AN ENHANCED HIERARCHY FOR (ROBUST) CONTROLLED INVARIANCE

1. Example parameters

Section IV: Example 1.

Dynamical system of a truck with N trailers:

$$\begin{cases} \dot{d}_{i} = v_{i-1} - v_{i} \\ \dot{v}_{0} = \frac{k_{s}}{m} d_{1} - \frac{k_{d}}{m} v_{0} + \frac{k_{d}}{m} v_{1} + u \\ \dot{v}_{i} = \frac{k_{s}}{m} (d_{i} - d_{i+1}) + \frac{k_{d}}{m} (v_{i-1} - 2v_{i} + v_{i+1}) \\ \dot{v}_{N} = \frac{k_{s}}{m} d_{N} - \frac{k_{d}}{m} v_{M} + \frac{k_{d}}{m} v_{N-1} \end{cases}, \quad \text{with constraints:} \begin{cases} -0.5 \le d_{i} \le 0.5 \\ 0 \le v_{0} \le 10 \\ 0 \le v_{i} \le 10 \end{cases}, i = 1, \dots, N,$$

where v_0, v_1, \ldots, v_N are the respective velocities, and d_1, \ldots, d_N the corresponding distances. We use the following parameters: $m_0 = 500 \text{kg}$, m = 1000 kg, $k_s = 4500 \text{N/kg}$, and $k_d = 4600 \text{Ns/m}$. We sample with $T_s = 0.4 \text{s}$ to obtain a discrete-time linear system with state $x = [d_1, \ldots, d_N, v_0, \ldots, v_N]$ corresponding to the above system.

Section IV: Example 2. Consider the continuous-time lateral dynamics of a vehicle:

$$\begin{cases} \dot{y} &= v + u\Delta\Psi, \\ \dot{v} &= -\frac{C_{\alpha f} + C_{\alpha r}}{mu}v + \left(\frac{bC_{\alpha r} - aC_{\alpha f}}{mu} - u\right)r + \frac{C_{\alpha f}}{m}\delta, \\ \dot{\Delta}\Psi &= r - r_d, \\ \dot{r} &= -\frac{aC_{\alpha f} + bC_{\alpha r}}{I_z u}v - \frac{b^2C_{\alpha r} + a^2C_{\alpha f}}{I_z u}r + a\frac{C_{\alpha f}}{I_z}\delta, \end{cases}$$
, with constraints:
$$\begin{cases} -0.9 \leq y \leq 0.9 \\ -1.2 \leq v \leq 1.2 \\ -0.3 \leq \Delta\Psi \leq 0.3 \\ -0.05 \leq r \leq 0.05 \\ -\frac{\pi}{2} \leq \delta \leq \frac{\pi}{2} \end{cases}$$

We use the following parameters u=30, $C_{\alpha f}=133000$, $C_{\alpha r}=98800$, m=1650, $I_z=2315$, a=1.11, b=1.59 are constants. Discretize with sampling time T_s to obtain a discrete-time linear system. We discretize the model with a sampling time of $T_s=0.1$ seconds assuming piecewise constant inputs.

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