** good chip using divide and conquer paradigm.

Discussion 4

2. Divide and Conquer: Stock Investment (AD Ch. 5 Solved Exercise 2)

You're consulting for a small computation-intensive investment company, and a typical instance of the problem you will encounter is the following. They're doing a simulation in which they look at n consecutive days of a given stock, at some point in the past. Lets number the days $i = 1, 2, ..., n$; for each day i, they have a price $p(i)$ per share for the stock on that day. Suppose during this time period, they wanted to buy 1,000 shares on some day and sell all these shares on some (later) day. They want to know: When should they have bought and when should they have sold in order to have made as much money as possible? (If there was no way to make money during the n days, you should report this instead.)

For example, suppose $n = 3$, $p(1) = 9$, $p(2) = 1$, $p(3) = 5$. Then you should return buy on 2, sell on 3 (buying on day 2 and selling on day 3 means they would have made $4 per share, the maximum possible for that period).

Show how to find the correct numbers i and j in time $O(n \lg n)$.

Hint: Recall how we find the minimum and maximum values in an array s of length n by divide and conquer in homework 1. Can we apply the same strategy?