ECS 122A
April 3, 2019

Discussion 1

1. Mathematical Induction

Example: Prove $T(n) = \lg n + 1$ is the solution of the recurrence relation:

$$T(n) = \begin{cases} 1 & \text{if } n = 1 \\ T(\frac{n}{2}) + 1 & \text{otherwise} \end{cases}$$

Assume that n $= 2^k$, $k \geq 0$.

1. Base Case:

2. Inductive Hypothesis:

3. Inductive Step:

4. Conclusion: We can prove by induction that ...

2a. Write pseudocode to compute the n-th Fibonacci number using recursion.

```
Fib(n)
// Base Case


// Recursive Call


```

2b. Draw a recursion tree for n = 4.

3. Running Time Analysis: Merge Sort

MERGE-SORT(A, i, j)
if $i < j$
    $k = (i + j)/2$
    MERGE-SORT(A, i, k)
    MERGE-SORT($A, k + 1, j$)
    MERGE(A, i, k, j)

Let us denote $T(n)$ as the maximum number of comparisons of merge sort on a list of size $n = j - i + 1$. What is the recurrence relation of $T(n)$?

Likewise, We can construct a recursion tree for merge sort.