A SIMPLE HIERARCHY FOR COMPUTING CONTROLLED INVARIANT SETS

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1. Example parameters

Section 2: Example 1 (same with Section 5: Example 2).

$$x^+ = \begin{bmatrix} 1.5 & 1 \\ 0 & 1 \end{bmatrix} x + \begin{bmatrix} 0.5 \\ 0.25 \end{bmatrix} u, \text{ with } u \in [-1,1], \text{ and the safe set: } \quad D = \left\{ x \in \mathbb{R}^2 \middle| \begin{bmatrix} 0.9147 & -0.5402 \\ 0.2005 & 0.6213 \\ -0.8193 & 0.9769 \\ -0.4895 & -0.8200 \\ 0.7171 & -0.3581 \\ 0.8221 & 0.0228 \\ 0.3993 & -0.8788 \end{bmatrix} x \leq \begin{bmatrix} 0.5566 \\ 0.8300 \\ 0.7890 \\ 0.3178 \\ 0.4522 \\ 0.7522 \\ 0.1099 \end{bmatrix} \right\}.$$

The above polyhedron was randomly generated in MATLAB.

Section 4: Example for comment (3).
Consider the double integrator system $x_1^+ = x_2$, $x_2^+ = u$, with $u \in \mathbb{R}$, and the safe set $D = \{x \in \mathbb{R}^2 | 0 \leq x_i \leq 1,\ i = 1,2\}$. By inspection one can see that D is itself controlled invariant. Our algorithm returns D as the computed controlled invariant set for any $L \geq 1$.

Section 4: Example for comment (4). Figures $5b, 5c$.
Consider the triple integrator system:

$$x^+ = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u, \text{ with } u \in \mathbb{R}, \text{ and the safe set: } \quad D = \left\{ x \in \mathbb{R}^3 \middle| \begin{bmatrix} 0.3 & -0.35 & -0.3 \\ -0.5 & 0.3 & -0.7 \\ -0.9 & -0.25 & 0.35 \\ 0.8 & 0.4 & -0.2 \\ 0.35 & 0.6 & 0.01 \end{bmatrix} x \leq \begin{bmatrix} 0.45 \\ 0.9 \\ 0.25 \\ 0.4 \\ 0.8 \end{bmatrix} \right\}.$$

Section 5: Example 3.
Dynamical system of a truck with N trailers:

$$\begin{cases} \dot{d}_i = v_{i-1} - v_i \\ \dot{v}_0 = \frac{k_s}{m} d_1 - \frac{k_d}{m} v_0 + \frac{k_d}{m} v_1 + u \\ \dot{v}_i = \frac{k_s}{m}(d_i - d_{i+1}) + \frac{k_d}{m}(v_{i-1} - 2v_i + v_{i+1}) \\ \dot{v}_N = \frac{k_s}{m} d_N - \frac{k_d}{m} v_M + \frac{k_d}{m} v_{N-1} \end{cases}, \qquad \text{with constraints: } \begin{cases} -0.5 \leq d_i \leq 0.5 \\ 0 \leq v_0 \leq 10 \\ 0 \leq v_i \leq 10 \end{cases}, i = 1, \ldots, N,$$

where $v_0, v_1, \ldots, v_N$ are the respective velocities, and $d_1, \ldots, d_N$ the corresponding distances. We use the following parameters: $m_0 = 500$kg, $m = 1000$kg, $k_s = 4500$N/kg, and $k_d = 4600$Ns/m. We sample with $T_s = 0.4$s to obtain a discrete-time linear system with state $x = [d_1, \ldots, d_N, v_0, \ldots, v_N]$ corresponding to the above system.

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