Discussion 3

1. Review: Polynomials, Exponentials, Logarithms, and their Derivatives

a. Polynomials

$p(n) = \sum_{i=0}^{d} a_i n^i$ is called a polynomial in n of degree d where $a_d \neq 0$.

A polynomial is asymptotically positive if and only if $a_d > 0$, and we have $p(n) = \Theta(n^d)$.

b. Exponentials

a^n is the basic form of an exponential, where a is called base.

$(a^n)^m = a^{mn} = (a^m)^n$.

For all real constants a and b such that $a > 1$,

$$\lim_{x \to \infty} \frac{n^b}{a^n} = 0$$

from which we can conclude that any exponential function with a base strictly greater than 1 grows faster than any polynomial function.

c. Logarithms

$\lg n = \log_2 n$; $\ln n = \log_e n$ (natural logarithm).

For all real $a > 0$, $b > 0$, $c > 0$, and n, we have the following properties:

i. $a = b^{\log_b a}$

ii. $\log_c(ab) = \log_c a + \log_c b$

iii. $\log_b a^n = n \log_b a$

iv. $\log_b a = \frac{\log_c a}{\log_c b}$

d. Derivatives

$f(x) = ax^n$, $\dfrac{\mathrm{d}f}{\mathrm{d}x} = anx^{n-1}$.

$f(x) = a^x$, $\dfrac{\mathrm{d}f}{\mathrm{d}x} = a^x \ln a$.

$f(x) = \ln x$, $\dfrac{\mathrm{d}f}{\mathrm{d}x} = \dfrac{1}{x}$, $x > 0$; $f(x) = \log_a x$, $\dfrac{\mathrm{d}f}{\mathrm{d}x} = \dfrac{1}{x \ln a}$, $x > 0$.

2. Asymptotic Notation: O, Ω, Θ

Example: We have $f(n) = n \lg n$, $g(n) = n^{1.5}$, what is their relation represented in asymptotic notation? There are two ways.

How: By definition and By using limits (L'Hopital's rule)

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3. Solving Recurrence Relation

Recall the recurrence relation of merge sort: $T(n) = 2T(\frac{n}{2}) + n, n >= 2$.
How: By Substitution Method, By Recurrence Tree(talked) and By Master Theorem

Followup: Which one should we use?

1. Substitution always works.
2. Master Theorem works only when $T(n) = aT(\frac{n}{b}) + f(n)$.
   Think about $T(n) = 2T(n-1) + n$.

4. Divide and Conquer: Chip Testing (CLRS Problem 4-5)

Professor Bai has n identical integrated-circuit chips that are capable of testing each other. A good chip always reports accurately whether the other chip is good or bad, but the professor cannot trust the answer of a bad chip. Thus, we have four different outcomes:

Chip A says	Chip B says	Conclusion
B is good	A is good	both are good or both are bad
B is good	A is bad	at least one is bad
B is bad	A is good	at least one is bad
B is bad	A is bad	at least one is bad

Show that the good chips can be identified with $\Theta(n)$ pairwise tests, assuming that more than $n/2$ of the chips are good. Give and solve the recurrence that describes the number of tests.

Hint: First think about how to find a **single** good chip using divide and conquer paradigm.