

# Description for MATSIRO6

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# 1 Introduction

Minimal Advanced Treatments of Surface Interaction and RunOff (MATSIRO) is a land surface parameterization formulated for application to the atmospheric general circulation model developed by the Center for Climate System Research at the University of Tokyo and the National Institute for Environmental Studies (CCSR/NIES AGCM), as well as to other global climate models. It has been designed to be primarily used for integral climate calculations such as those involving long time scales from one month to several hundred years coupled with the atmospheric model at grid resolutions of tens of kilometers or more. The main objective in its development was to represent all of the important water and energy circulation processes between land and atmosphere as fully and accurately as possible (i.e., *advanced* treatment) in such time and spatial scales, while modeling them as simply as possible (i.e., *minimal* treatment) so as to allow the results to be easily interpreted.

MATSIRO was developed based on the land surface submodel of CCSR/NIES AGCM5.4g coupled with the parameterization for a vegetated surface (canopy) by Watanabe (1994), while at the same time improving certain processes such as those related to snow and runoff. Subsequently, with modifications in the structure of AGCM, changes were made dealing with flux couplers and parallel processing so as to make it compatible with the current AGCM5.6. With regard to the physiological processes of vegetation, a Jarvis-type function was initially used for stomatal resistance. Later, however, the Farquhar-type photosynthesis scheme, which now serves as a de facto standard in the world due to the progress of studies on climate-ecosystem interactions in recent years, was ported from SiB2 code.

## 1.1 Structure

MATSIRO is divided into the following two parts: the flux calculation section, and the land surface integration section. Then, each part is divided into the land and lake parts.

In the flux calculation section, the calculations are conducted separately for snow-covered and snow-free portions. For each snow-free portion ( $l = 1$ ) and snow-covered portion ( $l = 2$ ), the subroutines for various processes are called, the fluxes are calculated, and the ground surface temperature and canopy temperature are updated. Specifically, the following subroutines are called in the order shown below:

- (a) MATLAI: vegetation type parameter (LAI, vegetation height) set
- (b) MATRAD: calculation of radiation parameters (albedo, vegetation transmissivity, etc.)
- (c) MATBLK: calculation of turbulence parameters (bulk coefficients) (momentum and heat)
- (d) MATRST: calculation of stomatal resistance, bare soil surface evaporation resistance, etc.
- (e) MATBLQ: calculation of turbulence parameters (bulk coefficients) (vapor)
- (f) MATFLX: calculation of surface flux
- (g) MATGHC: calculation of soil heat conductivity
- (h) MATSHB: solution of energy balance at ground surface and canopy

Then, fluxes from lake surface are calculated separately for ice-covered and ice-free portions. The following subroutines are called in the order shown below:

- (a) LAKEBC: calculation of lake surface conditions (albedo, roughness, etc.)
- (b) SFCFLX: calculation of surface flux
- (c) RADSFCL: calculation of radiation flux (downward and upward shortwave radiation)
- (d) LAKEHB: solution of energy balance at lake surface

In the land integration section, the subroutines for various processes are called and land surface prognostic variables are updated. Specifically, the following subroutines are called in the order shown below:

- (a) MATCNW: calculation of canopy water balance
- (b) MATSNW: calculation of snow water equivalent, snow temperature, and snow albedo
- (c) MATROF: calculation of runoff
- (d) MATGND: calculation of soil temperature, soil moisture, and frozen soil

Finally, the lake modules are called and the related prognostic variables are updated.

- (a) SETSCNV: calculation of convergence of shortwave radiation
- (b) LAKEIC: calculation of lake ice
- (c) LAKEPO: calculation of lake water and temperature
- (d) PUTDEFF: sending lake water deficit

## 1.2 Prognostic variables

MATSIRO has the following internal variables:

Variable	Description	Units
$T_{s(l)}$ ( $l = 1, 2$ )	surface temperature	[K]
$T_{c(l)}$ ( $l = 1, 2$ )	Canopy temperature	[K]
$T_{g(k)}$ ( $k = 1, \dots, K_g$ )	Soil temperature	[K]
$w_{(k)}$ ( $k = 1, \dots, K_g$ )	soil moisture content	[m <sup>3</sup> /m <sup>3</sup> ]
$w_{i(k)}$ ( $k = 1, \dots, K_g$ )	Frozen soil moisture content	[m <sup>3</sup> /m <sup>3</sup> ]
$w_c$	Water content in the canopy	[m]
$S_n$	amount of snowfall	[kg/m <sup>2</sup> ]
$T_{Sn(k)}$ ( $k = 1, \dots, K_{Sn}$ )	snow temperature	[K]
$\alpha_{Sn(b)}$ ( $b = 1, 2, 3$ )	snow albedo	[−]
$A_I$	Lake ice concentration	[−]
$h_I$	Mean lake ice thickness over ice-covered part of a grid	[cm]
$h_S$	Mean snow depth over lake ice	[cm]

where  $l = 1, 2$  denotes snow-free and snow-covered portions, respectively;  $k$  is the vertical layer number of the soil or snow (the uppermost layer is 1, with the number increasing as the layer becomes deeper);  $K_g$  is the number of soil layers;  $K_{Sn}$  is the number of snow layers; and  $b = 1, 2, 3$  denotes the bands of visible, near infrared, and infrared wavelengths, respectively.

As a standard, the soil has six layers whose thicknesses are defined by the depth boundaries of 5, 20, 75, 100, 200, and 1000 cm from the surface. The definition points of soil temperature, soil moisture, and frozen soil moisture are the same.

The number of snow layers is variable, increasing with the increase of snow water equivalent. As a standard, the maximum number is three layers.

The ground surface temperature and canopy temperature are so-called skin temperatures whose heat capacity is zero; however, they take the form of prognostic variables. (The current calculation method depends on the values of the preceding step because the stability, etc. assessed by the values of the preceding step are used. If the stability, etc. were to be assessed by updated values and calculation iterated to the point of convergence, perfect diagnostic variables would be obtained that would not depend on the values of the preceding step.) The other variables are all prognostic variables that always require the values of the preceding step.

The ground surface temperature and canopy temperature are updated in the flux calculation section. All of the other variables (original prognostic variables) are updated in the land surface integration section.

### 1.3 Input data

The following variables are input in the flux calculation section:

Variable	Description	Units
$u_a$	Atmospheric 1st layer east-west wind	[m/s]
$v_a$	First layer of atmospheric north-south wind	[m/s]
$T_a$	Atmospheric 1st layer temperature	[K]
$q_a$	Specific humidity in the first layer of the atmosphere	[kg/kg]
$P_a$	Atmospheric 1st layer pressure	[Pa]
$P_s$	surface pressure	[Pa]
$R_{(d,b)}^\downarrow$ ( $d = 1, 2; b = 1, 2, 3$ )	Surface downward radiation flux	[W/m <sup>2</sup> ]
$\cos \zeta$	cosine of the solar zenith angle	[—]

where  $d = 1, 2$  denotes direct and diffuse, respectively; and  $b = 1, 2, 3$  denotes the bands of visible, near infrared, and infrared wavelengths, respectively.

The following variables are input in the land surface integration section:

Variable	Description	Units
$Pr_c$	Convective rainfall flux	[kg/m <sup>2</sup> /s]
$Pr_l$	Layered Rainfall Flux	[kg/m <sup>2</sup> /s]
$P_{Snc}$	Convective snowfall flux	[kg/m <sup>2</sup> /s]
$P_{Snl}$	Layered Snowfall Flux	[kg/m <sup>2</sup> /s]

Variable	Description	Units
$F_{g(1/2)}$	Surface Heat Transfer Flux	[W/m <sup>2</sup> ]
$F_{Sn(1/2)}$	Heat Transfer Flux for Snow	[W/m <sup>2</sup> ]
	Surface	
$Et_{(i,j)}$ ( $i = 1, 2; j = 1, 2, 3$ )	Evapotranspiration components	[kg/m <sup>2</sup> /s]
$\Delta F_{conv}$	surface energy convergence	[W/m <sup>2</sup> ]
$F_{root(k)}$ ( $k = 1, \dots, K_g$ )	Root sucking flux	[kg/m <sup>2</sup> /s]
$LAI$	leaf area index	[m <sup>2</sup> /m <sup>2</sup> ]
$A_{Snc}$	Canopy freezing area ratio	[–]

## 1.4 Output data

The following variables are output from the flux calculation section:

Variable	Description	Units
$\tau_x$	surface-to-west wind stress	[N/m <sup>2</sup> ]
$\tau_y$	surface-to-south wind stress	[N/m <sup>2</sup> ]
$H$	surface sensible heat flux	[W/m <sup>2</sup> ]
$E$	Surface Water Vapor Flux	[kg/m <sup>2</sup> /s]
$R_S^\uparrow$	Upward Bound Shortwave	[W/m <sup>2</sup> ]
	Radiation Flux	
$R_L^\uparrow$	Upward Bound Longwave	[W/m <sup>2</sup> ]
	Radiation Flux	
$\alpha_{s(b)}$ ( $b = 1, 2, 3$ )	surface albedo	[–]
$T_{sR}$	surface radiation temperature	[K]
$F_{g(1/2)}$	Surface Heat Transfer Flux	[W/m <sup>2</sup> ]
$F_{Sn(1/2)}$	Heat Transfer Flux for Snow	[W/m <sup>2</sup> ]
	Surface	
$Et_{(i,j)}$ ( $i = 1, 2; j = 1, 2, 3$ )	Evapotranspiration components	[kg/m <sup>2</sup> /s]
$\Delta F_{conv}$	surface energy convergence	[W/m <sup>2</sup> ]
$F_{root(k)}$ ( $k = 1, \dots, K_g$ )	Root sucking flux	[kg/m <sup>2</sup> /s]
$LAI$	leaf area index	[m <sup>2</sup> /m <sup>2</sup> ]
$A_{Snc}$	Canopy freezing area ratio	[–]

where  $i = 1, 2$  denotes liquid and solid evapotranspiration, respectively; and  $j = 1, 2, 3$  denotes evaporation from the bare soil surface (forest floor), transpiration, and canopy water evaporation, respectively. Other indexes are the same as described earlier.

The following variable is output from the land surface integration section:



Header0	Header1	Header2
<i>Ro</i>	runoff	[kg/m <sup>2</sup> /s\blind}

Runoff is used as an input variable for the river channel network model.

## 1.5 External parameters

The external parameters necessary for the execution of MATSIRO are broadly divided into two types: parameters whose values for each grid cell are given by horizontal distribution (map), and parameters whose values are given by land cover type or soil type tables. The land cover types and soil types are among the parameters given by map, and through this, each parameter given by table is allocated to individual grid cells; that is,

parameter given by map:

$$\phi(i, j) \quad (1.1)$$

parameter given by table:

$$\psi(I), I = I_L(i, j) \quad (1.2)$$

or

$$\psi(I), I = I_S(i, j), \quad (1.3)$$

where  $(i, j)$  are indexes of the grid horizontal location,  $I_L$  is the land use type, and  $I_S$  is the soil type.

The types of external parameters given by map are as follows:

Variable	Description	Temporal resolution	Units
$I_L$	Land cover type	constant	[—]
$I_S$	Soil Type	constant	[—]
$LAI_0$	Leaf Area Index (LAI)	every month	[m <sup>2</sup> /m <sup>2</sup> ]
$\alpha_{0(b)} (b = 1, 2, 3)$	Ground surface (forest floor) albedo	constant	[—]
$\tan \beta_s$	Tangent of the mean surface slope	constant	[—]
$\sigma_z$	elevation standard deviation	constant	[m]

The types of external parameters given by table for each land cover type are as follows:

Variable	Description	Units
$h_0$	vegetation height	[m]
$h_{B0}$	Height of the bottom of the canopy	[m]
$r_{f(b)}$ ( $b=1,2$ )	Reflectivity of individual leaves	[−]
$t_{f(b)}$ ( $b=1,2$ )	Transmittance of individual leaves	[−]
$f_{root(k)}$ ( $k = 1, \dots, K_g$ )	Percentage of root presence	[−]
$c_d$	Momentum exchange coefficient between the individual leaves and the atmosphere	[−]
$c_h$	Heat Exchange Coefficient between individual leaves and the atmosphere	[−]
$f_V$	vegetation coverage	[−]
$V_{max}$	Rubisco Reaction Capacity	[m/s]
$m$	$A_n-g_s$ Slope of the relationship	[−]
$b$	$A_n-g_s$ relationship intercepts	[m/s]
$\epsilon_3, \epsilon_4$	Photosynthetic efficiency per photon	[m/s/mol]
$\theta_{ce}$	Coupling factor between $w_c$ and $w_e$	[−]
$\theta_{ps}$	Coupling factor between $w_p$ and $w_s$	[−]
$f_d$	respiratory coefficient	[−]
$s_2$	Critical temperature of high temperature suppression	[K]
$s_4$	Critical temperature of cryogenic suppression	[K]

The types of external parameters given by table for each soil type are as follows:

Header0	Header1	Header2
$c_{g(k)}$ ( $k = 1, \dots, K_g$ )	Specific Heat of Soil	[J/m <sup>3</sup> ]
$k_{g(k)}$ ( $k = 1, \dots, K_g$ )	Thermal Conductivity of Soil	[W/m/K]
$w_{sat(k)}$ ( $k = 1, \dots, K_g$ )	Soil Porosity	[m <sup>3</sup> /m <sup>3</sup> ]
$K_{s(k)}$ ( $k = 1, \dots, K_g$ )	Saturated Permeability of Soil	[m/s]
$\psi_{s(k)}$ ( $k = 1, \dots, K_g$ )	Soil Saturation Moisture Potential	[m]
$b_{(k)}$ ( $k = 1, \dots, K_g$ )	Index of Soil Moisture Potential Curve	[−]

## 2 Vegetation type parameters

The leaf area index (LAI), vegetation height, etc. are set as vegetation type parameters.

Seasonally changing horizontal distributions are loaded as external parameters for LAI, and the values according to land use type are loaded as external parameters for the heights of the canopy top and bottom. When there is snow, only the vegetation above the level of the snow depth is taken into consideration and the type parameters are corrected as follows:

$$h = \max(h_0 - D_{Sn}, 0) \quad (2.1)$$

$$h_B = \max(h_{B0} - D_{Sn}, 0) \quad (2.2)$$

$$LAI = LAI_0 \frac{h - h_B}{h_0 - h_{B0}} \quad (2.3)$$

where  $h$  is the height of the canopy top (vegetation height),  $h_B$  is the height of the canopy bottom (height of clear length),  $LAI$  is the leaf area index, and  $h_0$ ,  $h_{B0}$ , and  $LAI_0$  are the respective values when there is no snow.  $D_{Sn}$  is the snow depth.  $LAI$  is approximated on the assumption that it is uniformly distributed vertically between the canopy top and bottom.

In terms of coding, next the mean values of the snow-free and snow-covered portions are solved by weighting with the snow-covered ratio ( $A_{Sn}$ ), etc., as follows:

$$h = A_{Sn}h + (1 - A_{Sn})h_0 \quad (2.4)$$

However, because the snow-free portion and snow-covered portion are respectively calculated, it should be noted that  $A_{Sn}$  takes the value of either 0 (snow-free portion) or 1 (snow-covered portion), so no mixing of values occurs (similar cases are also seen later).

### 3 Radiation parameters

Next, the radiation parameters (albedo, vegetation transmissivity, etc.) are calculated.

#### 3.1 Calculation of ground surface (forest floor) albedo

The horizontal distributions of the ground surface (forest floor) albedo  $b = 1, 2$  are loaded as external parameters, with  $b = 1, 2$  denoting the wavelength bands of visible and near infrared, respectively. The infrared ground surface albedo ( $\alpha_{0(3)}$ ) is set to a fixed value (horizontal distributions can also be prepared if desired).

With regard to the ice-sheet portion and snow-covered portion, the dependence of the incidence angle of albedo is considered by the following function form:

$$\alpha_{0(d,b)} = \hat{\alpha}_{0(b)} + (1 - \hat{\alpha}_{0(b)}) \cdot 0.4(1 - \cos \psi_{in(d)})^5 \quad (3.1)$$

where  $b = 1, 2$  are wavelength bands;  $d = 1, 2$  are direct and diffuse, respectively; and  $\hat{\alpha}_{0(b)}$  is the value of albedo when the incidence angle is 0 (from directly overhead). The cosine of the incidence angle  $\cos \psi_{in(d)}$  is expressed as

$$\cos \psi_{in(1)} = \cos \zeta, \quad \cos \psi_{in(2)} = \cos 50^\circ \quad (3.2)$$

for direct insolation and diffuse radiation, respectively, where  $\zeta$  is the solar zenith angle.

With regard to regions other than the ice-sheet portion and snow-covered portion, the zenith angle dependence is not taken into consideration for the albedo of the ground surface (forest floor) and the same values are given to direct insolation and diffuse radiation; that is,

$$\alpha_{0(d,b)} = \alpha_{0(b)} \quad (d = 1, 2; b = 1, 2) \quad (3.3)$$

Moreover, in the case of the infrared wavelength only diffuse radiation needs to be considered. The value of the infrared albedo for all ground surfaces is given independently of the zenith angle, as follows:

$$\alpha_{0(2,3)} = \alpha_{0(3)} \quad (3.4)$$

### 3.2 Calculation of canopy albedo and transmissivity

The calculation of canopy albedo and transmissivity is based on the calculation of radiation within a canopy layer proposed by Watanabe and Ohtani (1995).

Considering the canopy as vertically uniform and making use of several assumptions for simplification, the transfer equations of insolation within the canopy and the boundary condition are expressed as

$$\frac{dS_d^\downarrow}{dL} = -F \sec \zeta S_d^\downarrow \quad (3.5)$$

$$\frac{dS_r^\downarrow}{dL} = -F(1 - t_{f(b)})d_f S_r^\downarrow + F t_{f(b)} \sec \zeta S_d^\downarrow + F r_{f(b)} d_f S_r^\uparrow \quad (3.6)$$

$$\frac{dS_r^\uparrow}{dL} = F(1 - t_{f(b)})d_f S_r^\uparrow - F r_{f(b)}(d_f S_r^\downarrow + \sec \zeta S_d^\downarrow) \quad (3.7)$$

$$S_d^\downarrow(0) = S_d^{top} \quad (3.8)$$

$$S_r^\downarrow(0) = S_r^{top} \quad (3.9)$$

$$S_r^\uparrow(LAI) = \alpha_{0(1,b)} S_d^\downarrow(LAI) + \alpha_{0(2,b)} S_r^\downarrow(LAI) \quad (3.10)$$

where  $S_d^\downarrow$  is the downward direct insolation;  $S_r^\uparrow$  and  $S_r^\downarrow$  are the upward and downward diffuse radiation, respectively;  $L$  is the leaf area cumulatively added downward from the canopy top;  $d_f$  is the diffusivity factor ( $= \sec 53^\circ$ ),  $r_{f(b)}$ ;  $t_{f(b)}$  are the leaf albedo and transmissivity, respectively (the same value is used for diffuse radiation and direct insolation); and  $F$  is a factor denoting the direction of the leaves with respect to the radiation. Here, the distribution of the direction of the leaves is assumed to be random ( $F = 0.5$ ) for simplicity.

These can be solved analytically, giving the following solutions:

$$S_d^\downarrow(L) = S_d^{top} \exp(-F \cdot L \cdot \sec \zeta) \quad (3.11)$$

$$S_r^\downarrow(L) = C_1 e^{aL} + C_2 e^{-aL} + C_3 S_d^\downarrow(L) \quad (3.12)$$

$$S_r^\uparrow(L) = A_1 C_1 e^{aL} + A_2 C_2 e^{-aL} + C_4 S_d^\downarrow(L) \quad (3.13)$$

where

$$a = Fd_f[(1 - t_{f(b)})^2 - r_{f(b)}^2]^{1/2} \quad (3.14)$$

$$A_1 = \{1 - t_{f(b)} + [(1 - t_{f(b)})^2 - r_{f(b)}^2]^{1/2}\}/r_{f(b)} \quad (3.15)$$

$$A_2 = \{1 - t_{f(b)} - [(1 - t_{f(b)})^2 - r_{f(b)}^2]^{1/2}\}/r_{f(b)} \quad (3.16)$$

$$A_3 = (A_1 - \alpha_{0(2,b)})e^{aLAI} - (A_2 - \alpha_{0(2,b)})e^{-aLAI} \quad (3.17)$$

$$C_1 = \{-(A_2 - \alpha_{0(2,b)})e^{-aLAI}(S_r^{top} - C_3S_d^{top}) + [C_3\alpha_{0(2,b)} + \alpha_{0(1,b)} - C_4]S_d^\downarrow(LAI)\}/A_3 \quad (3.18)$$

$$C_2 = \{(A_1 - \alpha_{0(2,b)})e^{aLAI}(S_r^{top} - C_3S_d^{top}) - [C_3\alpha_{0(2,b)} + \alpha_{0(1,b)} - C_4]S_d^\downarrow(LAI)\}/A_3 \quad (3.19)$$

$$C_3 = \frac{\sec \zeta [t_{f(b)} \sec \zeta + d_f t_{f(b)} (1 - t_{f(b)}) + d_f r_{f(b)}^2]}{d_f^2 [(1 - t_{f(b)})^2 - r_{f(b)}^2] - \sec^2 \zeta} \quad (3.20)$$

$$C_4 = \frac{r_{f(b)}(d_f - \sec \zeta) \sec \zeta}{d_f^2 [(1 - t_{f(b)})^2 - r_{f(b)}^2] - \sec^2 \zeta} \quad (3.21)$$

Albedo  $\alpha_s$  at the canopy top is expressed as

$$S_r^\uparrow(0) = \alpha_{s(1,b)}S_d^\downarrow(0) + \alpha_{s(2,b)}S_r^\downarrow(0) \quad (3.22)$$

therefore,

$$\alpha_{s(2,b)} = \{A_2(A_1 - \alpha_{0(2,b)})e^{aLAI} - A_1(A_2 - \alpha_{0(2,b)})e^{-aLAI}\}/A_3 \quad (3.23)$$

$$\alpha_{s(1,b)} = -C_3\alpha_{s(2,b)} + C_4 + (A_1 - A_2)(C_3\alpha_{0(2,b)} + \alpha_{0(1,b)} - C_4)e^{-F \cdot LAI \cdot \sec \zeta}/A_3 \quad (3.24)$$

are obtained.

If the canopy transmissivity ( $\mathcal{T}_c$ ) (specifically, the ratio of incident insolation absorbed by the forest floor to the incident insolation of the canopy top) is defined by

$$\mathcal{T}_{c(2,b)} = \{(1 - A_2)(A_1 - \alpha_{0(2,b)}) - (1 - A_1)(A_2 - \alpha_{0(2,b)})\}/A_3 \quad (3.25)$$

$$\begin{aligned} \mathcal{T}_{c(1,b)} &= -C_3\mathcal{T}_{c(2,b)} \\ &+ \{(C_3\alpha_{0(2,b)} + \alpha_{0(1,b)} - C_4)((1 - A_1)e^{aLAI} - (1 - A_2)e^{-aLAI})/A_3 + C_3 - C_4 + 1\}e^{-F \cdot LAI \cdot \sec \zeta} \end{aligned} \quad (3.26)$$

the following are obtained:

$$S_d^\downarrow(LAI) + S_r^\downarrow(LAI) - S_r^\uparrow(LAI) = \mathcal{T}_{c(1,b)}S_d^\downarrow(0) + \mathcal{T}_{c(2,b)}S_r^\downarrow(0) \quad (3.27)$$

The above calculations are performed for  $b = 1, 2$  (visible and near infrared), respectively.

The leaf albedo  $r_f$  and transmissivity  $t_f$  are loaded as external parameters for each land cover type; however, the following two modifications are made before these parameters are used in the above calculations.

## 1. Snow (ice) effect on leaf surface

When the canopy temperature does not exceed 0° C, the canopy water is regarded as snow (ice). In this case, using the snow albedo ( $\alpha_{Sn(b)}$ ) and canopy water ( $w_c$ ), the following assumptions are made:

$$r_{f(b)} = (1 - f_{cwet})r_{f(b)} + f_{cwet}\alpha_{Sn(b)} \quad (3.28)$$

$$f_{cwet} = w_c/w_{c,cap} \quad (3.29)$$

where  $w_{c,cap}$  is the canopy water capacity. With regard to transmissivity, the following assumption is made for convenience so that the absorptivity  $1 - r_f - t_f$  does not take a negative value:

$$t_{f(b)} = (1 - f_{cwet})t_{f(b)} + f_{cwet}t_{Sn(b)}, \quad t_{Sn(b)} = \min(0.5(1 - \alpha_{Sn(b)}), t_{f(b)}) \quad (3.30)$$

When the canopy water is in the liquid state, the resultant change in the leaf radiation parameter is ignored. Moreover, although the cases of snowfall after interception by the canopy (snow cover) and of frozen water in the canopy (ice) can be considered, with each having different radiation characteristics, the same albedo of snow on the forest floor is used for all cases here.

## 2. Effect considering the directions of reflection and transmission

In solving the above equations, all reflected light is assumed to return in the direction of incidence. However, if, for example, the diffusion of only a portion of the reflected light in the direction of incidence is taken into consideration, the leaf radiation parameters can be replaced as follows (Watanabe, personal communication):

$$r_{f(b)} = 0.75r_{f(b)} + 0.25t_{f(b)} \quad (3.31)$$

$$t_{f(b)} = 0.75t_{f(b)} + 0.25r_{f(b)} \quad (3.32)$$

The above calculations are performed for  $b = 1, 2$  (visible and near infrared), respectively.

In addition, in consideration of the uneven distribution of vegetation (such as savanna) in parts of the grid cells, before calculating the albedo, etc., the LAI (taking the original LAI as the grid mean value) of the vegetation-covered portion is calculated as follows:

$$LAI = LAI/f_V \quad (3.33)$$

and this is used for the calculation for albedo, etc. mentioned above. ( $R_{(d,b)}^\downarrow$ ) is the vegetation-covered ratio in the grid cell. After the albedo, etc. are calculated, the area-weighted mean of the vegetation-covered portion and non- vegetation-covered portion are obtained as

$$\alpha_{s(d,b)} = f_V\alpha_{s(d,b)} + (1 - f_V)\alpha_{0(d,b)} \quad (3.34)$$

$$\mathcal{T}_{c(d,b)} = f_V\mathcal{T}_{c(d,b)} + (1 - f_V)(1 - \alpha_{0(d,b)}) \quad (3.35)$$

### 3.3 Calculation of surface radiation flux, etc.

Using the surface downward radiation flux ( $R_{(d,b)}^\downarrow$ ) and albedo calculated above, the following radiation fluxes are calculated:

$$R_S^\downarrow = \sum_{b=1}^2 \sum_{d=1}^2 R_{(d,b)}^\downarrow \quad (3.36)$$

$$R_S^\uparrow = \sum_{b=1}^2 \sum_{d=1}^2 \alpha_{s(d,b)} R_{(d,b)}^\downarrow \quad (3.37)$$

$$R_L^\downarrow = R_{(2,3)}^\downarrow \quad (3.38)$$

$$R_S^{gnd} = \sum_{b=1}^2 \sum_{d=1}^2 \mathcal{T}_{s(d,b)} R_{(d,b)}^\downarrow \quad (3.39)$$

$$PAR = \sum_{d=1}^2 R_{(d,1)}^\downarrow \quad (3.40)$$

where  $R_S^\downarrow$  and  $R_S^\uparrow$  are the downward and upward shortwave radiation flux, respectively;  $R_L^\downarrow$  is the downward longwave flux;  $R_S^{gnd}$  is the shortwave flux absorbed by the forest floor; and  $PAR$  is the downward photosynthesis active radiation (PAR) flux.

The canopy transmissivity of shortwave and longwave radiation, and the emissivity of longwave radiation, are then calculated as follows:

$$\mathcal{T}_{cS} = R_S^{gnd} / (R_S^\downarrow - R_S^\uparrow) \quad (3.41)$$

$$\mathcal{T}_{cL} = \exp(-F \cdot LAI \cdot d_f) \quad (3.42)$$

$$\epsilon = 1 - \alpha_{s(2,3)} \quad (3.43)$$

## 4 Turbulence parameters (bulk coefficient)

Next, the turbulence parameter (bulk coefficient) is calculated.

### 4.1 Calculation of roughness with respect to momentum and heat

The calculation of roughness is based on Watanabe (1994). In that study, using the results of a multilayer canopy model by Kondo and Watanabe (1992) as a function form for the roughness of a bulk model best fitting those results, Watanabe (1994) proposed the following:

$$\left(\ln \frac{h-d}{z_0}\right)^{-1} = \left[1 - \exp(-A^+) + \left(-\ln \frac{z_{0s}}{h}\right)^{-1/0.45} \exp(-2A^+)\right]^{0.45} \quad (4.1)$$

$$\left(\ln \frac{h-d}{z_T^\dagger}\right)^{-1} = \frac{1}{-\ln(z_{Ts}/h)} \left[\frac{P_1}{P_1 + A^+ \exp(A^+)}\right]^{P_2} \quad (4.2)$$

$$\left(\ln \frac{h-d}{z_0}\right)^{-1} \left(\ln \frac{h-d}{z_T}\right)^{-1} = C_T^\infty \left[1 - \exp(-P_3 A^+) + \left(\frac{C_T^0}{C_T^\infty}\right)^{1/0.9} \exp(-P_4 A^+)\right]^{0.9} \quad (4.3)$$

$$h-d = h[1 - \exp(-A^+)]/A^+ \quad (4.4)$$

$$A^+ = \frac{c_d LAI}{2k^2} \quad (4.5)$$

$$\frac{1}{C_T^0} = \ln \frac{h-d}{z_0} \ln \frac{h-d}{z_T^\dagger} \quad (4.6)$$

$$C_T^\infty = \frac{-1 + (1 + 8F_T)^{1/2}}{2} \quad (4.7)$$

$$P_1 = 0.0115 \left(\frac{z_{Ts}}{h}\right)^{0.1} \exp\left[5 \left(\frac{z_{Ts}}{h}\right)^{0.22}\right] \quad (4.8)$$

$$P_2 = 0.55 \exp\left[-0.58 \left(\frac{z_{Ts}}{h}\right)^{0.35}\right] \quad (4.9)$$

$$P_3 = [F_T + 0.084 \exp(-15F_T)]^{0.15} \quad (4.10)$$

$$P_4 = 2F_T^{1.1} \quad (4.11)$$

$$F_T = c_h/c_d \quad (4.12)$$

where  $z_0$  and  $z_T$  are the roughness of the overall canopy with respect to momentum and heat, respectively;  $z_{0s}$  and  $z_{Ts}$  are the roughness of the ground surface (forest floor) with respect to momentum and heat, respectively;  $c_d$  and  $c_h$  are the exchange coefficient between an individual leaf and the atmosphere with respect to momentum and heat, respectively;  $h$  is the vegetation height;  $d$  is the zero-plane displacement; and  $LAI$  is LAI.  $z_T^\dagger$  is the roughness with respect to heat when assuming no transfer of heat to or from the leaf surface, and is used when solving the coefficient of heat transfer from the forest floor.

$z_{0s}$  and  $z_{Ts}$  are given as external data for each land cover type. Their values ( $z_{0s} = 0.05$  m,  $z_{Ts} = 0.005$  m) are fixed as standards. However, the following modifications are made with respect to the snow-covered portion:

$$z_{0s} = \max(f_{Sn} z_{0s}, z_{0Sn}) \quad (4.13)$$

$$z_{Ts} = \max(f_{Sn} z_{0s}, z_{TSn}) \quad (4.14)$$

$$f_{Sn} = 1 - D_{Sn}/z_{0s} \quad (4.15)$$

where  $D_{Sn}$ ,  $z_{0Sn}$  and  $z_{TSn}$  are the roughness of the snow-covered portion with respect to momentum and heat, respectively.

$c_d$  and  $c_h$  are parameters determined by the leaf shape, and are given as external data for each land cover type.



## 4.2 Calculation of bulk coefficient with respect to momentum and heat

After Watanabe (1994), the bulk coefficient is also calculated using Monin-Obukhov similarity as

$$C_M = k^2 \left[ \ln \frac{z_a - d}{z_0} + \Psi_m(\zeta) \right]^{-2} \quad (4.16)$$

$$C_H = k^2 \left[ \ln \frac{z_a - d}{z_0} + \Psi_m(\zeta) \right]^{-1} \left[ \ln \frac{z_a - d}{z_T} + \Psi_h(\zeta) \right]^{-1} \quad (4.17)$$

$$C_{Hs} = k^2 \left[ \ln \frac{z_a - d}{z_0} + \Psi_m(\zeta_g) \right]^{-1} \left[ \ln \frac{z_a - d}{z_T^\dagger} + \Psi_h(\zeta_g) \right]^{-1} \quad (4.18)$$

$$C_{Hc} = C_H - C_{Hs} \quad (4.19)$$

where  $C_M$  and  $C_H$  are the bulk coefficients of the overall canopy (leaf surface + forest floor) with respect to momentum and heat, respectively;  $C_{Hs}$  is the bulk coefficient of the ground surface (forest floor) flux with respect to heat;  $C_{Hc}$  is the bulk coefficient of the canopy (leaf surface) flux with respect to heat;  $\Psi_m$  and  $\Psi_h$  are Monin-Obukhov shear functions with respect to momentum and heat, respectively; and  $z_a$  is the reference height of the atmosphere (height of the troposphere). Using the Monin-Obukhov lengths  $\zeta$  and  $\zeta_g$  related to the overall canopy and ground surface (forest floor), respectively,  $L$  and  $L_g$  are respectively expressed as:

$$\zeta = \frac{z_a - d}{L} \quad (4.20)$$

$$\zeta_g = \frac{z_a - d}{L_g} \quad (4.21)$$

and the Monin-Obukhov lengths are expressed as:

$$L = \frac{\Theta_0 C_M^{3/2} |V_a|^2}{kg(C_{Hs}(T_s - T_a) + C_{Hc}(T_c - T_a))} \quad (4.22)$$

$$L_s = \frac{\Theta_0 C_M^{3/2} |V_a|^2}{kg C_{Hs}(T_s - T_a)} \quad (4.23)$$

where  $\Theta_0 = 300\text{K}$ ;  $|V_a|$  is the absolute value of the surface wind speed;  $k$  is the Karman constant;  $g$  is the gravitational acceleration; and  $T_a$ ,  $T_c$  and  $T_s$  are the temperature of the troposphere, canopy (leaf surface), and ground surface (forest floor), respectively.

Since the bulk coefficient is necessary for calculation of the Monin-Obukhov length, and the Monin-Obukhov length is necessary for calculation of the bulk coefficient, the calculation is iterated (twice as a standard) with a neutral bulk coefficient as the initial value.

Prior to this calculation, the snow depth in the snow-covered portion is added to the zero-plane displacement. However, the upper limit is set so that the zero-plane displacement does not exceed the value of  $z_a$ :

$$d = \min(d + D_{Sn}, f_{\max} \cdot z_a) \quad (4.24)$$

As a standard,  $f_{\max}$  is set at 0.5.

### 4.3 Calculation of bulk coefficient with respect to vapor

This calculation is performed after the calculation of stomatal resistance, described later.

When the stomatal resistance ( $r_{st}$ ) and ground surface evaporation resistance ( $r_{soil}$ ) have been solved, the bulk coefficient with respect to vapor is solved as:

$$C_{Ec}|V_a| = [(C_{Hc}|V_a|)^{-1} + r_{st}/LAI]^{-1} \quad (4.25)$$

$$C_{Es}|V_a| = [(C_{Hs}|V_a|)^{-1} + r_{soil}]^{-1} \quad (4.26)$$

(Previously, this parameter was solved by converting stomatal resistance, etc. into a decrease of the exchange coefficient via roughness. However, since this approach seems to be problematic, a simpler method had been adopted in its place.)

In addition, when there is no stomatal resistance, etc. (such as evaporation from wet surfaces), the same value as for the bulk coefficient of heat is used for the bulk coefficient of vapor.

## 5 Stomatal resistance

For the calculation of stomatal resistance, a photosynthesis-stomatal model based on Farquhar et al. (1980), Ball (1988), and Collatz et al. (1990, 1991, 1992) is used. The code of SiB2 (Sellers et al., 1996) is used virtually unchanged, with the exception of the method for solving the resistance of the overall canopy. A Jarvis-type empirical equation could be used instead; however, the explanation of this point is omitted here.

### 5.1 Calculation of soil moisture stress factor

Soil moisture stress with respect to transpiration is solved. By solving the soil moisture stress factor in each soil layer, and weighting with the root distribution in each layer, the stress factor of the overall soil is calculated.

Referring to SiB2 (Sellers et al., 1996), the soil moisture stress in each layer is evaluated by the following equation:

$$f_{w(k)} = [1 + \exp(0.02(\psi_{cr} - \psi_k))]^{-1} \quad (k = 1, \dots, K_g) \quad (5.1)$$

The stress factor of the overall soil is then obtained by

$$f_w = \sum_{k=1}^{K_g} f_{w(k)} f_{root(k)} \quad (5.2)$$

where  $f_{root(k)}$  is the root distribution fraction in each layer, which is an external parameter for each land cover type.  $\sum_{k=1}^{K_g} f_{root(k)} = 1$ .

Furthermore, the weight of transpiration distributed to the root uptake flux in each layer is expressed as

$$f_{rootup(k)} = f_{w(k)} f_{root(k)} / f_w \quad (k = 1, \dots, K_g) \quad (5.3)$$

Note that  $\sum_{k=1}^{K_g} f_{rootup(k)} = 1$  here.

## 5.2 Calculation of amount of photosynthesis

The amount of photosynthesis is calculated after SiB2 (Sellers et al., 1996).

The amount of photosynthesis is considered to be regulated by the following three upper limits:

$$A \leq \min(w_c, w_e, w_s) \quad (5.4)$$

where  $w_c$  is the upper limit set by the efficiency of photosynthesis enzymes (Rubisco), and  $w_e$  is the upper limit set by photosynthetically active radiation.  $w_s$  is the upper limit of the efficiency of use of photosynthate (sink) in the case of C3 vegetation, or the upper limit set by CO<sub>2</sub> concentration in the case of C4 vegetation (Collatz et al., 1991, 1992).

The respective magnitudes are estimated as follows:

$$w_c = \begin{cases} V_m \left[ \frac{c_i - \Gamma^*}{c_i + K_c(1 + O_2/K_O)} \right] & \text{(in case of } C_3 \text{ vegetation)} \\ V_m & \text{(in case of } C_4 \text{ vegetation)} \end{cases} \quad (5.5)$$

$$(5.6)$$

$$w_e = \begin{cases} PAR \cdot \epsilon_3 \left[ \frac{c_i - \Gamma^*}{c_i + 2\Gamma^*} \right] & \text{(in case of } C_3 \text{ vegetation)} \\ PAR \cdot \epsilon_4 & \text{(in case of } C_4 \text{ vegetation)} \end{cases} \quad (5.7)$$

$$(5.8)$$

$$w_s = \begin{cases} V_m/2 & \text{(in case of } C_3 \text{ vegetation)} \\ V_m c_i/5 & \text{(in case of } C_4 \text{ vegetation)} \end{cases} \quad (5.9)$$

$$(5.10)$$

where  $V_m$  is the Rubisco reaction capacity,  $c_i$  is the partial pressure of CO<sub>2</sub> in the stoma,  $O_2$  is the partial pressure of oxygen in the stoma, and  $PAR$  is the photosynthetically active radiation (PAR).  $\Gamma^*$  is the CO<sub>2</sub> compensation point, which is expressed by  $\Gamma^*$  is the compensation point of CO<sub>2</sub> and is represented by  $\Gamma^* = 0.5O_2/S$   $K_c$ ,  $K_O$ , and  $S$  are functions of temperature, whose function form is shown later.  $\epsilon_3$  and  $\epsilon_4$  are constants determined by the vegetation type.

In order to express a smooth transition between the different upper limits, 5.4 is actually solved as

$$\beta_{ce}w_p^2 - w_p(w_c + w_e) + w_cw_e = 0 \quad (5.11)$$

$$\beta_{ps}A^2 - A(w_p + w_s) + w_pw_s = 0 \quad (5.12)$$

The amount of net photosynthesis  $A_n$  can be obtained when solving the two equations in order while selecting the smaller of the two solutions for each equation.  $\beta_{ce}, \beta_{ps}$  are constants determined by the vegetation type. Note that when  $\beta = 1$ , coincidence is achieved with a simple minimum-value operation.

When the amount of photosynthesis has been solved,  $(A_n)$  is solved as

$$A_n = A - R_d \quad (5.13)$$

where  $R_d$  is the amount of respiration, expressed as

$$R_d = f_d V_m \quad (5.14)$$

Here,  $f_d$  is a constant determined by the vegetation type.

$V_m$ , etc. depend on the temperature and soil moisture, as follows (note that although the temperature dependence differs according to the term in which  $V_m$  appears, the value is expressed by the same  $V_m$ ):

$$V_m = V_{\max} f_T(T_c) f_w \quad (5.15)$$

$$K_c = 30 \times 2.1^{Q^T} \quad (5.16)$$

$$K_O = 30000 \times 1.2^{Q^T} \quad (5.17)$$

$$S = 2600 \times 0.57^{Q^T} \quad (5.18)$$

$$f_T(T_c) = \begin{cases} 2.1^{Q^T} / \{1 + \exp[s_1(T_c - s_2)]\} & \text{(when } w_c \text{ and } w_e \text{ for } C_3) \\ 1.8^{Q^T} / \{1 + \exp[s_3(s_4 - T_c)]\} & \text{(when } w_s \text{ is for } C_3) \\ 2.1^{Q^T} / \{1 + \exp[s_1(T_c - s_2)]\} / \{1 + \exp[s_3(s_4 - T_c)]\} & \\ \quad \text{(when } w_c \text{ and } w_e \text{ are for } C_4) & \\ 1.8^{Q^T} & \text{(when } w_s \text{ is for } C_4) \\ 2^{Q^T} / \{1 + \exp[s_5(T_c - s_6)]\} & \text{(when } R_d) \end{cases} \quad (5.19)$$

$$Q^T = (T_c - 298)/10 \quad (5.20)$$

where  $V_{\max}, s_1, \dots, s_6$  are constants determined by the vegetation type.

If  $V_{\max}, PAR, c_i, T_c$  and  $f_w$  are given by the above, the amount of photosynthesis in an individual leaf can be calculated. In reality, these values can be considered to be distributed unevenly even in the same canopy; however,  $c_i, T_c$ , and  $f_w$  are approximated here as being the same for all leaves, whereas vertical distribution is taken into consideration in the case of  $V_{\max}$

and  $PAR$ .  $PAR$  is greater at the top of the canopy, and the lower the position in the canopy, the more it is attenuated.  $V_{\max}$  is also considered to conform with this property of  $PAR$  and to have a similar distribution.

The average vertical distribution of  $PAR$  (and therefore the vertical distribution of  $V_{\max}$ ) is expressed as

$$PAR(L) = PAR^{top} \exp(-f_{atn}aL) \quad (5.21)$$

where  $L$  is the leaf area added cumulatively from the canopy top,  $PAR^{top}$  is  $PAR$  at the canopy top,  $a$  is the attenuation coefficient defined in 3.15, and  $f_{atn}$  is a constant for adjustment. Using this, the factor  $f_{avr}$  expressing the average value of  $PAR$ , is defined as follows:

$$f_{avr} = \int_0^{LAI} PAR(L)dL / (LAI \cdot PAR^{top}) = \frac{1 - \exp(-f_{atn}aL)}{f_{atn}a} \quad (5.22)$$

Since each of the terms  $A_n(w_c, w_s, w_e, R_d)$  is proportional to  $V_{\max}$  or  $PAR$ , based on the assumption that the vertical distributions of  $V_{\max}$  and  $PAR$  are proportional, by multiplying  $A_n$ , which was solved using the values of  $V_{\max}$  and  $PAR$  at the top of the canopy, by  $f_{avr}$ , the average amount of photosynthesis of leaf ( $\overline{A_n}$ ) can be solved:

$$\overline{A_n} = f_{avr}A_n \quad (5.23)$$

This parameter is expressed as  $A_n$  hereafter.

### 5.3 Calculation of stomatal resistance (2)

The net photosynthesis ( $A_n$ ) and stomatal conductance ( $g_s$ ) are related by the semiempirical equation of Ball (1988) as follows:

$$g_s = m \frac{A_n}{c_s} h_s + b f_w \quad (5.24)$$

where  $c_s$  is the molar fraction of  $CO_2$  (number of mol of  $CO_2$  per 1 mol of air) at the leaf surface,  $f_w$  is the soil moisture stress factor, and  $m$  and  $b$  are constants determined by the vegetation type.

$h_s$  is the relative humidity at the leaf surface and is defined as

$$h_s = e_s / e_i \quad (5.25)$$

where  $e_s$  is the molar fraction of vapor at the leaf surface,  $e_i$  is the molar fraction of vapor in the stoma, and  $e_i = e^*(T_c)$  is the mole fraction of water vapor in the stomata.  $e^*$  denotes the molar fraction of saturated vapor.

Assuming that the vapor flux from the inside of the stoma to the leaf surface is equal to the vapor flux from the leaf surface to the atmosphere (i.e., that there is no convergence and divergence of vapor at the leaf surface),

$$g_s(e_i - e_s) = g_l(e_s - e_a) \quad (5.26)$$

from which we obtain

$$e_s = (g_l e_a + g_s e_i) / (g_l + g_s) \quad (5.27)$$

where  $e_a$  is the molar fraction of vapor in the atmosphere and  $g_l$  is the conductance from the leaf surface to the atmosphere.  $g_l$  is expressed by  $g_l = C_{Hc}|V_a|/LAI$  using the bulk coefficient.

Similarly, assuming that there is no convergence and divergence of  $\text{CO}_2$  at the leaf surface,

$$A_n = g_l(c_a - c_s)/1.4 = g_s(c_s - c_i)/1.6 \quad (5.28)$$

from which we obtain

$$c_s = c_a - 1.4A_n/g_l \quad (5.29)$$

$$c_i = c_s - 1.6A_n/g_s \quad (5.30)$$

where  $c_a$  and  $c_i$  are the molar fractions of  $\text{CO}_2$  in the atmosphere and in the stoma, respectively. The numerical values 1.4 and 1.6 are constants that appear due to the difference in the diffusion coefficients of vapor and  $\text{CO}_2$ .

If we order the equations by substituting 5.24, the following equation is obtained for  $g_s$ :

$$Hg_s^2 + (Hg_l - e_i - Hbf_w)g_s - g_l(Hbf_w + e_a) = 0 \quad (5.31)$$

However, since

$$H = (e_i c_s) / (mA_n) \quad (5.32)$$

5.30 is used for  $c_s$ .

Among the two solutions of 5.30 if  $g_s$  is solved. That is,  $A_n$  is necessary in order to solve  $g_s$ , whereas  $c_i$ , namely  $g_s$ , is necessary in order to solve  $A_n$ . Iterative calculation is therefore required.

The algorithm for the iterative calculation is ported from SiB2, which uses the method of quickening the convergence by iterating six times and putting the errors in decreasing order to estimate the next solution.

Lastly, using stomatal conductance, the stomatal resistance is expressed as

$$r_{st} = 1/g_{st} \quad (5.33)$$

## 5.4 Calculation of ground surface evaporation resistance

The ground surface evaporation resistance ( $r_{soil}$ ) and relative humidity of the uppermost soil layer ( $h_{soil}$ ) are calculated as follows:

$$r_{soil} = a_1(1 - W_{(1)})/(a_2 + W_{(1)}) \quad (5.34)$$

$$h_{soil} = \exp\left(\frac{\psi_{(1)}g}{R_{air}T_{g(1)}}\right) \quad (5.35)$$

where  $W_{(1)} = w_{(1)}/w_{sat(1)}$  is the degree of saturation of the uppermost soil layer,  $\psi_1$  is the moisture potential of the uppermost soil layer,  $g$  is the gravitational acceleration,  $R_{air}$  is the gas constant of the air, and  $T_{g(1)}$  is the temperature of the uppermost soil layer.  $a_1$  and  $a_2$  are constants, with  $a_1 = 800$ ,  $a_2 = 0.2$ . as standard values.

## 6 Surface energy balance

### 6.1 Calculation of surface turbulent fluxes

The turbulent fluxes at the ground surface are solved by bulk formulae as follows. Then, by solving the surface energy balance, the ground surface temperature ( $T_s$ ) and canopy temperature ( $T_c$ ) are updated, and the surface flux values with respect to those values are also updated. The solutions obtained here are temporary values. In order to solve the energy balance by linearizing with respect to  $T_s$  and  $T_c$ , the differential with respect to  $T_s$  and  $T_c$  of each flux is calculated beforehand.

- Momentum flux

$$\tau_x = -\rho C_M |V_a| u_a \quad (6.1)$$

$$\tau_y = -\rho C_M |V_a| v_a \quad (6.2)$$

where  $\tau_x$  and  $\tau_y$  are the momentum fluxes (surface stress) of the zonal and meridional directions, respectively.

- Sensible heat flux

$$H_s = c_p \rho C_{Hs} |V_a| (T_s - (P_s/P_a)^\kappa T_a) \quad (6.3)$$

$$H_c = c_p \rho C_{Hc} |V_a| (T_c - (P_s/P_a)^\kappa T_a) \quad (6.4)$$

$$\partial H_s / \partial T_s = c_p \rho C_{Hs} |V_a| \quad (6.5)$$

$$\partial H_c / \partial T_c = c_p \rho C_{Hc} |V_a| \quad (6.6)$$

where  $H_s$  and  $H_c$  are the sensible heat flux from the ground surface (forest floor) and canopy (leaf surface), respectively;  $\kappa = R_{air}/c_p$  and  $R_{air}$  are the gas constants of air; and  $c_p$  is the specific heat of air.

- Bare soil surface (forest floor) evaporation flux

$$Et_{(1,1)} = (1 - A_{Sn})(1 - f_{ice}) \cdot \rho \widetilde{C}_{Es} |V_a| (h_{soil} q^*(T_s) - q_a) \quad (6.7)$$

$$Et_{(2,1)} = (1 - A_{Sn}) f_{ice} \cdot \rho \widetilde{C}_{Es} |V_a| (h_{soil} q^*(T_s) - q_a) \quad (6.8)$$

$$\partial Et_{(1,1)} / \partial T_s = (1 - A_{Sn})(1 - f_{ice}) \cdot \rho \widetilde{C}_{Es} |V_a| h_{soil} \cdot dq^* / dT|_{T_s} \quad (6.9)$$

$$\partial Et_{(2,1)} / \partial T_s = (1 - A_{Sn}) f_{ice} \cdot \rho \widetilde{C}_{Es} |V_a| h_{soil} \cdot dq^* / dT|_{T_s} \quad (6.10)$$

where  $Et_{(1,1)}$  and  $Et_{(2,1)}$  are the water evaporation and ice sublimation fluxes at the bare soil surface, respectively;  $q^*(T_s)$  is the saturation specific humidity at the ground surface temperature;  $h_{soil}$  is the relative humidity at the soil surface layer;  $A_{Sn}$  is the snow-covered ratio; and  $f_{ice}$  is the ratio of ice in the uppermost soil layer, expressed as

$$f_{ice} = w_{i(1)} / w_{(1)} \quad (6.11)$$

Since the snow-free portion and snow-covered portion are calculated separately, it should be noted that  $A_{Sn}$  takes the value of either 0 (snow-free portion) or 1 (snow-covered portion). When the flux is downward (i.e., dew formation), there is no soil moisture resistance; therefore, the bulk coefficient is taken as:

$$\widetilde{C}_{Es} = \begin{cases} C_{Es} (h_{soil} q^*(T_s) - q_a > 0) \\ C_{Hs} (h_{soil} q^*(T_s) - q_a \leq 0) \end{cases} \quad (6.12)$$

- Transpiration flux

$$Et_{(1,2)} = (1 - f_{cwet}) \cdot \rho \widetilde{C}_{Ec} |V_a| (q^*(T_c) - q_a) \quad (6.13)$$

$$Et_{(2,2)} = 0 \quad (6.14)$$

$$\partial Et_{(1,2)} / \partial T_c = (1 - f_{cwet}) \cdot \rho \widetilde{C}_{Ec} |V_a| \cdot dq^* / dT|_{T_c} \quad (6.15)$$

$$\partial Et_{(2,2)} / \partial T_c = 0 \quad (6.16)$$

where  $Et_{(1,2)}$  and  $Et_{(2,2)}$  are transpiration of water and ice, respectively; and  $Et_{(2,2)}$  is always 0.  $f_{cwet} = w_c / w_{c,cap}$  is the wet fraction of the canopy. When the flux is downward, which is considered to be dew formation on the dry part of the leaf, the bulk coefficient is taken as:

$$\widetilde{C}_{Ec} = \begin{cases} C_{Ec} & (q^*(T_c) - q_a > 0) \\ C_{Hc} & (q^*(T_c) - q_a \leq 0) \end{cases} \quad (6.17)$$

- Canopy evaporation flux



When  $T_c \geq 0^\circ \text{C}$ :

$$Et_{(1,3)} = f_{cwet} \cdot \rho C_{Hc} |V_a| (q^*(T_c) - q_a) \quad (6.18)$$

$$Et_{(2,3)} = 0 \quad (6.19)$$

$$\partial Et_{(1,3)} / \partial T_c = f_{cwet} \cdot \rho C_{Hc} |V_a| \cdot dq^*/dT|_{T_c} \quad (6.20)$$

$$\partial Et_{(2,3)} / \partial T_c = 0 \quad (6.21)$$

when  $T_c < 0^\circ \text{C}$  In case of C:

$$Et_{(1,3)} = 0 \quad (6.22)$$

$$Et_{(2,3)} = f_{cwet} \cdot \rho C_{Hc} |V_a| (q^*(T_c) - q_a) \quad (6.23)$$

$$\partial Et_{(1,3)} / \partial T_c = 0 \quad (6.24)$$

$$\partial Et_{(2,3)} / \partial T_c = f_{cwet} \cdot \rho C_{Hc} |V_a| \cdot dq^*/dT|_{T_c} \quad (6.25)$$

where  $Et_{(1,3)}$  and  $Et_{(2,3)}$  are the evaporation of water and the sublimation of ice at the canopy surface, respectively.

- Snow sublimation flux

$$E_{Sn} = A_{Sn} \cdot \rho C_{Hs} |V_a| (q^*(T_s) - q_a) \quad (6.26)$$

$$\partial E_{Sn} / \partial T_s = A_{Sn} \cdot \rho C_{Hs} |V_a| \cdot dq^*/dT|_{T_s} \quad (6.27)$$

where  $E_{Sn}$  is the snow sublimation flux. Since the snow-free portion and snow-covered portion are calculated separately, it should also be noted here that  $A_{Sn}$  takes the value of either 0 (snow-free portion) or 1 (snow-covered portion).

## 6.2 Calculation of heat conduction fluxes

The heat conduction fluxes in the snow-free and snow-covered portions are calculated. Similarly to the turbulent fluxes, when the energy balance is solved later and the surface temperature is updated, the heat conduction flux values are updated with respect to that value.

In addition, it should also be noted here that since the snow-free portion and snow-covered portion are calculated separately,  $A_{Sn}$  takes the value of either 0 (snow-free portion) or 1 (snow-covered portion).

- Heat conduction flux in the snow-free portion

$$F_{g(1/2)} = (1 - A_{Sn}) \cdot k_{g(1/2)} / \Delta z_{g(1/2)} (T_{g(1)} - T_s) \quad (6.28)$$

$$\partial F_{g(1/2)} / \partial T_s = -(1 - A_{Sn}) \cdot k_{g(1/2)} / \Delta z_{g(1/2)} \quad (6.29)$$

where  $F_{g(1/2)}$  is the heat conduction flux,  $k_{g(1/2)}$  is the soil heat conductivity,  $\Delta z_{g(1/2)}$  is the thickness from the temperature definition point of the uppermost soil layer to the ground surface, and  $T_{g(1)}$  is the temperature of the uppermost soil layer.

- Heat conduction flux in the snow-covered portion

$$F_{Sn(1/2)} = A_{Sn} \cdot k_{Sn(1/2)} / \Delta z_{Sn(1/2)} (T_{Sn(1)} - T_s) \quad (6.30)$$

$$\partial F_{Sn(1/2)} / \partial T_s = -A_{Sn} \cdot k_{Sn(1/2)} / \Delta z_{Sn(1/2)} \quad (6.31)$$

where  $F_{Sn(1/2)}$  is the heat conduction flux,  $k_{Sn(1/2)}$  is the snow heat conductivity,  $\Delta z_{Sn(1/2)}$  is the thickness from the temperature definition point of the uppermost snow layer to the ground surface, and  $T_{Sn(1)}$  is the temperature of the uppermost snow layer.

### 6.3 Solution of energy balance at ground surface and canopy

The energy balance is solved for two cases: (1) when there is no melting at the ground surface, and (2) when there is melting at the ground surface. In case (2), the solution is obtained by fixing the ground surface temperature ( $T_s$ ) at 0 ° C, and the energy available for use in melting is diagnosed from the energy balance. Snowmelt on vegetation is treated by correction later on; therefore, that case is not solved separately here. Moreover, the case of the snow completely melting within the time steps is also treated by correction later on.

#### 6.3.1 Energy balance at ground surface and canopy

The energy divergence at the ground surface (forest floor) is

$$\Delta F_s = H_s + R_s^{net} + lEt_{(1,1)} + l_s(Et_{(2,1)} + E_{Sn}) - F_{g(1/2)} - F_{Sn(1/2)} \quad (6.32)$$

where  $l$  and  $l_s$  are the latent heat of evaporation and sublimation, respectively; and  $R_s^{net}$  is the net radiation divergence at the ground surface, given by

$$R_s^{net} = -(R_s^\downarrow - R_s^\uparrow)\mathcal{T}_{cS} - \epsilon R_L^\downarrow \mathcal{T}_{cL} + \epsilon \sigma T_s^4 - \epsilon \sigma T_c^4 (1 - \mathcal{T}_{cL}) \quad (6.33)$$

where  $\sigma$  is the Stefan-Boltzmann constant.

The energy divergence at the canopy (leaf surface) is

$$\Delta F_c = H_c + R_c^{net} + l(Et_{(1,2)} + Et_{(1,3)}) + l_s(Et_{(2,2)} + Et_{(2,3)}) \quad (6.34)$$

where  $R_c^{net}$  is the net radiation divergence at the canopy, given by

$$R_c^{net} = -(R_S^\downarrow - R_S^\uparrow)(1 - \mathcal{T}_{cS}) - \epsilon R_L^\downarrow (1 - \mathcal{T}_{cL}) + (2\epsilon \sigma T_c^4 - \epsilon \sigma T_s^4)(1 - \mathcal{T}_{cL}) \quad (6.35)$$

### 6.3.2 Case 1: When there is no melting at the ground surface

When there is no melting at the ground surface,  $\Delta F_s = \Delta F_c = 0$  are solved so that  $T_s$  and  $T_c$  holds true for the energy balance at the ground surface and canopy.

The energy balance equation linearizing each term with respect to  $T_s$  and  $T_c$  can be expressed as

$$\begin{pmatrix} \Delta F_s \\ \Delta F_c \end{pmatrix}^{current} = \begin{pmatrix} \Delta F_s \\ \Delta F_c \end{pmatrix}^{past} + \begin{pmatrix} \partial \Delta F_s / \partial T_s & \partial \Delta F_s / \partial T_c \\ \partial \Delta F_c / \partial T_s & \partial \Delta F_c / \partial T_c \end{pmatrix} \begin{pmatrix} \Delta T_s \\ \Delta T_c \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (6.36)$$

The part with *past* on the right-hand side is where the fluxes calculated 6.35 using the values of  $T_s$  and  $T_c$  obtained in the previous step.

The differential terms are as follows:

$$\frac{\partial \Delta F_s}{\partial T_s} = \frac{\partial H_s}{\partial T_s} + \frac{\partial R_s^{net}}{\partial T_s} + l \frac{\partial Et_{(1,1)}}{\partial T_s} + l_s \left( \frac{\partial Et_{(2,1)}}{\partial T_s} + \frac{\partial E_{Sn}}{\partial T_s} \right) - \frac{\partial F_{g(1/2)}}{\partial T_s} - \frac{\partial F_{Sn(1/2)}}{\partial T_s} \quad (6.37)$$

$$\frac{\partial \Delta F_s}{\partial T_c} = \frac{\partial R_s^{net}}{\partial T_c} \quad (6.38)$$

$$\frac{\partial \Delta F_c}{\partial T_s} = \frac{\partial R_c^{net}}{\partial T_s} \quad (6.39)$$

$$\frac{\partial \Delta F_c}{\partial T_c} = \frac{\partial H_c}{\partial T_c} + \frac{\partial R_c^{net}}{\partial T_c} + l \left( \frac{\partial Et_{(1,2)}}{\partial T_c} + \frac{\partial Et_{(1,3)}}{\partial T_c} \right) + l_s \left( \frac{\partial Et_{(2,2)}}{\partial T_c} + \frac{\partial Et_{(2,3)}}{\partial T_c} \right) \quad (6.40)$$

where

$$\frac{\partial R_s^{net}}{\partial T_s} = \epsilon 4\sigma T_s^3 \quad (6.41)$$

$$\frac{\partial R_s^{net}}{\partial T_c} = -(1 - \mathcal{T}_{cL}) \epsilon 4\sigma T_c^3 \quad (6.42)$$

$$\frac{\partial R_c^{net}}{\partial T_s} = -(1 - \mathcal{T}_{cL}) \epsilon 4\sigma T_s^3 \quad (6.43)$$

$$\frac{\partial R_c^{net}}{\partial T_c} = 2(1 - \mathcal{T}_{cL}) \epsilon 4\sigma T_c^3 \quad (6.44)$$

Using the above equations, Eq. (140) is solved for  $T_s$  and  $T_c$ .

### 6.3.3 Case 2: When there is melting at the ground surface

When either there is snow on the ground surface or the land cover type is ice sheet, and also the ground surface temperature solved in case 1,  $T_s^{current} = T_s^{past} + \Delta T_s$ , is higher than 0 ° C, melting at the ground surface occurs. When there is melting at the ground surface, the ground surface temperature is fixed at 0 ° C. That is:

$$\Delta T_s = \Delta T_s^{melt} = T_{melt} - T_s^{past} \quad (6.45)$$

where  $T_{melt}$  is the melting point ( $0^\circ \text{C}$ ) of ice.

With  $T_c$  known,  $\Delta T_s$  is solved by the following equation similarly to 6.36:

$$\Delta T_c = \left( -\Delta F_c^{past} - \frac{\partial \Delta F_c}{\partial T_s} \Delta T_s^{melt} \right) / \frac{\partial \Delta F_c}{\partial T_c} \quad (6.46)$$

Thus,  $\Delta T_s$  and  $\Delta T_c$  are determined, and the energy convergence at the ground surface to be used for melting is solved by the following equation:

$$\Delta F_{conv} = -\Delta F_s^{current} = -\Delta F_s^{past} - \frac{\partial \Delta F_s}{\partial T_s} \Delta T_s^{melt} - \frac{\partial \Delta F_s}{\partial T_c} \Delta T_c \quad (6.47)$$

### 6.3.4 Conditions for solutions

Several conditions are set for the solution of the ground surface energy balance. After solving the energy balance, if any of the conditions are not followed, the flux that has contravened the conditions is fixed at the limit value that satisfies the conditions, and the energy balance is solved again.

1. Vapor in the troposphere should not be excessively removed.

Due to the instability of temporal calculations, it is possible that large downward latent heat is produced. The conditions are set so that even in such a case, the vapor in the troposphere from the surface is not completely removed; that is,

$$Et_{(i,j)}^{current} > -q_a(P_s - P_a)/(g\Delta t) \quad (i = 1, 2; j = 1, 2, 3) \quad (6.48)$$

$$E_{Sn}^{current} > -q_a(P_s - P_a)/(g\Delta t) \quad (6.49)$$

where  $g$  is the gravitational acceleration and  $\Delta t$  denotes the time steps of the atmospheric model. For the values of  $Et$  etc. to be used for judgment, the updated flux values (*current*) with respect to the values of  $T_s$  and  $T_c$  that have been updated so as to satisfy the energy balance are used. The same applies to all of the other conditions listed below. Updating of the flux values is described later.

2. Soil moisture should not take a negative value.

Soil moisture should not take a negative value due to transpiration; that is,

$$Et_{(1,2)}^{current} < \sum_{k \in \text{rootzone}} \rho_w w_k \Delta z_{g(k)} / \Delta t_L \quad (6.50)$$

where  $\rho_w$  is the water density and  $\Delta t_L$  denotes the time steps of the land surface model.

3. Canopy water should not take a negative value.

Canopy water should not take a negative value due to evaporation; that is,

$$Et_{(i,3)}^{current} < \rho_w w_c / \Delta t_L \quad (i = 1, 2) \quad (6.51)$$

4. The snow water equivalent should not take a negative value.

The snow water equivalent should not take a negative value due to sublimation of snow; that is,

$$E_{Sn}^{current} < Sn / \Delta t_L \quad (6.52)$$

### 6.3.5 Updating of ground surface and canopy temperatures

The ground surface temperature and canopy temperature are updated as follows:

$$T_s^{current} = T_s^{past} + \Delta T_s \quad (6.53)$$

$$T_c^{current} = T_c^{past} + \Delta T_c \quad (6.54)$$

Based on the updated canopy temperature, the canopy water is diagnosed in advance as being either liquid or solid. This information is used when treating freezing and melting of the canopy water, as follows:

$$A_{Snc} = \begin{cases} 0 & (T_c \geq T_{melt}) \\ 1 & (T_c < T_{melt}) \end{cases} \quad (6.55)$$

where  $A_{Snc}$  is the frozen fraction on the canopy.

### 6.3.6 Updating of flux values

The flux values are updated with respect to the updated values of  $T_s$  and  $T_c$ . When  $F$  denotes any given flux, updating of the values is performed as follows:

$$F^{current} = F^{past} + \frac{\partial F}{\partial T_s} \Delta T_s + \frac{\partial F}{\partial T_c} \Delta T_c \quad (6.56)$$

Using the updated flux values, the fluxes output into the atmosphere, etc. are calculated as follows:

$$H = H_s + H_c \quad (6.57)$$

$$E = \sum_{j=1}^3 \sum_{i=1}^2 Et_{(i,j)} + E_{Sn} \quad (6.58)$$

$$R_L^\uparrow = \mathcal{T}_{cL} \epsilon \sigma T_s^4 + (1 - \mathcal{T}_{cL}) \epsilon \sigma T_c^4 + (1 - \epsilon) R_L^\downarrow \quad (6.59)$$

$$T_{sR} = (R_L^\uparrow / \sigma)^{1/4} \quad (6.60)$$

where  $T_{sR}$  is the radiation temperature at the ground surface.

The root uptake flux in each soil layer is then calculated as follows:

$$F_{root(k)} = f_{rootup(k)} Et_{(1,2)} \quad (k = 1, \dots, K_g) \quad (6.61)$$

where  $F_{root(k)}$  is the root uptake flux and  $f_{rootup(k)}$  is the weighting for distribution of the transpiration to the root uptake flux in each layer.

## 7 Canopy Water Balance

The canopy water balance is calculated.

### 7.1 Diagnosis of canopy water phase

With regard to canopy water, the liquid phase (intercepted rainfall, dew formation, and frozen water content that has melted) and solid phase (intercepted snow, icing, and liquid water content that has frozen) are considered separately and the coexistence of the two phases is allowed. The only prognostic variable is the water content ( $w_c$ ) encompassing both the liquid and solid phases, and depending on whether the canopy temperature ( $T_c$ ) is higher or lower than  $T_{melt} = 0^\circ \text{ C}$ , it is diagnosed as liquid or solid, respectively. The reason why the liquid and solid phases can coexist is that  $T_c$  in snow-covered and snow-free portions is calculated separately. That is, the frozen fraction on the canopy ( $A_{Snc}$ ) is defined (in actuality, it is obtained as a result of spatial averaging by the coupler) as follows:

$$A_{Snc} = \begin{cases} 0 & (T_{c(1)} \geq T_{melt}, T_{c(2)} \geq T_{melt}) \\ (1 - A_{Sn}) & (T_{c(1)} < T_{melt}, T_{c(2)} \geq T_{melt}) \\ A_{Sn} & (T_{c(1)} \geq T_{melt}, T_{c(2)} < T_{melt}) \\ 1 & (T_{c(1)} < T_{melt}, T_{c(2)} < T_{melt}) \end{cases} \quad (7.1)$$

where  $w_{cl} = w_c(1 - A_{Snc})$  and  $w_{ci} = w_c A_{Snc}$  are the liquid and solid water content of the canopy, respectively.

For  $A_{Snc}$ , the value updated in the flux calculation section  $A_{Snc}^{\tau+1}$  is given by the coupler, but the value of the previous step  $A_{Snc}^\tau$  is stored in MATCNW.  $\tau$  denotes the time steps. This is solved from the initial values of  $T_c$  and  $Sn$  at the time of initiating the calculation, and therefore does not become a new prognostic variable.

### 7.2 Prognosis of canopy water

The prognostic equations for the canopy water in the liquid and solid phases are given respectively as

$$\rho_w \frac{w_{cl}^{\tau+1} - w_{cl}^{\tau}}{\Delta t_L} = P_{Il} - E_l - D_l + M_c \quad (7.2)$$

$$\rho_w \frac{w_{ci}^{\tau+1} - w_{ci}^{\tau}}{\Delta t_L} = P_{Ii} - E_i - D_i - M_c \quad (7.3)$$

where  $P_{Il}$  and  $P_{Ii}$  are the precipitation interception in the respective cases,  $E_l$  and  $E_i$  are the evaporation (sublimation),  $D_l$ ,  $D_i$  are the dripping, and  $M_c$  is the melting. Note that here, the values before the updated  $w_{cl}^{\tau}$  and  $w_{ci}^{\tau}$  are defined using  $A_{Snc}^{\tau}$  before it is updated, as follows:

$$w_{cl}^{\tau} = w_c^{\tau} (1 - A_{Snc}^{\tau}) \quad (7.4)$$

$$w_{ci}^{\tau} = w_c^{\tau} A_{Snc}^{\tau} \quad (7.5)$$

### 7.2.1 Evaporation (sublimation) of canopy water

First, by subtracting the evaporation (sublimation), the canopy water is partially updated as follows. The evaporation (sublimation) has already been solved in the flux calculation section.

$$w_{cl}^* = w_{cl}^{\tau} - E_l \Delta t_L / \rho_w \quad (7.6)$$

$$w_{ci}^* = w_{ci}^{\tau} - E_i \Delta t_L / \rho_w \quad (7.7)$$

$$E_l = Et_{(1,3)} \quad (7.8)$$

$$E_i = Et_{(2,3)} \quad (7.9)$$

Then, if either  $w_{cl}$  or  $w_{ci}$  become negative in value, it is supplemented by the other until the value returns to 0, and the melting (negative value in the case of frozen water) that is assumed to be produced is then inserted in  $M_c$ .

### 7.2.2 Interception of precipitation by the canopy

The precipitation interception and dripping are considered by separating the places of convective precipitation and nonconvective precipitation. The fraction of the convective precipitation area ( $A_c$ ) is assumed to be uniform (0.1 as a standard value). Stratiform precipitation is also assumed to be uniform.

$$P_{Il}^c = f_{int}(Pr_c/A_c + Pr_l) \quad (7.10)$$

$$P_{Il}^{nc} = f_{int}Pr_l \quad (7.11)$$

$$P_{Ii}^c = f_{int}(P_{Snc}/A_c + P_{Snl}) \quad (7.12)$$

$$P_{Ii}^{nc} = f_{int}P_{Snl} \quad (7.13)$$

where  $P_{ll}^c$  and  $P_{li}^c$  denote the interception in the convective precipitation area, and  $P_{ll}^{nc}$  and  $P_{li}^{nc}$  denote the interception in the nonconvective precipitation area.  $f_{int}$  is the interception efficiency, and is simply given by

$$f_{int} = \begin{cases} LAI & (LAI < 1) \\ 1 & (LAI \geq 1) \end{cases} \quad (7.14)$$

By adding the intercepted precipitation, the canopy water is further partially updated as follows:

$$w_{cl}^{c*} = w_{cl}^* + P_{ll}^c \Delta t_L / \rho_w \quad (7.15)$$

$$w_{cl}^{nc*} = w_{cl}^* + P_{ll}^{nc} \Delta t_L / \rho_w \quad (7.16)$$

$$w_{ci}^{c*} = w_{ci}^* + P_{li}^c \Delta t_L / \rho_w \quad (7.17)$$

$$w_{ci}^{nc*} = w_{ci}^* + P_{li}^{nc} \Delta t_L / \rho_w \quad (7.18)$$

### 7.2.3 Dripping of the canopy water

For dripping, dripping due to the canopy water capacity being exceeded and natural dripping due to gravity are considered, as follows:

$$D_l^c = \max(w_{cl}^{c*} - w_{c, cap}, 0) + D_g(w_{cl}^{c*}) \quad (7.19)$$

$$D_l^{nc} = \max(w_{cl}^{nc*} - w_{c, cap}, 0) + D_g(w_{cl}^{nc*}) \quad (7.20)$$

$$D_i^c = \max(w_{ci}^{c*} - w_{c, cap}, 0) + D_g(w_{ci}^{c*}) \quad (7.21)$$

$$D_i^{nc} = \max(w_{ci}^{nc*} - w_{c, cap}, 0) + D_g(w_{ci}^{nc*}) \quad (7.22)$$

where the canopy water capacity ( $w_{c, cap}$ ) is, from the water capacity per unit leaf area ( $w_{c, max}$ ) and LAI, assumed to be

$$W_{c, cap} = W_{c, max} LAI \quad (7.23)$$

$W_{c, max}$  is set at 0.2 mm as a standard value, and the same value is used with respect to the liquid and solid phases.

The natural dripping due to gravity  $D_g$  is, after Rutter et al. (1975), assumed to be

$$D_g(w_c) = D_1 \exp(D_2 w_c) \quad (7.24)$$

$D_1 = 1.14 \times 10^{-11}$  and  $D_2 = 3.7 \times 10^3$  are standard values, and the same values are used with respect to the liquid and solid phases.

By subtracting the dripping, the values are updated as follows:



$$w_{cl}^{c**} = w_{cl}^{c*} - D_{Il}^c \Delta t_L / \rho_w \quad (7.25)$$

$$w_{cl}^{nc**} = w_{cl}^{nc*} - D_{Il}^{nc} \Delta t_L / \rho_w \quad (7.26)$$

$$w_{ci}^{c**} = w_{ci}^{c*} - D_{Ii}^c \Delta t_L / \rho_w \quad (7.27)$$

$$w_{ci}^{nc**} = w_{ci}^{nc*} - D_{Ii}^{nc} \Delta t_L / \rho_w \quad (7.28)$$

### 7.2.4 Updating and melting of canopy water

Moreover, by taking the average of the convective precipitation area and nonconvective precipitation area, the canopy water can be updated as follows:

$$w_{cl}^{**} = A_c w_{cl}^{c**} + (1 - A_c) w_{cl}^{nc**} \quad (7.29)$$

$$w_{ci}^{**} = A_c w_{ci}^{c**} + (1 - A_c) w_{ci}^{nc**} \quad (7.30)$$

$$w_c^{\tau+1} = w_{cl}^{**} + w_{ci}^{**} \quad (7.31)$$

However, if updating of the frozen fraction ( $A_{Snc}$ ) is considered,

$$w_{cl}^{\tau+1} = w_c^{\tau+1} (1 - A_{Snc}^{\tau+1}) \quad (7.32)$$

$$w_{ci}^{\tau+1} = w_c^{\tau+1} A_{Snc}^{\tau+1} \quad (7.33)$$

The melting  $M_c$  is therefore diagnosed as

$$M_c = -\rho_w (w_{ci}^{\tau+1} - w_{ci}^{**}) / \Delta t_L \quad (7.34)$$

When the melting is produced during evaporation, that portion is added.

Here, the canopy temperature should be changed due to the latent heat of melting; however, it is impossible because we are ignoring the heat capacity of the canopy. Moreover, although it would be advantageous to change the temperature of the surrounding atmosphere, this is also not possible in view of the need for agreement with the calculation in the land surface integration section. Hence, for convenience, in order to conserve the energy of the system, the latent heat of melting is given as the heat flux to the soil (or snow).

### 7.3 Fluxes given to the soil, snow, and runoff process

The water flux  $F_w$  given to the snow or the runoff process after interception by the canopy is respectively expressed with respect to the convective precipitation area and nonconvective precipitation area, and the liquid and solid phases, as follows:

$$F_{wl}^c = (1 - f_{int})(Pr_c/A_c + Pr_l) + D_l^c \quad (7.35)$$

$$F_{wl}^{nc} = (1 - f_{int})Pr_l + D_l^{nc} \quad (7.36)$$

$$F_{wi}^c = (1 - f_{int})(P_{Snc}/A_c + P_{Snl}) + D_i^c \quad (7.37)$$

$$F_{wi}^{nc} = (1 - f_{int})P_{Snl} + D_i^{nc} \quad (7.38)$$

For the calculation of runoff, convective rainfall and stratiform rainfall are given separately, while snowfall is consolidated because separation is not necessary, as follows:

$$Pr_c^* = Ac(F_{wl}^c - F_{wl}^{nc}) \quad (7.39)$$

$$Pr_l^* = F_{wl}^{nc} \quad (7.40)$$

$$P_{Sn}^* = A_c F_{wl}^c + (1 - A_c) F_{wl}^{nc} \quad (7.41)$$

where  $Pr_c^*$ ,  $Pr_l^*$ , and  $P_{Sn}^*$  are the convective precipitation, the stratiform precipitation, and the snowfall after interception by the canopy, respectively.

The energy flux correction portion for the soil or the snow is

$$\Delta F_{c,conv} = -l_m M_c \quad (7.42)$$

where  $l_m$  is the latent heat of melting.

## 8 Snow

The snow cover fraction, snow water equivalent, snow temperature and snow albedo are calculated here.

Most of the processes are included in SUBROUTINE MATSNW in matsnw.F, but the ice albedo is calculated in SUBROUTINE ICEALB in matice.F.

Tables below show the important arguments of SUBROUTINE MATSNW classified by the statements of “modified”, “output” and “input”.

Modified

Name in code	Variable	Description	Unit
GLSNW	$Sn$	Amount of snowfall	kg/m <sup>2</sup>
GLTSN	$T_{Sn(k)}$ ( $k = 1, 2, 3$ )	Snow temperature of the $k$ th layer	K
GLASN	$\alpha_b$ ( $b = 1, 2, 3$ )	Snow albedo for band $b$	-
GLRSN	$A_{Sn}$	Snow fraction	-
GLSDA	-	Snow accumulation	kg/m <sup>2</sup>
GLSDM	$M_{Sn}$	Accumulation of the snow melt	kg/m <sup>2</sup>
WINPC	$Pr_c$	Convective precipitation flux	kg/m <sup>2</sup> /s
WINPL	$Pr_l$	Layered precipitation flux	kg/m <sup>2</sup> /s
CDST	$\rho_{(k)}$	Dust density in the $k$ th layer	ppmv

Output

Name in code	Variable	Description	Unit
GGLACR	$Ro_{gl}$	Glacier formation	$\text{kg/m}^2/\text{s}$

Input

Name in code	Variable	Description	Unit
SNFAL	$P_{Sn}$	Snow fall	$\text{kg/m}^2/\text{s}$
SNSUB	$E_{Sn}$	Snow sublimation	$\text{kg/m}^2/\text{s}$
SNFLXS	$F_{Sn(1/2)}$	Snow surface heat flux	$\text{W/m}^2$
GLG	$T_{g(k)}$	Soil temperature of the $k$ th layer	T
GLW	$w_{g(k)}$	Soil moisture	$\text{m}^3/\text{m}^3$
GLWC	$w_c$	Canopy water	m
SNRATC	$A_{Snc}$	Canopy snow ratio	-
DSTFAL	$D$	Dust fall	$\text{ppmv/s}$
GRZSD	-	Standard deviation of topography	m
T2HIST	-	Annual mean temperature over the latest 30 years	K
ILSFC	-	Index of the surface condition	-
ILSOIL	-	Soil type	-

## 8.1 Diagnosis of snow cover fraction

MATSIRO has two ways of calculation of the snow cover fraction, and the user can switch them with the option OPT\_SSNOWD.

### 8.1.1 Case 1: When OPT\_SSNOWD is active

The snow cover fraction is diagnosed in the SUBROUTINE SSNOWD.DRV, a driver of a Subgrid SNOW Distribution (SSNOWD) submodel developed by Liston (2004), with a physically based parameterization of sub-grid snow distribution considering various factors such as differences in topography, the time of snowfall or snow melting, etc (Nitta et al., 2014, Tatebe et al., 2019).

The snow cover fraction is formulated for accumulation and ablation seasons separately. For the accumulation season, snowfall occurs uniformly and the snow cover fraction is assumed to be equal in the grid cell. For the ablation season, the snow cover fraction decreases based on the sub-grid distribution of the snow water equivalent. Under the assumption of uniform melt depth  $D_m$ , the sum of snow-free and snow-covered fraction equals unity:

$$\int_0^{D_m} f(D)dD + \int_{D_m}^{\infty} f(D)dD = 1, \quad (8.1)$$

where  $D$  is the snow water equivalent depth and  $f(D)$  is the probability distribution function (PDF) of snow water equivalent depth within the grid cell. The snow depth distribution within each grid cell is assumed to follow a lognormal distribution:

$$f(D) = \frac{1}{D\zeta\sqrt{2\pi}} \exp \left[ -\frac{1}{2} \left( \frac{\ln(D) - \lambda}{\zeta} \right)^2 \right], \quad (8.2)$$

where

$$\lambda = \ln(\mu) - \frac{1}{2}\zeta^2, \quad (8.3)$$

$$\zeta^2 = \ln(1 + CV^2). \quad (8.4)$$

Here  $\mu$  is the accumulated snowfall and  $CV$  is the coefficient of variation.  $CV$  is diagnosed from the standard deviation of the subgrid topography, coldness index and vegetation type that is a proxy for surface winds. For coldness index, the annually averaged temperature over the latest 30 years using the time relaxation method of Krinner et al. (2005), in which the timescale parameter is set to 16 years. The temperature threshold for a category diagnosis is set to 0 and 10 °C.

The snow amount  $Sn$  is given by

$$Sn(D_m) = \int_0^{D_m} 0[f(D)]dD + \int_{D_m}^{\infty} (D - D_m)[f(D)]dD, \quad (8.5)$$

and this equation is rewritten to

$$Sn(D_m) = \frac{1}{2} \exp \left( \lambda + \frac{\zeta^2}{2} \right) \operatorname{erfc} \left( \frac{z_{D_m} - \xi}{\sqrt{2}} \right) - \frac{1}{2} D_m \operatorname{erfc} \left( \frac{z_{D_m}}{\sqrt{2}} \right), \quad (8.6)$$

where  $\xi = (1 - \sqrt{2})z$ ,  $z = \frac{\ln(D) - \lambda}{\zeta}$ ,  $z_{D_m}$  is the value of  $z$  when  $D = D_m$  and  $\operatorname{erfc}$  is the complementary error function.

$D_m$  is calculated from this equation and the snow amount  $Sn$  using Newton-Raphson methods (in SUBROUTINE SSNOWD.ITR in ssnowd.F).

Then, the snow cover fraction  $A_{Sn}(D_m)$  is calculated by

$$A_{Sn}(D_m) = 1 - \int_0^{D_m} f(D)dD = \frac{1}{2} \operatorname{erfc} \left( \frac{z_{D_m}}{\sqrt{2}} \right). \quad (8.7)$$

### 8.1.2 Case 2: When OPT\_SSNOWD is inactive

The snow cover fraction is diagnosed in SUBROUTINE SNWRAT. The snow cover fraction is formulated as a function of the snow amount  $Sn$ :

$$Sn(D_m) = \min(\sqrt{Sn/Sn_c}), \quad (8.8)$$

where  $Sn_c$  is 100 kg/m<sup>2</sup>.

## 8.2 Vertical division of snow layers

In order to express the vertical distribution of the snow temperature, when the snow water equivalent is large, the snow is divided into multiple layers and the temperature is defined in each layer. The number of snow layers can be varied, with the number of layers increasing as the snow water equivalent becomes larger. A minimum of one layer and a maximum of three layers are set as a standard.

This process is treated in SUBROUTINE SNWCUT in matsnw.F.

The number of layers and the mass of each layer are determined uniquely by the snow water equivalent. Consequently, the mass of each layer does not become a new prognostic variable.

As a standard, the mass of each layer ( $\Delta\widetilde{S}n_{(k)} (k = 1, 2, 3)$ ) is determined as follows ( $k = 1$  is the uppermost layer):

$$\begin{aligned}\widetilde{S}n_{(1)} &= \begin{cases} \widetilde{S}n & (\widetilde{S}n < 20) \\ 0.5\widetilde{S}n & (20 \leq \widetilde{S}n < 40) \\ 20 & (\widetilde{S}n \geq 40) \end{cases} \\ \widetilde{S}n_{(2)} &= \begin{cases} 0 & (\widetilde{S}n < 20) \\ \widetilde{S}n - \Delta S n_{(1)} & (20 \leq \widetilde{S}n < 60) \\ 0.5(\widetilde{S}n - 20) & (60 \leq \widetilde{S}n < 100) \\ 40 & (\widetilde{S}n \geq 100) \end{cases} \\ \widetilde{S}n_{(3)} &= \begin{cases} 0 & (\widetilde{S}n < 60) \\ \widetilde{S}n - (\Delta S n_{(1)} + \Delta S n_{(2)}) & (\widetilde{S}n \geq 60) \end{cases}\end{aligned}\tag{8.9}$$

where

$$\widetilde{S}n = S n / A_{S n}\tag{8.10}$$

$S n$  is the grid-mean snow water equivalent, and  $\widetilde{S}n$  is the snow water equivalent in the snow-covered portion. Note that the mass of each layer ( $\Delta\widetilde{S}n_{(k)}$ ) is also the value of the snow-covered portion, not the grid-mean value. The unit is  $\text{kg}/\text{m}^2$ .

From the above, it can be clearly seen that the number of snow layers ( $K_{S n}$ ) is as follows, as a standard:

$$K_{S n} = \begin{cases} 0 & (\widetilde{S}n = 0) \\ 1 & (0 < \widetilde{S}n < 20) \\ 2 & (20 \leq \widetilde{S}n < 60) \\ 3 & (\widetilde{S}n \geq 60) \end{cases}\tag{8.11}$$

## 8.3 Calculation of snow water equivalent

The prognostic equation of the snow water equivalent is given by

$$\frac{S n^{\tau+1} - S n^{\tau}}{\Delta t_L} = P_{S n}^* - E_{S n} - M_{S n} + Fr_{S n}\tag{8.12}$$

where  $P_{Sn}^*$  is the snowfall flux after interception by the canopy,  $E_{Sn}$  is the sublimation flux,  $M_{Sn}$  is the snowmelt, and  $Fr_{Sn}$  is the refreeze of snowmelt or the freeze of rainfall.

### 8.3.1 Sublimation of snow

First, by subtracting the sublimation, the snow water equivalent is partially updated:

$$Sn^* = Sn^\tau - E_{Sn}\Delta t_L \quad (8.13)$$

$$\Delta\widetilde{Sn}_{(1)}^* = \Delta\widetilde{Sn}_{(1)}^\tau - E_{Sn}/A_{Sn}\Delta t_L \quad (8.14)$$

In a case where the sublimation is larger than the snow water equivalent in the uppermost layer, the remaining amount is subtracted from the layer below. If the amount in the second layer is insufficient for such subtraction, the remaining amount is subtracted from the layer below that.

### 8.3.2 Snowmelt

Next, the snow heat conduction is calculated to solve the snowmelt. The method of calculating the snow heat conduction is described later. The updated snow temperature incorporating the heat conduction is assumed to be  $T_{Sn(k)}^*$ . When the temperature is calculated and the temperature of the uppermost snow layer becomes higher than  $T_{melt} = 0^\circ\text{C}$ , the temperature of the uppermost layer is fixed at  $T_{melt}$  and the calculation is performed again. In this case, the energy convergence  $\Delta\widetilde{F}_{conv}$  in the uppermost layer is calculated. This is not the grid-mean value but the value of the snow-covered portion. The snowmelt in the uppermost layer is

$$\widetilde{M}_{Sn(1)} = \min(\Delta\widetilde{F}_{conv}/l_m, \Delta\widetilde{Sn}_{(1)}^*/\Delta t_L) \quad (8.15)$$

With regard to the second layer and below, if the temperature is higher than  $T_{melt}$ , it is put back to  $T_{melt}$  and the internal energy of that temperature change portion is applied to the snowmelt. That is, it is assumed to be

$$T_{Sn(k)}^{**} = T_{melt} \quad (8.16)$$

$\Delta\widetilde{F}_{conv}$  is newly defined by

$$\Delta\widetilde{F}_{conv} = (T_{Sn(k)}^* - T_{melt})c_{pi}\Delta\widetilde{Sn}_{(k)}^*/\Delta t_L \quad (8.17)$$

and the snowmelt is solved as in 8.15.

By subtracting the snowmelt, the mass of each layer is updated:

$$\Delta \widetilde{S}n_{(k)}^{**} = \Delta \widetilde{S}n_{(k)}^* - \widetilde{M}_{Sn_{(k)}} \quad (8.18)$$

During these calculations, when a certain layer is fully melted, the remaining amount of  $\Delta \widetilde{F}_{conv}$  is given to the layer below to raise the temperature in that layer; that is,

$$\Delta \widetilde{F}_{conv}^* = \Delta \widetilde{F}_{conv} - l_m \widetilde{M}_{Sn_{(k)}} \quad (8.19)$$

$$T_{Sn_{(k+1)}}^{**} = T_{Sn_{(k+1)}}^* + \Delta \widetilde{F}_{conv}^* / (c_{pi} \Delta \widetilde{S}n_{(k+1)}^*) \Delta t_L \quad (8.20)$$

where  $c_{pi}$  is the specific heat of snow (ice). When all of the snow is melted,  $\Delta \widetilde{F}_{conv}^*$  is given to the soil.

The snowmelt of the overall snow is the sum of the snowmelt in each layer (note, however, that it is the grid-mean value):

$$M_{Sn} = \sum_{k=1}^{K_{Sn}} \widetilde{M}_{Sn_{(k)}} A_{Sn} \quad (8.21)$$

By subtracting the snowmelt, the snow water equivalent is partially updated:

$$Sn^{**} = Sn^* - M_{Sn} \Delta t_L \quad (8.22)$$

### 8.3.3 Freeze of snowmelt water and rainfall in snow

The freeze of snowmelt water and rainfall in the snow is calculated next. With regard to the snowmelt water, consideration is given to the effect of the liquid water produced by the snowmelt in the upper layer refreezing in the lower layer. The retention of liquid water content in the snow is not considered, and the entire amount is treated whether it has frozen in the snow or percolated under the snow.

The liquid water flux at the snow upper boundary in the snow-covered portion is

$$\widetilde{F}_{wSn(1)} = Pr_c^* + Pr_l^* + M_{Sn} / A_{Sn} \quad (8.23)$$

Here, the melted portion in the second layer of the snow and below is also assumed to have percolated from the snow upper boundary (in actuality, snowmelt in the second layer or below rarely occurs).

It is reasonable to assume the temperature of the snowmelt water as 0 °C, and the temperature of rainfall on the snow is also assumed to be 0 °C for convenience. The temperature of the snow increases due to the latent heat of the freezing of water; however, when the temperature of the snow in a certain layer is increased to 0 °C, any additional water is assumed to be unable to

freeze and to percolate to the layer below. In addition, an upper limit is set on the ratio of water that can be frozen compared with the mass of snow in the layer. The amount of freeze in a given layer  $\widetilde{Fr}_{Sn(k)}$  is solved by

$$\widetilde{Fr}_{Sn(k)} = \min \left( \widetilde{F}_{wSn(k)}, \frac{c_{pi}(T_{melt} - T_{Sn(k)}^{**})}{l_m} \frac{\Delta \widetilde{Sn}_{(k)}^{**}}{\Delta t_L}, f_{Fmax} \frac{\Delta \widetilde{Sn}_{(k)}^{**}}{\Delta t_L} \right) \quad (8.24)$$

where  $F_{wSn(k)}$  is the liquid water flux percolated from the upper boundary of the  $k$ th layer of the snow.  $\widetilde{F}_{wSn(k)}$  is the liquid water flux flowing from the top of the  $k$ th layer of snow cover.  $f_{Fmax}$  is assumed to be 0.1 as a standard value.

The snow temperature change is updated by

$$T_{Sn(k)}^{***} = \frac{l_m \widetilde{Fr}_{Sn(k)} \Delta t_L + c_{pi}(T_{Sn(k)}^{**} \Delta \widetilde{Sn}_{(k)}^{**} + T_{melt} \widetilde{Fr}_{Sn(k)} \Delta t_L)}{c_{pi}(\Delta \widetilde{Sn}_{(k)}^{**} + \widetilde{Fr}_{Sn(k)} \Delta t_L)}, \quad (8.25)$$

and the mass is updated as follows:

$$\Delta \widetilde{Sn}_{(k)}^{***} = \Delta \widetilde{Sn}_{(k)}^{**} + \widetilde{Fr}_{Sn(k)} \Delta t_L. \quad (8.26)$$

The amount of freeze in the overall snow is the sum of the amounts of freeze in each layer (note, however, that it is the grid-mean value):

$$Fr_{Sn} = \sum_{k=1}^{K_{Sn}} \widetilde{Fr}_{Sn(k)} A_{Sn}. \quad (8.27)$$

By adding the amount of freeze, the snow water equivalent is partially updated as follows:

$$Sn^{***} = Sn^{**} + Fr_{Sn} \Delta t_L. \quad (8.28)$$

The liquid water that has percolated from the snow to the lower boundary is given to the soil.

### 8.3.4 Snowfall

Lastly, by adding the snowfall after interception by the canopy, the finally updated snow water equivalent is obtained:

$$Sn^{\tau+1} = Sn^{***} + P_{Sn}^* \Delta t_L \quad (8.29)$$

However, when the temperature of the uppermost soil layer is 0 °C or more, the snowfall is assumed to melt on the ground. In this case, the energy of the latent heat of melting is taken from the soil.



When snow is produced by snowfall in a grid where no snow was formerly present, the snow-covered ratio ( $A_{Sn}$ ) is newly diagnosed as described in the section 8.1 and the snow temperature ( $T_{Sn(1)}$ ) is assumed to be equal to the temperature of the uppermost soil layer.

The snowfall is added to the mass of the uppermost layer:

$$\Delta \widetilde{Sn}_{(k)}^{\tau+1} = \Delta \widetilde{Sn}_{(k)}^{***} + P_{Sn}^* \Delta t_L / A_{Sn}. \quad (8.30)$$

### 8.3.5 Redivision of snow layer and rediagnosis of temperature

When the snow water equivalent is updated, the snow-covered ratio is rediagnosed as described in the section 8.1 and the mass of each layer is redivided as described in the section 8.2. The temperature in each redivided layer is rediagnosed so that the energy is conserved as follows:

$$T_{Sn(k)}^{\text{new}} = \left( \sum_{l=1}^{K_{Sn}^{\text{old}}} f_{(l^{\text{old}} \in k^{\text{new}})} T_{Sn(l)}^{\text{old}} \Delta \widetilde{Sn}_{(l)}^{\text{old}} A_{Sn}^{\text{old}} \right) / (\Delta \widetilde{Sn}_{(k)}^{\text{new}} A_{Sn}^{\text{new}}). \quad (8.31)$$

It should be noted that the variables with the index “old” and “new” are those before and after redivision, respectively.  $f_{(l^{\text{old}} \in k^{\text{new}})}$  is the ratio of the mass of the  $k$ th layer after redivision to the mass of the  $l$ th layer before redivision.

## 8.4 Calculation of snow heat conduction

### 8.4.1 Snow heat conduction equations

The prognostic equation of the snow temperature due to snow heat conduction is as follows:

$$c_{pi} \Delta \widetilde{Sn}_{(k)} \frac{T_{Sn(k)}^* - T_{Sn(k)}^\tau}{\Delta t_L} = \widetilde{F}_{Sn(k+1/2)} - \widetilde{F}_{Sn(k-1/2)} \quad (k = 1, \dots, K_{Sn}) \quad (8.32)$$

with the heat conduction flux  $\widetilde{F}_{Sn}$  given by

$$\widetilde{F}_{Sn(k+1/2)} = \begin{cases} (F_{Sn(1/2)} - \Delta F_{conv}) / A_{Sn} - \Delta F_{c,conv} & (k = 0) \\ k_{Sn(k+1/2)} \frac{T_{Sn(k+1)} - T_{Sn(k)}}{\Delta z_{Sn(k+1/2)}} & (k = 1, \dots, K_{Sn} - 1) \\ k_{Sn(k+1/2)} \frac{T_{Sn(B)} - T_{Sn(k)}}{\Delta z_{Sn(k+1/2)}} & (k = K_{Sn}) \end{cases}, \quad (8.33)$$

where  $k_{Sn(k+1/2)}$  is the snow heat conductivity, assigned the fixed value of 0.3 W/m/K as a standard.  $\Delta z_{Sn(k+1/2)}$  is the thickness of each snow layer, defined by

$$\Delta z_{Sn(k+1/2)} = \begin{cases} 0.5 \Delta \widetilde{Sn}_{(1)} / \rho_{Sn} & (k = 1) \\ 0.5 (\Delta \widetilde{Sn}_{(k)} + \Delta \widetilde{Sn}_{(k+1)}) / \rho_{Sn} & (k = 2, \dots, K_{Sn} - 1) \\ 0.5 \Delta \widetilde{Sn}_{(K_{Sn})} / \rho_{Sn} & (k = K_{Sn}) \end{cases} \quad (8.34)$$

where  $\rho_{Sn}$  is the snow density, assigned the fixed value of  $300\text{kg/m}^3$  as a standard. The snow density and heat conductivity are considered to change with the passage of time due to compaction and changes in properties (aging), but the effect of such changes is not considered here.

In 8.33, the snow upper boundary flux  $\tilde{F}_{Sn(1/2)}$  is given using the heat conduction flux from the snow to the ground surface solved in the ground surface energy balance  $F_{Sn(1/2)}$ , the ground surface energy convergence produced when the ground surface temperature is solved by the snowmelt condition  $\Delta F_{conv}$ , and the energy correction produced when a change has occurred in the phase of the canopy water  $\Delta F_{c,conv}$ .  $\Delta F_{conv}$  is assumed to be given only to the snow-covered portion, while  $\Delta F_{c,conv}$  is given uniformly to the grid cells. Since the sign of the flux is taken as upward positive, the convergence has a negative sign.

In the equation for the snow lower boundary flux  $\tilde{F}_{Sn(K_{Sn}+1/2)}$ ,  $T_{Sn(B)}$  is the temperature of the snow lower boundary (the boundary surface of the snow and the soil). However, since the flux from the uppermost soil layer to the snow lower boundary is

$$\tilde{F}_{g(1/2)} = k_{g(1/2)} \frac{T_{g(1)} - T_{Sn(B)}}{\Delta z_{g(1/2)}} \quad (8.35)$$

there is assumed to be no convergence at the snow lower boundary, and by putting

$$\tilde{F}_{Sn(K_{Sn}+1/2)} = \tilde{F}_{g(1/2)} \quad (8.36)$$

$T_{Sn(B)}$  is solved. When this is substituted into 8.33, the following is obtained:

$$\tilde{F}_{Sn(K_{Sn}+1/2)} = \left[ \frac{\Delta z_{g(1/2)}}{k_{g(1/2)}} + \frac{\Delta z_{Sn(K_{Sn}+1/2)}}{k_{Sn(K_{Sn}+1/2)}} \right]^{-1} (T_{g(1)} - T_{Sn(K_{Sn})}) \quad (8.37)$$

#### 8.4.2 Case 1: When snowmelt does not occur in the uppermost layer

The implicit method is used to treat the temperature from the uppermost snow layer to the lowest snow layer, as follows:

$$\begin{aligned}
\tilde{F}_{Sn(k+1/2)}^* &= \tilde{F}_{Sn(k+1/2)}^\tau + \frac{\partial \tilde{F}_{Sn(k+1/2)}}{\partial T_{Sn(k)}} \Delta T_{Sn(k)} + \frac{\partial \tilde{F}_{Sn(k+1/2)}}{\partial T_{Sn(k+1)}} \Delta T_{Sn(k+1)}, \\
\tilde{F}_{Sn(k+1/2)}^\tau &= \begin{cases} (F_{Sn(1/2)} - \Delta F_{conv}) / A_{Sn} - \Delta F_{c,conv} & (k=0) \\ \frac{k_{Sn(k+1/2)}}{\Delta z_{Sn(k+1/2)}} (T_{Sn(k+1)}^\tau - T_{Sn(k)}^\tau) & (k=1, \dots, K_{Sn}-1) \\ \left[ \frac{\Delta z_{g(1/2)}}{k_{g(1/2)}} + \frac{\Delta z_{Sn(K_{Sn}+1/2)}}{k_{Sn(K_{Sn}+1/2)}} \right]^{-1} (T_{g(1)} - T_{Sn(K_{Sn})}^\tau) & (k=K_{Sn}) \end{cases}, \\
\frac{\partial \tilde{F}_{Sn(k+1/2)}}{\partial T_{Sn(k)}} &= \begin{cases} -\frac{k_{Sn(k+1/2)}}{\Delta z_{Sn(k+1/2)}} & (k=1, \dots, K_{Sn}-1) \\ -\left[ \frac{\Delta z_{g(1/2)}}{k_{g(1/2)}} + \frac{\Delta z_{Sn(K_{Sn}+1/2)}}{k_{Sn(K_{Sn}+1/2)}} \right]^{-1} & (k=K_{Sn}) \end{cases}, \\
\frac{\partial \tilde{F}_{Sn(k+1/2)}}{\partial T_{Sn(k+1)}} &= \begin{cases} 0 & (k=0) \\ \frac{k_{Sn(k+1/2)}}{\Delta z_{Sn(k+1/2)}} & (k=1, \dots, K_{Sn}-1) \end{cases}
\end{aligned} \tag{8.38}$$

and 8.32 is treated as

$$\begin{aligned}
c_{pi} \Delta \tilde{S}_{n(k)} \frac{\Delta T_{Sn(k)}}{\Delta t_L} &= \tilde{F}_{Sn(k+1/2)}^* - \tilde{F}_{Sn(k-1/2)}^* \\
&= \tilde{F}_{Sn(k+1/2)}^\tau + \frac{\partial \tilde{F}_{Sn(k+1/2)}}{\partial T_{Sn(k)}} \Delta T_{Sn(k)} + \frac{\partial \tilde{F}_{Sn(k+1/2)}}{\partial T_{Sn(k+1)}} \Delta T_{Sn(k+1)} \\
&\quad - \tilde{F}_{Sn(k-1/2)}^\tau - \frac{\partial \tilde{F}_{Sn(k-1/2)}}{\partial T_{Sn(k-1)}} \Delta T_{Sn(k-1)} - \frac{\partial \tilde{F}_{Sn(k-1/2)}}{\partial T_{Sn(k)}} \Delta T_{Sn(k)}
\end{aligned} \tag{8.39}$$

and solved by the LU factorization method as  $\Delta T_{Sn(k)}$  ( $k=1, \dots, K_{Sn}$ ) simultaneous equations with respect to  $K_{Sn}$ . At this juncture, it should be noted that the flux at the snow upper boundary is fixed as the boundary condition, the snow lower boundary condition is the temperature in the uppermost soil layer, and the snow lower boundary flux is treated explicitly with regard to the temperature of the uppermost soil layer. The snow temperature is partially updated by

$$T_{Sn(k)}^* = T_{Sn(k)}^\tau + \Delta T_{Sn(k)} \tag{8.40}$$

#### 8.4.3 Case 2: When snowmelt occurs in the uppermost layer

When the temperature of the uppermost snow layer solved in case 1 is higher than 0degC, snowmelt occurs in the uppermost snow layer. In this case, the temperature of the uppermost snow layer is fixed at 0 °C. The flux from the second snow layer to the uppermost snow layer is then expressed as

$$\tilde{F}_{3/2}^* = \frac{k_{Sn(3/2)}}{\Delta z_{Sn(3/2)}} (T_{Sn(2)}^\tau - T_{melt}) + \frac{\partial \tilde{F}_{Sn(3/2)}}{\partial T_{Sn(2)}} \Delta T_{Sn(2)} \tag{8.41}$$

and solved similarly to case 1 (when there is only one snow layer, the snow temperature is similarly fixed in the flux from the soil to the snow).

The energy convergence used for melting in the uppermost snow layer is given by:

$$\Delta\tilde{F}_{conv} = (\tilde{F}_{3/2}^* - \tilde{F}_{1/2}) - c_{pi}\Delta\tilde{S}n_{(1)}\frac{T_{melt} - T_{Sn(1)}^*}{\Delta t_L} \quad (8.42)$$

Even if the temperature of the second snow layer and below is higher than  $T_{melt}$ , the calculation is not iterated and the snowmelt is corrected accordingly.

## 8.5 Fluxes given to the soil or the runoff process

The heat flux given to the soil through the snow process is

$$\Delta F_{conv}^* = A_{Sn}(\Delta\tilde{F}_{conv}^* - \tilde{F}_{Sn_{K_{Sn}}}) - l_m P_{Sn,melt}^*, \quad (8.43)$$

where  $\Delta\tilde{F}_{conv}^*$  is the energy convergence remaining when all of the snow has melted,  $\tilde{F}_{Sn_{K_{Sn}}}$  is the heat conduction flux at the lowest snow layer, and  $P_{Sn,melt}^*$  is the snowfall that melts immediately when it reaches the ground.

Since the energy of the snow-free portion is given to the soil as it is, the energy correction term due to the phase change of the canopy water is as follows:

$$\Delta F_{c,conv}^* = (1 - A_{Sn})\Delta F_{c,conv}. \quad (8.44)$$

The water flux given to the runoff process through the snow process is then expressed as

$$\begin{aligned} Pr_c^{**} &= (1 - A_{Sn})Pr_c^*, \\ Pr_l^{**} &= (1 - A_{Sn})Pr_l^* + A_{Sn}\tilde{F}_{wSn}^* + P_{Sn,melt}^*, \end{aligned} \quad (8.45)$$

where  $\tilde{F}_{wSn}^*$  is the flux of the rainfall or snowmelt water that has percolated through the lowest snow layer.

## 8.6 Glacier formation

In this case, the maximum value is set for the snow water equivalent, and the portion exceeding the maximum value is considered to become glacier runoff:

$$\begin{aligned} Ro_{gl} &= \max(Sn - Sn_{\max}, 0)/\Delta t_L, \\ Sn &= Sn - Ro_{gl}\Delta t_L, \\ \Delta\tilde{S}n_{(K_{Sn})} &= \Delta\tilde{S}n_{(K_{Sn})} - Ro_{gl}/A_{Sn}\Delta t_L, \end{aligned} \quad (8.46)$$

where  $Ro_{gl}$  is the glacier runoff. The mass of this portion is subtracted from the lowest snow layer.  $Sn_{\max}$  is uniformly assigned the value of 1000kg/m<sup>2</sup> as a standard.

## 8.7 Dust in snow

The amount of dust on the snow cover and in the snow layers are calculated in SUBROUTINE DSTCUT in matsnw.F.

### 8.7.1 Dust fall on the snow cover

The dust fall is added to the top layer:

$$M_{d(1)}^{\tau+1} = M_{d(1)}^{\tau} + D, \quad (8.47)$$

where  $M_{d(k)}$  is the amount of snow on the  $k$ th layer and  $D$  is the dust fall.

### 8.7.2 Redistribution of dust

The amount of dust in each layer is calculated in SUBROUTINE DSTCUT based on the results of snow layer recutting (SUBROUTINE SNWCUT).

Snow mass of  $k$ th layer after updating of snow mass and before snow layer recutting  $Sn_{(k)}^{\tau+1/2}$  is calculated in

$$Sn_{(k)}^{\tau+1/2} = Sn_{(k)}^{\tau} A_{Sn}^{\tau} / A_{Sn}^{\tau+1} \quad (k = 1, 2, 3), \quad (8.48)$$

where  $\tau$  and  $\tau + 1$  represent before and after recutting of snow layer, respectively.

When  $Sn_{(1)}^{\tau+1} > Sn_{(1)}^{\tau+1/2}$ , the amount of dust in the 1st layer increases due to increase in the snow mass in this layer. This is calculated as

$$M_{d(1)}^{\tau+1} - M_{d(1)}^{\tau} = \begin{cases} \rho_{d(2)} Sn_{(2)}^{\tau+1/2} + \rho_{d(3)} \left( Sn_{(1)}^{\tau+1} - Sn_{(1)}^{\tau+1/2} - Sn_{(2)}^{\tau+1/2} \right) & \left( Sn_{(1)}^{\tau+1} - Sn_{(1)}^{\tau+1/2} > Sn_{(2)}^{\tau+1/2} \right) \\ \rho_{d(2)} \left( Sn_{(1)}^{\tau+1} - Sn_{(1)}^{\tau+1/2} \right) & \left( Sn_{(1)}^{\tau+1} - Sn_{(1)}^{\tau+1/2} \leq Sn_{(2)}^{\tau+1/2} \right) \end{cases} \quad (8.49)$$

where  $\rho_{d(k)}$  is the density of dust in the  $k$ th layer.

When  $Sn_{(1)}^{\tau+1} \leq Sn_{(1)}^{\tau+1/2}$ , the amount of dust in the 1st layer decreases, and thus

$$M_{d(1)}^{\tau+1} - M_{d(1)}^{\tau} = -\rho_{d(1)} \left( Sn_{(1)}^{\tau+1/2} - Sn_{(1)}^{\tau+1} \right). \quad (8.50)$$

It leads to

$$M_{d(1)}^{\tau+1} - M_{d(1)}^{\tau} = \Delta M_{d(1)}^{+} - \Delta M_{d(1)}^{-}, \quad (8.51)$$

where

$$M_{d(1)}^{+} = \rho_{d(2)} \min \left( \max \left( \Delta Sn_{(1)}, 0 \right), Sn_{(2)}^{\tau+1/2} \right) + \rho_{d(3)} \max \left( \max \left( \Delta Sn_{(1)}, 0 \right) - Sn_{(2)}^{\tau+1/2}, 0 \right) \quad (8.52)$$

$$M_{d(1)}^- = \rho_{d(1)} \max(-\Delta S n_{(1)}, 0), \quad (8.53)$$

$$\Delta S n_{(1)} = S n_{(1)}^{\tau+1} - S n_{(1)}^{\tau+1/2}. \quad (8.54)$$

The change in the dust amount in the 3rd layer is determined similarly, and thus in the 2nd layer it is calculated as follows:

$$M_{d(2)}^{\tau+1} - M_{d(2)}^\tau = \Delta M_{d(1)}^- - \Delta M_{d(1)}^+ + \Delta M_{d(3)}^- - \Delta M_{d(3)}^+. \quad (8.55)$$

## 8.8 Albedo of snow and ice

### 8.8.1 Albedo of snow

The albedo of the snow is calculated in SUBROUTINE SNWALB in matsnw.F.

The albedo of the snow is large in fresh snow, but becomes smaller with the passage of time due to compaction and changes in properties as well as soilage. In order to take these effects into consideration, the albedo of the snow is treated as a prognostic variable.

The nondimensional age of snow at the time step of  $\tau$ ,  $A_g^\tau$ , is formulated in

$$A_g^\tau = \frac{f_{alb}}{1 - f_{alb}}, \quad (8.56)$$

where

$$f_{alb} = \min \left( \frac{\alpha_{vis}^\tau - \alpha_{vis,new}}{\alpha_{vis,old} - \alpha_{vis,new}}, 0.999 \right). \quad (8.57)$$

$\alpha_b^\tau$  is the albedo of the snow for band  $b$  at the time step of  $\tau$ . Three bands of wavelength, visible (vis), near infrared (nir) and infrared (ifr) are considered in MATSIRO, and here the factors for visible band are used.  $\alpha_{b,new}$  is the albedo of newly fallen snow for band  $b$  and  $\alpha_{b,old}$  is that of old snow. In default,  $\alpha_{vis,new}$ ,  $\alpha_{nir,new}$ ,  $\alpha_{ifr,new}$ ,  $\alpha_{vis,old}$ ,  $\alpha_{nir,old}$  and  $\alpha_{ifr,old}$  are set to 0.9, 0.7, 0.01, 0.65 (or 0.4), 0.2 and 0.1, respectively.

The age of snow at the next time step  $\tau + 1$  is, after Yang et al. (1997), assumed to be given by the following equation:

$$A_g^{\tau+1} = A_g^\tau + (f_{age} + f_{age}^{10} + r_{dirt}) \Delta t_L / \tau_{age}, \quad (8.58)$$

where

$$f_{age} = \exp \left[ f_{ageT} \left( \frac{1}{T_{melt}} - \frac{1}{T_{Sn(1)}} \right) \right], \quad (8.59)$$

$$f_{ageT} = 5000, \quad \tau_{age} = 1 \times 10^6 \text{ s}, \quad T_{melt} = 273.15 \text{ K}. \quad (8.60)$$

$T_{Sn(1)}$  is the temperature of the first layer of snow.

$r_{dirt}$  represents the effect of dirt and soot. When the option OPT\_SNWALB is inactive,

$$r_{dirt} = \begin{cases} r_{dirt,c} & (\text{over continental ice}) \\ r_{dirt,0} & (\text{elsewhere}) \end{cases}, \quad (8.61)$$

where  $r_{dirt,c} = 0.01$  and  $r_{dirt,0} = 0.3$ . When this option is active, the density of the dirt is considered as

$$r_{dirt} = \begin{cases} \min(r_{dirt,c} + r_{dirt,s}\rho_{d(1)}, 1000) & (\text{over continental ice}) \\ \min(r_{dirt,0} + r_{dirt,s}\rho_{d(1)}, 1000) & (\text{elsewhere}) \end{cases}, \quad (8.62)$$

where  $r_{dirt,s}$  is the dirt factor for slope with a constant value of 0.1 and  $\rho_{d(1)}$  is the dirt density of the first layer.

Using this, the albedo of the snow at the time step of  $\tau + 1$ ,  $\alpha_b^{\tau+1}$ , is solved by

$$\alpha_b^{\tau+1} = \alpha_{b,new}^{\tau+1} + \frac{A_g^{\tau+1}}{1 + A_g^{\tau+1}}(\alpha_{b,old} - \alpha_{b,new}), \quad (8.63)$$

When snowfall has occurred, the albedo is updated to the value of the fresh snow in accordance with the snowfall:

$$\alpha_b^{\tau+1} = \alpha_b^{\tau+1} + \min\left(\frac{P_{Sn}^* \Delta t_L}{\Delta S n_c}, 1\right) (\alpha_{b,new} - \alpha_b^{\tau+1}). \quad (8.64)$$

$\Delta S n_c$  is the snow water equivalent necessary for the albedo to fully return to the value of the fresh snow.

## 8.8.2 Albedo of ice

The albedo of the ice sheet,  $\alpha_{b,surf}$ , is calculated in ENTRY ICEALB in matice.F.

This is expressed in a following function of the water content above the ice according to Bougamont et al. (2005):

$$\alpha_{b,surf} = \alpha_{b,wet} - (\alpha_{b,wet} - \alpha_{b,ice}) \exp\left(-\frac{w_{surf}}{w^*}\right), \quad (8.65)$$

where  $\alpha_{b,ice}$  is the land ice albedo without surface water,  $\alpha_{b,wet}$  is the one with surface water,  $w_{surf}$  is the thickness of surface water and  $w^*$  is the characteristic scale for surficial water.  $b$  represents the three bands of wavelength, visible (vis), nearinfrared (nir) and infrared (ifr), similar to ice albedo. In default,  $\alpha_{vis,ice}$ ,  $\alpha_{nir,ice}$  and  $\alpha_{ifr,ice}$  are set to 0.5, 0.3 and 0.05, respectively, and  $\alpha_{b,wet}$  is set to 0.15 for all bands.

## 9 Runoff

SUBROUTINE: MATROF in matrof.F.

The surface runoff and groundwater runoff are solved using a simplified TOPMODEL (Beven and Kirkby, 1979).

### 9.1 Outline of TOPMODEL

In TOPMODEL, the horizontal distribution of a water table along the slope in a catchment basin is considered. The downward groundwater flow at a certain point on the slope is assumed to be equal to the accumulated groundwater recharge in the upper part of the slope above that point (quasi-equilibrium assumption). Then, the groundwater flow must be greater in the lower part of the slope. Under another assumption described later, for the groundwater flow to be greater, the water table needs to be shallow. Thus, the distribution is derived such that the lower the slope, the shallower the water table. When the mean water table is shallower than a certain level, the water table rises to the ground surface at an area lower than a certain point in the slope to form a saturated area. In this way, TOPMODEL is characterized by the mean water table, the size of the saturated area, and the groundwater flow velocity, which are important concepts for estimating the runoff, being physically connected in a coherent manner.

TOPMODEL contains the following major assumptions:

1. The soil saturation hydraulic conductivity is attenuated toward the depth of the soil in the manner of an exponential function.
2. The gradient of the water table is in close agreement locally with the gradient of the slope.
3. The downward groundwater flow at a certain point on the slope is equal to the accumulated groundwater recharge in the upper slope above that point.

The usage of the symbols below is in accordance with the usual practice in descriptions of TOPMODEL (Sivapalan et al., 1987; Stieglitz et al., 1997).

Assumption 1 can be expressed as

$$K_s(z) = K_0 \exp(-fz) \quad (9.1)$$

where  $K_s(z)$  is the soil saturation hydraulic conductivity at depth  $z$ ,  $K_0$  is the saturation hydraulic conductivity at the ground surface, and  $f$  is the attenuation coefficient.

When the depth of the water table at a certain point  $i$  is designated as  $z_i$ , the downward groundwater flux on the slope at that point  $q_i$  is

$$q_i = \int_{z_i}^Z K_s(z) dz \cdot \tan \beta = \frac{K_0}{f} \tan \beta [\exp(-fz_i) - \exp(-fZ)] \quad (9.2)$$

where  $\beta$  is the gradient of the slope, and assumption 2 is applied here.  $Z$  is the depth of the impervious surface; normally, however,  $Z$  is assumed to be sufficiently deep compared with  $\frac{1}{f}$ ,



so the term  $\exp(-fZ)$  is omitted. Moreover, since the slope direction soil moisture flux in the unsaturated zone above the water table is small, it is ignored.

If the groundwater recharge rate  $R$  is assumed to be horizontally uniform, assumption 3 is expressed as

$$aR = \frac{K_0}{f} \tan \beta \exp(-fz_i) \quad (9.3)$$

where  $a$  is the total upstream area (per unit contour line length at point  $i$  with respect to point  $i$ ).

When this is solved for  $z_i$ , the following is obtained:

$$z_i = -\frac{1}{f} \ln \left( \frac{faR}{K_0 \tan \beta} \right) \quad (9.4)$$

The averaged water table depth  $\bar{z}$  in domain  $A$  is

$$\bar{z} = \frac{1}{A} \int_A z_i dA = -\Lambda - \frac{1}{f} \ln R \quad (9.5)$$

$$\Lambda \equiv \frac{1}{A} \int_A \ln \left( \frac{fa}{K_0 \tan \beta} \right) dA \quad (9.6)$$

The recharge rate  $R$  can then be expressed as a function of the mean water table depth  $\bar{z}$  as follows:

$$R = \exp(-f\bar{z} - \Lambda) \quad (9.7)$$

Under assumption 3, this is exclusively the groundwater runoff discharged from domain  $A$ .

Next, if  $R$  is substituted into 9.4, the following relationship of  $z_i$  and  $\bar{z}$  is obtained:

$$z_i = \bar{z} - \frac{1}{f} \left[ \ln \left( \frac{fa}{K_0 \tan \beta} \right) - \Lambda \right] \quad (9.8)$$

The domain that satisfies  $z_i \leq 0$  is the surface saturated area.

## 9.2 Application of TOPMODEL assuming simplified topography

Normally, when TOPMODEL is used, detailed topographical data on the target area is required. Here, however, the average shape of the slope in a grid cell is roughly estimated from the data on the average inclination and the standard deviation of the altitude in the grid (this estimation method is temporary at this stage, and further study is required).

The topography in the grid cell is represented by the slope with uniform gradient  $\beta_s$  and the distance from the ridge to valley  $L_s$ .

$L_s$  is estimated using the standard deviation of altitude ( $\sigma_z$ ) as follows:

$$L_s = \frac{2\sqrt{3}\sigma_z}{\tan \beta_s} \quad (9.9)$$

where  $2\sqrt{3}\sigma_z$  is the altitude difference between the ridge and valley in serrate topography such that the standard deviation of altitude is  $\sigma_z$ .

The x-axis is taken from the ridge toward the valley on the horizontal surface. Then, the total upstream area at point  $x$  is  $x$ , and 9.4 becomes

$$z(x) = -\frac{1}{f} \ln \left( \frac{fxR}{K_0 \tan \beta_s} \right) \quad (9.10)$$

where depth of  $z''$  is 2m. Using this, from 9.5 the mean water table is

$$\bar{z} = \frac{1}{L_s} \int_0^{L_s} z(x) dx = -\frac{1}{f} \left[ \ln \left( \frac{fL_s R}{K_0 \tan \beta_s} \right) - 1 \right] \quad (9.11)$$

from 9.7 the groundwater recharge rate is

$$R = \frac{K_0 \tan \beta_s}{fL_s} \exp(1 - f\bar{z}) \quad (9.12)$$

and from 9.8, the relationship between the water table at point  $x$  and the mean water table is

$$z(x) = \bar{z} - \frac{1}{f} \left( \ln \frac{x}{L_s} + 1 \right) \quad (9.13)$$

If  $z(x) \leq 0$  is solved for  $x$ , the following are obtained:

$$x \geq x_0 \quad (9.14)$$

$$x_0 = L_s \exp(f\bar{z} - 1) \quad (9.15)$$

Therefore, the fraction of the saturated area is solved as

$$A_{sat} = \frac{L_s - x_0}{L_s} = 1 - \exp(f\bar{z} - 1) \quad (9.16)$$

However, when  $A_{sat} \geq 0$  and  $\bar{z} > \frac{1}{f}$ , no saturated area exists.

### 9.3 Calculation of runoff

Four types of runoff mechanisms are considered, and the total of the runoffs  $Ro$  by each mechanism is assumed to be the total runoff from the grid cell:

$$Ro = Ro_s + Ro_i + Ro_o + Ro_b \quad (9.17)$$

where  $Ro_s$  is the saturation excess runoff (Dunne runoff),  $Ro_i$  is the infiltration excess runoff (Horton runoff), and  $Ro_o$  is the overflow of the uppermost soil layer, these three being classified as the surface runoff; and  $Ro_b$  is the groundwater runoff.

However, when taking snow-fed wetland into account (Nitta et al., 2017), part of the surface water will be stored in a surface tank and runoff to rivers will be delayed, which leads to an increase in land surface wetness and hence evaporation in water-limited regimes. Please refer to Wetland section for the details.

#### 9.3.1 Estimation of mean water table depth

The soil moisture is examined from the lowest soil layer. A layer is assumed to be the  $k_{WT}$ th layer when it becomes unsaturated for the first time, the mean water table depth ( $\bar{z}$ ) is estimated by:

$$\bar{z} = z_{g(k_{WT}-\frac{1}{2})} - \psi_{k_{WT}} \quad (9.18)$$

This is equivalent to considering the moisture potential on the upper boundary of the unsaturated layer as  $\psi_{k_{WT}}$ , and the soil moisture distribution as being in the equilibrium state underneath (i.e., the state in which gravity and the capillary force are in equilibrium). Under saturation condition that  $\psi_{k_{WT}}$  exceeds soil layer thickness, water table will generate at the upper boundary of soil layer.

When  $\bar{z} > z_{g(k_{WT}-\frac{1}{2})}$ , in case  $k_{WT}$  is the lowest layer, the water table is assumed to not exist; when  $k_{WT}$  isn't the lowest layer, the layer below (the uppermost layer among the saturated layers) is assumed to be  $k_{WT}$  and the above equation is applied.

When there is a frozen soil surface in the middle of the soil, estimation of the water table depth is performed from above the frozen soil surface.

#### 9.3.2 Calculation of groundwater runoff

From the quasi-equilibrium assumption, the groundwater runoff is equal to the groundwater recharge rate in 9.12. In the latest version of MATSIRO, Hirabayashi (2004) changed  $K_0$  to  $K_{s0}$  in groundwater runoff calculation, which denotes a saturation hydraulic conductivity at depth of 2m:

$$K_{s0} = \exp(z''f)K_0E_p \quad (9.19)$$

where  $z''$  is the depth of 2m,  $E_p$  denotes the effect of macropore on groundwater runoff. It's also worth noting that the value of  $\frac{1}{f}$  has changed from 0.6 to 0.33 in current version of MATSIRO. Therefore, calculation of groundwater runoff will become:

$$Ro_b = \frac{K_{s0} \tan \beta_s}{f L_s} \exp(1 - f \bar{z}) \quad (9.20)$$

However, when a frozen soil surface exists under the water table, referring to the case of not omitting the term  $\exp(-fZ)$  in 9.2, it is assumed that

$$Ro_b = \frac{K_{s0} \tan \beta_s}{f L_s} [\exp(1 - f \bar{z}) - \exp(1 - f z_f)] \quad (9.21)$$

$z_f$  is the depth of frozen soil surface. Although other relations in TOPMODEL should also be changed in such a case, the other relations are not changed here for the sake of simplification.

When there is an unfrozen layer under the frozen soil surface and a water table exists, the groundwater runoff from there is added by a similar calculation.

The water content from the groundwater runoff is removed from the  $k_{WT}$ th soil layer:

$$Ro_{(k_{WT})} = Ro_b \quad (9.22)$$

where  $Ro_{(k)}$  denotes the runoff flux from the  $k_{WT}$ th soil layer.

### 9.3.3 Calculation of surface runoff

All of the rainfall that falls on the surface saturated area runs off as is (saturation excess runoff):

$$Ro_s = (Pr_c^{**} + Pr_l^{**}) A_{sat} \quad (9.23)$$

The fraction of the surface saturated area  $A_{sat}$  is given by 9.16. Here, the correlation between the rainfall distribution of the subgrid and topography is ignored.

With regard to rainfall that falls on the surface unsaturated area, only the portion that exceeds the soil infiltration capacity runs off (infiltration excess runoff). The soil infiltration capacity is given by the saturation hydraulic conductivity of the uppermost soil layer for simplification. The convective precipitation is considered to fall locally, and the fraction of the precipitation area ( $A_c$ ) is assumed to be uniform (0.1 as a standard value). The stratiform precipitation is also assumed to be uniform.

$$Ro_i^c = \max\left(\frac{Pr_c^{**}}{A_c} + Pr_l^{**} - K_{s(1)}, 0\right)(1 - A_{sat}) \quad (9.24)$$

$$Ro_i^{nc} = \max(Pr_l^{**} - K_{s(1)}, 0)(1 - A_{sat}) \quad (9.25)$$

$$Ro_i = A_c Ro_i^c + (1 - A_c) Ro_i^{nc} \quad (9.26)$$

where  $Ro_i^c$  and  $Ro_i^{nc}$  are  $Ro_i$  in the convective precipitation area and nonconvective precipitation area, respectively; and  $K_{s(1)}$  is the saturation hydraulic conductivity in the uppermost soil layer.

The overflow of the uppermost soil layer, allowing a small amount of ponding  $w_{str}$  (1 mm as a standard value), is assumed to be

$$Ro_o = \frac{\max(w_{(1)} - w_{sat(1)} - w_{str}, 0) \rho_w \Delta z_{g(1)}}{\Delta t_L} \quad (9.27)$$

This portion is subtracted from the uppermost soil layer later, and therefore should be remembered as the runoff from the uppermost layer, as follows.

$$Ro_{(1)} = Ro_{(1)} + Ro_o \quad (9.28)$$

When calculating surface runoff  $R_s$ , glacial runoff  $Ro_{gl}$  should also be considered. Then the  $R_s$  calculated by MATSIRO will be:

$$Rs = Ro + Ro_{gl} - Ro_b = Ro_s + Ro_i + Ro_o + Ro_{gl} \quad (9.29)$$

When snow-fed wetlands scheme is considered:

$$Rs = (Ro_s + Ro_i + Ro_o) \alpha + Ro_{gl} \quad (9.30)$$

here  $\alpha$  determines the inflow rate into surface tank and is specified in Wetland section.

## 9.4 Water flux given to soil

The water flux given to the soil through the runoff process is

$$P_r^{***} = P_r^{**} + P_l^{**} - Ro_s - Ro_i \quad (9.31)$$

## 9.5 Appendix

### 9.5.1 Output variables

Variable	Description	Code	Dimension	Units
$Ro$	Total runoffs	RUNOFF	FLDIM,	$kg/m^2/s$
			KRVMAX	
$Ro_{(k)}$	Runoff flux from the $k$ th soil layer	RUNOFF	FLDIM,	$kg/m^2/s$
			KWMAX	
$P_r$	Water flux given to the soil through the runoff process	WINPT	FLDIM	$kg/m^2/s$

### 9.5.2 Input variables

Variable	Description	Code	Dimension	Units
$Pr_c$	Convective rainfall	WINPC	IJLDIM	$kg/m^2/s$
$Pr_l$	Nonconvective rainfall	WINPL	IJLDIM	$kg/m^2/s$
$M_{sn}$	Snow melt	SNMLT	IJLDIM	$(kg/m^2/s)$
$T_{g(k)}$	Soil temperature	GLG	IJLDIM, KGMAX	$K$
$w_{(k)}$	Soil moisture	GLW	IJLDIM, KWMAX	$m^3/m^3$
$\theta_{i(k)}$	Soil ice	GLFRS	IJLDIM, KWMAX	$m^3/m^3$
$(Ro_{gl})$	Glacier formation	GLACR	IJLDIM	$(kg/m^2/s)$
$\Delta t$	Time step	DELT	-	$s$
$X_s$	Surface condition	ILSFC	IJLDIM	-
-	Soil type	ILSOIL	IJLDIM	-

### 9.5.3 Internal work variables

Variable	Description	Code	Dimension	Units
$Ro_s$	Saturation excess runoff	RUNOFF	IJLDIM	$kg/m^2/s$
$Ro_i$	Infiltration excess runoff	RUNOFF	IJLDIM	$kg/m^2/s$
$Ro_o$	Surface storage overflow	RUNOFF	IJLDIM	$kg/m^2/s$
$Ro_b$	Base runoff	RUNOFF	IJLDIM	$kg/m^2/s$
$R_s$	Surface runoff (CMIP5)	SRUNOFF	IJLDIM	$kg/m^2/s$
$w_{sat(k)}$	Saturated soil moisture	GWS	IJLDIM, KWMAX	$m^3/m^3$
$K_{s(k)}$	Saturated hydrological conductivity from each layer	-	IJLDIM, KWMAX	$m/s$
$K_0$	Saturation hydraulic conductivity at the ground surface	DFWS	IJLDIM	$m/s$
$K_{s0}$	Saturation hydraulic conductivity at the depth of 2m	DFWST	IJLDIM	$m/s$
$\psi_{(k)}$	Matric potential	GPSI	IJLDIM, KWMAX	$m$
-	d(Psi)/d(W)	DPDWI	IJLDIM, KWMAX	
$z(x)$	Water table depth	WTABD	IJLDIM	$m$
$A_{sat}$	Surface saturation fraction	ASSATI	IJLDIM	-
$z_f$	Frost table level	KFTAB	IJLDIM	$m$
-	Water table level	KWTAB	IJLDIM	$m$
-	Flag for saturation	ISAT	IJLDIM	-

#### 9.5.4 Internal parameters

PARAMETER	Description	Code	Dimension	Initial value
$\tan\beta_s$	Tangent of mean surface slope	GRTANS	IJLDIM	-
$L_s$	Mean length of surface slope	GRLENS	IJLDIM	-
$\alpha$	Inflow rate into surface tank	FRACSR	IJLDIM	-
$\tau$	Outflow rate from surface tank	TAUSS	IJLDIM	-
$\frac{1}{f}$	Critical water table depth	WTCRIT	-	0.333
$S$	Surface water storage	SFCSTR	-	0.001
-	Maximum water table depth	WTBMAX	-	-
-	Epsilon for soil moisture	EPSGW	-	0.001
$A_{cum}$	Convective storm area for runoff	FRACCR	-	0.1
$E_p$	Effect of macropore on base-runoff	EFFMP	-	1
$z''$	Ground depth for groundwater runoff calculation	-	-	2

## 10 Soil

The soil temperature, the soil moisture, and the frozen soil are calculated next.

### 10.1 Calculation of soil heat conduction

#### 10.1.1 Soil heat conduction equations

The prognostic equation for the soil temperature by soil heat conduction is

$$C_{g(k)} \frac{T_{g(k)}^* - T_{g(k)}^\tau}{\Delta t_L} = F_{g(k+1/2)} - F_{g(k-1/2)} \quad (k = 1, \dots, K_g) \quad (10.1)$$

with  $C_{g(k)}$ , the soil heat capacity, defined by

$$C_{g(k)} = (c_{g(k)} + \rho_w c_{pw} w_{(k)}) \Delta z_{g(k)} \quad (10.2)$$

where  $c_{g(k)}$  is the specific heat of the soil, and is given as a parameter for each soil type;  $c_{pw}$  is the specific heat of the water;  $w_{(k)}$  is the soil moisture (volumetric moisture content); and  $\Delta z_{g(k)}$  is the thickness of the  $k$  th soil layer. When including the heat capacity of the soil moisture in the soil heat capacity in this way, unless the heat transfer accompanying the soil moisture movement is considered, the energy is not conserved. The heat transfer accompanying the soil moisture movement is not considered in the MATGND soil submodel at present, and its introduction is under study. However, it should be noted that unless the heat capacity of such elements as vapor in the atmosphere, rainfall, etc. is considered, the conservation of energy is disrupted in certain respects.

The heat conduction flux  $F_g$  is given by

$$F_{g(k+1/2)} = \begin{cases} F_{g(1/2)} - \Delta F_{conv}^* - \Delta F_{c,conv}^* & (k = 0) \\ k_{g(k+1/2)} \frac{T_{g(k+1)} - T_{g(k)}}{\Delta z_{g(k+1/2)}} & (k = 1, \dots, K_g - 1) \\ 0 & (k = K_g) \end{cases} \quad (10.3)$$

with  $k_{g(k+1/2)}$ , the soil heat conductivity, expressed as

$$k_{g(k+1/2)} = k_{g0(k+1/2)}[1 + f_{kg} \tanh(w_{(k)}/w_{kg})] \quad (10.4)$$

where  $k_{g0(k+1/2)}$  is the heat conductivity when the soil moisture is 0, and  $f_{kg} = 6$  and  $w_{kg} = 0.25$  are constants.

$\Delta z_{g(k+1/2)}$  is the thickness between the soil temperature definition points of the  $k + 1$ th layer and the  $k + 1$ th layer (when  $k = 0$ , the thickness between the uppermost layer temperature definition point and the soil upper boundary; when  $k = K_g$ , the thickness between the lowest layer temperature definition point and the soil lower boundary).

In 10.3, the value given to the soil upper boundary condition ( $F_{g(1/2)}$ ) is the value obtained at the time of solving the ground surface energy balance, with the addition of the energy convergence at the snow lower boundary (including the heat conduction flux at the snow lower boundary) as well as the allotment to the snow-free portion of the energy correction term due to phase change of the canopy water. The flux takes an upward (positive) direction, so when the amount of convergence is added it has a negative sign. The soil lower boundary condition  $F_{g(K_g+1/2)}$  is assumed to be zero flux.

### 10.1.2 Solution of heat conduction equations

These equations are solved using the implicit method with regard to the soil temperature from the uppermost layer to the lowest layer. That is, for  $k = 1, \dots, K_g - 1$ , the heat conduction flux is expressed as

$$F_{g(k+1/2)}^* = F_{g(k+1/2)}^\tau + \frac{\partial F_{g(k+1/2)}}{\partial T_{g(k)}} \Delta T_{g(k)} + \frac{\partial F_{g(k+1/2)}}{\partial T_{g(k+1)}} \Delta T_{g(k+1)} \quad (10.5)$$

$$F_{g(k+1/2)}^\tau = \frac{k_{g(k+1/2)}}{\Delta z_{g(k+1/2)}} (T_{g(k+1)}^\tau - T_{g(k)}^\tau) \quad (10.6)$$

$$\frac{\partial F_{g(k+1/2)}}{\partial T_{g(k)}} = - \frac{k_{g(k+1/2)}}{\Delta z_{g(k+1/2)}} \quad (10.7)$$

$$\frac{\partial F_{g(k+1/2)}}{\partial T_{g(k+1)}} = \frac{k_{g(k+1/2)}}{\Delta z_{g(k+1/2)}} \quad (10.8)$$

and 10.1 is treated as



$$\begin{aligned}
C_{g(k)} \frac{\Delta T_{g(k)}}{\Delta t_L} &= F_{g(k+1/2)}^* - F_{g(k-1/2)}^* \\
&= F_{g(k+1/2)}^\tau + \frac{\partial F_{g(k+1/2)}}{\partial T_{g(k)}} \Delta T_{g(k)} + \frac{\partial F_{g(k+1/2)}}{\partial T_{g(k+1)}} \Delta T_{g(k+1)} \\
&\quad - F_{g(k-1/2)}^\tau - \frac{\partial F_{g(k-1/2)}}{\partial T_{g(k-1)}} \Delta T_{g(k-1)} - \frac{\partial F_{g(k-1/2)}}{\partial T_{g(k)}} \Delta T_{g(k)} \quad (10.9)
\end{aligned}$$

and solved by the LU factorization method as  $K_g$  simultaneous equations with respect to  $\Delta T_{g(k)}$  ( $k = 1, \dots, K_g$ ). At this juncture, it should be noted that the equations are solved with the fluxes at the soil upper boundary and lower boundary fixed as the boundary conditions:

$$T_{g(k)}^* = T_{g(k)}^\tau + \Delta T_{g(k)} \quad (10.10)$$

The soil temperature is partially updated by the above equation. By this, as well as through correction of the phase change in the soil moisture mentioned later, the soil temperature is completely updated.

## 10.2 Calculation of soil moisture movement

### 10.2.1 Soil moisture movement equations

The equation for soil moisture movement (Richards equation) is given by

$$\rho_w \frac{w_{(k)}^{\tau+1} - w_{(k)}^\tau}{\Delta t_L} = \frac{F_{w(k+1/2)} - F_{w(k-1/2)}}{\Delta z_{g(k)}} + S_{w(k)} \quad (k = 1, \dots, K_g) \quad (10.11)$$

The soil moisture flux  $F_w$  is given by

$$F_{w(k+1/2)} = \begin{cases} Pr^{***} - Et_{(1,1)} & (k = 0) \\ K_{(k+1/2)} \left( \frac{\psi_{(k+1)} - \psi_{(k)}}{\Delta z_{g(k+1/2)}} - 1 \right) & (k = 1, \dots, K_g - 1) \\ 0 & (k = K_g) \end{cases} \quad (10.12)$$

in which  $K_{(k+1/2)}$  is the soil hydraulic conductivity that, referring to Clapp and Hornberger (1978), is expressed as

$$K_{(k+1/2)} = K_{s(k+1/2)} (\max(W_{(k)}, W_{(k+1)}))^{2b_{(k)}+3} f_i \quad (10.13)$$

where  $K_{s(k+1/2)}$  is the saturation hydraulic conductivity and  $b_{(k)}$  is the index of the moisture potential curve, which are given as external parameters for each soil type.  $W_{(k)}$  is the degree of saturation considered excluding the frozen soil moisture, given by

$$W_{(k)} = \frac{w_{(k)} - w_{i(k)}}{w_{sat(k)} - w_{i(k)}} \quad (10.14)$$

where  $w_{sat(k)}$  is the porosity of the soil, which is also given as a parameter for each soil type.  $f_i$  is a parameter that denotes that soil moisture movement is suppressed by the existence of frozen soil. Although further study of this point is required, at present it is given by

$$f_i = (1 - W_{i(k)}) (1 - W_{i(k+1)}) \quad (10.15)$$

where  $W_{i(k)} = w_{i(k)} / (w_{sat(k)} - w_{i(k)})$ .

The soil moisture potential  $\psi$  is given as follows from Clapp and Hornberger:

$$\psi_{(k)} = \psi_{s(k)} W_{(k)}^{-b(k)} \quad (10.16)$$

where  $\psi_{s(k)}$  is given as an external parameter for each soil type.

In 10.11,  $S_{w(k)}$  is a source term which, considering the root uptake and the runoff, is given by

$$S_{w(k)} = -F_{root(k)} - Ro_{(k)} \quad (10.17)$$

In 10.12, the soil upper boundary condition  $F_{w(1/2)}$  is the difference between the moisture flux through the runoff process ( $P^{***}$ ) and the evaporation flux from the soil ( $Et_{(1,1)}$ ). Separately from this, the sublimation flux portion is subtracted from the frozen soil moisture of the uppermost layer before calculation of the soil moisture movement:

$$w_{i(k)}^\tau = w_{i(k)}^\tau - Et_{(2,1)} \Delta t_L / (\rho \Delta z_{g(1)}) \quad (10.18)$$

$$w_{(k)}^\tau = w_{(k)}^\tau - Et_{(2,1)} \Delta t_L / (\rho \Delta z_{g(1)}) \quad (10.19)$$

### 10.2.2 Solution of soil moisture movement equations

These equations are solved by using the implicit method for the soil moisture from the uppermost layer to the lowest layer. For  $k = 1, \dots, K_g - 1$ , the soil moisture flux is

$$F_{w(k+1/2)}^{\tau+1} = F_{w(k+1/2)}^\tau + \frac{\partial F_{w(k+1/2)}}{\partial w_{(k)}} \Delta w_{(k)} + \frac{\partial F_{w(k+1/2)}}{\partial w_{(k+1)}} \Delta w_{(k+1)} \quad (10.20)$$

$$F_{w(k+1/2)}^\tau = K_{(k+1/2)} \left( \frac{\psi_{(k+1)}^\tau - \psi_{(k)}^\tau}{\Delta z_{g(k+1/2)}} - 1 \right) \quad (10.21)$$

$$\frac{\partial F_{w(k+1/2)}}{\partial w_{(k)}} = -\frac{K_{(k+1/2)}}{\Delta z_{g(k+1/2)}} \left[ -b_{(k)} \frac{\psi_{s(k)}}{w_{sat(k)} - w_{i(k)}} W_{(k)}^{-b(k)-1} \right] \quad (10.22)$$

$$\frac{\partial F_{w(k+1/2)}}{\partial w_{(k+1)}} = \frac{K_{(k+1/2)}}{\Delta z_{g(k+1/2)}} \left[ -b_{(k)} \frac{\psi_{s(k+1)}}{w_{sat(k+1)} - w_{i(k+1)}} W_{(k+1)}^{-b(k)-1} \right] \quad (10.23)$$

and 10.11 is treated as

$$\begin{aligned}
\rho_w \Delta z_{g(k)} \frac{\Delta w_{(k)}}{\Delta t_L} &= F_{w(k+1/2)}^{\tau+1} - F_{w(k-1/2)}^{\tau+1} + S_{w(k)} \Delta z_{g(k)} \\
&= F_{w(k+1/2)}^{\tau} + \frac{\partial F_{w(k+1/2)}}{\partial w_{(k)}} \Delta w_{(k)} + \frac{\partial F_{w(k+1/2)}}{\partial w_{(k+1)}} \Delta w_{(k+1)} \\
&\quad - F_{w(k-1/2)}^{\tau} - \frac{\partial F_{w(k-1/2)}}{\partial w_{(k-1)}} \Delta w_{(k-1)} - \frac{\partial F_{w(k-1/2)}}{\partial w_{(k)}} \Delta w_{(k)} + S_{w(k)} \Delta z_{g(k)}
\end{aligned} \tag{10.24}$$

and solved by the LU factorization method as  $K_g$  simultaneous equations with respect to  $\Delta T_{g(k)}$  ( $k = 1, \dots, K_g$ ). At this juncture, it should be noted that the equations are solved with the fluxes at the soil upper boundary and lower boundary fixed as the boundary conditions.

The soil moisture is updated by

$$w_{(k)}^{\tau+1} = w_{(k)}^{\tau} + \Delta w_{(k)} \tag{10.25}$$

As a result of this calculation, if a part appears where the soil moisture become supersaturated, it is adjusted in the vertical direction to eliminate supersaturation. The reason why such a supersaturated portion is not considered as runoff is that this supersaturation is artificially produced because the vertical movement of the soil moisture is solved without saturation data. First, from the second soil layer downwards, the supersaturated portion of the soil moisture is given to the layer below. Next, from the lowest soil layer upwards, the supersaturated portion of the soil moisture is given to the next layer up. With this operation, when the soil moisture is large enough, a saturated layer around the lowest soil layer is formed and the water table of Eq. (278) can be defined.

### 10.3 Phase change of soil moisture

As a result of calculating the soil heat conductivity, when the temperature in the layer containing liquid water is lower than  $T_{melt} = 0 \text{degC}$ , or when the temperature in the layer containing solid water is higher than  $T_{melt}$ , the phase change of the soil moisture is calculated. If the amount of freeze (adjustment portion) of the soil moisture in the  $k$ th layer is assumed to be  $\Delta w_{i(k)}$ ,

when  $T_{g(k)}^* < T_{melt}$  and  $w_{(k)}^{\tau+1} - w_{i(k)}^{\tau} > 0$  (frozen):

$$\Delta w_{i(k)} = \min \left( \frac{C_{g(k)}(T_{melt} - T_{g(k)}^*)}{l_m \rho_w \Delta z_{g(k)}}, w_{(k)}^{\tau+1} - w_{i(k)}^{\tau} \right) \tag{10.26}$$

when  $T_{g(k)}^* > T_{melt}$  and  $w_{i(k)}^{\tau} > 0$  (melting):

$$\Delta w_{i(k)} = \max \left( \frac{C_{g(k)}(T_{melt} - T_{g(k)}^*)}{l_m \rho_w \Delta z_{g(k)}}, -w_{i(k)}^{\tau} \right) \tag{10.27}$$

The frozen soil moisture and the soil moisture are then updated as follows:

$$w_{i(k)}^{\tau+1} = w_{i(k)}^{\tau} + \Delta w_{i(k)} \quad (10.28)$$

$$T_{g(k)}^{\tau+1} = T_{g(k)}^* + l_m \rho_w \Delta z_{g(k)} \Delta w_{i(k)} / C_{g(k)} \quad (10.29)$$

### 10.3.1 Ice sheet process

When the land cover type is ice sheet, if the soil temperature exceeds  $T_{melt}$ , it is returned to  $T_{melt}$ :

$$T_{g(k)}^{\tau+1} = \min(T_{g(k)}^*, T_{melt}) \quad (10.30)$$

The rate of change of the ice sheet  $F_{ice}$  is then diagnosed as

$$F_{ice} = -Et_{(2,1)} - \frac{C_{g(k)} \max(T_{g(k)}^* - T_{melt}, 0)}{l_m \Delta t_L} \quad (10.31)$$

## 11 Lake

Lake is treated in MATSIRO (lakesf.F, lakeic.F, and lakepo.F), as well as land.

Up to and including the calculation of the surface flux (section 11.1 and 11.2), the method is derived from the land surface model MATSIRO, while the calculation below the lake ice (section 11.3 and 11.4) is derived from the ocean model COCO (COCO-OGCM). lakeic.F and lakepo.F were based on the COCO-OGCM, and the ENTRY statement are used so as to keep the structure of the original program. For practical use, note, for example, that the unit of temperature is K until flux calculation (section 11.1 and 11.2), while it is ° C after the ice and inter lake (section 11.3 and 11.4). It is also noted that because the second half part is based on the old version of COCO, hence it is slightly different from the MIROC6-AOGCM and Hasumi (2015).

Dimensions of the lake scheme is defined in `include/zkg21c.F`. `KLMAX` is the number of vertical layers set to 5 in MIROC6/MATSIRO6. `NLTDIM` is the number of tracers, 1:temperature 2:salt. Since the vertical layers are actually from `KLSTR=2` to `KLEND=KLMAX+1`, `NLZDIM = KLMAX+KLSTR` exists as a parameter for management.

Minimum depth of lake is given in `matdrv.F` as  $10 \times 10^2$ [cm], hence any lakes cannot be disappeared even in severe conditions.

### 11.1 Calculation of lake surface conditions

In `ENTRY[LAKEBC]` (in `SUBROUTINE:[LSFBCS]` of lakesf.F) lake surface albedo, roughness, and heat flux are calculated. They are calculated supposing ice-free conditions, then modified. While the albedo of snow is a pronostic variable, the lake surface albedo considering with ice and snow above is a diagnostic variable. The aging effect of the snow is differently treated. These methods are actually same with an old version of COCO-OGCM. The newest version of COCO, which is

going to be coupled to MIROC7-AOGCM, has been applied a melt pond scheme a snow aging scheme which is basically the same with the treatment in the current land surface (Komuro, the GCM meeting on 22nd Feb, 2021).

- Output variables

Variable	Variable in source		Units
	code	Description	
$\alpha_{Lk}$	GRALB	Lake surface albedo	[−]
$z_{Lk0}$	GRZ0	Lake surface roughness	[m]
$G$	FOGFLX	Heat flux	[W/m <sup>2</sup> ]
$\frac{\partial G}{\partial T}$	DFGT	Heat diffusion coefficient	[W/m <sup>2</sup> /K]

- Input variables

Variable	Variable in source codes	Description	unit
$T_s$	GRTS	surface temperature	[K]
$T_b$	GRTB	ice base temperature	[K]
$Ic$	GRICE	lake ice amount	[kg/m <sup>2</sup> ]
$SnLk$	GRSNW	snow amount	[kg/m <sup>2</sup> ]
$R_{IcLk}$	GRICR	lake ice concentration	[−]
$u_a$	GDUA	u surface wind	[m/s]
$v_a$	GDVA	v surface wind	[m/s]
$\cos\zeta$	RCOSZ	cos(solar zenith)	[−]

First, let us consider the lake albedo. The lake level  $\alpha_{Lk(d,b)}$ ,  $b = 1, 2, 3$  represent the visible, near-infrared, and infrared wavelength bands, respectively. Also,  $d = 1, 2$  represents direct and scattered light, respectively. The albedo for the visible bands are calculated in SUBROUTINE: [LAKEALB], supposing ice-free conditions. The albedo for near-infrared is set to same as the visible one. The albedo for infrared is uniformly set to a constant value.

When lake ice is present, the albedo is modified to take into account the ice concentration.

$$\alpha'_{Lk} = \alpha_{Lk} + (\alpha_{IcLk} - \alpha_{Lk})R_{IcLk} \quad (11.1)$$

where  $\alpha_{IcLk}$  is the lake ice albedo, and  $R_{IcLk}$  is the lake ice concentration, respectively. In addition, we want to consider the albedo change due to snow cover. Assuming that the snow albedo depends on the skin temperature, we can calculate a function  $F$  below.

$$F(T_s) = \frac{T_s - T_m^{min}}{T_m^{max} - T_m^{min}} \quad , \quad (0 \leq F(T_s) \leq 1) \quad (11.2)$$

where  $T_s$  is the skin temperature, and  $T_m^{min}$  and  $T_m^{max}$  are the minimum and the maximum temperature for the albedo change, respectively.

Then, the albedo can be modified by

$$\alpha''_{Lk} = \alpha_{Lk(1,b)} + (\alpha_{SnLk(2,b)} - \alpha_{SnLk(1,b)})F(T_s) \quad (11.3)$$

$$\alpha_{Lk} = \alpha'_{Lk} + (\alpha''_{Lk} - \alpha'_{Lk})R_{SnLk} \quad (11.4)$$

where the  $\alpha_{SnLk(d,b)}$  is the snow albedo covering the lake, and  $R_{SnLk}$  is the snow coverage, respectively.

Second, let us consider the lake surface roughness. The roughnesses of for momentum, heat and vapor are calculated in SUBROUTINE: [LAKEZOF], based on Miller et al. (1992), same with COCO-OGCM (Hasumi 2015), supposing the ice-free conditions, then modified.

When lake ice is present, each roughness is modified to take into account the lake ice concentration ( $R_{IcLk}$ )

$$z'_{Lk0} = z_{Lk0} + (R_{IcLk} - z_{Lk0})R_{IcLk} \quad (11.5)$$

where  $z_{Lk0}$  is surface roughness.

Then, taking into account the snow coverage ( $R_{SnLk}$ ), we can express it as

$$z_{Lk0} = z'_{Lk0} + (R_{SnLk} - z'_{Lk0})R_{SnLk} \quad (11.6)$$

Third, the heat flux is considered with the temperature differences between the snow surface and the ice bottom, because the difference should be zero in the ice-free conditions.

If the lake ice exists, the heat diffusion coefficient is described as

$$\left(\frac{\partial G}{\partial T}\right)_{IcLk} = \frac{D_{IcLk}}{R_{IcLk}} \quad (11.7)$$

where  $D_{IcLk}$  is the coefficient of lake ice.

If the snow exists, the heat diffusion coefficient of snow covered area is

$$\left(\frac{\partial G}{\partial T}\right)_{SnLk} = \frac{D_{IcLk}D_{SnLk}}{D_{IcLk}R_{SnLk} + D_{SnLk}R_{IcLk}} \quad (11.8)$$

where  $D_{SnLk}$  is the coefficient of snow.

Therefore, the net heat diffusion coefficient is finally

$$\frac{\partial G}{\partial T} = \left(\frac{\partial G}{\partial T}\right)_{IcLk}(1 - R_{SnLk}) + \left(\frac{\partial G}{\partial T}\right)_{SnLk}R_{SnLk} \quad (11.9)$$

The temperature differences between the snow surface ( $T_S$ ) and the ice bottom ( $T_B$ ) is saved as heat flux( $G$ ).

$$G = \frac{\partial G}{\partial T}(T_B - T_S) \quad (11.10)$$

### 11.1.1 Calculation of lake surface albedo

Contents of SUBROUTINE: [LAKEALB] is the same with SUBROUTINE [SEAAALB] (in p-sfc.md). For lake surface level albedo  $\alpha_{Lk,L(d)}$ ,  $d = 1, 2$  represents direct and scattered light, respectively.

Using the solar zenith angle at latitude  $\zeta$  ( $\cos\zeta$ ), the albedo for direct light is presented by

$$\alpha_{Lk,L(1)} = e^{(C_3 A^* + C_2) A^* + C_1} \quad (11.11)$$

where  $A = \min(\max(\cos\zeta, 0.03459), 0.961)$ , and  $C_1, C_2, C_3$  is  $-0.7479, -4.677039, 1.583171$  respectively.

On the other hand, the albedo for scattered light is uniformly set to a constant parameter.

$$\alpha_{Lk,L(2)} = 0.06 \quad (11.12)$$

### 11.1.2 Lake surface roughness

Contents of SUBROUTINE: [LAKEZOF] is the same with SUBROUTINE: [SEAZOF] (of pgocn.F).

The roughness variation of the lake surface is determined by the friction velocity  $u^*$ .

$$u^* = \sqrt{C_{M_0}(u_a^2 + v_a^2)} \quad (11.13)$$

We perform successive approximation calculation of  $C_{M_0}$ , because  $F_u, F_v, F_\theta, F_q$  are required.

$$z_{Lk0,M} = z_{0,M_0} + z_{0,M_R} + \frac{z_{0,M_R} u^{*2}}{g} + \frac{z_{0,M_S} \nu}{u^*} \quad (11.14)$$

$$z_{Lk0,H} = z_{0,H_0} + z_{0,H_R} + \frac{z_{0,H_R} u^{*2}}{g} + \frac{z_{0,H_S} \nu}{u^*} \quad (11.15)$$

$$z_{Lk0,E} = z_{0,E_0} + z_{0,E_R} + \frac{z_{0,E_R} u^{*2}}{g} + \frac{z_{0,E_S} \nu}{u^*} \quad (11.16)$$

where,  $\nu = 1.5 \times 10^{-5} [\text{m}^2/\text{s}]$  is the kinetic viscosity of the atmosphere,  $z_{0,M}, z_{0,H}$  and  $z_{0,E}$  are surface roughness for momentum, heat and vapor, and  $z_{0,M_0}, z_{0,H_0}$  and  $z_{0,E_0}$  are base of them, and  $z_{0,M_R}, z_{0,M_R}$  and  $z_{0,E_R}$  are rough factor for them, and  $z_{0,M_S}, z_{0,M_S}$  and  $z_{0,E_S}$  are smooth factor for them, respectively.

## 11.2 Solution of energy balance at lake surface

In SUBROUTINE: [LAKEHB] (of lakesf.F), the energy balance at lake surface is solved.

- Output variables

Variable	Variable in source code	Description
$W_{free/ice}$	WFLUXS	surface water flux
$LW^\uparrow$	RFLXLU	upward long wave
$F$	SFLXBL	flux balance

- Input variables

Variable	Variable in source code	Description
$\frac{\partial H}{\partial T_s}$	DTFDS	sensible heat flux coefficient
$\frac{\partial E}{\partial T_s}$	DQFDS	latent heat flux coefficient
$\frac{\partial G}{\partial T_s}$	DGFDS	surface heat flux coefficient
$SW^\downarrow$	RFLXSD	downward SW radiation
$SW^\uparrow$	RFLXLU	upward SW radiation
$LW^\downarrow$	RFLXLD	downward LW radiation
$\alpha_{Lk}$	GRALBL	lake surface albedo
$R_{IcLk}$	GRICR	lake ice concentration

*The comments for some variables say “soil”, but this is because the program was adapted from a land surface scheme, and has no particular meaning.*

Downward radiative fluxes are not directly dependent on the condition of the lake surface, and their observed values are simply specified to drive the model. Shortwave emission from the lake surface is negligible, so the upward part of the shortwave radiative flux is accounted for solely by reflection of the incoming downward flux. Let  $\alpha_{Lk,SW}$  be the lake surface albedo for shortwave radiation. The upward shortwave radiative flux ( $SW^\uparrow$ ) is represented by

$$SW^\uparrow = -\alpha_{Lk,SW}SW^\downarrow \quad (11.17)$$

where  $SW^\downarrow$  is the downward shortwave radiation flux, and  $\alpha_{Lk,SW}$  is lake surface albedo for shortwave radiation in the ice-free area, respectively. On the other hand, the upward longwave radiative flux has both reflection of the incoming flux and emission from the lake surface. Let  $\alpha_{Lk}$  be the lake surface albedo for longwave radiation and  $\epsilon$  be emissivity of the lake surface relative to the black body radiation. The upward shortwave radiative flux is represented by

$$LW^\uparrow = -\alpha_{Lk}LW^\downarrow + \epsilon\sigma T_s^4 \quad (11.18)$$



where  $\sigma$  is the Stefan-Boltzmann constant, and  $T_s$  is surface temprature, respectively. If lake ice exists, snow or lake ice temperature is considered by fractions. When radiative equilibrium is assumed, emissivity becomes identical to co-albedo:

$$\epsilon = 1 - \alpha_{Lk} \quad (11.19)$$

The net surface flux is presented by

$$F^* = H + (1 - \alpha_{Lk})\sigma T_s^4 + \alpha_{Lk}LW^\uparrow - LW^\downarrow + SW^\uparrow - SW^\downarrow \quad (11.20)$$

The heat flux into the lake surface is presented, with the surface heat flux ( $G$ ) calculated in SUBROUTINE: [SFCFLX] (in matdrv.F).

$$G^* = G - F^* \quad (11.21)$$

where  $G^*$  is the net incoming flux (the opposite direction with  $F^*$ ).

The temperature derivative term is

$$\frac{\partial G^*}{\partial T_s} = \frac{\partial G}{\partial T_s} + \frac{\partial H}{\partial T_s} + \frac{\partial R}{\partial T_s} \quad (11.22)$$

When the lake ice exists, the sublimation flux ( $l_s E$ ) is considered

$$G_{IcLk} = G^* - l_s E \quad (11.23)$$

The temperature derivative term is

$$\frac{\partial G_{IcLk}}{\partial T_s} = \frac{\partial G^*}{\partial T_s} + l_s \frac{\partial E}{\partial T_s} \quad (11.24)$$

Finally, we can update the skin temperature with the lake ice concentration with  $\Delta T_s = G_{IcLk} \left( \frac{\partial G_{IcLk}}{\partial T_s} \right)^{-1}$

$$T_s = T_s + R_{IcLk} \Delta T_s \quad (11.25)$$

Then, the sensible heat flux ( $E_{IcLk}$ ) and latent heat flux ( $E_{IcLk}$ ) on the lake ice is updated.

$$E_{IcLk} = E + \frac{\partial E}{\partial T_s} \Delta T_s \quad (11.26)$$

$$H_{IcLk} = H + \frac{\partial H}{\partial T_s} \Delta T_s \quad (11.27)$$

When the lake ice does not exist, otherwise, the evaporation flux ( $l_c E$ ) is added to the net flux.

$$G_{freeLk} = F^* + l_c E \quad (11.28)$$

Finally each flux is updated.

For the sensible heat flux ( $H$ ), the temperature change on the lake ice is considered.

$$H = H + R_{IcLk} H_{IcLk} \quad (11.29)$$

where  $H_{IcLk}$  is the sensible heat flux on the lake ice. Then, the heat used for the temperature change is saved as:

$$F = R_{IcLk} H_{IcLk} \quad (11.30)$$

For the upward longwave radiative flux ( $LW^\uparrow$ ), the temperature change on the lake ice is considered.

$$LW^\uparrow = LW^\uparrow + 4 \frac{\sigma}{T_s} R_{IcLk} \Delta T_s \quad (11.31)$$

For the surface heat flux, the lake ice concentration is considered.

$$G = (1 - R_{IcLk}) G_{freeLk} + R_{IcLk} G_{IcLk} \quad (11.32)$$

For the latent heat flux, the lake ice concentration is considered.

$$E = (1 - R_{IcLk}) E + R_{IcLk} E_{IcLk} \quad (11.33)$$

Each term above are saved as freshwater flux.

$$W_{freeLk} = (1 - R_{IcLk}) E \quad (11.34)$$

$$W_{IcLk} = R_{IcLk} E_{IcLk} \quad (11.35)$$

### 11.3 Calculation of lake ice

In this section, the lake ice calculation is described. There are three prognostic variables in the lake ice model described herein: lake ice concentration  $A_I$ , which is area fraction of a grid covered by lake ice and takes a value between zero and unity; mean lake ice thickness  $h_I$  over ice-covered part of a grid; mean snow depth  $h_S$  over lake ice. Horizontal flow of ice is not

considered in the lake parts, differently from the COCO-OGCM. Let us consider here a case that the model is integrated from the  $n$ -th time level to the  $(n+1)$ -th time level.  $A_I$ ,  $h_I$  and  $h_S$  are incrementally modified.

The model also calculates temperature at snow top (lake ice top when there is no snow cover)  $T_I$ , which is a diagnostic variable. Density of lake ice ( $\rho_I$ ) and snow ( $\rho_S$ ) are assumed to be constant. Lake ice is assumed to have nonzero salinity, and its value  $S_I$  is assumed to be a constant parameter.

### 11.3.1 Calculation of heat flux and growth rate

In ENTRY: [FIHEATL] (in SUBROUTINE: [FIHSTL] of lakeic.F), heat flux in lake ice and its growth rate is calculated.

- Output variables

Variable	Variable in source code	Description	unit
$W_{AO}$	WAO	Lake ice growth rate in ice-free area	[cm/s]
$W_{AS}$	WAS	snow growth rate due to heat imbalance	[cm/s]
$W_{IO}$	WIO	basal growth rate of lake ice	[cm/s]

- Input variables

Variable in source		Description	unit
Variablecode			
$A_I$	A	lake ice concentration	[—]
$Q_{AI}$	QAI	air-ice heat flux multiplied by the factor of lake ice concentration	[—]
$Q_{IO}$	QIO	vertical heat flux through lake ice and snow	[W/m <sup>2</sup> ]
$SW^A$	SWABS	Shortwave radiation absorbed at ice-free lake surface, with the factor of ice-free area multiplied	[W/m <sup>2</sup> ]
$T, S$	T(NLTDIM)	Lake temperature / Salinity	[°C/m <sup>2</sup> ]/ [psu]
$\Delta t$	TS	time step	[s]

- Internal works

Variable	Variable in source code	Description	unit
$\Delta T$	TDEV	freezing point depression	[°C]
$W_{FZ}$	WFRZ	lake ice growth rate	[cme.w./s]

Temperature at lake ice base is taken to be the lake model's top level temperature  $T(k = 2)$ . In this model, lake ice exists only when and where  $T(k = 2)$  is at the freezing point  $T_f$ , which is a decreasing function of salinity ( $T_f = -0.0543S$  is used here, where temperature and salinity are measured by °C and psu, respectively). In heat budget calculation for snow and lake ice, only latent heat of fusion and sublimation is taken into account, and heat content associated with temperature is neglected. Therefore, temperature inside lake ice and snow are not calculated, and  $T_I$  is estimated from surface heat balance.

Nonzero minimum values are prescribed for  $A_I$  and  $h_I$ , which are denoted by  $A_I^{min}$  and  $h_I^{min}$ , respectively. These parameters define a minimum possible volume of lake ice in a grid. If a predicted volume  $A_I h_I$  is less than that minimum,  $A_I$  is reset to zero, and  $T_1$  is lowered to compensate the corresponding latent heat. In this case, the lake model's top level is kept at a supercooled state. Such a state continues until the lake is further cooled and the temperature becomes low enough to produce more lake ice than that minimum by releasing the latent heat corresponding to the supercooling.

Surface heat flux is separately calculated for each of air-lake and air-ice interfaces in one grid.

The skin temperature of lake ice  $T_I$  is determined such That

$$Q_{AI} = Q_{IO} \quad (11.36)$$

is satisfied, where  $Q_{IO}$  is corresponding to  $G + SW^\downarrow$  and  $Q_{AI}$  is corresponding to  $G_{IceLk} - W_{IceLk}$ . However, When the estimated  $T_I$  exceeds the melting point of lake ice  $T_m$  (which is set to 0 °C for convenience),  $T_I$  is reset to  $T_m$  and  $Q_{AI}$  and  $Q_{IO}$  are re-estimated by using it. The heat imbalance between  $Q_{AI}$  and  $Q_{IO}$  is consumed to melt snow (lake ice when there is no snow cover). Snow growth rate due to this heat imbalance is estimated by

$$W_{AS} = \frac{Q_{AI} - Q_{IO}}{\rho L_f} \quad (11.37)$$

where  $\rho_O$  is density of lakewater and  $L_f$  is the latent heat of fusion (the same value is applied to snow and lake ice). This growth rate is expressed as a change of equivalent liquid water depth per time. It is zero when  $T_I < T_m$  and negative when  $T_I = T_m$ . Note that  $W_{AS}$  is weighted by lake ice concentration.

Although it is assumed that  $T(2) = T_f$  when lake ice exists,  $T_1$  could deviated from  $T_f$  due to a change of salinity or other factors. Such deviation should be adjusted by forming or melting lake ice. Under a temperature deviation of the top layer of lake,

$$\Delta T = T(k = 2) - T_f S(k = 2) \quad (11.38)$$

lake ice growth rate necessary to compensate it in the single time step is given by

$$W_{FZ} = -\frac{C_{po}\Delta T\Delta z_1}{L_f\Delta t} \quad (11.39)$$

where  $C_{po}$  is the heat capacity of lake water and  $\Delta z_1 = 100\text{cm}$  is the thickness of the lake model's top level (uniformly set to constant in case of the current lake model.) This growth rate is estimated at all grids, irrespective of lake ice existence, for a technical reason. As described below, this growth rate first estimates negative ice volume for ice-free grids, but the same heat flux calculation procedure as for ice-covered grids finally results in the correct heat flux to force the lake. Basal growth rate of lake ice is given by

$$W_{IO} = A_I W_{FZ} + \frac{Q_{IO}}{\rho_O L_f} \quad (11.40)$$

where  $W_{IO}$  is weighted by lake ice concentration.

Lake ice formation could also occur in the ice-free area. Let us define  $Q_{AO}$  by

$$Q_{AO} = (1 - A_I)[Q - (1 - \alpha_{Lk,SW})SW^\downarrow] \quad (11.41)$$

i.e., air-lake heat flux except for shortwave, multiplied by the factor of the fraction of ice-free area. Here,  $Q$  is air-ice heat flux. Shortwave radiation absorbed at ice-free lake surface, with the factor of ice-free area multiplied, is represented by

$$SW^A = (1 - A_I)(1 - \alpha_{Lk,SW})SW^\downarrow \quad (11.42)$$

Lake ice growth rate in ice-free area is calculated by

$$W_{AO} = (1 - A_I)W_{FZ} + \frac{Q_{AO} + I(k=2)SW^A}{\rho_O L_f} \quad (11.43)$$

where  $I(k=2)$  denotes the fraction of  $SW^A$  absorbed by the lake model's top level, which is calculate in SUBROUTINE: [SVTSETL] of lakepo.F.

Finally, the heat flux for freshwater is

$$G_{lake} = \Delta z_1 \frac{\Delta T}{\Delta t} \quad (11.44)$$

### 11.3.2 Sublimation and freshwater flux for lake

In ENTRY [FWATERL] (in SUBROUTINE: [FWASTL] of lakeic.F), sublimation (freshwater) flux, which is practically come from the land ice runoff, is calculated or prescribed over lake ice cover.

- Output variables (Prognostic variables)

Variable	Variable	Description	unit
$A'_I$	AX	lake ice fraction	$[-]$
$h'_I$	HIX	lake ice thickness	$[\text{cm}]$
$h'_S$	HSX	snow depth	$[\text{cm}]$

- Input variables

Variable	Variable	Description	unit
$F_W^{EV}$	WEV	latent heat flux of evaporation	$\text{cm/s}$
$F_W^{SB}$	WSB	latent heat flux of sublimation	$\text{cm/s}$
$S_{off}$	SOFF	overflow snow flux	$\text{cm/s}$

- Parameters

Variable	Variable	Description	unit	value
$\rho_S$	rhos	density of snow	$[\text{g/cm}^3]$	0.33
$\rho_I$	rhoi	density of lake ice	$[\text{g/cm}^3]$	0.9
$R_{\rho_S}$	rrs	Ratio of density (ocean/snow)	$[-]$	$\rho_O/\rho_s$
$R_{\rho_I}$	rri	Ratio of density (ocean/ice)	$[-]$	$\rho_O/\rho_I$
$h_I^{min}$	himin	Minimum thickness of ice	$[\text{cm}]$	$1.0 \times 10^1$

The flux is first consumed to reduce snow thickness in n-th time step:

$$h'_S = h_S^n - \frac{\rho_O F_W^{SB} \Delta t}{\rho_S A_I^n} \quad (11.45)$$

If  $h'_S$  becomes less than zero, it is reset to zero. Then, the melted snow flux is added to  $F_W^{SB}$ .  $F_W^{SB}$  is redefined by

$$F_W^{SB'} = F_W^{SB} + \frac{\rho_S A_I^n (h'_S - h_S^n)}{\rho_O \Delta t} \quad (11.46)$$

Where there no remains snow, but  $F_W^{SB'}$  is not zero, The remain flux is consumed to reduce lake ice thickness:

$$h'_I = h_I^n - \frac{\rho_O F_W^{SB'} \Delta t}{\rho_I A_I^n} \quad (11.47)$$

If  $h'_I$  becomes less than  $h_I^{min}$ , it is reset to zero. Then, the melted iceflux is added to  $F_W^{SB'}$ .  $F_W^{SB'}$  is redefined by

$$F_W^{SB''} = F_W^{SB'} - A_I^n \frac{\rho_S(h_I^n - h_I')}{\rho_O \Delta t} \quad (11.48)$$

Finally, nonzero  $F_W^{SB''}$  is consumed to reduce lake ice concentration:

$$A_I' = A_I^n - \frac{R_{\rho_I} F_W^{SB''} \Delta t}{h_I^{min}} \quad (11.49)$$

if  $A_I'$  becomes less than 0, it is reset to zero. Even if  $A_I'$  becomes less than  $A_I^{min}$ , on the other hand, it is not adjusted here. If  $A_I'$  is adjusted to zero, it means that the sublimation flux is not used up by eliminating snow and lake ice.

The remaining part is consumed to reduce lake water, so the evaporation flux  $F_W^{EV}$  is modified as

$$F_W^{EV} = F_W^{EV} + F_W^{SB} + \frac{(A_I' - A_I^n) h_I^{min}}{R_{\rho_I} \Delta t} \quad (11.50)$$

The later two terms cancel out if the adjustment does not take place.

If there is no lake ice, evaporation flux is just as

$$F_W^{EV'} = F_W^{EV} + F_W^{SB} \quad (11.51)$$

The adjusted evaporation flux is saved

$$\Delta F_W^{EV} = F_W^{EV'} - F_W^{EV} \quad (11.52)$$

When sublimation flux is consumed to reduce lake ice amount, salt contained in lake ice has to be added to the remaining lake ice or the underlying water. Otherwise, total salt of the ice-lake system is not cosereved. Here, it is added to underlying water, and the way of this adjustment is described later. Note that lake ice tends to gradually drain high salinity water contained in brine pockets in reality. Thus, such an adjustment is not very unreasonable. When  $A_I'$  is adjusted to zero, on the other hand, the remaining sublimation flux is consumed to reduce lake water. In this case, difference between the latent heat of sublimation and evaporation has to be adjusted, which is also described later.

If the ice and/or snow is too thick, they are converted to snow flux. Here, the overflow snow flux  $S_{off}$  is added to  $F_W^{SN}$

$$F_W^{SN} = F_W^{SN} + S_{off} \quad (11.53)$$

$S_{off}$  is actually calculated in SUBROUTINE [MATDRV] (of matdrv.F), then handed to ENTRY: [FWATER].

### 11.3.3 Updating lake ice fraction

- Input variables

Variable	Variable in source code	Description	unit
$h_I$	HIX	Thickness of lake ice a	[cm]
$W_{AO}$	WAO	snow growth rate in ice-free area	[cm/s]

In ENTRY: [PCMPCTL] (in SUBROUTINE: [CMPSTL] of lakeic.F), the lake ice fraction is updated, using the lake ice thickness ( $h_I$ ) and the growth (retreat) rate in ice-free area ( $W_{AO}$ ):

$$A_I^{n+1} = A'_I + \frac{\rho_O}{\rho_I h_I \phi W_{AO} \Delta t} \quad (11.54)$$

If  $A_I^{n+1}$  becomes greater than 1, it is reset to 1, and if  $A_I^{n+1}$  becomes smaller than zero, it is reset to zero.

#### 11.3.4 Growth and Melting

In ENTRY: [PTHICKL] (in SUBROUTINE: [OTHKSTL] of lakeic.F), the lake ice growth and melting are calculated. The variables in the (n+1)-th time level are finally determined here.

- Variables

Variable	Variable in source code	Description
$A_I^{n+1}$	AX	lake ice fraction
$V_I$	AXHIX	lake ice volume
$V_S, V'_S, V_S^*$	AXHSX	lake snow volume
$V_I^{n+1}$	AXHIXN	lake ice volume
$V_S^{n+1}$	AXHSXN	lake snow volume
$h'_I$	HIX	lake ice thickness
$h'_S$	HSX	Snow depth
$h_S^n$	HSZ	Snow depth
$h_I^n$	HIZ	lake ice thickness
$W_{AS}$	WAS	snow growth rate due to heat imbalance
$W_{AI}$	WAI	lake ice growth rate due to heat imbalance
$W_{res}$	WRES	Reduced heat flux
$W_{IO}$	WIO	basal growth rate of lake ice
$F_W^{SN}, F_W^{SN'}$	SNOW	snow fall flux
$W_{AO}$	WAO	Lake ice growth rate in ice-free area
$F_W^{PR}$	PREC	precipitation flux
$L_e$	EVAP	latent heat flux of evaporation
–	SUBI	latent heat flux of sublimation
$A'_I$	AZ	Lake ice concentration
$H_{lake}$	FT	lake heat flux



Variable	Variable in source code	Description
$\Delta t$	TS	time step
–	ROFF	–
–	ADJLAT	–
–	FS	–

The lake ice volume ( $V_I'$ ) and snow volume ( $V_S'$ ) before the snow and ice growth are presented by

$$V_I' = A_I' h_I^n \quad (11.55)$$

$$V_S' = A_I' h_S^n \quad (11.56)$$

From here, let us consider the contribution of snowfall and freshwater fluxes to the growth.

Changes of snow depth due to snow fall (freshwater) flux ( $F_W^{SN}$ ) (expressed by negative values to be consistent with other freshwater flux components) is first taken into account.  $F_W^{SN}$  is not weighted by lake ice concentration or ice-free area are fraction, as snowfall take place for both regions.

If the newly predicted (in **ENTRY: [PCMPCTL]**) lake ice concentration ( $A_I^{n+1}$ ) is zero, the amount of snow existed before the growth is added to the snowfall flux.

$$F_W^{SN'} = F_W^{SN} + \frac{\rho_S V_S'}{\rho_O \Delta t} \quad (11.57)$$

Snow depth and amount is set to zero:

$$h_S' = 0, \quad V_S^* = 0 \quad (11.58)$$

Otherwise, snowfall accumulates over the ice covered region. Snow depth is modified by

$$h_S^* = \frac{V_S'}{A_I^{n+1}} + \frac{\rho_O F_W^{SN} \Delta t}{\rho_S} \quad (11.59)$$

And the snow amount is also modified by

$$V_S^* = A_I^{n+1} h_S^* \quad (11.60)$$

The snowfall flux is reduced by that amount:

$$F_W^{SN'} = (1 - A_I^{n+1}) F_W^{SN} \quad (11.61)$$

Then, the snowfall flux is put together with the precipitation flux.

$$F_W^{PR} = F_W^{PR} + F_W^{SN'} \quad (11.62)$$

When the lake ice is not existed ( $A_I^{n+1} = 0$ ), the snow amount grown above is converted to ice. Growth rate of the lake ice is presented by:

$$W_{AI}^* = \frac{\rho_O V_S}{\rho_S \Delta t} \quad (11.63)$$

When the lake ice existed, if the snow growth rate

$$W_{AS}^* = W_{AS} + \frac{\rho_O V_S}{\rho_S \Delta t} \quad (11.64)$$

is positive, the energy is used for the snow growing. Otherwise,  $W_{AS}^*$  is assumed to reduce the lake ice.

$$W_{AI}^* = W_{AS}^* \quad (11.65)$$

and deficient flux is come from the snow amount changes.

$$W_{AS}^* = -\frac{\rho_O V_S}{\rho_S \Delta t} \quad (11.66)$$

Then, the snow depth is modified with the accumulation.

$$h_S^* = \frac{V_S + \rho_O W_{AS} \Delta t}{\rho_S A_I^{n+1}} \quad (11.67)$$

if  $h_S'$  is less than 0, it is reset to zero.

Freshwater flux for ice growth is also considered. When the lake ice is not existed, the flux is just handed to the lake surface.

$$W_{IO}^* = W_{IO} + W_{AI}^* \quad (11.68)$$

When the lake ice exists ( $A_I^{n+1} > 0$ ), if the ice growth rate

$$W_{AI}^* = W_{AI} + \frac{V_I}{R_{\rho_I} \Delta t} \quad (11.69)$$

is negative, the flux is handed to the lake surface.

$$W_{IO}^* = W_{IO} + W_{AI}^* \quad (11.70)$$

and deficient flux is come from the lake amount changes.

$$W_{AI}^* = -\frac{V_I}{R_{\rho_I \Delta t}} \quad (11.71)$$

The amount of the ice is then updated,

$$V_I^* = V_I' + \frac{\rho_O W_{AI} \Delta t}{\rho_I} \quad (11.72)$$

Then, the snow amount in the new time step (n+1) is,

$$V_S^{n+1} = A_I^{n+1} h_S^* \quad (11.73)$$

For the ice amount,

$$V_I^{n+1} = V_I^* + \frac{\rho_O (W_{IO} + W_{AO}) \Delta t}{\rho_I} \quad (11.74)$$

If  $V_I^{n+1}$  is equal or less than 0, lake ice fraction is set to zero ( $A_I^{n+1} = 0$ ) and its thickness is set to  $h_I^{min}$ . Otherwise,

$$h_I^{***} = \frac{V_I^{n+1}}{A_I^*} \quad (11.75)$$

If  $h_I^{***}$  is smaller than  $h_I^{min}$ , it is set to  $h_I^{min}$  and the lake ice concentration is adjusted.

$$A_I^{n+1} = \frac{V_I^{n+1}}{h_I^{min}} \quad (11.76)$$

If the  $A_I^{n+1}$  is less than  $A_I^{min}$ , it is set to 0. If the  $A_I^{n+1}$  is larger than  $A_I^{max} = 1$ , it is set to  $A_I^{max}$ .

Let us consider the case of the ice is very thick. Here, the remained volume, which is not covered by ice is considered.

$$V_a^{free} = (A_{max} - A_{min}) h_I^{min} \quad (11.77)$$

$$V^{free} = (A_{max} - A_I^{n+1}) h_I^{***} \quad (11.78)$$

If  $V^{free} > V_a^{free}$ , the ice thickness is increased, by adding  $V^{free}$

$$h_I^{***} = V^{free} + \frac{A_I^{n+1} h_I^{min}}{A_I^{max}} \quad (11.79)$$

The deficient water is come from the snow. The snow depth is now updated

$$h_S^{***} = A_I^{n+1} \frac{h_S^{***}}{A_{max} - \frac{V_a^{free}}{h_I^{***}}} \quad (11.80)$$

Finally, check if the snow is under water.

$$h_S^{n+1} = \min(h_S^{***}, \frac{\rho_O - \rho_I}{\rho_S} h_I^{***}) \quad (11.81)$$

and the ice thickness is also updated.

$$h_I^{n+1} = h_I^{***} + \frac{\rho_S}{\rho_I} (h_S^{***} - h_S^{n+1}) \quad (11.82)$$

The growth rate of the lake ice is

$$W_I^n = \frac{\rho_S A_I^{n+1} h_I^{n+1} - V_I'}{\rho_I \Delta t} \quad (11.83)$$

$$W_I^{n+1} = \frac{\rho_S A_I^{n+1} h_I^{n+1} - V_I^{n+1}}{\rho_I \Delta t} \quad (11.84)$$

The growth rate of the snow is

$$W_S^n = \frac{\rho_S A_I^{n+1} h_S^{n+1} - V_S'}{\rho_S \Delta t} \quad (11.85)$$

$$W_S^{n+1} = \frac{\rho_S A_I^{n+1} h_S^{n+1} - V_S^{n+1}}{\rho_S \Delta t} \quad (11.86)$$

The sublimation flux  $F_S$  is

$$F_S = S_I (W_I - F_W^{SB''}) \quad (11.87)$$

The freshwater flux ( $F_W(1)$ ) is

$$F_W(1) = -F_W(1) + \frac{L_f}{C_p} (W_I^{n+1} + W_S^{n+1} - S_n + \Delta F_W^{EV}) \quad (11.88)$$

The salinity flux ( $F_W(2)$ ) is

$$F_W(2) = F_W^{EV} - F_W^{PR} - R_{off} + W_S^n + W_I^n \quad (11.89)$$

## 11.4 Physical formulation and process

This sub-section [lakepo.F] introduces the vertical heat and salinity diffusion and convection of the lake water.

### 11.4.1 Setting the vertical diffusion coefficients

#### Input

Meaning	Character	In code	Dimension	Unit
The default value of vertical diffusion coefficient	$K_{V0}$	AHVL0	NZ	-

#### Output

Meaning	Character	In code	Dimension	Unit
Vertical diffusion coefficient	$K_V$	AHVL	IJLDIM, NLZDIM	-

ENTRY: [VDIFFL] (in SUBROUTINE: [LAKEPO] of lakepo.F)

In this part, the vertical diffusion coefficients are first determined for each lake layer.

In the current version of MATSIRO 6, the diffusive flux of tracer follows Fick's law. In Fick's first law, the vertical diffusive tracer flux  $J$  of unit time is:

$$J = K_V \frac{\partial T}{\partial z} \quad (11.90)$$

where  $T$  represents temperature (temperature is used as an example here, this equation also applies to salinity), and  $K_V$  represents the diffusion coefficient, and  $z$  represents the vertical distance. The vertical diffusion coefficients for all lake layers are set as the default value, which is  $K_{V0}=1$ . Therefore, the vertical diffusion coefficient  $K_{V0}$  of each layer is simply set as:

$$K_V(k) = K_{V0} \quad (11.91)$$

It is known that the surface wind and vertical temperature stratification will also influence the vertical diffusion coefficients, and they will be considered and added in the future version.

### 11.4.2 Estimate the diffusion terms of the tracer equations

#### Input

Meaning	Character	In code	Dimension	Unit
Vertical diffusion coefficient	$K_V$	AHVL	IJLDIM, NLZDIM	-
Water temperature	$T$	TX(1)	IJLDIM, NLZDIM, NLTDIM	$^{\circ}C$
Water depth	$h$	HX	IJLDIM	m

## Output

Meaning	In		Dimension	Unit
	Character	code		
The vertical component of diffusive tracer flux	$F_D$	ADT	IJLDIM, NLZDIM, NLTDIM	$^{\circ}\text{C}/\text{s}$

## Internal variables

Meaning	Character	In code	Dimension	Unit
The water depth excluding the first layer	$h'$	HZBOT	IJLDIM	m
Depth of each lake layer	$D$	DZ	IJLDIM, NLZDIM	cm

ENTRY: [FLXTRCL] (in SUBROUTINE: [LAKEPO] of lakepo.F)

This part introduces the calculation of the vertical diffusive tracer flux. Temperature is shown here as an example, and the tracer equation of salinity is identical.

The thickness of each lake layer is first calculated. The water column is divided into 5 layers ( $k=2, 3, 4, 5, 6$ ) in the current version, and the  $k=1$  and  $k=7$  represent the top surface and the bottom surface of the water column, respectively. The layer of the water column is shown in Fig. 11-1.

Firstly, the thickness of the first layer ( $k=2$ ) is fixed and represented as  $D_1$ , and  $D_1$  is set to 100 cm. Therefore, the thickness of the first layer can be described as:

$$D(2) = D_1 \quad (11.92)$$

The thicknesses of the other layers ( $k=3, 4, 5, 6$ ) are calculated based on the ratio of the remaining thickness of the water column. The ratio ( $S(k)$ ) of layer 3 to 6 are 0.1, 0.1, 0.2, and 0.6, respectively. Therefore, using  $h$  to represent the total depth of the water column, and the thickness of the remaining water column ( $h'$ ) excluding the top layer is presented by:

$$h' = h - D_1 \quad (11.93)$$

and thickness ( $D(k)$ ) of the remaining layer ( $k=3, 4, 5, 6$ ) can be represented as:

$$D(k) = S(k)h' \quad (11.94)$$

Therefore, the thermal change of the  $k$ -th layer by vertical diffusion can be represented as the flux difference between upper layer  $J_{k-1,k}$  and lower layer  $J_{k,k+1}$ :

$$F_D(k) = J_{k-1,k} - J_{k,k+1} = K_V(k) \frac{T(k-1) - T(k)}{\frac{D(k-1)+D(k)}{2}} - K_V(k+1) \frac{T(k) - T(k+1)}{\frac{D(k)+D(k+1)}{2}} \quad (11.95)$$

The above equations apply to salinity as well.

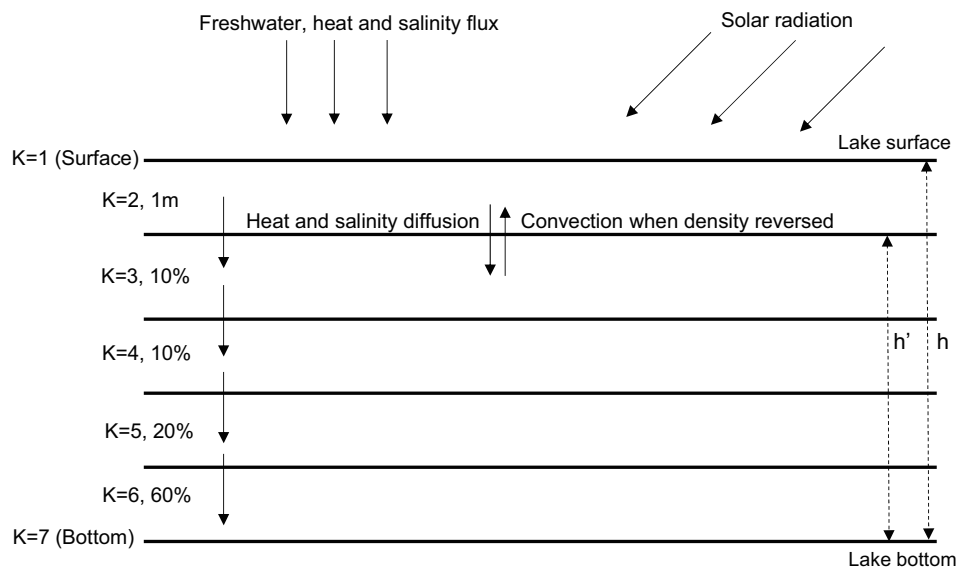


图 1: The layer of the water column

### 11.4.3 Time integration of the tracer equations

#### Input

Meaning	Character	In code	Dimension	Unit
The vertical component of diffusive tracer flux	$F_D$	ADT	IJLDIM, NLZDIM, NLTDIM	$^{\circ}C/s$
Minimum depth of the lake	$h_{min}$	HXMINI	IJLDIM, NLZDIM	m
Heat flux	$F_T$	FT	IJLDIM (1)	$^{\circ}C/s$
Freshwater flux	$F_W$	FT(2)	IJLDIM	cm/s
Absorbed shortwave solar radiation	$S_r$	SWABS	IJLDIM	
Salt flux	$F_S$	FS	IJLDIM	
Timestep	$\Delta t$	TS		s
Surface-type fraction (lake)	$R_{lake}$	LKFRAC	IJLDIM	-

### Output

Meaning	Character	In code	Dimension	Unit
Lake water deficiency	$V_D$	XHD	IJLDIM	cm

ENTRY: [SLVTRCL] (in SUBROUTINE: [LAKEPO] of lakepo.F)

This part introduces the update of water temperature and salinity of each layer due to the diffusion, freshwater flux, and absorption of solar radiation.

Heat diffusion is first considered. According to Fick's second law, the temperature (salinity as well) change of k-th layer follows:

$$\frac{\partial T}{\partial t} = K_V(k) \frac{\partial^2 T}{\partial z^2} \quad (11.96)$$

where  $z$  is the vertical distance. The equation is implicitly (backward-in-time) integrated, and obtains:

$$\frac{T^{n+1}(k) - T^n(k)}{\Delta t} = \frac{K_V(k) \frac{T^{n+1}(k-1) - T^{n+1}(k)}{\frac{D(k-1)+D(k)}{2}} - K_V(k+1) \frac{T^{n+1}(k) - T^{n+1}(k+1)}{\frac{D(k)+D(k+1)}{2}}}{D(k)} \quad (11.97)$$

The Tridiagonal Matrix Algorithm (also known as the Thomas Algorithm) is used to solve the equation, the following equation is constructed:

$$A_A(k)F_D(k-1) + A_B(k)F_D(k) + A_C(k)F_D(k+1) = F_D(k) \quad (11.98)$$

where

$$A_A(k) = -\frac{K_V(k)}{\frac{D(k-1)+D(k)}{2}} \Delta t \quad (11.99)$$

$$A_C(k) = -\frac{K_V(k+1)}{\frac{D(k)+D(k+1)}{2}} \Delta t \quad (11.100)$$



$$A_B(k) = D(k) - A_A(k) - A_C(k) \quad (11.101)$$

Then, the new coefficient of the Tridiagonal Matrix Algorithm can be constructed:

$$A'_c(k) = \begin{cases} \frac{A_C}{A_B}, k = 2 \\ \frac{A_C}{A_B - A_A(k)A'_C(k-1)}, k = 3, 4, 5, 6 \end{cases} \quad (11.102)$$

$$A'_D(k) = \begin{cases} \frac{F_D(k)}{A_B(k)}, k = 2 \\ \frac{F_D(k) - A_A(k)A'_D(k-1)}{A_B - A_A(k)A'_C(k-1)}, k = 3, 4, 5, 6 \end{cases} \quad (11.103)$$

Then, the vertical diffusion term can be obtained:

$$F_D(k) = \begin{cases} A'_D(k), k = 6 \\ A'_D(k) - A'_C(k)F_D(k+1), k = 2, 3, 4, 5 \end{cases} \quad (11.104)$$

The water temperature of each layer is updated as:

$$T_k = T(k) + F_D(k)\Delta t \quad (11.105)$$

After solving diffusive changes of tracers, the effect of heat, salinity, and freshwater fluxes at the lake surface is taken into account. Since the height of the water column changes with the freshwater flux, tracers at each layer should be re-estimated. It worth noting that a minimum depth threshold  $h_{min}$  is set, which means the lake depth can not be lower than this value. When surface freshwater flux  $F_W$  (positive upward, i.e., the lake level is lowered when  $F_W > 0$ ) is imposed, and the change of the water column for each time step can be represented as:

$$h_D = -F_W\Delta t \quad (11.106)$$

and the lake water deficiency (when lake depth  $h$  is smaller than the minimum threshold  $h_{min}$ ) can be represented as:

$$V_D = \max(h_{min} - h - h_D, 0) \times R_{lake} \quad (11.107)$$

where  $R_{lake}$  is the lake fraction of the grid. Therefore, the change of the lake depth is finalized as:

$$h_D = \max(h_D, h_{min} - h) \quad (11.108)$$

and the lake depth is updated:

$$h = h + h_D \quad (11.109)$$

Due to the added freshwater, the depth of each lake layer has been changed, so the water temperature of each layer is updated as well. The update of depth and water temperature starts from the bottom layer. When  $h_D > 0$ , the bottom of the  $k$ -th layer in the  $z$  coordinate is raised by:

$$\Delta z_B^k = (1 - \sum_{l=2}^k S(k))h_D \quad (11.110)$$

where  $S(k)$  is the vertical depth proportion of the  $k$ -th layer, and its top is raised by:

$$\Delta z_T^k = (1 - \sum_{l=2}^{k-1} S(l))h_D \quad (11.111)$$

where the following equation holds:

$$\Delta z_k^T = \Delta z_{k-1}^B \quad (11.112)$$

Therefore, the temperature of the  $k$ -th ( $k=3, 4, 5, 6$ ) layer becomes:

$$T(k) = \frac{T(k)D(k) - T(k)\Delta z_k^B + T(k-1)\Delta z_k^T}{D(k)} \quad (11.113)$$

The temperature of the first layer ( $k=2$ ) becomes:

$$T(2) = \frac{T(2)D(2) - T(2)\Delta z_2^B + T(2)\Delta z_2^T + F_T\Delta t}{D(2)} \quad (11.114)$$

where  $F_T$  is the heat flux of the freshwater added to the lake, and for salinity, the salinity flux is  $F_S$ .

When  $h_D < 0$ , on the other hand, the bottom of the  $k$ -th level is lowered by  $\Delta z_k^B$ , and its top is lowered by  $\Delta z_k^T$ . In this case, the above tracer equations for the  $k$ -th layer ( $k=3, 4, 5, 6$ ) and the first layer ( $k=2$ ) become:

$$T(k) = \frac{T(k)D(k) - T(k)\Delta z_k^T + T(k+1)\Delta z_k^B}{D(k)} \quad (11.115)$$

and

$$T(2) = \frac{T(2)D(2) - T(2)\Delta z_2^T + T(3)\Delta z_2^B + F_T\Delta t}{D(2)} \quad (11.116)$$

Finally, the absorption of solar radiation is considered. The temperature of each layer is updated as:

$$T(k) = T(k) + \frac{S_r \times C_{sr}(k)}{D(k)\rho_0 C_{P0}} \Delta t \quad (11.117)$$

where  $S_r$  represents the solar radiation, and  $C_{sr}$  represents the absorption proportion of each layer.  $\rho_0$  and  $C_{P0}$  are density and specific heat capacity of water, and their values are set as 1 g/cm<sup>3</sup> and  $3.99 \times 10^7$  erg/g/K, respectively.  $C_{sr}$  is calculated based on the depth of each layer, the transitivity  $T_{rs}(k)$  at depth  $D_t(k)$  can be represented as:

$$T_{rs} = R_r \times e^{-\frac{D_t(k)}{z_1}} + (1 - R_r) \times e^{-\frac{D_t(k)}{z_2}} \quad (11.118)$$

where  $R_r$ ,  $z_1$ , and  $z_2$  are three parameters, and their values are 0.58, 35, and 2300, respectively.

$D_t(k)$  represent the total depth of the bottom of  $k$ -th layer:

$$D_t(k) = \sum_{l=2}^k D(l) \quad (11.119)$$

Therefore,  $C_{sr}$  can be represented as:

$$C_{sr}(k) = \begin{cases} 1 - T_{rs}(k), & k = 2 \\ T_{rs}(k-1) - T_{rs}(k), & k = 3, 4, 5 \\ 1 - \sum_{l=2}^5 C_{sr}(l), & k = 6 \end{cases} \quad (11.120)$$

#### 11.4.4 The vertical convection

ENTRY: [OVTURNL] (in SUBROUTINE: [LAKEPO] of lakepo.F)

This part introduced the vertical convection of the water between different layers.

A classical, still widely used method-convective adjustment, which unstable water column is artificially homogenized with conserving heat and salt. MATSIRO 6 employs the convective adjustment for the standard choice, but its algorithm is not the pairwise adjustment. It is summarized as follows:

- (1) Set the index  $K_s=2$  (the first layer), indicating the start level of the convective adjustment, and set the index  $K_e=3$  (the second layer), indicating the level where instability is currently judged.
- (2) Compare potential density ( $\rho$ ) of the  $(K_e-1)$ -th and  $K_e$ -th layers. If instability ( $\rho_{K_e-1} > \rho_{K_e}$ ) is found, go to step 3, and if instability is not found, go to step 4.
- (3) Mix the water column between the  $K_s$ -th layer and  $K_e$ -th layer. Increase  $K_e$  by 1, and if the (new)  $K_e$  is greater than 6 (the bottom layer), the convection procedure ends. If not, go back to step 2 with the new  $K_e$ .
- (4) Set  $K_s=K_e$ , and increase  $K_e$  by 1. Increase  $K_e$  by 1, and if the (new)  $K_e$  is greater than 6 (the bottom layer), the convection procedure ends. If not, go back to step 2 with new  $K_s$  and  $K_e$ .

The water density is calculated by:

$$\rho = 999.842594 + 6.793952 \times 10^{-2} \times T - 9.095290 \times 10^{-3} \times T^2 + 1.001685 \times 10^{-4} \times T^3 - 1.120083 \times 10^{-6} \times T^4$$

where  $T$  represents temperature. When instability happens, the water column between the  $K_s$  layer and  $K_e$  will convect and mix, and the water temperature of these layers will be identical and are updated as:

$$T(K_s, \dots, K_e) = \frac{\sum_{k=K_s}^{K_e} D(k)T(k)}{\sum_{k=K_s}^{K_e} D(k)} \quad (11.122)$$

## 12 Snow-fed Wetland

[#ifdef OPT\_SW\_STORAGE] (in SUBROUTINE: [MATROF] in matrof.F)

### 12.1 Outline of wetland scheme

A snow-fed wetland scheme, in which snowmelt can be stored with consideration of sub-grid terrain complexity, is incorporated as a sub-module of TOPMODEL in MATSIRO 6 to represent the wetland-related process in the middle and high latitudes grid with snowmelt (Nitta et al., 2015, 2017) (Fig. 12-1). The wetland scheme has two major effects: 1) the storage of part of the surface water and delay of runoff to rivers, 2) an increase in land surface wetness thus enhancing the evaporation in water-limited regimes.

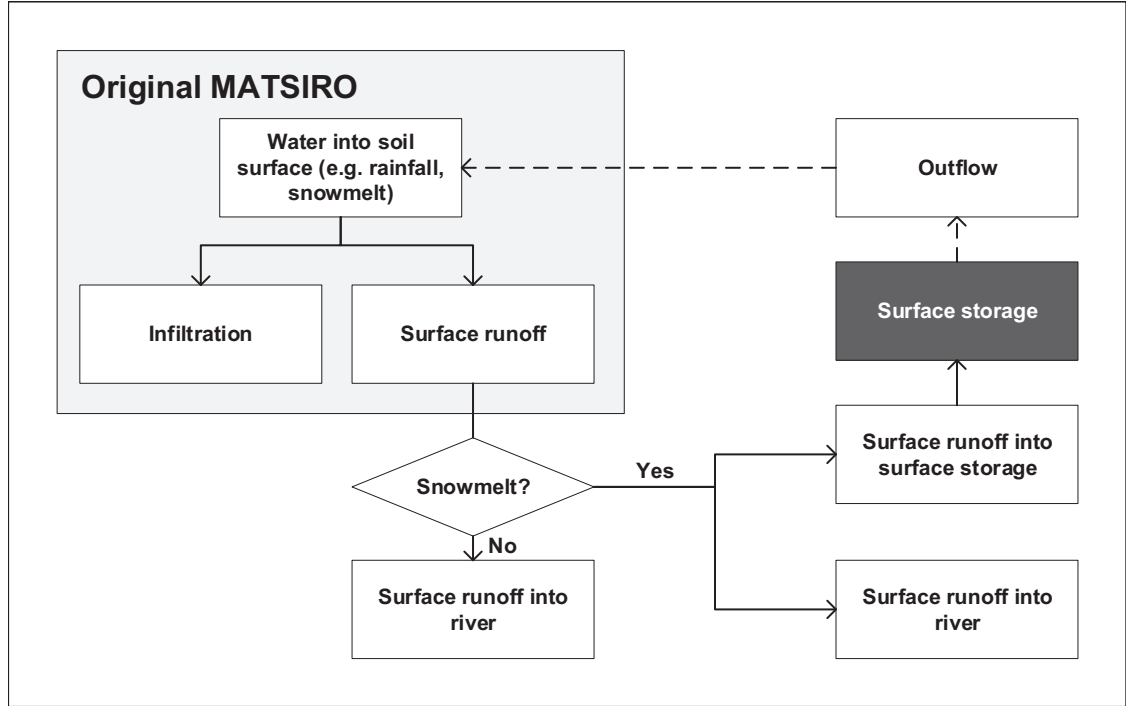


Fig 2: Flowchart of the wetland scheme in the MATSIRO 6

With the wetland scheme, when snowmelt occurs, instead of all the generated surface runoff flows directly to the rivers, only a part of the surface runoff flows into the rivers and the remaining part of the surface runoff is stored by the added tank (also known as the snow-fed wetland). Then, the stored water in the wetland is then re-added to the water input of soil combining with other kinds of water inputs (Fig. 12-1). In the current version, only snow-fed wetlands are considered, and more types of wetland schemes will be added in the future version.

## 12.2 Inflow and outflow of the wetland

The inflow of the wetland comes from the fraction of the surface runoff, and its amount is determined by the tunable parameter  $\alpha$ . The outflow from the wetland is calculated using a time constant  $\beta$  and the wetland storage  $S$ , consequently flowing into the soil surface. Therefore, the update of the wetland storage  $S$  at each time step can be represented as:

$$\frac{S^{\tau+1} - S^{\tau}}{\Delta t} = -\frac{S}{\beta} + (1 - \alpha)R_s \quad (12.1)$$

where  $R_s$  is surface runoff calculated as a total of  $Ro_s$  (saturation excess runoff),  $Ro_i$  (infiltration excess runoff), and  $Ro_o$  (overflow of the uppermost soil layer);  $\tau$  is time; and  $\alpha$  and  $\beta$  are parameters related to the inflow and outflow of the wetland storage, respectively.

$\beta$  is a spatially dependent time constant, and can be calculated using a function of the standard deviation of elevation above sea level:

$$\beta = \max(\beta_0(1 - \min(\sigma_z(x), \sigma_{zmax})/\sigma_{zmax}), \Delta t) \quad (12.2)$$

where  $\beta_0$  is the maximum of the time constant,  $\sigma_z$  is the standard deviation of elevation above sea level within each grid at point  $x$ , and  $\Delta t$  is the time step of the model. Parameter  $\sigma_z$  is a physical parameter calculated by a topography dataset, with a higher spatial resolution than the simulation, and  $\beta_0$ ,  $\sigma_{zmax}$ , and  $\alpha$  are tunable parameters. These parameter values were determined based on sensitivity simulations using an offline land model with perturbed parameters; 1 month, 200m, and 0.1 were chosen as the most appropriate values for  $\beta_0$ ,  $\sigma_{zmax}$ , and  $\alpha$ , respectively (Nitta et al., 2015).

### 12.3 Storage of the surface runoff

The ratio of total surface runoff that flows directly to the rivers is controlled by parameter  $\alpha$ . Therefore, the actual runoff flows into rivers  $Ro$  changes to:

$$Ro = (Ro_s + Ro_i + Ro_o) \times \alpha + Ro_b \quad (12.3)$$

where  $\alpha$  is the inflow parameter (see 12.1);  $Ro_s$  is the saturation excess runoff (Dunne runoff),  $Ro_i$  is the infiltration excess runoff (Horton runoff), and  $Ro_o$  is the overflow of the uppermost soil layer, and all these three kinds of runoff make up the total surface runoff, and  $Ro_b$  is the groundwater runoff (section 7.3).

### 12.4 Water input of soil surface

The outflow from the wetland storage is re-added to the water input of the soil surface, combining with the original water input (e.g. precipitation that passes through canopy gaps, water drops from the canopy, and snowmelt water). Therefore, the updated soil water input  $WI_{soil,total}$  of each time step can be represented as:

$$WI_{soil,total} = WI_{soil,original} + \frac{S}{\beta} \Delta t \quad (12.4)$$

where  $WI_{soil,original}$  is the original soil water input,  $S$  represents the wetland storage,  $\beta$  represents the outflow parameter of the wetland, and  $\Delta t$  is the time step.

## 13 Tile scheme

ENTRY:[LNDFLX] (in SUBROUTINE: [MATSIRO] of matdrv.F)

In the latest version of MATSIRO, a tile treatment of the land surface has been introduced to represent the subgrid fraction of land surface types, so as to partially mimic the behavior at a higher resolution.

Basically, one land surface grid is divided into three tiles in the control run: lake, potential vegetation and cropland. All the prognostic and diagnostic variables are calculated in each tile, and the fluxes at the land surface  $F$  are averaged:

$$F = F_{lake} f_{lake} + \sum_{i=1}^n F_i f_i (1 - f_{lake}) \quad (13.1)$$

$$\sum_{i=1}^n f_i = 1 \quad (13.2)$$

where  $n$  is 2,  $F_{lake}$ ,  $F_1$  and  $F_2$  denote fluxes at the land surface of lake, potential vegetation and cropland,  $f_{lake}$ ,  $f_1$  and  $f_2$  denote their corresponding fractional weights, respectively.

By default, tile scheme is applied in land surface type, but it can be used for multiple purposes.

### 13.1 Lake

The surface heat and water fluxes over lakes have been calculated as one of the tiles in a grid. The water temperature and mass are predicted for the surface layer (minimum thickness of 1 m) and four subsurface layers, based on the thermal diffusion and mass conversion, considering vertical overturning, evaporation, precipitation, and in-flow from and outflow to rivers.

### 13.2 Potential Vegetation and Cropland

Both potential vegetation and cropland tiles consist of six soil layers, up to three snow layers, and a single canopy layer, driving predictions of the temperature and amount of water in the canopy, soil, and snow.

Potential vegetation is defined according to the vegetation types of the Simple Biosphere Model 2 (SiB2; Sellers et al. 1996) scheme and has 10 categories including land ice. There is no wetland category for land cover in the original SiB2 vegetation types or soil types.

### 13.3 Appendix

#### 13.3.1 Output variables

Variable	Description	Code	Dimension	Units
$F$	Fluxes at the land surface	MATFLX		-
$F_{lake}$	Fluxes at the land surface of lake	-	IJLDIM	-
$F_i$ ( $i = 1, 2$ )	Fluxes at the land surface of potential vegetation and cropland	-	IJLDIM, MULTCY	

#### 13.3.2 Parameters

PARAMETER	Description	Code	Dimension	Initial values	Units
$f_{lake}$	Fractional weight of lake	LKFRAC	IJLDIM	-	-

PARAMETER	Description	Code	Dimension	Initial values	Units
$f_i$ ( $i = 1, 2$ )	Fractional weight of potential vegetation and cropland	SFFRAC	CILDIM, MULTCY		-

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