Discussion 8

1. Graph Basics

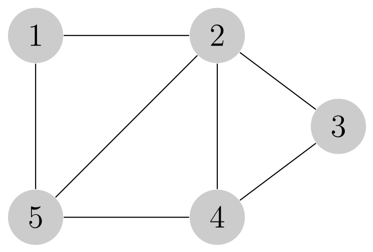


Figure 1: An undirected graph G_1 with 5 vertices and 7 edges

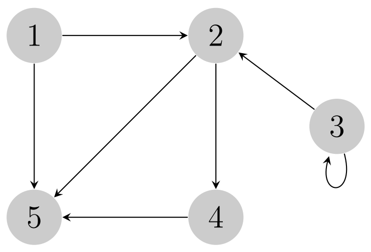
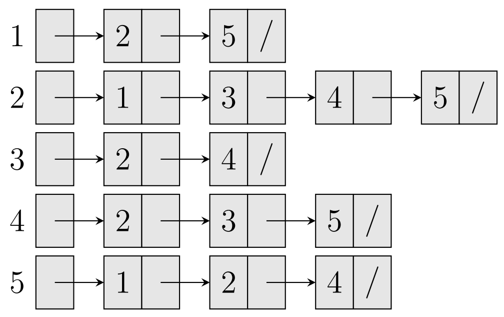


Figure 2: A directed graph G_2 with 5 vertices and 7 edges.

a. Representation: Adjacency List, Adjacency Matrix

What is the adjacency list and adjacency matrix of G_1 and G_2, respectively?



	1	2	3	4	5
1	0	1	0	0	0
2	1	0	1	1	1
3	0	1	0	1	0
4	0	1	1	0	1
5	1	1	0	1	0

Figure 3: Adjacency list and adjacency matrix representation of G_1

How about G_2? It's your turn now!

Discussion 8

1. Adjacency matrix is of size $|V| \times |V|$ while adjacency list needs $\Theta(|V| + |E|)$ space.

2. If G is undirected, its adjacency matrix A is symmetric. Namely, $A^T = A$. Further, the main diagonal entries of A are all zeros.

3. **Self-loops**—edges from a vertex to itself—are possible in a directed graph, but are forbidden in an undirected graph.

b. Degree

1. $\sum_{u \in V}$ degree$(u) = 2|E|$, where G is an undirected graph.

2. $\sum_{u \in V}$ out-degree$(u) = \sum_{u \in V}$ in-degree$(u) = |E|$, where G is a directed graph.

3. degree(u) = out-degree(u) + in-degree(u), where $u \in V$ and G is a directed graph.

4. A vertex whose degree is 0 is **isolated**.

c. Path, Connected Component

1. A **path** of length k from a vertex u to a vertex u' in a graph $G =< V, E >$ is a sequence $< v_0, v_1, v_2, ..., v_k >$ of vertices such that $u = v_0$, $u' = v_k$.

2. An undirected graph is **connected** if every vertex is reachable from all other vertices.

3. A directed graph is **strongly connected** if every two vertices are reachable from each other.

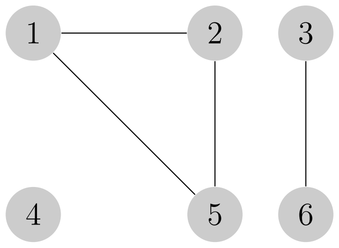


Figure 4: An undirected graph G_3 with 3 connected components: $\{1, 2, 5\}$, $\{3, 6\}$ and $\{4\}$

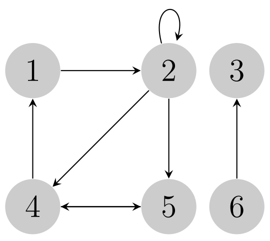


Figure 5: A directed graph G_4 with 3 strongly connected components: $\{1, 2, 4, 5\}$, $\{3\}$ and $\{6\}$

2. BFS and DFS

```
BFS($G, s$)
// $G$: input graph (sorted in alphabetical/ascending order);
// $s$: source vertex
for each vertex $u \in V - \{s\}$
    $d[u] = +\text{INFTY}$
$d[s] = 0$

// create FIFO queue
$Q = \text{EMPTY}$
ENQUEUE($G, s$)
while $Q$ not EMPTY
    $u = $ DEQUEUE($G$)
    for each $v \in Adj[u]$
        if $d[v] = +\text{INFTY}$
            $d[v] = d[u] + 1$
            ENQUEUE($G, v$)
return $d$
```

Let's run BFS on graph G_1!

Three-color is used to indicate search status of vertices

- WHITE = a vertex is undiscovered
- GRAY = a vertex is discovered, but its processing is incomplete
- BLACK = a vertex is discovered, and its processing is complete

Classification of edges (When we explore the edge, line 7-9 in DFS-VISIT(u)):

- T = Tree edge = encounter new vertex (GRAY to WHITE)
- B = Back edge = from descendant to ancestor (GRAY to GRAY)
- F = Forward edge = from ancestor to descendant (GRAY to BLACK)
- C = Cross edge = any other edges (between trees and subtrees) (GRAY to BLACK)

```
DFS($G$)
1  // $G$: input graph (sorted in alphabetical/ascending order);
2  for each vertex $u \in V$
3      $u.color$ = WHITE
4  $time = 0$
5  for each vertex $u \in V$
6      if $u.color$ = WHITE
7          // recursive routine/function
8          DFS-VISIT($u$)
```

```
DFS-VISIT($u$)
 1  // white vertex $u$ has just been discovered
 2  $time = time + 1$
 3  $u.discover = time$
 4  $u.color$ = GRAY
 5
 6  // explore edge $(u, v)$
 7  for each vertex $v \in Adj[u]$
 8      if $v.color$ = WHITE
 9          DFS-VISIT($v$)
10
11  // blacken $u$, it is finished
12  $u.color$ = BLACK
13  $time = time + 1$
14  $u.finish = time$
```

Try DFS on graph G_2!