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Discussion 2

ECS 122A
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1. Review: Polynomials, Exponentials, Logarithms, and their Derivatives

a. Polynomials

$p(n) = \sum_{i=0}^{d} a_i n^i$ is called a polynomial in n of degree d where $a_d \neq 0$.

A polynomial is asymptotically positive if and only if $a_d > 0$, and we have $p(n) = \Theta(n^d)$.

b. Exponentials

a^n is the basic form of an exponential, where a is called base.

$(a^n)^m = a^{mn} = (a^m)^n$.

For all real constants a and b such that $a > 1$,

$$\lim_{x \to \infty} \frac{n^b}{a^n} = 0$$

from which we can conclude that any exponential function with a base strictly greater than 1 grows faster than any polynomial function.

c. Logarithms

$\lg n = \log_2 n$; $\ln n = \log_e n$ (natural logarithm).

For all real $a > 0$, $b > 0$, $c > 0$, and n, we have the following properties:

i. $a = b^{\log_b a}$
ii. $\log_c(ab) = \log_c a + \log_c b$
iii. $\log_b a^n = n \log_b a$
iv. $\log_b a = \frac{\log_c a}{\log_c b}$

d. Derivatives

$f(x) = ax^n$, $\dfrac{\mathrm{d}f}{\mathrm{d}x} = anx^{n-1}$.

$f(x) = a^x$, $\dfrac{\mathrm{d}f}{\mathrm{d}x} = a^x \ln a$.

$f(x) = \ln x$, $\dfrac{\mathrm{d}f}{\mathrm{d}x} = \dfrac{1}{x}$, $x > 0$; $f(x) = \log_a x$, $\dfrac{\mathrm{d}f}{\mathrm{d}x} = \dfrac{1}{x \ln a}$, $x > 0$.

2. Asymptotic Notation: O, Ω, Θ

Example: We have $f(n) = \lg n$, $g(n) = \ln n$, what is their relation represented in asymptotic notation? There are two ways.

a. By definition

b. By using limits (L'Hopital's rule)

3. Solving Recurrence Relation

Recall the recurrence relation of merge sort: $T(n) = 2T(\frac{n}{2}) + n - 1, n >= 2$.

a. Substitution Method

b. Master Theorem

Followup: Which one should we use?

1. Substitution always works.
2. Master Theorem works only when $T(n) = aT(\frac{n}{b}) + f(n)$. Think about $T(n) = 2T(n - 1) + n$.