Estimating Dynamic Functional Network Connectivity in fMRI Data

Bayesian Approach

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December 16, 2021

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DISIA DIPARTIMENTO DI STATISTICA INFORMATICA, APPLICAZIONI "GIUSEPPE PARENTI"

Outline

1. Background
2. Bayesian Approach
3. Conclusion

Graphical Models[1]

A graphical model represent conditional dependence relationships among random variables. It is composed by two parts:

- A set $\boldsymbol{X} = \{X_1, \ldots, X_p\}$ of random variables with distribution $P(\boldsymbol{X})$;
- A graph $G = (V, E)$ in which each element of V is associated with one of the random variables in $\boldsymbol{X}$, and the element of E is the conditional relationships among this.

[1] Lauritzen 1996.

Gaussian Graphical Models

In GGMs we have

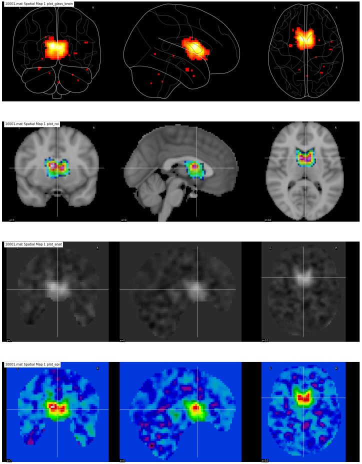
$$\boldsymbol{x}_i \sim N_p(\boldsymbol{\mu}, \boldsymbol{\Omega}^{-1}) \quad i = 1, \ldots n$$

The conditional independence relationships correspond to constraints on the precision matrix $\boldsymbol{\Omega} = \boldsymbol{\Sigma}^{-1}$.

Specifically, the precision matrix $\boldsymbol{\Omega}$ is a symmetric positive definite matrix with off-diagonal entry ω_{ij} equal to zero if there is no edge in G between vertex i and vertex j.

Functional magnetic resonance imaging Data (Booth et al. 2021)

- Indirect measure of neuronal activity by changes in blood oxygenation;
- Time series of blood oxygenation level dependent (BOLD) responses are collected at each location;
- Learn *effective* and *functional* connectivity.



Regression on *HRF*-convolved Stimulus (Warnick et al. 2018)

Let $\boldsymbol{Y}_t = (Y_{t1}, ..., Y_{tV})^T$ be the vector of *fMRI BOLD* responses.

Assuming an experiment with K distinct stimuli,

$$\boldsymbol{Y}_t = \boldsymbol{\mu} + \sum_{k=1}^{K} X_t^k \circ \beta_k + \epsilon_t \tag{1}$$

where X_t^k is the vector for the k-th stimulus, $\boldsymbol{\mu}$ the global mean and $\beta_k = (\beta_{1k}, \ldots, \beta_{Vk})^T$ the stimulus-specific vector coefficients.

Then we model X_t^k as the convolution,

$$X_{\upsilon t}^k = (x_k * h_\lambda)(t) = \int_0^t x_k(\tau) h_\lambda(t-\tau) d\tau$$

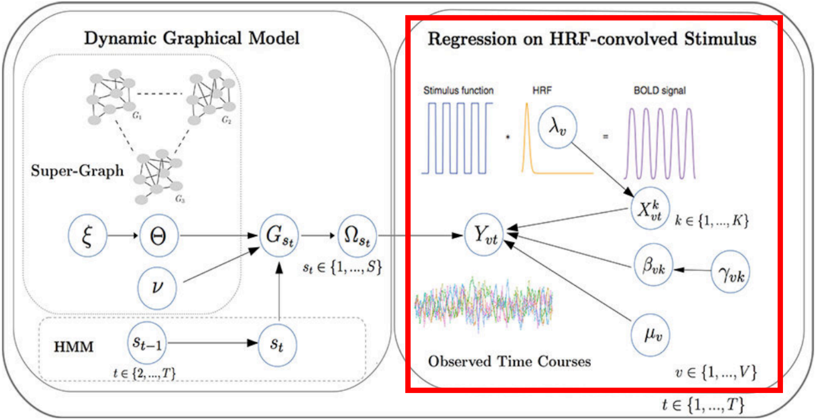


Figure 1: In the red rectangle the Regression with HRF (1).

Functional networks and dynamic connectivity (Peterson, Stingo, and Vannucci 2015)

Let $\boldsymbol{s} = (s_1, ..., s_T)^T$ denoting the connectivity state at time t,

$$(\epsilon_t | s_t = s) \sim N_p(0, \Omega_s)$$

where Ω_s is the precision matrix.

The *G-Wishart* prior density $W_G(b, D)$ can be written as

$$p(\boldsymbol{\Omega} \mid G, b, D) = I_G(b, D)^{-1} \mid \boldsymbol{\Omega} \mid^{\frac{b-2}{2}} exp(-\frac{1}{2} tr(\boldsymbol{\Omega} D))$$

where b > 2 is the degrees of freedom parameter, D is a $p \times p$ positive definite symmetric matrix, I_G is the normalizing constant.

Hidden Markov Model (Fine, Singer, and Tishby 1998)

Let X_n and Y_n be discrete-time stochastic processes, the pair (X_n, Y_n) is a hidden Markov model, if:

- X_n is a Markov process with *Hidden* states;
- $p(Y_n \in A \mid X_1 = x_1, \dots, X_n = x_n) = p(Y_n \in A \mid X_n = x_n)$

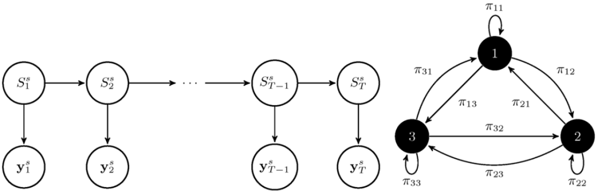


Figure 2: Schematic of an HMM with $N = 3$. Left shows the temporal sequence. Each S_t^s could be one of N states (Eavani et al. 2013).

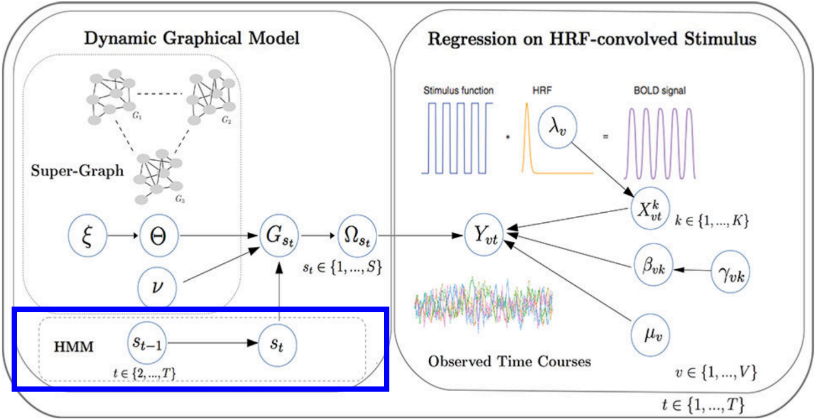


Figure 3: Modeling the connectivity states via Hidden Markov Model.

Joint modeling of the connectivity states (Peterson, Stingo, and Vannucci 2015)

Let $\boldsymbol{g}_{ij} = (g_{ij,1}, \ldots, g_{ij,S})^T$ with $g_{ij,s}$ indicating the presence or absence of edge (i, j) in graph G_s.

The *super-graph*, defines the presence of an edge across graphs through a Markov Random Field (MRF) prior,

$$p(\boldsymbol{g_{ij}} \mid \upsilon_{ij}, \boldsymbol{\Theta}) = C(\upsilon_{ij}, \boldsymbol{\Theta})^{-1} exp(\upsilon_{ij} \mathbf{1}^T \boldsymbol{g_{ij}} + \boldsymbol{g_{ij}}^T \Theta \boldsymbol{g_{ij}})$$

with υ_{ij} is a sparsity parameter specific and $\boldsymbol{\Theta}$ is a symmetric matrix which captures relatedness among networks.

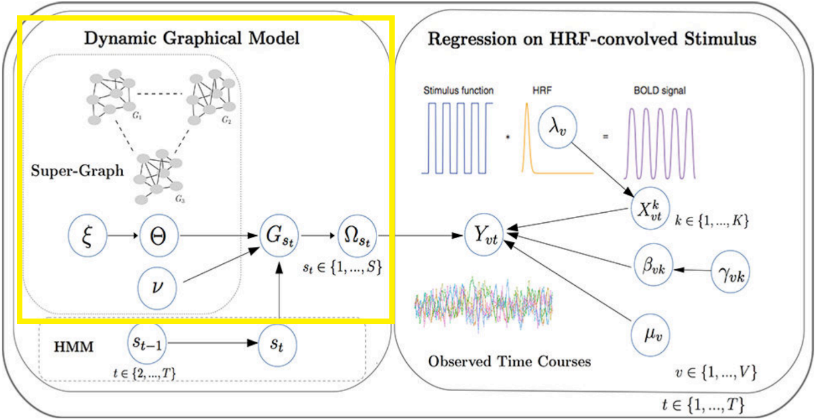


Figure 4: Super-Graph and Dynamic model.

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Conclusion

- In the Bayesian model it is assumed the existence of a structure called *super-graph* which models the relationship between the various temporal states;
- Setting of the priors including an **MRF** for the super-graph and a G-**Wishart** for the precision matrix on each graph;
- With the knowledge of the data obtain a posteriori and so the distribution of the parameters of interest.

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